AN

INTRODUCTION

TO

PRACTICAL ASTRONOMY.

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§ I PRELIMINARY REMARKS

1. No subject has supplied matter so interesting to the contemplative mind of man, as the structure of the Universe, and no study is more pleasing than that which searches into the means employed by the almighty Creator, in his admirable plan of providence, for perpetuating the motions of the heavenly bodies, and at the same time for affording sustenance to every species of created beings, however numerous or however various. We need not then be surprised that authors have frequently indulged in extravagant expressions of admiration, when contemplating the works of creation, who yet had very imperfect notions of the laws by which the great Upholder of our planetary system regulates the motions of the heavenly bodies. They are indeed persuaded that he created them for wise and benevolent purposes, but with respect to the globe even which they inhabit, they frequently have but little knowledge of the beautifully simple means employed, to produce that periodical succession of day and night, summer and winter, on which the changes of temperature and succession of crops, that administer to the wants of man, are made to depend. Yet they perceive that the genial rays of the sun are variously distributed over the surface of the globe, which is destined for man's temporary habitation, and they acknowledge the bounty of a divine providence in thus making the different climates co-operate in the production of food for the support of all its inhabitants. It is the business of practical astronomy to examine, investigate, and explain the nature of those phenomena which excite the untutored sensations of the poet; and to satisfy the dubious mind, that the deductions of the Newtonian philosophy, being confirmed by the most accurate observations, are founded in truth. The majestic appearance, on a fine night, of the boundless expanse, studded with sparkling gems of different apparent dimensions, is indeed a spectacle that will ever excite the admiration of the beholder; and we are disposed to unite in the feelings of every one, who on such an occasion looks "through nature up to nature's God"; but a casual impression is not conviction; one evening's inspection may amuse, but not instruct. When we behold the moon, for instance, constantly varying the appearance of her orb from night to night, and riding to all appearance unsupported in her monthly circuit from star to star, and apparently visiting other stars in her

next periodic course, when we observe that after the lapse of about nineteen years she revisits the same stars in the same succession, that her measured diameter is not the same at each return of her entire orb, and that she is deprived of her light at a time when it is most resplendent in certain lunations only; our own senses tell us, not only that this luminary changes her place, and shines by borrowed light reflected from her body, but also that her distance from us is constantly changing, while yet she continues to attend the earth, and, in the absence of the sun, to enlighten our darkness. These obvious changes in the appearance of a large body at a distance from our globe, stimulate the mental faculties of man to enquire into the nature of these visible alternations of aspect. Recorded observations of former astronomers on these periodic aspects, have furnished data for computing the future changes that may be expected to take place to the end of time, supposing the laws to continue by which the variable motions are now regulated and preserved. In meditating on the periodic return of all the lunar phenomena that arise out of the variable changes of distance, direction, and velocity of motion, we are led into the enquiry, not *why*, but *how* the invisible hand of an almighty Power directs the path of so large a body in open space, and the enquiry that is thus directed, embraces the same consideration with reference to the other moving bodies that compose our planetary system. By the union of philosophical researches with mathematical investigation, the *nature* of the aerial paths, formed agreeably to the properties or laws of planetary motion, and continued by the power that gave them existence, has fortunately been disclosed, and the only danger that is now likely to affect the pride of man is, lest he should mistake the law of nature for the Lawgiver himself. While we were in ignorance, we thought we traced the finger of God in the works of creation; but now that our powers are enlarged, and the light of heaven hath shined on us*, the wisdom of man that traces the effects arising from secondary causes, in some few instances, will not condescend to refer those secondary causes or means to the agency of an omnipotent Being, to the nature of whose existence he cannot apply his mathematics and his philosophy. We leave therefore the sceptic to contemplate the heavenly motions as he would those of a mill actuated by the wind, the agency of which he only perceives from the effects that are invisibly occasioned thereby.

2. Our business in the first volume was to tabulate the corrections to be applied to the heavenly bodies, as arising out of theories that have been devised by men of the first-rate talents, and that have been found applicable to, and explicable of, the various phenomena of the heavenly bodies, in order to convert their apparent into their mean places, and the contrary. Our intention in this volume is to describe the most useful instruments that are serviceable in making the requisite observations, to which the aforesaid corrections may be applied, for the purpose of obtaining the results with accuracy; and to show the uninstructed observer how to adjust and use his instruments with success. The terror which the numerous pages of crowded figures in the first volume may have excited in the mind of the juvenile amateur in astronomy, will, it is hoped, gradually subside, when he finds that great mathematical skill is not required as an indispensable qualification in even a good observer. When he has made himself acquainted with the powers of his instrument, and has been successful in obtaining a few correct observations, his desire to apply these

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* "Nature and Nature's laws lay hid in night,
God said—let Newton be! and all was light."

observations to some useful purpose, and to be able to make such application by means within his own command, will naturally stimulate him to become acquainted with the arithmetical work of several, if not all of the most interesting problems connected with practical astronomy. If our labours should eventually produce this effect, in any considerable degree, our object will be attained; for the man who has interesting and rational amusement in his own abode, even though he may not be so fortunately skilful as to benefit science or society by his discoveries, will yet at least benefit himself and his select companions, by cultivating and promoting a taste for one of the most refined attainments.

3. When the inexperienced possessor of a telescope first points his instrument to the sky, provided the heavenly bodies are visible, his first astonishment is, that the star he sees is *in motion*, though he had been taught to consider the stars at rest; and he is induced to ask, why he has entertained an erroneous notion in this respect? But a little consideration will suggest to him, that this apparent motion may arise from either of two causes; the star may be in motion, or the telescope may be carried across the star so as to make the latter *appear* to be the moving body; and when he recollects that the earth is said to rotate on its axis, he becomes satisfied from this *visible proof* that the earth really moves. This first glance carries conviction, which is strengthened at every subsequent inspection of any heavenly body; but after he has directed his instrument to different points in the firmament, he observes that some of the stars appear to move faster through the telescope than others, and that, if he happens to examine the pole-star among the rest, its motion is the slowest of all, and that it will occupy the field of his telescope for nearly a couple of hours. This fact leads to another important inference, that he has found nearly that point of the heavens towards which the earth's axis is directed as it rotates. These are not matters depending on imagination, but are astronomical facts that may be relied on, and that lead to an important method of classification and arrangement of a catalogue of stars. Hence he will perceive the necessity of having instruments that will enable him to measure the angular distances of the stars from the polar point, and also the time corresponding to the intervals of the passages of different successive stars, or what is called their relative right ascensions. Thus on a sudden he finds himself initiated in the *practice* of astronomy, and he becomes more ardent in his pursuit as he gains possession of new facts. When a number of accurate observations has been made on the apparent places of the stars, a comparison of these places with those determined in the last century, shows that changes have taken place either in the heavens, or in the position of the earth; but on reference to the corrections arising out of precession, aberration of light, and nutation of the earth's axis solar and lunar, the differences are reconciled, and the reasons are comprehended, why the stars are not in reality in the very points of the celestial regions where they *appear* to be. The daily changes in the meridian altitude of the sun are indeed observable without an instrument; but the sensible change in his apparent diameter from day to day, requires the assistance of both optical and mechanical means to detect and to measure. With the former of these the vicissitude of the seasons is connected, and from the latter we know that the distance of the earth from the sun is not always the same. The admirable contrivance of giving the earth a motion round the sun, agreeably to a law common to all the planetary bodies, while the parallelism of her inclined axis of rotation is constantly preserved throughout her orbit, occasions all the phenomena of the seasons, by bringing each pole gradually and alternately within the reach of the sun's rays, which, by this simple device, fall with more or less obliquity on the dif-

ferent parts of the globe as the year advances, or as the earth proceeds in her annual tour. The succession of day and night is produced by means still more simple, namely, the rotation of the earth round its polar axis by an equable velocity. The annual revolution of the earth, however, is not performed by an equable motion, as we know from the difference observable in the length of the summer and winter half-years; but this difference is explicable by reference to the same cause that produces the apparent changes in the sun's diameter, namely, the variable distance of the earth from the sun, during her periodic revolution in her orbit, which is thus known to be, like that of the other planets, excentric. An union of the earth's annual revolution and diurnal rotation, would not however produce all the beneficent changes of climate and of the ever-varying length of the days in those climates, which are favourable to a plentiful supply of seasonable produce, if either the angle of inclination of the earth's axis, or the due preservation of its parallelism, did not form a part of the divine mechanism by which this system of benevolence is supported. If the angle of inclination had been materially different from what it is, the corresponding change of seasons would have rendered our existence on the earth probably less comfortable, by occasioning a different distribution of light and heat: for instance, if the earth's axis had been made perpendicular to the plane of her orbit, notwithstanding the parallelism, there would have been no change of seasons at all, and the days and nights would have been of equal length all over the world throughout the year; for the sun would have had no declination, and the earth's yearly motion could have been detected only by the sun's apparent change of angular distance from star to star, during her progress. On the contrary, if the earth's axis had been parallel to the plane of her orbit, and always pointing to the sun, one half of the globe would have had constant day, and the other half constant night to endless ages: but if, in this position, we conceive the parallelism of the axis to be preserved during the revolution round the sun, there would indeed have been a change of seasons, but of such a nature, that the whole northern and southern hemispheres would alternately have had perpetual day and night, instead of the regions only within the arctic and antarctic circles, and the transitions from heat to cold, and the reverse, would have been severely felt all over the globe. We have therefore great reason to rejoice in the provision that the Almighty has kindly made for the support and well being of all his creatures, by his appointment of a favourable position to the earth's axis, and by the continual preservation of that position, as it has reference to the celestial pole, to which it always points with such slight deviations, as are occasioned only by the periodical attractions of other bodies that form a part of the solar system.

4. Thus impressed with a sense of the wisdom and goodness of the omnipotent Creator and Preserver of the universe, we are naturally induced to inquire, whether this our system is the only one that is subjected to the laws of planetary motion, separated as it is from the stars at an immeasureable, or at least at such an immense distance, that the earth's whole orbit is only a point as it regards that distance? The enquiry affords ample scope for the exercise of our largest and most perfect instruments, and the discoveries we may make will enlarge our circle of knowledge, and teach us probably the relation in which our system stands to other systems, which are also governed by an union of power and goodness such as transcends the grasp of our limited conceptions. One of the most probable means of enlarging our knowledge of the immensity of God's works, is the observing the apparent changes that are taking place among those contiguous stars which are usually denominated *double* and *treble*; for the periodic variations of ap-

parent distance, and of the relative positions of different pairs, situated in various parts of the heavens, may at length determine in what direction our system is moving among the infinitude of bodies that compose the universe. With this view the subject has been taken up, and successfully pursued by several eminent astronomers, whose labours have been recently sanctioned by the marked approbation of various scientific societies.

5. Another plan has lately been proposed, and partly carried into execution, for ascertaining whether any new bodies may hereafter visit our system, or any of the known ones depart from it, by means of a general examination and mapping of all the visible stars, down to the ninth magnitude inclusive. If such map were finished with correctness, it would become a document of reference to future astronomers, whenever a second examination of the same kind might be deemed desirable, or whenever the supposed loss, gain, or alteration of any star may require some standard of comparison.

6. A still more satisfactory document of reference would be an improved and extended catalogue of the mean places of the stars that are visible in both hemispheres, taken by the best instruments, and carefully reduced; including the register of their apparent magnitudes, respective colours, or other remarkable appearances, particularly of those that are known or suspected to be changeable.

7. But whatever changes may hereafter be detected in the transposition of the solar system in the expanse of infinite space, we are well satisfied, that the system itself will not perish from any tendency to dissolution; for, as far as our present knowledge goes, the elements of computation, that enable us to account for all the observable alterations taking place in the position of the earth's axis, and of the phenomena depending on it, are of a kind that *recur* after they have attained their limit; and therefore we are convinced, that the laws by which the Almighty upholds the system will be co-extensive with his will. We feel fully aware that "the heavens declare the glory of God," and that "the firmament showeth his handy-work."

§ II. ON THE SITUATION, STRUCTURE, AND FURNITURE OF AN OBSERVATORY.

1. When a theoretic astronomer has made up his mind to purchase instruments and to become a practical observer, his first object will be to look out for a suitable situation for the site of his Observatory, and if practicable to make it his own, before he incurs the expense of erecting a building; or, if he should be so fortunate as to meet with a house adapted for his purpose already built, before he makes such alterations and additions, as will most probably be necessary, to render it in all respects commodious for the erection of his apparatus. The leading feature that must guide his choice of the spot where his observations are to be made, will be a visible horizon, particularly to the north and south, and if attainable, all round him: such a situation may require to be a little elevated above surrounding objects, and should be at a distance from manufactories, or other buildings that emit much smoke; it should also be at a distance from swampy ground or valleys that are liable to be covered with fogs or exhalations: the ground should be on gravel or other solid stratum, and not too close to a public road, particularly if it be paved, and frequented by carriages. A large town is therefore the least desirable of any place.

2. Another consideration of importance will be, that the astronomer have access to some distant field where he may be permitted to erect a pillar on which to fix his meridian mark, if there happen not to be a building already standing in his meridian line, either to the north or south. When the new observatory at Cambridge had the foundation laid, the committee availed themselves of Granchester Church steeple, which affords the means of fixing a southern mark of the most permanent kind, and also at a good distance for a large transit-instrument; which circumstance has been advantageous in more respects than one. The distance at which a meridian mark ought to be placed, will depend mainly upon the length of the telescope, the condition being, that the object shall be visible with the solar focus of the instrument; by the small telescope of a portable transit-instrument, a mark may be distinctly seen at about a quarter of a mile without altering the focal length, which in this instrument should never be disturbed; but for a longer telescope the distance may be required to be half a mile, or a mile, or, as at Greenwich, several miles, according to the length of the telescope. Whenever circumstances prevent the mark being placed at a sufficient distance to obtain distinct vision with the solar adjustment for focal distance, the aperture may be diminished to an inch or less, so as to admit only the central rays to form the image, and then the solar focus will not be limited to an exact length, and objects will be visible beyond and short of the true focal adjustment; but as the line of collimation may be affected by a change of the aperture, it is better to avoid this expedient. There is, however, one inconvenience in having the mark at a great distance, which is, that on a sunny day it will become very tremulous by the rays passing over a long stratum of the lowest part of the atmosphere, particularly if ploughed fields intervene, or other grounds that furnish a plentiful supply of exhalations. The best rule for ascertaining the proper distance will be, to view several objects at different distances, in the morning or evening, with the solar focal distance of the identical telescope intended to be used, before the place of the mark be finally fixed upon; for the nearest distance that will admit of distinct vision at the limited length of the telescope, will generally be the most convenient, as well as the best; particularly in a climate that is subject to mists and sudden atmospheric changes. When the place for a meridian mark has been obtained, by the pole-star, to the north of the proposed site for an observatory, it will be an object of equal interest, that a corresponding mark be placed also to the south, and therefore respect must be had to the same considerations that have been stated, as they apply to the southern side of the intended observatory. If nature has afforded facilities, and good neighbourhood* given its sanction, the aid of the architect may be resorted to, but not until the astronomer has chosen his instruments, and also ordered them to be executed; for their size, number, and quality must be submitted as a guide for the plan that is to be adopted. It is an error that public bodies have fallen into, that they have expended too large a portion of their finances in brick and mortar, and too small a sum in suitable instruments and stipends. When the situation has a command of the hemisphere, and of distant meridian marks, the nearer to the ground the instruments are fixed, the better; the main objects being steadiness and permanency of position of the pillars or piers on which the instruments are destined to stand; for every unknown deviation occasioned by changes of temperature, or settlements in the bases, will produce corresponding errors in the adjustments of the instruments, and

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* Meridian marks have sometimes been injured or destroyed through malice or ignorance

consequently in the observations made by them, if not detected at the moment and corrected. A small building, or even the wings of a building of ordinary dimensions for a family, may be constructed so as to become a most useful private observatory.

3. With respect to the furniture of an observatory, which must regulate the dimensions of the room or rooms appropriated to the making of observations, a transit-instrument and a good clock are indispensable; and if no other instruments should be used, they will supply constant employment to an active observer, whose object is to determine the right ascensions of a catalogue of stars, or to observe the daily meridian passages of the planets and comets that may at any time be visible. If he has a small divided circle to give the elevation of the telescope as a finder, it will give the altitudes or zenith distances near enough, to furnish arguments for the necessary reductions to the mean places. Astronomy would probably be more benefited if individual observers would confine their labours to individual instruments, and pursue a certain object for a given time with undivided attention. The use of a transit-instrument will require an opening in the roof of the room in which it stands, both towards the north and towards the south, as well as in the zenith, and down the walls to the horizon and meridian marks: these openings should not be too narrow, so as to occasion currents of air into the room, which will be detrimental to the vision of the telescope. From experience it has been found that nine inches wide is too little, and fifteen inches not too much, when the doors are made to fit well, to open conveniently, and to exclude the rain and snow. The number of doors, and mode of hanging them, will depend on the length of the apertures and height of the roof. In a room only twelve feet wide we have a single door only at each side of the roof, nine feet long and fifteen inches and a half wide, which opens at once, and the lower end of it is within reach from the stone that lies horizontally at the bottom of the narrow door in the wall, at each side; so that it is opened by the hand with perfect convenience by a person standing on the said stone, as a step eighteen inches from the floor; the vertical doors, in place of windows, are of the same breadth as the openings in the roof, and five feet high above the solid stone. The rafters covered by the edges of the inclined doors in the roof, and to one of which the door is fixed with three similar hinges, is grooved and covered with lead to allow the water to run down; the north door that shuts last, being terminated with a ridge of tin that lies over the first or south door, excludes the rain; and the plan is simple, economical, and effectual. There is no objection to the whole door opening at once, in a small room, because it soon acquires and preserves the temperature of the external air; and when the evening is calm, both doors may be opened with advantage. The clock must have a niche in its own pier: and if a circular instrument of any description is to be used for measuring altitudes, zenith distances, or polar distances, a second opening across the roof, and down the north and south walls, similar to the one described, will be necessary for the use of this instrument, unless a transit-circle be used for both purposes, similar to the one formerly used by Mr. Groombridge, in which case one opening will suffice, as both right ascensions and altitudes, or zenith distances, will be taken at the same time. Should an altitude and azimuth circle, or an equatorial instrument be made choice of, they will require a revolving roof with openings and doors on two opposite sides, to enable the observer to follow a heavenly body, out of the meridian, across all the cardinal points. This roof should not be larger than necessary for giving room to the observer and to the instrument turning under it, lest its bulk and consequent weight should impede its easy motion, and re-

quire an assistant to move it. Such roof would also be useful when a simple telescope is directed to the heavens, either with or without an equatorial motion, whenever the planets or double stars are the objects to be observed out of the meridian. If the telescope, of either the refracting or reflecting kind, is unusually large, it will be not only most commodiously, but most beneficially used on a corresponding stand, placed on a platform in the open air. Instruments for making zenith and polar observations must also be well fixed on substantial masonry, but will require only small apertures in the roof, that may be opened and closed by corresponding trap-doors.

4. For portable instruments tripods of different heights, that may be moved into any convenient situation, will be found very serviceable on various occasions, both in and out of doors, as useful temporary supports. A fire would be detrimental to the uniform temperature of an observing-room, by causing currents through the doors; it will therefore be found desirable to have an adjoining room to be used as a library or computing-room, which may communicate with the observing-room, and which will contribute greatly to comfort in cold weather as a room of occasional retirement, during the intervals of waiting, or during the temporary intervention of clouds, which frequently annoy the assiduous observer.

5. It is hardly necessary to caution the young observer against the purchase of old and inferior instruments, which will occupy more of his time than good modern instruments, and to his great mortification will afford imperfect results. The money that is spent in the purchase of two inferior instruments will buy one good one, as far as its dimensions go; and the use of such a single instrument, when the true time is known, will always afford pleasure, which is one of the motives, if not the principal motive, that induces the independent amateur to devote his time to the rational pursuit.

6. In erecting a public observatory there is another consideration which should influence the choice of its situation, namely the locality with respect to its longitude and latitude. In the present state of astronomy, observers that have first-rate instruments compete with one another in accuracy, and results may be derived more advantageously from the labours of different observers, when the situations of their observatories differ more in latitude than in longitude. If one observatory were in a northern, and the other in a southern latitude in the same meridian, or nearly so, the observations there made would supply data for many useful purposes, besides those which are their immediate objects: the south polar distances of stars, as observed at one observatory, added to the north polar distances taken at the other, would check both the observations and refractions; for when the corrections shall have been applied, it is evident that the sum should be in every case 180°. Again, the distance between the observatories, or their difference of latitude, might be taken as an element in computing the parallax of a planet, as Mars, when observed at the same time at both places: likewise many stars would be seen at one place which are never seen at the other.

7. When however there exists a material difference of longitude, comets may be seen in one situation, which would be above the horizon in the day-time at another: and even in the same kingdom where clouds frequently occur, eclipses, occultations, and other phenomena will frequently be visible to one observer, which are invisible to another at no great distance; so that the practical astronomer is not under the necessity of quitting his home to render his observations useful.

§ III. ROTATIVE DOME. [Plate I.]

1. In our last section we recommended the use of a rotative dome, when instruments are used to observe stars and planets out of the meridian. Such dome may be made to turn round on a circular bed, placed in a horizontal position, on either a wooden frame, or on brick-work, as the building may render most convenient. There are two methods of producing a circular motion in azimuth; the first that which was contrived by the ingenious Smeaton, and the second that which was first constructed by ourselves, at the suggestion of Troughton, at East Sheen in Surrey, and which has been since copied at the observatories at Edinburgh and Cambridge, and at one of the military academies, the last of which is an exact copy executed by Cubitt. We will first describe Smeaton's construction by reference to a drawing made of a conical roof, constructed under his direction for the late Mr. Aubert at Highbury, but which is now standing on the top of a brick summer-house in the rectory-garden of South Kilworth.

2. Figure 1. of Plate I. is a bird's eye view of the conical roof, when viewed perpendicularly downwards; it shows the different triangular boards of wainscot put compactly together, which compose the external face, and which, after several years' exposure to the weather, is yet quite sound. The letters *a* and *a* are placed on two oblong doors that meet at the apex of the cone, and a piece of sheet-copper bent over the upper end of the door which shuts last, keeps the rain from entering at the place of junction. These doors are nine inches wide, but would have been more favourable for the good performance of a telescope looking through, if they had been wider: they turn each upon four pairs of brass hinges, and fall back on being opened outwards. Fig. 2. is a section of this conical dome, the plane of which lies along the opening under the doors, and *a a* in this figure are the rafters that form the cheeks of the doors: the dome is represented as standing on a wooden frame supporting a flat, to show that the building may be much larger than the base of the dome, if required: the interior diameter is 8 feet 4 inches. The wooden plate *b b*, which appears a straight line, is a circular broad ring to which the covering wainscot boards are made fast above the eaves, and *c c* is a similar ring forming the wall-plate or gang-way, on which the dome rests and revolves. The piece *d d* that carries ten brass rollers is also a ring of wood of the same dimensions as the two former ones, between which it lies. This intermediate ring is shown separately in fig. 3. with sections of its equi-distant rollers, some of which are seen in fig. 2. more clearly. The intention of this ring is to diminish the friction, by giving a double velocity to the revolving motion of the conical dome, as compared with the motion of the ring itself that holds the rollers, and also by taking all the weight of the dome from the pivots of the rollers, which pivots are made comparatively small as they regard the diameter of the rollers, which are each five inches in diameter, and three fourths of an inch thick. If the ring *d d* had rested on wedges interposed between its face and that of the circular wall-plate *c c*, so that the rollers could not touch this lower plate, the superincumbent weight would have been carried by the pivots of the rollers, and the rollers themselves, while turning round, would have remained stationary, when the dome was made to revolve, and an ordinary mechanic

would have thought this mode of fixing the rollers sufficient for the purpose of giving easy motion; but Smeaton knew better, and therefore diminished the friction a second time, by making the rollers bear the weight on their circumferences, while they revolve in a groove on the wall-plate *c c*; the consequence of this contrivance is, that the dome moves with just double the velocity of the ring *d d*; for while it is carried once round by this ring, on which it rides as in a carriage, it derives an additional motion from the action of the upper sides of the rollers, which may be considered as toothless pinions impelling *b b* as a large wheel; and thus the friction is only one half of what it would have been, if the ring *b b* had been stationary. This ring is divided into five arcs by as many hinges, that, in case of any part shrinking or swelling with the changes of weather, the weight may still press alike on all the rollers. The other rings are made into two semicircles, and united in their places at the time of the erection of the building. The under face of the ring *b b* is also grooved to receive the upper edges of the rollers, and a few bars of iron are attached to it, which descend and turn under a circular projection on the edge of the ring or plate *c c*, to prevent the dome being overturned by the wind. Fifteen handles within the moveable ring *d d*, which are shown within the figure, are for the purpose of pushing this ring round, and with it the whole dome, into the position that any observation may require. If the instrument to be used within this dome be of an ordinary size, it may stand on a pillar rising from the ground; but if a large refracting telescope be required for any extraordinary purpose, its object-end may pass through the opening between the two halves of the cone, and a support may easily be made to slide up the cheeks of the door by means of a pulley, so as to give it any degree of elevation, while the lower end may rest on an adjustable stand within. Fig. 4. gives an enlarged section of the rings and of one of the rollers and cranked iron bars, together with one of the rafters, or door cheek, from which a correct idea may be entertained of the connexion that the different parts have with each other without further explanation. The central part of the cone, distinguished in both the figures 1. and 2., is covered with copper, which is nailed to a solid interior ring of wainscot, to which the upper ends of the triangular pieces are made fast, and near this small ring a rod of metal unites the cheeks of each door, and ties the two halves into one cone. The principal objection to this construction is, that the grooves formed in the rings *b b* and *c c* are liable to alterations from the swelling and shrinking of the wood, in different states of the weather, which cause them sometimes to set the rollers fast, and to prevent the power of one man from moving the dome round, which unfavourable circumstance led to the contrivance of the following construction.

3. Figure 5 represents a section of the rotatory dome at East Sheen, which turns round on three detached spheres of lignum vitæ, in a circular bed, formed partly by the dome and partly by the cylindrical frame work, which surrounds the circular room of nine feet diameter: a section of this bed forms a square which the sphere just fills, so as to have a small play to allow for shrinking; and when the dome is carried round, the spheres, having exactly equal diameters of about four inches and a quarter each, when placed at equal distances from one another, keep their relative places, and move together in a beautifully smooth manner. These spheres act as friction rollers in two directions at the four points of contact, in case any obstacle is opposed to their progressive motion by the admission of dirt, or by change of figure of the wood, that composes the rings of the dome and of the gang-way. Here no groove is made, but what the weight of the roof resting on the hard spheres occasions; and the third intermediate ring, that keeps

Smeaton's rollers apart, is entirely omitted, though, as in his construction, the dome itself moves twice round for the balls once, and has therefore its friction in the same manner diminished. The wood of this dome is covered by Wyatt's patent copper, one square foot of which weighs upwards of a pound, the thinnest kind weighing exactly a pound; and the copper is so turned over the nails that fix it at the parts of junction, that not a single nail is seen in the whole dome. This covering will no doubt render the dome more permanent than it would have been if made of wood alone, but it is found too hot in summer for an instrument that is graduated to stand in, and harbours insects which breed in great quantities between the warm metal covering and the wood, which shrinks with the heat, and makes crevices for the admission of flies. The dome at Cambridge, being made chiefly of iron, may be less liable to this inconvenience. In the dome at Edinburgh we have been informed that lateral rollers are superadded; but why they were deemed necessary we have not learned. We have put the same letters of reference to figure 5 that we put to Smeaton's dome, that the corresponding rings and doors might be understood without a repetition of the description. The pillar in the middle of this roof ascends 36 feet from the ground, through two stories without touching any floor, but its height was found too great for the dimensions of its base; which circumstance would have been still more objectionable, if it had been exposed to the solar rays. The two halves of the dome are united by brass rods passing through the door-cheeks of wainscot at *a* and *a*, by means of nuts that screw upon their ends as seen in the figure, which union allows the dome to be separated into two parts when there may be occasion to displace it.

4. Figure 6 shows a small door that lies over the summit of the dome, and may be separately opened for zenith observations; the rod of metal, with a ring at the lower end passing through it, serves to open and shut this door by, and at the same time carries upon its upper end a large ball, that falls back on the roof when the door is open, and keeps the door in a situation to be acted on by the hook of a handle that is used for this purpose: the doors *a a* being curved, are made to open in two halves, the upper one being opened first, on account of its covering the end of the other; and the observer may open one, two, or more doors as may best suit his purpose. The weight of this dome is such that a couple of wedges, inserted by a gentle blow between the rings *b b* and *c c*, will keep it in its situation under the influence of the strongest wind, and a Gregorian telescope of six inches aperture, turning on pivots between the cheeks of the doors, was as manageable and as steadily mounted, as on the best stand that can be constructed; for a lever with one end formed into a wedge shape, and inserted between the fixed and moveable rings *c c* and *d d*, on being raised gently and moved gradually to the right or left, gave an uniform slow motion to the telescope, which enabled an observer to follow a star or planet in azimuth with great ease, and through any distance. A long refracting telescope might also have its object-end passed through the opening of this dome, and supported at any height, as was stated to be practicable in Smeaton's. When the dome is elevated considerably above the floor, its doors may require some additional mechanism by pulleys or crank-pieces, to render the opening convenient; but the means that will best suit the case will naturally occur to the mechanic who is employed in the construction. It is to be regretted that this beautiful dome is no longer used for the purposes of astronomy, the property of the estate having been transferred.

§ IV. REFRACTING TELESCOPES WITH CELESTIAL EYE-PIECES.

1. A good telescope is an essential constituent of every useful astronomical instrument, and when well mounted on a firm and convenient stand, may be employed without any appendages, in the examination of various celestial phenomena that otherwise might pass unobserved. By the aid of a simple telescope the immersions and emersions of Jupiter's satellites, the occultation of stars and planets by the moon, and the aspects of comets, stellar nebulæ, and double stars may be observed with the utmost precision: and when the instrument is furnished with a good micrometer, results of a most interesting nature may be obtained thereby. Telescopes may be divided into two classes, refracting and reflecting, which will require separate explanations: and as the former class is better adapted for an union with graduated appendages, on account of the lightness, compactness, and permanent adjustment of the parts, than the heavy structure of the latter, it demands our first consideration.

2. The refracting telescope was formerly of an unwieldy length, and very inconvenient for the purpose of magnifying celestial objects, that were invisible to the unassisted eye; but since the achromatic object-glass was invented, and brought to its present state of perfection, its tube is reduced to a portable length, without diminution of the amplifying property; and the prismatic colours, occasioned by a single lens, are nearly annihilated by one of the most ingenious and beautiful contrivances that human ingenuity ever invented. It is not within the scope of this work, to enter into a mathematical detail of all the properties of lenses, with faces of different curvatures, and composed of different materials, either in their simple or combined state, with a view of developing the achromatic theory, which is the province of dioptrics; but to give a popular description and explanation of the several instruments as they come out of the maker's hands, and to assist the young astronomer to understand them, and to use them in the most advantageous manner.

3. It is scarcely necessary to tell the reader, that the first refracting telescope, as constructed by Galileo, was composed of two simple lenses; a convex lens that formed the image of an object at its focus, and a concave eye-lens, which rendered that image visible by the eye under an enlarged visual angle, and in an erect position. This contrivance, simple as it appears to be, excited the astonishment of the whole civilized world, by giving enlarged powers to the human eye, and by removing the obstacles opposed to natural vision, arising out of distance immeasurable and incalculable. Secondary revolving bodies till then unknown, and thousands of lucid points that stud the firmament, unnoticed by mortal eye, were then displayed for the first time. Man's curiosity was roused to devise means of rendering the aërial telescope a manageable instrument; and it is only from the influence of habit, that we cease to admire the wonderful effect produced by the powers of the telescope, when we direct and adjust it to distant objects. Afterwards a convex lens was substituted for the concave at the eye, and the image was both inverted and reversed; but the power of enlarging the visual angle remained the same, while the quotient of the focal length of the object-lens, divided by

the focal length of the eye-lens, was unaltered. In both constructions the prismatic colours produced by the rays of light, that were differently refrangible, tinged the images of all objects thus viewed, and pointed out the defect arising from the use of single lenses so circumstanced. Another bad effect produced by the spherical figure of the lenses was a distortion of the image, arising out of the aberration of the extreme rays, that were unequally refracted from different points of the transmitting surface: this defect was however presently obviated by the adoption of additional lenses, accordingly as the image was desired to be in a direct or inverted position; the latter of which is as useful in celestial observations as the former, since most of the celestial bodies appear spherical.

4. Galileo's telescope was contrived about the year 1610, and it was not before the year 1630 that Scheiner substituted the convex for the concave eye-lens, as suggested by Kepler. It has been asserted that Scheiner afterwards contrived a combination of two lenses for an eye-piece that gave an erect image; and Martin has explained, that this will be the case, if the second lens be placed at double its focal distance from the focal image; for then the image will be repeated in a contrary position, at the same distance on the other side of the lens, which second image may be viewed by the eye-lens.

5. Rheita, however, afterwards produced the erect position of an image by three similar lenses placed from each other at double their focal distances respectively, which arrangement greatly diminished the aberration, but did not alter the power of magnifying; this eye-piece obtained, and still retains the denomination of the *terrestrial* eye-piece, by way of distinction from the inverting or celestial eye-piece.

6. Little further improvement was made in the construction of the refracting telescope, till we come down to the time of Dollond and Ramsden, almost within our own recollection; when both parts of this instrument were brought nearly at the same time to almost a state of perfection. While Dollond was occupied in exterminating the prismatic colours from the compound object-glass, by means of a concave lens of flint glass interposed between two convex lenses of crown glass, Ramsden, his brother-in-law, succeeded in diminishing the aberrations both in the terrestrial and celestial eye-pieces; so that an union of the two improvements has rendered the achromatic refracting telescope as perfect as the materials of which it is composed will admit. It is matter of much regret that large discs of flint-glass of uniform texture, of good colour, and free from veins, are extremely difficult to be acquired, notwithstanding the pretensions of certain foreign manufacturers to the secret of producing perfect specimens; nor indeed have we many opticians, whose skill is equal to the task of computing and completing the true curves, for specimens of glass of different refractive and dispersive powers, particularly when the diameter is required to be above the ordinary dimensions.

7. In France and in Germany object-glasses have been attempted of eight, ten, or even more inches diameter, but we cannot affirm, otherwise than by report, that the best of these afford perfect specimens of telescopic vision. In England good object-glasses of five inches diameter have long been made by Dollond and Tulley respectively, and the latter artist has lately finished one of seven inches aperture and twelve feet focal length, of foreign glass, for the Astronomical Society of London, which performs most admirably*; the art is certainly equal

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* Mr. Dollond, we understand, is engaged in making an object-glass of eight inches aperture of similar glass.

to the attempt, when the materials prove perfect. In the present state of optical skill one convex lens and one concave are found adequate to the purpose of composing a good object-glass, whenever suitable discs of both kinds of glass can be procured; but good flint-glass of useful dimensions is so scarce, that a perfect object-glass is considered a valuable acquisition.

8. Hence it becomes an object of considerable importance to the cultivator of practical astronomy, that he should be enabled to judge of the qualities of his telescopes, as well as of their magnifying powers; inasmuch as that the accuracy of his observations will be materially affected thereby, as well as his pleasure in observing. Generally speaking, no one can be expected to be so good a judge of a fine object-glass as the skilful manufacturer himself, who not only computed the curves suitable for the refractive and dispersive powers of the respective pieces of glass used, but who, during the manipulation and gradual formation of the necessary curves, knew what final strokes were required to be given by his finishing tool, before he was satisfied when to conclude his last delicate operation. Having been frequently present at the examination of an object-glass nearly finished, and having been favoured with permission to witness the modes of examination in the different progressive stages of the work, we may be allowed to offer to the reader, who may not have had the same opportunity, some directions that may prove serviceable, in guiding his judgment in the choice of a refracting telescope.

9. Let us suppose, that an achromatic refracting telescope of $3\frac{1}{2}$ feet focal length, and $3\frac{1}{4}$ inches aperture, be offered for sale, and that it be required to ascertain whether the object-glass, on which its excellence principally depends, is a good one, and duly adjusted: some opinion may be formed by laying the tube of the telescope in a horizontal position on a tripod, or other support, about the height of the eye, and by placing a printed card, or a watch-glass vertically, but in an inverted position, against some wall or pillar at thirty or forty yards distance, so as to be exposed to a clear sky; then, when the telescope is directed to this object, and adjusted by the sliding eye-tube until distinct vision is obtained, the letters on the card, or the strokes and dots on the watch-glass, should appear clearly and sharply defined, without any colouration or mistiness; and if very small points appear well defined, great hopes may be entertained that the object-glass will turn out a good one. But this cursory examination will not always be sufficient for detecting slight imperfections, either in the substance or curves of the glasses, for a telescope may appear a good one, when viewing common terrestrial objects, to an eye unaccustomed to discriminate small deviations from perfect vision, though it may turn out to be an indifferent one when directed to certain celestial objects. Instead therefore of a printed card, fix a black board, or one half of a sheet of black paper, in a vertical position at the same distance, and a circular disc of white writing paper, about a quarter of an inch or less in diameter, on the centre of the black ground; then having directed the telescope to this object, and adjusted for the place of distinct vision, mark with a black-lead pencil the sliding eye-tube, at the end of the main tube, so that this position can always be known; and if this sliding tube be gradually drawn out, or pushed in, while the eye beholds the disc, it will gradually enlarge, and lose its colour, till its edges cease to be well defined. Now if the enlarged misty circle is observed to be concentric with the disc itself, the object-glass is properly centered, as it has reference to the tube; but if the misty circle goes to one side of the disc, the cell of the object-glass is not at right angles to the tube, and must have its screws removed, and its holes elongated by a rat-tailed file small enough to enter the holes. When this

has been done the cell may be replaced, and the disc examined a second time, and a slight stroke on one edge of the cell, by a wooden mallet, will show by the alteration made in the position of the misty portion of the disc, how the adjustment is to be effected, which is known to be right when a motion in the sliding tube will make the diluted disc enlarge in a circle concentric with the disc itself. This effect may be produced by giving the blow gently on that side of the cell towards which the diluted disc inclines during its enlargement: viz. if the disc enlarges toward the right, the cell must be forced gently inwards on its right hand edge, by a gentle blow, and the contrary. When the disc will enlarge so as to make a ring of diluted white light round its circumference, as the sliding tube holding the eye-piece is pushed in, or drawn out, the cell may be finally fixed by the screws passing through its elongated holes. When the object-glass is thus adjusted, which the best opticians will not neglect doing themselves, when the telescope is an expensive one, but which the ordinary makers of common telescopes do not understand, it may then be ascertained whether the curves of the respective lenses, composing the object-glass, are well formed, and suitable for each other. If a small motion of the sliding tube of about one tenth of an inch, in a 3½ feet telescope, from the point of distinct vision, will dilute the light of the disc and render the appearance confused, the figure of the object-glass is good; particularly if the same effect will take place at equal distances from the point of good vision, when the tube is alternately drawn out and pushed in. But if one of these distances be greater than the other, the parabolic figure is not perfect, but approaches towards a spherical or hyperbolical curve, accordingly as the inner or the outer space moved over is the greater. If in this respect there be an apparent defect, it must be cured by the workman himself. A telescope that will admit of much motion in the sliding tube, without sensibly affecting the distinctness of vision, will not define an object well at any point of adjustment, and must be considered as having an imperfect object-glass, inasmuch as that the spherical aberration of the transmitted rays is not duly corrected. The due adjustment of the convex lens, or lenses, to the concave one, will be judged of by the absence of colouration round the enlarged disc, and is a property distinct from the spherical aberration; the achromatism, depending on the relative focal distances of the convex and concave lenses, is regulated by the relative dispersive powers of the pieces of glass made use of, but the distinctness of vision depends on a good figure of the computed curves that limit the focal distances. When an object-glass is free from imperfection in both these respects, it may be called a good glass for terrestrial purposes.

10. How far such object-glass may be good for viewing a star or planet remains yet to be ascertained, and can only be known by actual observation of a heavenly body. When a good telescope is directed to the moon or to Jupiter, the achromatism may be judged of, by alternately pushing in, and drawing out the eye-piece, from the place of distinct vision; in the former case, a ring of purple will be formed round the edge; and in the latter, a ring of light green, which is the central colour of the prismatic spectrum; for these appearances show, that the extreme colours, red and violet, are corrected. Again, if one part of a lens employed have a different refractive power from another part of it; that is, if the glass, particularly flint-glass, is not homogeneous, a star of the first, or even of the second magnitude, will point out the natural defect by the exhibition of an irradiation, or what opticians call a *wing* at one side, which no perfection of figure or of adjustment will banish, and the greater the aper-

ture, the more liable is the evil to happen. Hence caps with different apertures are usually supplied with large telescopes, that the extreme parts of the glass may be cut off, in observations requiring a round and well defined image of the body observed.

11. Another method of determining both the figure and quality of the object-glass is by first covering its centre by a circular piece of paper, as much as one half of its diameter, and adjusting it for distinct vision of a given object, which may be the said disc, when the central rays are intercepted, and then trying if the focal length remains unaltered when the paper is taken away, and an aperture of the same size applied, so that the extreme rays may in their turn be cut off. If the vision remains equally distinct in both cases, without any new adjustment for focal distance, the figure is good, and the spherical aberration cured, and it may be seen by viewing a star of the first magnitude successively in both cases, whether the irradiation is produced more by the extreme or by the central parts of the glass; or, in case one half of the glass be faulty and the other good, a semicircular aperture, by being turned gradually round in trial, will detect what semicircle contains the defective portion of the glass, and if such portion should be covered, the only inconvenience that would ensue, would be the loss of so much light as is thus excluded.

When an object-glass produces radiations in a large star, it is unfit for the nicer purposes of astronomy, such as viewing double stars of the first class, or giving either the right ascension or declination of a star that is large enough to be affected. Indeed the smaller a large star appears in any telescope, the better is the figure of the object-glass, but if the image of the star be free from wings, the size of its disc is not an objection in practical observations, as it may be bisected without deflection from the small line by which the measure is to be taken.

12. Among the Parisian opticians a diaphragm is frequently inserted into the body of the large tube, to cut off the extreme rays coming from the object glass, when the figure is not good, instead of lessening the aperture by a cap, and when this is the case, a deficiency of light will be the consequence beyond what the apparent aperture warrants, and in measuring the amplifying power of such telescope by a dynameter, the measure far exceeds the quantity due to the length of the telescope. It will therefore always be prudent to examine that the diaphragm be not placed too near the object-glass, so as to intercept any of the useful rays, or indeed any of the rays at all, if the object-glass be good. In the large telescope recently finished by the Senior Tulley, the whole aperture of 6.8 inches is allowed to be effective, and yet the vision is as perfect as art can make it with imperfect materials. For this chef d'œuvre two convex lenses of different refractive powers were both adapted to the German flint-glass, when English plate was used, the focal length was obliged to be diminished $2\frac{1}{2}$ inches, by reason of its different dispersive power, as compared with that of the French plate, of which the other lens is formed. They perform however equally well, and afford an undoubted proof of the perfection of the workmanship.

13. An old Dollond's telescope of 63 inches focal length and $3\frac{3}{4}$ inches aperture, supposed to be an excellent one, was brought to Tulley to be examined, when we were present, and the result of the examination was, that its achromatism was not perfect. The imperfection was thus determined by experiment, a small glass globe was placed at 40 yards distance from the object-end of the telescope when the sun was shining, and the speck of light seen reflected from this globe formed a good substitute for a large star, as an object to be viewed. When the

focal length of the object-glass was adjusted to this luminous object, by the rack of the eye-piece of the telescope, no judgment could be formed of its prismatic aberrations, until the eye piece had been pushed in beyond the place of correct vision; but when the telescope was shortened a little, the luminous disc occasioned by such shortening was strongly tinged with *red* rays at its circumference. On the contrary when the eye-piece was drawn out, so as to lengthen the telescope too much, the disc thus produced was tinged with a small circle of *red* at its *centre*, thereby denoting that the convex lens had too short a focal length, and the ingenious optician observed, that if one or both of the curves of the convex lens were flattened, till the total focal length should be about four inches increased, it would render the telescope quite achromatic, provided in doing this the aberration should not be increased.

14 Whenever an object glass is under examination, it will be proper to have the object examined by it in the centre of the field of view, where there appears the least distortion of the object, otherwise the judgment may be misled, particularly when the eye-piece itself is not properly constructed and adjusted to the object-glass in question. While a single lens was used at the eye-end of the earlier telescopes, it was ascertained that the spherical aberration of a double convex lens was least when the radii of their curved faces were to each other as 1 : 6, with the face 1 turned to the radiant or object viewed: but with any single lens it was found that the object viewed was both distorted and coloured, at the extremities of the field of view; and therefore a combination of lenses, such as might diminish these imperfections in the most sensible manner, became an object of investigation.

15 Boscovick and Huygens proposed the construction of an eye-piece composed of two lenses each, that diminishes the spherical aberration about four times, which eye-pieces differ from each other only in this respect, that Boscovick used two similar lenses, and Huygens two lenses that had their focal distances to each other as 1 : 3, but in both cases the distance between the lenses was equal to half the sum of their respective focal distances. The latter construction has been found the best adapted of any for reflecting telescopes, and has therefore continued in use to the present day, but modern opticians have modified the position of the lenses, and have adapted the distance between them to suit the purposes of particular telescopes. When distinct vision is the principal object of an achromatic refracting telescope, the two lenses are usually both plano-convex and fixed with their curved faces towards the object-glass, at a distance from each other something less than half the sum of their focal lengths, the one next to the eye having about one third, more or less, shorter focus than the other; and a diaphragm cutting off the extreme rays of the inner or larger lens, called the field lens, is placed at the focus of the outer or eye-lens, where the image formed by the object glass falls; which circumstance allows this eye-piece to receive a divided piece of mother-of-pearl, as proposed by Cavallo, for a species of micrometer. This eye-piece having the image viewed by the eye behind the inner lens, is sometimes called the *negative* eye-piece, and is that which the instrument makers usually supply, of three or four different sizes for so many magnifying powers, for the ordinary purposes of simple vision.

16. Another modification of the lenses known by the name of the *positive* or Ramsden's eye-piece, is applied when wires or spider's lines are used in the common focus, and this affords equally good vision with the other eye-piece, in this construction the lenses are plano-convex, and nearly of the same focus, but are placed at a distance from each other less than the focal

distance of the lens next the eye, so that the image of the object viewed is beyond both the lenses, when measuring from the eye, hence the piece, containing the two lenses, can be taken out without disturbing the lines, and is adjustable for distinct vision, and whatever may be the measure of any object given by the wire micrometer, at the solar focus, it is not altered by a change of the magnifying power, when a second eye-piece of this construction is substituted. The flat faces of the two lenses are turned into contrary directions in this eye-piece, one facing the object-glass, and the other the eye of the observer, and as the image formed at the focus of the object glass, lies parallel to the flat face of the contiguous lens, every part of the field of view is distinct at the same adjustment, or, as the opticians say, there is a *flat field*, which, without a diaphragm, prevents distortion of the object One or other of these two kinds of celestial eye pieces is now made a part of every refracting telescope of the achromatic kind, and the choice depends upon the use to which the telescope is intended to be applied

17. In Vol. 48, part I, of the Philosophical Transactions of London (p. 103), is published a letter addressed to Short by John Dollond, which points out the cure for the spherical aberration of a lens used singly. This aberration, the author says, in a single lens is as the cube of the refracted angle, but if the refraction is caused by two lenses, the sum of the cubes of each half will be ¼ of the refracted angle; twice the cube of 1 being equal to ¼ the cube of 2. So, three times the cube of 1 is only one-ninth of the cube of 3, &c.; hence the indistinctness of the borders of the field of view of a telescope is diminished by increasing the number of lenses in an eye-piece. The second imperfection of an eye-piece, arising from the prismatic aberration and producing colouration, the same author observes, may also be cured by means of a second lens rightly formed and properly placed in the celestial eye piece

Ramsden's account of his improvement is given in Vol. 73, p. 94, of the Philosophical Transactions of London for the year 1783. It may appear a curious circumstance that any eye-piece which is good with a short telescope is also good with a long one, but that the reverse is not true, for it is more difficult to make a good eye-piece for a short than for a long focal distance of the object-glass.

18. Before we can determine the magnifying power of a refracting telescope with a celestial eye-piece theoretically, we must know what single lens is equivalent to the two lenses that compose it, and this cannot be known till their focal lengths and the exact distance between them are correctly ascertained by some practical measurement; when these elements of computation are known, the following dioptric formula will give the equivalent lens; $\frac{Ff}{F+f-d}=\varphi$, where F denotes the solar focal length of the inner, f that of the outer lens, d the distance between them, and φ the focal length of the equivalent lens; then if we put S for the solar focal distance of the object-glass, and T the focal distance of the same with diverging rays, or when viewing a near terrestrial object, the celestial magnifying power will be $\frac{S}{\varphi}$, and the terrestrial power $\frac{T}{\varphi}$ very nearly. If the rays that are incident on the inner lens of a celestial eye-piece were parallel after passing the solar focus, this formula would give the solar power truly, but as they must always be in a state of divergence in every telescope after passing the said focus, it gives φ too short, and consequently the power too great, which has been observed to be the

case in practice by Troughton, Tulley, and others. The better way of ascertaining the celestial power is by means of that beautiful small instrument called the dynameter, which we shall have occasion to describe and explain (§ XI.), or by some other of the mechanical methods which take no account of the focal distances of any of the lenses.

It may however be proper to give an example of the application of our formula to the determination of the lens ϕ, that may be substituted for any eye-piece of given dimensions. Suppose that an achromatic telescope, with any aperture, have the solar focal length of its object glass forty two inches, and that it has two eye pieces, one of the Huygenian construction, and the other of Ramsden's, of the following dimensions; viz. in the first, $F = 2.5$ inches, $f = 0.75$, and $d = 1.4$, and in the second, $F = 1.4$, $f = 1.2$,-and $d = 0.95$, and that the respective powers P and P be required?

For the power with the Huygenian eye-piece we have $Ff = 2.5 \times 0.75 = 1.875$, and $F + f - d = 2.5 + 0.75 - 1.4 = 1.85$, and $\frac{1.875}{1.85} = 1.0135 = \phi$; then $\frac{42}{1.0135} = 41.44 = P$ is the magnifying power again, in the Ramsden's eye-piece we have $Ff = 1.4 \times 1.2 = 1.68$, and $1.4 + 1.2 - 0.95 = 1.65$, and $\frac{1.68}{1.65} = 1.01818 = \phi$; then $\frac{42}{1.01818} = 41.25 = P$ will differ but little from the former determination, and though the focal distances and position of the lenses are very different in these two eye pieces, yet the magnifying powers are very nearly the same, and when they are applied in succession to the same telescope, an estimate may be formed of their comparative merits with respect to producing good vision. When either of these eye pieces is applied to a telescope of a longer focal length, say five feet, or sixty inches, the new power will be increased in the ratio of $\frac{60}{42}$.

19. It is not, indeed, an easy matter to measure exactly the focal length of an object-glass, on account of its being composed of more lenses than one; Mr. Troughton informs us that the measure should commence from the interior part of the convex lens, at a distance from its exterior surface equal to one-fifth of the thickness of the double compound object-glass.

An approximate measure may be obtained by first ascertaining the whole power with a given single lens as an eye-piece, by means of a dynameter (§ XI.), and then if we put $P =$ the power, and $f =$ the focal length of the single lens, which may be very nearly measured by the distance of its solar focal image, we shall have $\frac{P}{f} = F$, the focal distance of the object-glass, and when this is once determined, the power of any eye-piece used with the same object glass will always be had from the formula $\frac{P}{F} = \phi$, the focus of a single lens equivalent to the eye-piece in question.

20. With respect to the intensity of light in any telescope, it is in general considered to be directly proportional to the square of the diameter of the object-glass, and inversely as the magnifying power of the telescope, but this rule supposes all the light to enter the pupil of the observer, the quantity of which is limited by the size of the pupil, or by the small hole in the eye-piece, that usually cuts off some of the rays occasioned by aberration · Delambre's

rule may therefore be preferable in practice: this author observes (tome I. p. 20, *Astronomie*), that the surface of an object-glass being more considerable than that of an eye-lens, which has usually a short focus, it follows that all the rays falling on the surface $B\ D$ of the object glass is collected into the space $b\ d$, the diameter of the effective portion of the lens, or lenses at the eye-end; and that therefore the light so collected ought to be more bright than the light in its dispersed state when falling on the object-glass: he therefore proposes to take *unity* as the measure of the intensity of light entering the object-end, and $\left(\frac{B\,D}{b\,d}\right)^2$ as the intensity at the eye-hole, and hence it arises, that in general the objects that are most luminous are most easily seen in a telescope. If we suppose the diameter $B\ D$ of any telescope's object-glass to be three inches, and the diameter $b\ d$ of the eye hole to be two tenths, we shall have $\frac{3 \times 3}{.2 \times .2} = \frac{9.00}{.04} =$ 225 for the increased intensity, or the first and last intensities will be as 1 : 225, to an eye that has a pupil large enough to admit all the rays. This result points out the danger of viewing the sun through a telescope without a darkening glass before the eye, and, according to Mr. Fallows, it is even unpleasant to bear the light of the moon in this way, on a fine evening at the Cape of Good Hope, the atmosphere is so clear, and the moon frequently so high.

21. The field of view of every telescope, or, in other words, the number of minutes and seconds subtended by a distant object that will be visible with a given magnifying power, will depend partly on the focal length of the object-glass, and partly on the diameter of the diaphragm placed at its focus; when the telescope is long, and the aperture of the diaphragm small, the field will be confined, and the contrary: if we put d for the diameter of the diaphragm, and F for the focal length of the object-glass in terms of the same denomination; then to obtain the value of the field in seconds, according to Delambre's rule, we have $\frac{d}{F \sin 1''}$ for the measure required.

For instance, if in a telescope of 3½ feet focal length the diameter of the diaphragm of a negative eye-piece be 0.3, we shall have $42 \times .000004848 = .000203616$, and $\frac{0.3}{.000203616} = 1473'' = 24'\ 33''$ for the whole measure of the field.

In such a telescope the focal disc, when the sun is the object, will be 0.3665 of an inch when his diameter is 30′, for $\frac{42 \times 2 \times 3.1416}{720 \text{ half degrees}} = 0.3665$, then as $0.3665 : 0°, 30' :: .3 : 24'\ 33''$ as before, hence the diameter of the diaphragm ought to be 0.3665 to allow the whole disc of the sun to be visible in the telescope, which determination accords with the rule given by Delambre.

The same result may be obtained by a case in plane trigonometry, thus

As radius	10.000000
Is to the focal length 42 inches .	1.623249
So is half the sun's diameter 15′ .	7.639816
To half the disc 0.18327	9.263065

Then $0.18327 \times 2 = 0.36654$ as above determined.

These methods of determining the measure of the focal disc, or image of the sun, are founded upon the well known optical fact, that if two pencils of extreme rays proceeding from the opposite sides or ends of a distant object are both incident on the centre of the object-glass of a telescope, they will there cross one another, and pass on without refraction, till they form an inverted image of the said object, or rather of its extremities in the focus, so that the object and its image subtend the same angle at the centre of the object-glass.

22. When the object is near, and the telescope long, the object and its image will be placed in the relative conjugate foci of the object-glass, and the difference between the solar and conjugate foci will be considerable enough to afford data for determining the place of the remote conjugate focus, or distance of the object. If we put F to represent the length of the solar focus, F' for the length of the shorter conjugate focus where the image is formed in the telescope, and D for the distance of the object situated in the remote conjugate focus, the analogy will be

As $(F'-F) : F :: F' : D$, therefore we have $D.(F'-F) = F'F$, and $\frac{F'F}{F'-F} = D$; also $\frac{F'F}{D} = F'-F$, and $\frac{D.(F'-F)}{F'} = F$, so that when any three of these terms are given, the fourth will also be known. As an example, let the solar focal distance of a telescope be 12 feet or 144 inches, and the elongation just two inches, when it is brought to distinct vision of a near object by drawing out the tube holding the eye-piece, then we shall have $F = 144$, $(F'-F) = 2$, and consequently $F' = 146$, also $\frac{F'F}{F'-F} = \frac{146 \times 144}{146-144} = \frac{21024}{2} = 10512 = D$ in inches, or 876 feet, likewise we shall have $\frac{F'F}{D} = F'-F = \frac{21024}{10512} = 2$, and $\frac{D\,(F'-F)}{F'} = \frac{10512 \times 2}{146} = 144 = F$; hence we may obtain the solar focal distance of the telescope from a terrestrial object placed at a measured distance.

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§ V. DIAGONAL EYE-PIECES [PLATE II]

1. THE astronomical eye-pieces, which are described in the preceding chapter, both invert and reverse the object that is viewed, that is, they show the position as changed by the object-glass with respect to both altitude and azimuth, but when the body is spherical this is of no importance. In high altitudes, however, the head of the observer is obliged to be placed in a very inconvenient position when these eye-pieces are used, whatever may be the ordinary structure of the stand used; to obviate which inconvenience, eye-pieces have been invented, that will admit of the eye being applied at the side instead of the end, and when one of these is used, it is of no importance what may be the depression of the eye-end of the telescope. These eye-pieces are called *diagonal*, because a flat piece of polished speculum metal is usually applied between the two lenses of the eye-piece at an angle of 45°, which alters the direction of the converging rays, and forms an image which becomes erect with respect to altitude, but is still reversed with respect to azimuth. The same effect will be produced if either of the lenses be re-

moved, that is, whether the diagonal reflector be before or behind the lens used, and as two lenses in a celestial eye piece are substituted for one, to diminish the aberration, and as they jointly produce the same effect on the vision as one lens of higher power than either of them taken singly would do, there are three situations in which the diagonal reflector may be placed, before the eye-piece, behind it, and between the lenses, each of which position has its peculiar advantage

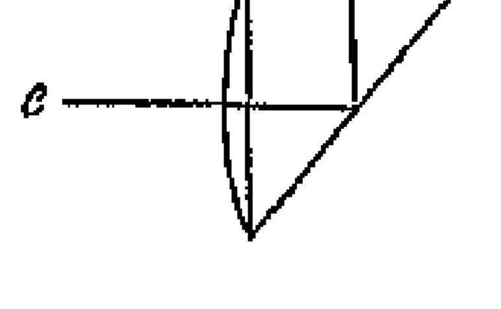
2. When the diagonal piece is fixed within the sliding tube that receives the eye-piece, as in fig. 3. of Plate II., it may be used with any eye-piece screwing into a hole made at the side of the said tube, and will admit of any power, which is perhaps the best construction, though not commonly adopted.

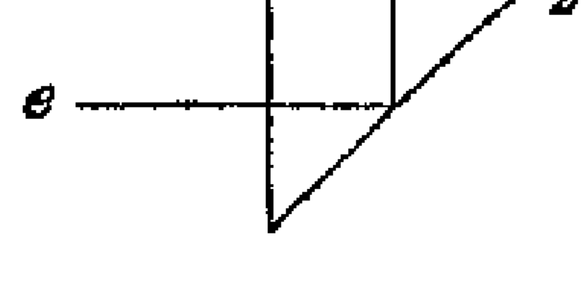
3 A small piece of reflecting metal is sometimes made to slide before the eye pieceat the requisite angle of reclination, in which application each eye-piece must necessarily have a groove to receive it, and the eye must be applied without a hole to direct it, but it may be put on and taken off without disturbing the adjustment for distinct vision, and is very simple in its application Great care is necessary in handling the piece of polished metal by the edges, lest it should be tarnished, and a small box is necessary to keep it in a fixed position when not used.

4. The third and most common position of the reflector is between the lenses, and this may be done in both the negative and positive eye-pieces, but as the distance between the two lenses is necessarily considerable, to make room for the diagonal position of the reflector, the magnifying power cannot be great, otherwise a diagonal eye-piece of this construction, which is shown by fig 4., remains always in adjustment, and is useful in all cases where the power of magnifying highly can be dispensed with. When a micrometer with spider's lines is used with a diagonal eye-piece, the eye-piece must necessarily be of the positive construction.

5. Instead of a piece of reflecting metal that requires a surface perfectly flat, which is not easily obtained, a rectangular prism of glass may be substituted, provided it be of a good quality and perfectly worked; for the rays of light are then bent by reflection from the second polished surface which ought to be *dry*, and undergo two refractions which achromatise them, and the same effect is thus produced as by the polished metal.

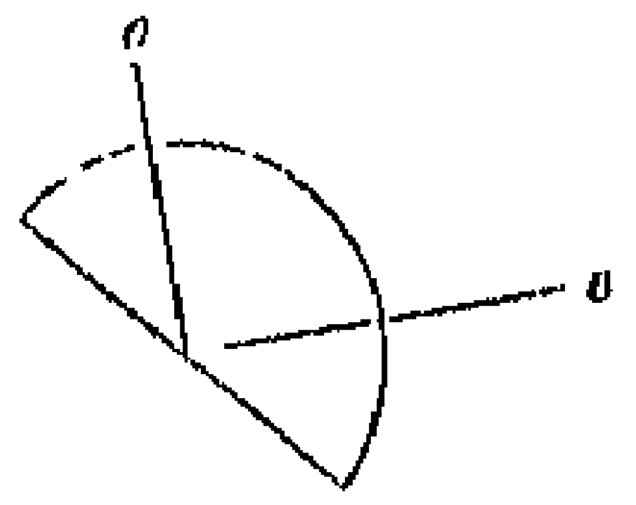
6. Ramsden sometimes gave one of the polished faces of a right-angle prism a curve, which prism served instead of a lens in an eye-piece, and also performed the office of a reflector, which plan has occasionally been adopted by his pupil, Jones of Charing Cross. This construction requires some attention to be paid to the position of the faces of the prism, as they regard the object and the eye. The younger W. Tulley discovered that, when the curve faces the object, and a flat side is presented to the eye, a large portion of the field of view will be darkened on the side next to the refracting angle, but if the flat face contiguous to the right angle be made to face the object, and the curved face at the other side of the included right angle be placed opposite the eye, the vision will be good. In the annexed figures, where *o* denotes the object and *e* the eye, the first position of a prismatic lens is good, but the second bad.



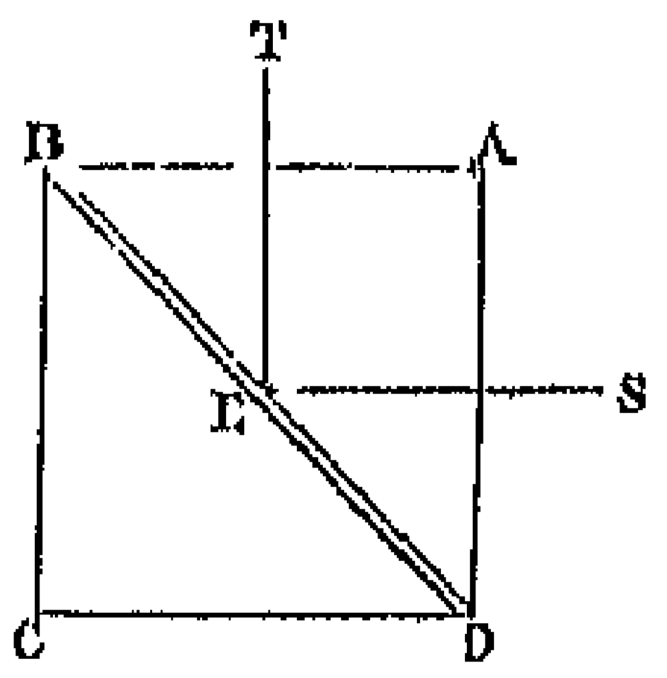


Whatever may be the construction of a diagonal eye-piece, there is a considerable loss of light occasioned by the reflection, and therefore when very small stars are observed, which is often the case in examining double ones, the observer should not study his own ease so much as the quantity of light he can retain with a high power, which object may be best attained with an ordinary eye-piece and a telescope of large aperture.

7. A semi-globe, or what is called a bull's eye, has also been used as a diagonal eye piece with a telescope, and when the curve is well formed, and the glass good, it is achromatic, and performs pretty well. The line forming the flat face must be so inclined in the tube, that the rays of light coming from an object o, may be reflected towards the eye at e, after two refractions have rendered them achromatic



8 We have said above, that the reflecting face of a prism that occasions interior reflection should be *dry*, and the same observation will apply to the flat face of the bull's eye, the reason of which may be thus explained:



If $A\ B\ C\ D$ be a cubical vessel divided by a pane of glass, so that the triangular half $B\ C\ D$ will hold water, and if a ray of light coming from S is made to fall on the said glass at any angle of incidence $A\ E\ S$ greater than 42°, while the portion $B\ C\ D$ is empty, the ray will be reflected towards T, but if it be filled with water, which has a greater attraction for light than air has, the ray will pass through the glass into the water, and suffer no reflection. When the angle of incidence is gradually lessened till the rays of light that are incident cease to be reflected, the red rays are the last to disappear, as being the least refrangible.

The description of the four-glassed diagonal eye-pieces will be given (§ VII. 11.) after the erect eye-pieces have been described.

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§ VI CELESTIAL EYE-PIECES WITH VARIABLE POWERS (Plate III)

1. We have hitherto supposed the distance between the two lenses of a celestial eye-piece to be constant, and consequently its power of magnifying the image, formed by the object-glass or large speculum of a telescope, to be invariable, and to obtain the best vision it is necessary that they should be so, because they admit of a diaphragm being inserted in the most advantageous situation, for excluding the devious rays, but occasions occur, when the power of the eye-piece requires to be changed, for limiting the field, or for modifying the light; and in the use of some of the most modern micrometers, a variable power forms one of the elements of computation of the measure. We shall therefore describe the principle on which a celestial eye-

moved, that is, whether the diagonal reflector be before or behind the lens used; and as two lenses in a celestial eye piece are substituted for one, to diminish the aberration, and as they jointly produce the same effect on the vision as one lens of higher power than either of them taken singly would do, there are three situations in which the diagonal reflector may be placed, before the eye-piece, behind it, and between the lenses, each of which position has its peculiar advantage.

2. When the diagonal piece is fixed within the sliding tube that receives the eye-piece, as in fig. 3, of Plate II., it may be used with any eye-piece screwing into a hole made at the side of the said tube, and will admit of any power, which is perhaps the best construction, though not commonly adopted.

3. A small piece of reflecting metal is sometimes made to slide before the eye piece at the requisite angle of inclination, in which application each eye-piece must necessarily have a groove to receive it, and the eye must be applied without a hole to direct it, but it may be put on and taken off without disturbing the adjustment for distinct vision, and is very simple in its application Great care is necessary in handling the piece of polished metal by the edges, lest it should be tarnished, and a small box is necessary to keep it in a fixed position when not used.

4. The third and most common position of the reflector is between the lenses, and this may be done in both the negative and positive eye-pieces, but as the distance between the two lenses is necessarily considerable, to make room for the diagonal position of the reflector, the magnifying power cannot be great, otherwise a diagonal eye-piece of this construction, which is shown by fig 4, remains always in adjustment, and is useful in all cases where the power of magnifying highly can be dispensed with. When a micrometer with spider's lines is used with a diagonal eye-piece, the eye-piece must necessarily be of the positive construction.

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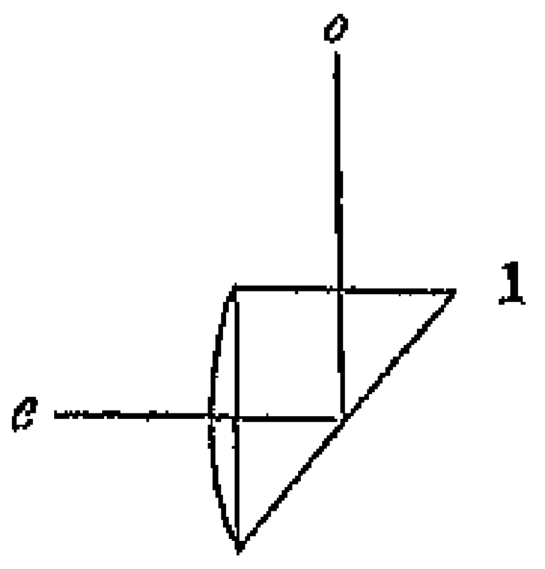

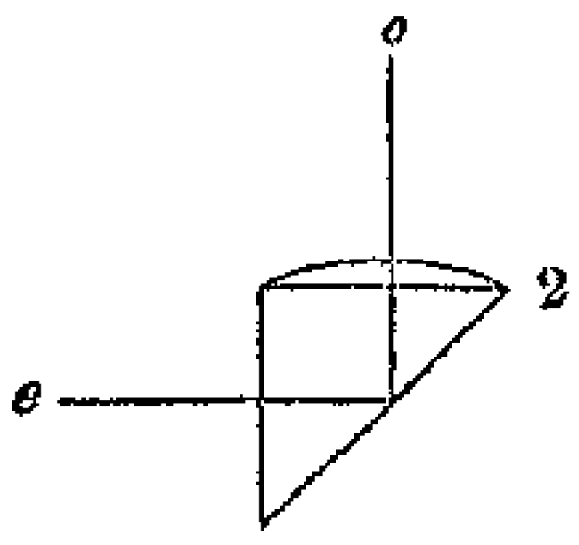

Whatever may be the construction of a diagonal eye-piece, there is a considerable loss of light occasioned by the reflection, and therefore when very small stars are observed, which is often the case in examining double ones, the observer should not study his own ease so much as the quantity of light he can retain with a high power, which object may be best attained with an ordinary eye-piece and a telescope of large aperture.

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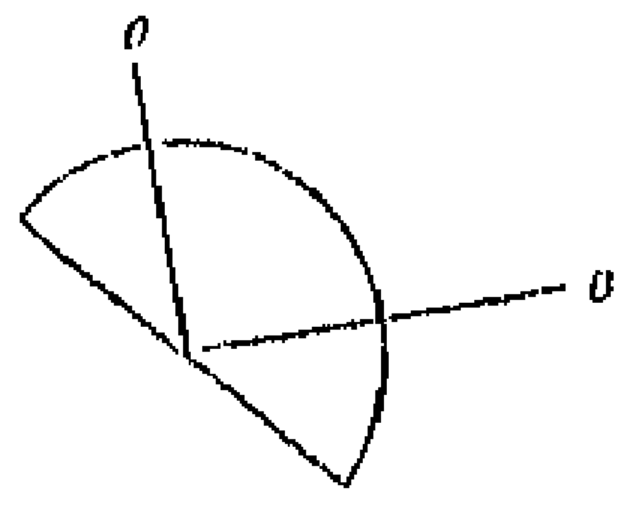

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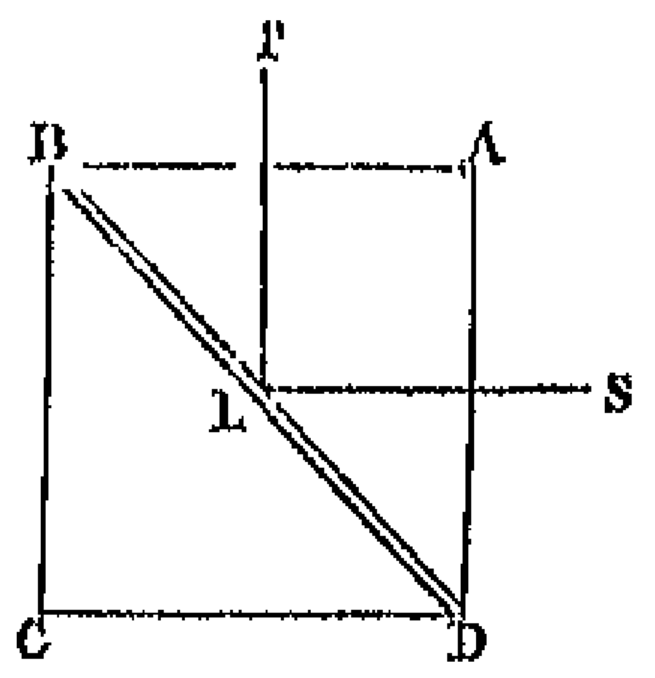

If *A B C D* be a cubical vessel divided by a pane of glass, so that the triangular half *B C D* will hold water, and if a ray of light coming from *S* is made to fall on the said glass at any angle of incidence *A E S* greater than 42°, while the portion *B C D* is empty, the ray will be reflected towards *T*, but if it be filled with water, which has a greater attraction for light than air has, the ray will pass through the glass into the water, and suffer no reflection. When the angle of incidence is gradually lessened till the rays of light that are incident cease to be reflected, the red rays are the last to disappear, as being the least refrangible.

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piece, affording a succession of variable magnifying powers, may be constructed; and when we come to describe the micrometers to which this principle is applied, we will enter more minutely into its practical application to the measurement of small angular subtenses

In the first volume of the Memoirs of the Astronomical Society of London, we have given an account of both the theory and construction of an astronomical eye-piece, with a variety of powers, to which we might refer the reader for particular information, but as that work may not be in the hands of persons who are not members of that society, we will give an outline of that communication in this place.

2 Whenever two lenses are used in an eye-piece, or otherwise, as a substitute for a single lens, for diminishing the aberrations, the focal length of that single lens, which would possess equal magnifying power, may be readily determined by a theorem in dioptrics; and as the distance between the two lenses used is an element in the calculation, there may be as many equivalent single lenses supposed to exist, as there are variable distances between the two lenses adopted.

Whatever may be the construction of the convex or plano-convex lenses used, with respect to their curves, if we put F for the focal length of the inner lens, f for the focal length of the outer lens, d for the distance between them, and φ for the focal length of the single imaginary or equivalent lens required, at any position of the two actual lenses, we shall have the formula $\frac{Ff}{F+f-d} = \varphi$, as we have before stated. (§ IV 18)

In this formula d is the only variable quantity, and the focal distance of the imaginary lens φ will decrease as d increases, but not in the same ratio If an equal scale be made to represent or indicate the quantities d, in the different situations of the two lenses, that scale will also indicate the corresponding magnifying powers that depend on the focal lengths of φ, for if we put S for the solar focal length of any object-glass, and φ for the focal length of a single lens equivalent to an eye-piece, we shall have $\frac{S}{\varphi} = P$, the magnifying power of a telescope, having that object-glass and eye-piece; therefore $\frac{S.(F+f-d)}{Ff} = P$, varies as $F+f-d$, which shows, that the increase or decrease of magnifying power depends upon the decrease or increase of the distance, at which the two lenses of the eye piece are placed from each other. The scale of distance between the two lenses cannot however be greater than the focal distance F of the inner lens, for if it were, the lenses would no longer form an eye piece, but would become an inverting opera-glass. In computing the length of a scale, that would admit of the power of a four-feet telescope to be doubled, when F was taken at two inches and f at one, it was found just an inch and a half, and the subjoined Table was computed for the purpose of showing, that the differences of the magnifying powers at equal intervals are equal parts, the focal length of the object-glass being assumed exactly at 48 inches.

3. TABLE OF THE *VARIABLE MAGNIFYING POWERS* OF A CELESTIAL EYE-PIECE, USED WITH A TELESCOPE OF FOUR FEET.

Distances	Equivalent single lens	Powers	Differences
0	.66	72.0	2.4
.1	.69	69.6	2.4
.2	.714	67.2	2.4
.3	.74	64.8	2.4
.4	.77	62.4	2.4
.5	.80	60.0	2.4
.6	.83	57.6	2.4
.7	.87	55.2	2.4
.8	.909	52.8	2.4
.9	.954	50.4	2.4
1.0	1.00	48.0	2.4
1.1	1.05	45.6	2.4
1.2	1.11	43.2	2.4
1.3	1.177	40.8	2.4
1.4	1.25	38.4	2.4
1.5	1.33	36.0	

From an inspection of this Table it is evident that, though the focal length of the equivalent single lens ϕ does not increase by equal parts, yet the powers depending on these varying focal lengths are quantities, that decrease by equal differences. If therefore we knew the first and last magnifying powers only, the modes of doing which will be shown hereafter in Section XI., we could fill up all the intermediate powers by the constant addition, or subtraction, of one-fifteenth of the whole difference, thus $\frac{72-36}{15}=2.4$ gives the constant difference in this case, to be continually applied to 36, till we arrive, after 15 additions, at 72; or if we begin at 72, we shall arrive at 36, by as many subtractions, and in this way any other Table of Powers belonging to a variable celestial eye-piece may be completed, whatever may be the number of intervals in the scale, and that without knowing either the relative or absolute values of F, f, S, or ϕ, provided the magnifying powers of the telescope used be ascertained previously, by any of the methods hereafter explained.

4. When the eye-piece is constructed on the most simple plan, a pair of short tubes of brass, sliding pretty closely over one another, and carrying each a lens, will be all the mechanism that is required. In Plate III. figures 13 and 14, represent an eye-piece of this kind, which has been found to answer its purpose very well. The screw at the end of the inner tube A, is adapted to the racked tube of a telescope, which gives it the necessary adjustment for vision; a scale, of upwards of an inch and three quarters, is divided into 180 equal parts, upon the exterior face of this tube, and a plano-convex lens *a* screws into its remote end, having its focal length very nearly two inches; round this tube the outer tube B slides, when a little force is applied to it, and carries another lens *b* of an inch focal length, which can be brought into contact with the lens *a*, or be removed from it to any distance, not exceeding the length of the scale, which is indicated by the interior end of the tube B; the lens *b* is also a plano-convex, and may be taken out at pleasure, by unscrewing it, when a lens of a different focal length, as ½ of an inch, may be substituted, which will give a new scale of powers, to be tabulated for any given telescope. When the second or eye lens has a short focal distance, and consequently a scale of large magnifying powers, a diaphragm may be placed at its focus in the tube B, which will improve the vision, but shorten the length of the scale. The only inconvenience attending the use of this eye-piece is, that the length of tube A, being added to the ordinary

length of a finished telescope, with common-eye pieces, diminishes the range of the racked tube in getting distinct vision, and this rack is wanted to be used again after every new position of the lens *b*, as it has relation to lens *a* This inconvenience, when it occurs, may however be removed, by shortening the main tube as much as the small tube A adds to the length of the telescope

5. Another and more convenient construction of this eye-piece is represented by figures 15, 16, and 17, of the same plate, with a circle on one end, graduated for taking angles of position also, which appendage will be described in another place This eye-piece, like the former one, is adapted to screw into the racked tube of a telescope, and has the scale engraved on the outer or visible tube, this tube has a longitudinal opening cut down a large portion of its length, and a shorter inner tube, seen in fig. 16, bears a rack, which is actuated by a pinion on the axis of the milled head, visible in both figures, and slides within the said outer tube, while it carries the second lens, *a*, at the end next the eye, and also a vernier piece, attached by a pair of screws passing through the oblong slit. The eye lens *b*, in this construction, is screwed fast into the end of the stationary tube, and may be changed for another at option. This eye-piece also lengthens the telescope a little, but is easier to manage, on account of the second rack, and will also admit of a diaphragm, when the eye-lens has a short focal distance. A succession of two or three eye-lenses used with the same inner lens, in either of these constructions, will extend the scale of magnifying powers to any limit that the telescope will bear. Each eye-piece is made to turn round, at the junction of the fixed tube with a screw that attaches it to the telescope, for the convenience of observing with those micrometers, of which the variable eye piece forms an essential part, and the screws at the eye-end are made to receive eye-caps, and other appendages that will be described hereafter.

6. As the distance between the two lenses is considerable when the magnifying power is small, and the reverse, it is evident that this eye-piece is sometimes a positive, or a Ramsden's eye-piece, and sometimes a Huygenian, or negative one, and affords all the gradations that can occur with the same lenses, in the different deviations from these two constructions, and therefore the vision is not equally good at every distance, though at none objectionable, when the proportion between the two focal lengths is from 2 : 1 to 3 : 1.

§ VII ERECT EYE-PIECES [Plate II]

1. We have already said, that the eye piece invented by Rheita, commonly called the *terrestrial* eye-piece of a refracting telescope, because it does not invert the object, was composed of three similar lenses placed from each other at double their focal distances respectively, and that they greatly diminished the spherical aberration, but did not alter the magnifying power from what it would have been with one of the said lenses. This construction is become obsolete, and a four-glassed eye-piece is substituted for it, which, besides improving the vision, increases the magnifying power, and also gives an erect position to the visible image of any object. During the progressive stages of improvement made in the construction of an erect

eye piece by the Dollonds and Ramsden, three and five lenses were successively introduced, but as four lenses have ultimately gained the preference, we will satisfy ourselves with a description of the most approved arrangement of the lenses, as adopted by modern opticians

2 In a telescope having a celestial or inverting eye-piece, the image that is formed in the focus of the object-glass is that which is seen magnified , but when a four-glassed or erect eye-piece is used, the image is repeated, and the *second* image, which is formed by the inner pair of lenses on an enlarged scale, is that which the pair of lenses at the eye end render visible, on a scale still more enlarged In fact, the modern terrestrial eye-piece is a compound microscope, consisting of an object-lens, an amplifying-lens, and an eye-piece composed of a pair of lenses, and its properties will be best understood by considering the first image of an object, which is formed in the focal place of the object-glass, as a small luminous object to be rendered visible, in a magnified state, by the said microscope. This plan of describing the eye-piece, we presume, will render the offices of the different lenses familiar to the reader, without a reference to an optical diagram, and will supersede the necessity of a geometrical investigation of the angles subtended by the two images in their relative positions. Such of our readers as wish for theoretic demonstrations, will of course have recourse to some treatise on optics for the fundamental information.

3. For the sake of perspicuity, we will call the lens nearest to the first image, or to the solar focus of the object-glass, the *object-lens*, the next to it the *amplifying-lens*, the third or inner lens of the *pair* at the eye end the *field-lens*, and the outermost the *eye-lens*. We will also call the first image or substitute for a small microscopic object, the *radiant*. Now it is well known, that if any single lens have the radiant in its principal or solar focus, the rays issuing from it, and falling in a state of divergence on the face of the said lens, will be so refracted by passing through it, as to become parallel after emerging from the second surface into air , but that if the radiant be brought nearer to the lens than its principal focus, the emerging rays will *diverge* , and on the contrary, if the radiant be put farther from the lens than its principal focal distance, the emerging rays will *converge* to a point or disc at a distance beyond the lens, which will depend on the quantity of convergence, or, in other words, of the distance of the radiant from the first face of the lens. In this place a second image, or image of the radiant, will be formed by the concurrence of the converging rays, but in a contrary position. These two points at the opposite sides of the lens, where the radiant and its image are posited, are called the two *conjugate foci* of the lens, and the same angle is formed at the centre of the lens, whether the radiant or its image be the subtense. Hence it will be perceived, that, whatever may be the places of the corresponding conjugate foci, the length of the image will exceed the length of the radiant, in the same proportion that the distance from the lens of the former exceeds that of the latter. This inference flows from the properties of similar triangles, which are familiar to every one; and hence it will be seen, that the linear enlargement of the image may always be had by dividing the longer conjugate focal distance by the shorter, as in the case of a magic lantern, or of a solar microscope.

4. This secondary image, or image of the radiant, is not well defined when only one lens is used, owing to the great spherical as well as prismatic aberrations, and therefore the amplifying lens is placed at the distance of the shorter conjugate focus, with an intervening diaphragm at the place of the principal focus, the uses of which second lens and diaphragm are, first to cut

off the coloured rays that are occasioned by the dispersive property of the object lens, and secondly to bring the rays to a shorter conjugate focus for the place of the image, than would have taken place with a single lens having only one refraction. Then it is evident, that, as the magnifying power of the object lens depends on the relative distances from it of the two conjugate foci, the second lens, by shortening the longer conjugate focal distance, lessens the magnifying power in the same proportion, but as the secondary image is thus much better defined and free from colouration, the addition of this lens is a great improvement to the vision, and the place of the second image can at any time be varied, by simply varying the distance of the radiant from the lens in a contrary direction, by means of the racked tube of adjustment for distinct vision. On this account the main tube may have its length varied at pleasure, without making any alteration in the relative position of the first and second lenses which we have described.

5. In the ordinary compound microscope the object-lens is very small, and the amplifying lens of a large diameter, and situated nearly half way between the two ends of the main tube, its principal object being to amplify the field of view, but in the erect eye-piece the first glass is usually larger than the second, which yet, by lessening the power, may be said relatively to amplify the field, otherwise the explanation of the properties of the two constructions is the same, and in our estimation the one we are describing defines the image of the radiant the more distinctly of the two, and forms the better microscope with a good pair of eye-glasses. The remaining portion of the magnifying power depends upon the eye glasses, which may be put together after the form of either of the celestial eye-pieces above described [§ IV 15, 16.], to produce what is technically called *a flat field*. In theory the whole magnifying power is gained at twice, the first portion may be had by dividing the focal length of the object glass by the focal length of the eye-lenses taken jointly as one equivalent lens, in the way the power is gained in a celestial telescope, and then the resulting power is increased in the ratio of the distances of the two conjugate foci from the object-lens, as has been explained. The theory by which the magnifying power of a telescope is determined, supposes that the thickness and focal distances of the respective lenses, and also the exact distances between them may all be perfectly measured, which in practice is no easy matter; we will not therefore trouble our readers with the various formulæ that the science of optics supplies, as explanatory of all the varieties that occur in the construction of instruments, and in the methods of appreciating their properties; but shall explain presently the practical methods of gaining the magnifying powers by mechanical means, much more correctly, and much sooner, than could be done from imperfect data by theoretic computation.

6. It will however be gratifying our readers, if we present them with a dioptric formula by means of which they may compute the place in the main tube, where the second image will be achromatic, and consequently the best defined with given lenses for the object and amplifying lenses placed at a given distance. If we call the focal length of object-lens a, of the amplifying lens b, and the distance between them d; and also D the distance of the second image from b, when that image is the best possible, we shall have $\frac{b\,d}{(d-a).\,(a-b)}=D$. For instance, if we take $a=1.75$, $b=2.1$, and $d=2.6$, as are nearly the dimensions in a four-glassed eye piece before us, we shall have $\frac{b\,d}{(d-a).(d-b)}=\frac{2.1\times2.6}{0.85\times0.5}=\frac{5.46}{0.425}=12.85=D$. But if we make only a

small change in d, while a and b remain the same, the distance D of the second image will be greatly altered thereby, thus suppose we substitute 3.1 for 2.6, then we shall have $\frac{b\,d}{(d-a).(d-b)}=\frac{2.1\times 3.1}{1.35\times 1.0}=\frac{6.51}{1.35}=4.82=D$ Hence, if the distance between the lenses a and b were made adjustable by a screw, or by two pieces of tubes sliding one within the other, the place of the achromatic image in any terrestrial eye-piece, might easily be made to have its place in the common focus of the two lenses of the eye-piece, with a given conjugate focus of the lens a, as the place of the radiant on one side of it, and of the lens b on the other, under which circumstances the vision would be the best that can be produced with the said lenses.

7 There are however two measures of d that will answer equally well for a given measure of D, and either of these may be taken that will fall within the proposed adjustment. Let the operation be reversed, and let D be given $=7$, to find d, while a and b remain as before, then by putting the operation into the form of a quadratic equation, we shall obtain two roots that will give $d=2.075\pm.794$, or 2.869, and 1.281.

For the first value of d we have $\frac{2.869\times 2.121}{1.12\times 0.77}=\frac{6.0249}{0.8624}=7$ very nearly And for the second value $\frac{2.1\times 1.281}{-.479\times -.819}=\frac{2.6901}{0.3923}=7$ also very nearly

8. The field of view in the reading microscopes of the modern graduated circles have usually a contracted field, as well as a distortion of the image at the edges, which might be cured by adopting four lenses We beg leave to insert here a small table of the focal lengths and distances between several pairs of lenses, such as would produce a length of tube suitable for such a construction, together with the corresponding measures of D, any one of which may be taken, that best suits the size or power of the intended microscope, and, by varying the data in any one or more of the three first columns, the table may be extended to any length.

TABLE OF DIMENSIONS FOR A READING MICROSCOPE.

a	b	d	D
1.75	2.1	3.1	4.8
0.8	1.0	1.5	4.3
0.7	0.9	1.4	3.6
0.7	0.9	1.5	2.8
0.7	0.9	1.6	2.3
1.4	1.4	1.75	2.0

9 From what has been above said, we may see that any variety of magnifying powers within given limits, may be obtained by attaching the pair of eye-lenses to an inner tube that will draw out and separate them from the pair of inner lenses, and Dr. Brewster has shown, in his *Treatise on New Philosophical Instruments* (Chap. VII.), that a scale divided upon this tube will be a scale of magnifying powers, if divided into equal intervals. Such an interior tube

was proposed and made by B Martin many years ago, and Tulley made several for us, which we described under the article TELESCOPE, in Dr Rees' Cyclopædia, antecedently to the contrivance of the *pancratic,* or omnipotent eye-piece of Dr Kitchener, which is the same thing effected by cutting the single tube into several parts, and by thus giving it the appearance of a new invention

10 We shall have occasion to explain, in a subsequent section, how the wire micrometer may have several new values given to a revolution of its screw, by applying it to the eye end of such a sliding tube, and by varying its distance from the pair of object and amplifying lenses When any micrometer is applied to measure the first image of an object, or that which is formed in the solar focus, an increase of power makes no difference in the measure, because the image measured and the scale of measurement are alike magnified, but when this image is made the radiant, and the second image is measured, a change of power, occasioned by the separation of the lenses, will produce a change in the scale of measurement, and this circumstance affords several constructions of micrometers that may be applied at the eye-end, which may have different values given them, all which will be explained in their respective places. The upper tube in fig 2. shows the common erect eye-piece, the lower one is a representation of one that has variable powers occasioned by the sliding of an inner tube.

11. Sometimes it is convenient to form the four-glassed eye piece into the diagonal form, to make observations in the zenith, or at high altitudes, without putting the body into a painful position Figures 5 and 6 show two constructions of such an eye-piece, the former of which has a solid piece of speculum metal, at the right hand end of the horizontal part at *a*, adjustable by screws; the object-lens is at the left hand end , the second, or amplifying lens, is near the reclining face of the polished speculum; and the pair of eye-lenses are in the interior vertical tube, and may be drawn out to vary the power as we have already described. The eye-piece represented by figure 6 is in every respect constructed like the other, except that, instead of speculum metal, the reflector, *a*, is a prism of glass, with the first face plane and the second formed into a curve, to answer the purpose of the amplifying-lens, and as it is placed in the best position (§ V 6.), its performance is unobjectionable, when well executed, and properly adjusted with respect to position. These eye-pieces were both constructed by the junior W Tulley, whose skill and perseverance in overcoming practical difficulties entitles him to share largely in the celebrity, which his father has justly obtained as a superior optician.

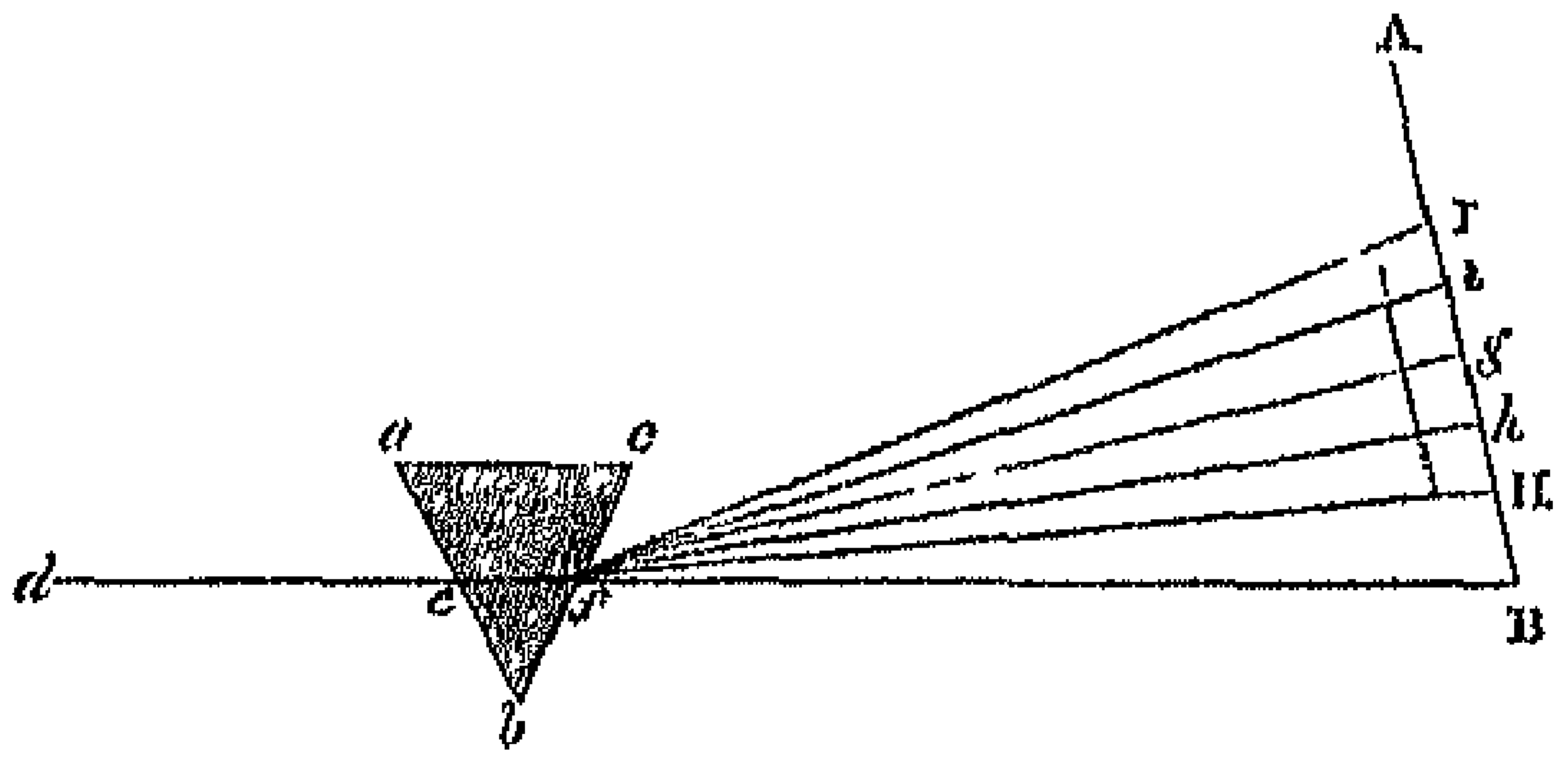
§ VIII A POPULAR EXPLANATION OF THE ACHROMATISM OF THE REFRACTING TELESCOPE

1. AT a time when the study of optics forms a part of a liberal education, it may appear to some of our readers unnecessary to introduce, into a Treatise on Practical Astronomy, a subject that seems to concern the instrument-maker rather than the astronomer; but as every practical man claims the privilege of judging of the merits of his own telescope, and as a popular explanation of the *achromatic principle* may be acceptable to several persons of this de-

scription, we have been induced to give a section on this subject, such as they may understand without much previous knowledge of mathematical science.

2. The word *achromatic*, which is derived from the Greek negative α and χρωμα colour, was first applied to the refracting telescope by Dr. Bevis, and is a proper term of distinction between the long refracting telescopes, which were formerly made with single object-lenses, and the shorter instruments with object-glasses, which are sometimes so combined of three, but of late more usually of two lenses, as to banish the Newtonian or prismatic colours.

3. It is well known that when a solar ray of light is admitted into a darkened room, and made to pass through a prism of glass, so as to be received on a white screen, the primary colours will be separated in such way, that the red and violet will be the extreme colours, and the length of the spectrum will depend, partly on the distance of the screen from the refracting prism, and partly on the quality of the glass. The explanation of this phenomenon will best be understood from a diagram.



Let *a b c* be the section of a triangular prism of transparent glass, and *d* a pencil of solar light, incident on the point *e* of the face *a b*, and coming in a direction parallel to the face *a c*, with a tendency to proceed in a straight line towards B; this pencil, on entering the glass, will be bent out of its course by refraction, towards the thick part of the prism, and will finally emerge at the point *f*, nearer to *c* than *b*, and will be *dispersed* into several rays of different colours, generally considered to be seven, but it will be sufficient for our purpose to consider only the two extreme rays and the middle one. Let A B be the screen that receives these coloured rays, and let the prism be of crown glass, then *f g* will be the mean ray, the *deviation* of which, as caused by the refraction, will be the angle *g e* B, or angle of mean refraction, *f h* will be the red ray with a smaller deviation, and *f i* the violet ray, with a greater deviation than that of the mean ray *f g*, in this case, *h i* will be the length of the *spectrum*, or the measure of the whole angle of *dispersion*, contained between the extreme red and violet rays. Let the screen remain, and substitute a similar prism of flint glass, in the same situation which the crown glass occupied, both as to distance and position, as they regard the screen, and the extreme rays will now be seen at I and H, instead of *i* and *h*, from which experiment it is clear, that the flint prism disperses the extreme rays considerably more than the prism of crown glass did; and it has been found that, in the *same* specimens of glass, the angle of *deviation* always bears the same proportion to the angle of *dispersion*, indeed, it was the opinion of Sir Isaac Newton, that this is the case in *all* specimens, which opinion has since been ascertained to be incorrect; for, if this had been strictly true, the telescope could not have been rendered *achromatic*. As the measure of dispersion of the crown glass was the length of the spectrum *h i*, so the length of the new and enlarged spectrum, H I, is the measure of the dispersion of the flint glass, and shows that the flint has greater refractive power than crown glass, which may also be said with reference to plate glass, though not in the same proportion. According to the determination of the ingenious B. Martin, the ratio of the dispersive powers of flint and crown glass is 5 : 3, but the elder Tulley has found that this ratio is not precisely the same with all specimens of flint glass, and he therefore varies it, according to circumstances, in the formation of the curves that form the faces of his lenses used for object glasses.

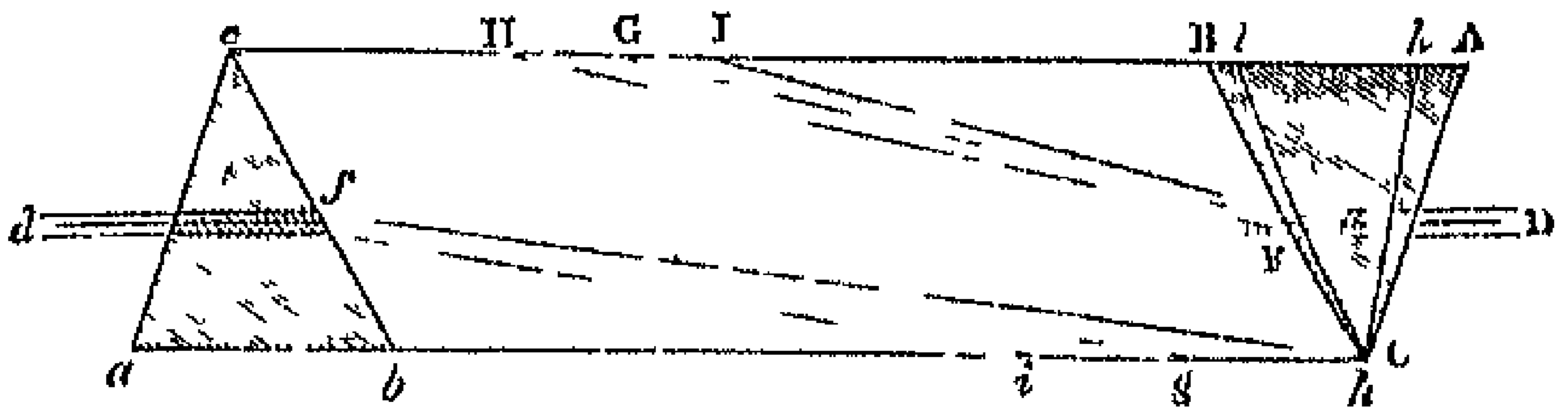
4. It has been asserted that Chester More Hall, of More Hall in Essex, employed different kinds of glass in the structure of a telescope, so long ago as in the year 1729, some of which have been preserved, and on examination have been found to be in a certain degree of the achromatic description, but it does not appear that he proceeded on any scientific principle, that entitles his contrivance to the appellation of an *invention*, and it was not till about the year 1747, that Euler, profiting by the discovery of Sir Isaac Newton, attempted to make an achromatic telescope by including water between two menisci, forming the ends of a box, so that the different refractive powers of the different media might counteract the dispersive powers, in such way as might destroy the colouration of the image formed in the focus of such a compound object-glass, but the experiment failed of success.

5. This attempt of Euler attracted the attention of the ingenious John Dollond, who was then a silk-weaver at Spital Fields, and who in the year 1753 addressed a letter to Short the optician, which was published in the Philosophical Transactions of London in the same year, "concerning a mistake in M. Euler's theorem for correcting the aberrations in the object-glasses of refracting telescopes," which theorem Short observes in his accompanying letter was "contrary to the established principles of optics." Dollond taking the great philosopher Sir Isaac for his pattern, conducted his researches by experiments, and after many trials ascertained, that the contrary refractions of certain prisms of glass and of water would make the colours disappear, when the refraction occasioned by the water was to that occasioned by the glass as 5 : 4. After this discovery in the year 1757, our experimentalist proceeded to make an object-glass composed of a deep double convex lens of pure water, united to a concave one of the same glass, and the image formed by such an object glass was free from colour, but was indistinct when magnified by an eye-glass, by reason of the spherical aberrations occasioned by the curves. It was in making these experiments that Dollond discovered the *principle* which laid the basis of the achromatic construction, he ascertained, in the case of an union of glass with water, that the *dispersive* power is not always *proportional* to the *mean refraction*, as Sir Isaac from similar experiments had concluded was the case. (Newton's Optics, p. 112. 3d edit.)

6. After this discovery the idea naturally suggested itself, to try what the dispersion would be in other diaphanous bodies. After some time different kinds of glass were obtained, and in the year above stated it was found, from trying prisms of various specimens of glass, that the dispersive power of chrystal or white *flint* glass was *greater* than that of the English *crown* glass, and also that the quality of the latter was very similar to that of the straw-coloured Venice glass, with regard to its *dispersive* property. A wedge of flint glass was so fitted to a wedge of crown glass, by reversing their refracting angles, and by grinding away a portion of the flint, that, their opposite refractions being equal, the pencil of incident rays proceeded through them both in a straight line, and when this counteraction took place, the angles of the two wedges were respectively 25° and 29°, and the sines of half their angles, or the indices of their

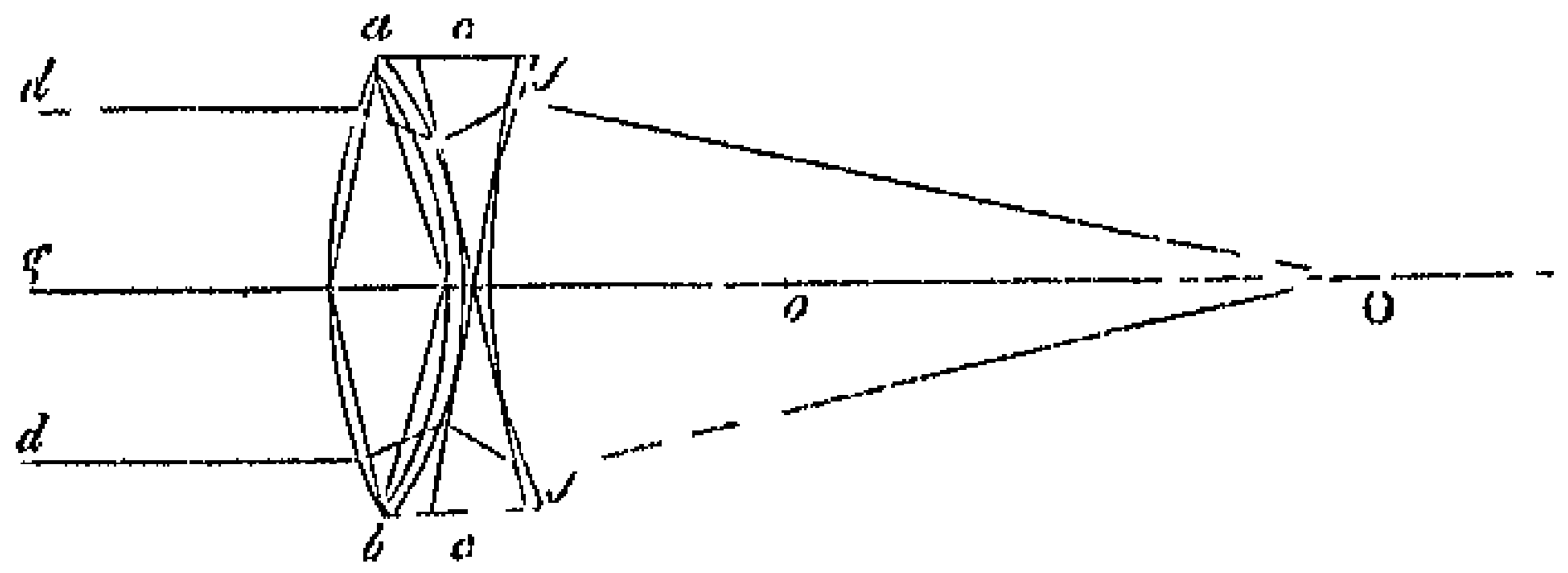
refractions, were 216 and 250, or nearly as 19 : 22. Though the direction of the pencil of light was now unchanged during the passage through the solid formed of the two unequal wedges, yet the compound rays had not all the same divergence, that is, some dispersion remained after the refractions were equalized, and the wedges had their proportions so altered a second time, as to produce a due opposition of dispersions to destroy the remaining colours. When this was done, the refractive powers of the wedges were found to be nearly as 2 : 3; and consequently the sines of half their angles 19 : 33, or nearly as 4 : 7. In this combination of the wedges, the rays which enter the crown glass parallel, emerge from the flint also parallel, notwithstanding the different refractions, and proceed afterwards without colour. From this experiment Dollond was led to conclude, that, in the construction of a double achromatic object-glass, the convex focus of the crown glass must be, to the concave focus of the flint nearly as 7 : 4, or in the ratio of their respective dispersive powers; which, as we have already said, Martin afterwards determined to be as 5 : 3. A telescope was constructed on this principle so early as the year 1758, in which an exact balance of the opposite dispersive powers of the crown and flint lenses made the colours disappear, while the predominating refraction of the thicker (crown) lens disposed the achromatic rays to meet at a distant focus.

7. The effects thus produced will be more clearly apprehended if we represent two prisms, in a diagram, opposed to each other at some distance, and show how the incident rays will be refracted and dispersed under different circumstances. Let *a b c* be the section of a prism of crown glass, and *A B C* that of a similar prism of flint glass; and let two pencils *d* and *D* enter these prisms respectively in opposite directions; then *g* and *G* will be the green rays of mean refraction, *h* and *H* the red, or those of least refraction, and *i* and *I* the violet, or those of greatest refraction. Since the refractive power of the flint prism *A B C* exceeds that of the crown prism *a b c*, the mean ray *G* in the first will fall nearer its prism than the ray *g*, or mean of the second to its prism; but the angle of dispersion opposed to *HI* will be greater than that opposed to *h i*, though the prisms have the same refracting angles at *C* and *c*. Now the refraction and dispersion in any prism, as has been said, will both bear the same proportion to the refracting angle of that prism, in any given specimen; therefore these prisms may both be reduced to any given quantity by a corresponding reduction of the refracting angle. Let the side *C A* of the prism of flint be ground down till it becomes *C k*, thereby making the angle *B C A* the original refracting angle, equal to *B C k* the new refracting angle, and let this second angle be to the first, as the refractive power of the crown is to the refractive power of the flint; that is, let the refracting angles at *C* and *c* be inversely as the refractive powers of the two specimens of glass formed into prisms; viz as *g f F* : *G F f*, in which case the mean ray *G* will be extended to *c*, and *F c* will be parallel to *f C*; and thus the *mean refraction* of the two lenses will be alike, the angle *G F f* being = the ∠ *g f F*, by being alternate. In this proportion of the refracting angles of the opposite prisms, the rays would both enter and emerge parallel as to *refraction*, if the prisms were in contact, and would therefore never come to a focal point; but with respect to *dispersion* that



of the flint would predominate, since the angle *HFI* would in a certain degree exceed the angle *hfi*, and therefore the colour would not be entirely destroyed. But these angles of dispersion are what we want to be equalized, by making the refracting angles at *C* and *c* of the two prisms in exact proportion to the dispersive powers, or to the spectra *h i* and *H I* in our preceding diagram to effect this purpose, let also the side *B C* of the flint prism be ground down a little to *l*, so that the refracting angle *l C h* of this flint may be to the refracting angle *a c b* of the crown in this ratio of the spectra. When this second reduction of the flint is made, the dispersive powers of the two prisms will become equal, but the refractive power of the crown will now be the greater, and its predominance will bring the achromatic rays to a distant focal point, for the formation of a colourless image.

8. In the next place let us conceive two crown prisms placed base to base, and two flint prisms, such as we have determined, placed point to point, as inscribed in the figure, the former



part within the convex lens *a b*, and the latter part within the concave lens *c c*, then such a figure will form a double achromatic object-glass, provided the curves are not of such a nature as to produce spherical aberrations that will occasion indistinctness of vision, which is a consideration of another nature.

9. If we consider the double convex lens *a b* as having equal radii of curvature on both sides, and trace the progress that the incident rays *d d* and *g* will have in passing through it singly, we shall find that, if they come parallel or from an object at an infinite distance, they will be refracted inwards, and meet at the point *o*, which is the geometrical centre of the first curve, provided the glass be of a quality similar to that used by Sir Isaac Newton, in which the sine of the angle of incidence was to the sine of the angle of refraction as 30   21, or nearly as 3   2 out of air into glass, but when the double concave lens of flint is applied close to the said convex lens, while the mean ray *g* proceeds in a straight line, the two extreme rays *d d*, will be refracted outwards, or in a contrary direction, and will emerge at the points *f* and *f*, and afterwards proceed with a less degree of convergence to the distant point *O*, which is the achromatic focus of the compound object-glass. When however the radii of curvature are alike for the two faces of the convex lens, its thickness and spherical aberration are too great to admit of distinct vision, and in practice it is found necessary to modify the curves of both lenses in such way that the bad effect of spherical aberrations may be counteracted, as well as the colours destroyed   The proportional focal distances of the two lenses must be nearly as 2   3 to produce an achromatic image, the convex lens having the shorter focus, and therefore the greater refraction, which, by exceeding the opposite refraction of the concave, bends the rays sufficiently to bring them to their achromatic focus. But a lens may be ground into various curves, as they respect each other, and yet the focal distance of the lens may remain the same; hence the optician has his choice of curves, in regulating his spherical aberrations, which curves will depend on the respective refractive and dispersive powers of the particular discs of glass,

that he adopts for the formation of his intended object-glass. When the refractive power of the convex is small in comparison of the refractive power of the concave, the radii of the convex curves will require to be shortened, to obtain the required focal distance, and the contrary; also when the proportion between the dispersive powers of the two kinds of glass is found to be extraordinary, the respective focal lengths of the two lenses will not be strictly as $2 : 3$, but in an altered ratio depending on the relative dispersions.

10. If we put the ratio of the sine of incidence to the sine of refraction as $m : n$ in any specimen of glass, as is usual in the theory of optics, this ratio may be determined practically in various ways, such as Dr. Wollaston's, or Dr. Brewster's methods, or, which is better, by grinding the glass into a lens, and finding the difference between the geometrical focus, and the focus determined by the actual refraction, which therefore has been called the *refracted* focus. The last method was practised first by Martin, and afterwards more successfully by the Senior Tulley, who, with a hard metallic tool of known curvature, ground at the same time, and partially polished, various specimens of glass to the same radius, and then compared together their respective solar focal lengths, which are always inversely as their refractive powers. After having done this with a specimen of both the crown (or plate) and flint glasses, this artist is able to compute his curves, and, by his peculiar dexterity of tact, to work the faces to such perfect figures, that all the sources of error will be cured by him, that art can accomplish with given specimens of glass.

11. The Astronomical Society of London have lately had a most convincing proof of the state of perfection to which the art of constructing achromatic object-glasses is now brought, when suitable discs of both sorts of glass can be obtained, which acquisition, as we have before said, is now a desideratum in the manufactures of this country.

12. With respect to the prismatic or chromatic aberrations of a lens, Dr. Smith has shown, that, if the common sine of incidence be to the sine of refraction of the *least* refrangible rays of light as $I : R$, and to the sine of refraction of the *most* refrangible rays as $I : S$; then the diameter of the least circular space into which heterogeneal parallel rays can be collected by a spherical surface, or by a plano convex lens, will be to the diameter of its aperture in the constant ratio of $(S-R)$ to $(S+R-2I)$; which ratio will be found as $1 : 55$. For if we suppose, with Newton, that the prismatic spectrum is divided into seven primary colours, and that the extreme red and violet rays have their sines of incidence and of refraction, I, R, and S respectively, as 50, 77, and 78; then we shall find that $(S-R) : (S+R-2I) :: 1 : 55$.

13. In like manner the diameter of the least circle that can receive the rays of any single colour, or of several contiguous colours, may be determined from the proportions of their sines. For instance, if the sines of the outermost orange, AR, and of the yellow, AS, be to the common sine of incidence respectively as $77\frac{1}{8}$ and $77\frac{1}{3}$ to 50, the diameter of the small containing circle, when a plano-convex lens is used, will be only one two hundred and sixtieth part of the whole aperture: but still this small circle must be considered as more than a *mere point*.

14. The same author has also inferred, that in different surfaces of plano convex lenses, the angles of prismatic aberration, R, A, S, are as the breadths of the apertures directly, and as the focal distances inversely. Hence, in the early construction of the refracting telescope, before the chromatic aberration was cured, the focal length of the lens forming the object-glass

was long, and the aperture small, and yet with such advantages the image would not bear to be much magnified by the eye piece.

15. In judging of the achromatism of an object glass by the method of drawing out and pushing in the eye-piece, to obtain the green and purple fringes occasioned by the remaining colours, when the red and violet are corrected (§ IV. 10.), it is necessary to employ a good eye piece of the negative construction, which generally gives a better field of view than the positive; for otherwise the supposed imperfection of the object-glass may be actually the imperfection of the eye-piece. If any fringes of red or yellow are observed on the edges of a white disc placed on black ground, when the telescope is adjusted for distinct vision, and the disc carried to nearly the edges of the field, this species of colouration indicates that the eye-piece is not sufficiently free from spherical aberrations; and if the curves of the lenses are suitable for each other, the cure is effected by an alteration in the distance between them, which must be finally adjusted by trial with a good object-glass. Indeed if the object-glass be ever so perfect, a bad eye-piece will greatly injure its performance.

16. An object-glass may, however, be perfectly achromatic, and the eye-piece may also be good, yet the aberrations arising from the relative curvatures of the lenses forming the object-glass, may occasion, and often do produce, a mistiness in the vision which renders it imperfect; and in the present state of optical workmanship, this cause of imperfect vision requires more skill to counteract, than to insure the achromatism. We have already given practical methods of detecting the spherical errors of an object-glass (§ IV. 9, 10, 11.) where they exist; but it is the province of mathematical investigation to determine such combinations of lenses as shall be both *achromatic* and *aplanatic*, or without spherical aberrations. This latter term has been with much propriety adopted by Mr. Herschel, in his ingenious paper "On the Aberrations of Compound Lenses and Object-glasses", read before the Royal Society of London, March 22, 1821, which contains much useful information both of a theoretic and practical nature, inasmuch as he has tabulated such results of his investigations, as enable the working optician to manufacture a good object-glass from the computed proportions of the radii of curvature, that depend on the different refractive and dispersive powers of various specimens of glass. An object-glass having three convex surfaces and only one concave, was constructed by Tulley from Mr. Herschel's computations, and though the formation of the flint lens was very different from what he had been accustomed to execute in his ordinary practice, yet he succeeded in accomplishing his object. The object glass turned out so good, that it was competent to separate double stars of the first class, and to exhibit minute objects very distinctly with an aperture of 3 25 inches, and a focal length of 45 0. The telescope is now in the possession of a Mr Moore of Lincoln.

17 When Mr. Herschel had computed the radii of curvature for the four surfaces of a double object-glass for different specimens of glass, he discovered two very important circumstances, resulting from his equations, that were so combined as to give the dimensions of an object glass free from aberrations both for celestial and terrestrial objects, which circumstances are, first, that the external, viz. the first and fourth, surfaces vary so imperceptibly in the different specimens of glass, that they may in practice be considered constant, and secondly, that the internal, or second and third, surfaces have very nearly the same radius of curvature, so as always to allow of a very near approach to contact. He has in fact come to this conclusion; "that a dou-

ble object-glass will be free from aberration, provided the radius of the exterior surface of the crown lens be 6.720, and of the flint 14.20, the focal length of the combination being 10.000, and the radii of the interior surfaces being computed from these data, by the formulæ given in all elementary works on optics, so as to make the focal lengths of the two glasses in the direct ratio of their dispersive powers." Object-glasses of any focal length, constructed according to these proportions, when made of ordinary specimens of glass, if well worked, will be free, or nearly free, from both the chromatic and spherical aberrations, or may be made so by slight alterations. When an object-glass is good, concentric rings will appear, beyond and short of the solar focus, when a star of the first or second magnitude is viewed; forming sections of the cones of rays that are differently refrangible, as they approach to or recede from the focus; if these rings are circular, free from mistiness, and alike at equal distances from the place of distinct vision, the workmanship, as well as the materials, may be considered excellent. Tulley's practical methods of proceeding may be seen under the article TELESCOPE, in the Cyclopædia already more than once referred to.

§ IX. STANDS FOR ACHROMATIC REFRACTING TELESCOPES. [PLATES II. V. VI.]

1. WHEN an achromatic refracting telescope is not of a size that admits of being carried in the pocket, it is usually packed in a box containing a brass tripod and different eye pieces, suitable for observing both celestial and terrestrial objects. The stem of the tripod has a motion in azimuth, which, in some of the better instruments, is regulated by quick or slow motion, as occasion may require, by the aid of circular rack-work and a compound, or Hooke's, joint at the extremity of the handle, while a joint at the top of the stem carries a bed for the tube, to which it is made fast at its centre of gravity by a pair of thumb screws. Where a simple view of a celestial body is all that is required, such a stand is competent to its purpose, provided it be placed in a situation not liable to produce vibrations that will disturb the vision. But if the telescope be large, and admit of great power of magnifying, it will not be sufficiently steady, unless perhaps counterpoised according to Fraunhofer's plan, hereafter explained, when supported only at one point, or centre of motion. Various stands have been contrived for supporting heavy telescopes at two or more points distant from each other, which afford the means of producing motion, both vertical and horizontal, in any given quantity, and with any requisite degree of velocity, to promote the convenience of the observer. We propose to describe in this section some of the most approved modern stands, by a reference to engravings that, we trust, will render the utility of their different parts sufficiently obvious.

2. IMPROVED STAND.—Fig. 1. of Plate II. exhibits a perspective view of a 3½ feet achromatic telescope, with an aperture of more than three inches, that will magnify from 60 to about 200 times, and that is very useful for observing eclipses, occultations of stars or planets, and immersions or emersions of Jupiter's satellites. The tube, *a b*, is tapering for the sake of lightness, and may be of either brass or mahogany, the stem, *c*, is a strong brass tube, also tapering a little, that will revolve smoothly round a stem of polished steel, which forms the

vertical part of the tripod, and each foot has a joint, which are of no other use but to facilitate the packing, by allowing the feet to approach one another. The top of the revolving tube, *c*, bears a cranked piece, on which the main tube of the telescope rests by means of a pair of levers, *n*, bearing each a pivot at the middle, one of which levers is concealed from sight in the figure, then a pair of tapped nuts with milled heads take hold of two screws, made fast to the main tube, and fix the lever to them by means of notches made on their remote ends, while the middle or third nut takes hold of the axis of vertical motion, and produces friction enough by pressure to hold the tube steady at any given elevation. At *d* is a small telescope, with but little power, and consequently an enlarged field of view, which has its axis of vision parallel to that of the larger telescope, the use of which is to find an object, when a high power is applied to the eye end of the main tube, which object, in this case, would otherwise be difficult to get into the field of view hence this secondary telescope is usually called the *finder*. Under the eye-end of the telescope is fixed a system of brass tubes, sliding into one another, the innermost of which has a joint at its upper extremity, and is racked to suit a small pinion fixed on the axis of the thumb piece, *f*, which pinion is used for producing slow motion in altitude, when the finder is previously directed upwards to a celestial object, and the clamping piece at *e*, that surrounds the outermost tube, has made it fast to the next contiguous tube, that has a joint at its lower extremity. In this way the telescope rests on three points, one at the eye-end of the tube, and one at each pivot of the axis of vertical motion, and thus all tremors are obviated, which are liable to occur when the main telescope is supported only at its centre. At *l*, the lower end of the revolving tube *c*, is made fast a circular rack, which admits an endless screw on the inner end of the handle *g*, that is furnished with a Hooke's joint, which screw can be detached from the rack, or brought into contact, by a spring-piece that will remain in either of two positions. The use of this contrivance is to give a quick horizontal or azimuthal motion without the handle, when the rack is detached, but a slow one, by means of the handle, when the endless screw is in action: and when the right hand holds the handle *g*, while the left holds the piece *f*, a compound motion in any given direction may be easily imparted to the telescope, while the eye applied near *a* is viewing the body in apparent motion, which may thus be followed, with the requisite velocity, during the time that it is under examination. The advantage that a long vertical axis has over the common one with ordinary joints is, that it remains free from shake in wear, as well as from tremors in use, and produces motions truly vertical and horizontal by means of the long bearings of the centres of motion, provided that the tripod be properly placed, and on an immoveable basis. When a micrometer is applied as an eye piece to this telescope, or a quadrant of altitude attached to the tube, it would be desirable to apply screws to the feet of the tripod, particularly if the quadrant is furnished with a level, in order to make the stem perfectly vertical. There are usually several celestial eye-pieces, one of which is represented by *h*, and a smaller piece, or higher power, is screwed into the inner small tube at *a*, which is made to project more or less from the main tube by the milled nut *m*, the axis of which carries a concealed pinion that drives the rack attached to the said tube, during the adjustment for distinct vision. It is convenient, when an observer has different telescopes, to be able to apply any of the different eye-pieces to the same individual telescope in succession, and the simple contrivance, represented at *i*, will answer the purpose of adapting the eye-piece of one telescope to the eye-tube of another. This contrivance is nothing more than a short piece of tube with a male screw on the outer face, and a female one on the inner, the former to fit the eye-tube, and the latter to suit the screw of the eye-piece. An eye-piece bearing an *adapter* of this description is seen attached at *h*, and a small cap, with a diminished central aperture for limiting the place of the eye, is usually screwed to the outer end of the eye-piece. A few of these caps, holding each a piece of coloured glass, of different shades, are necessary to protect the eye when the sun is the object of examination. More than one instance has occurred, where a want of due attention to this precaution has spoiled the eye of the observer incurably. A little experience will show that some states of the atmosphere are much more favourable for the good performance of a telescope than others, currents of air and great humidity are particularly unfavourable, the former producing tremors, and the latter moisture on the surface of the object-glass Generally speaking, a telescope performs best in the open air and on the solid ground, where the treading of the observer does not affect its steadiness of position · but, to admit of external observations, the sky must be serene and perfectly calm, which is mostly the case about sun set.

Besides the celestial pieces here noticed, a telescope of this description has generally a couple of four-glassed eye pieces, seen in fig 2, which show the objects erect for terrestrial purposes; and sometimes diagonal eye pieces for relieving the body in high elevations, seen in figs. 3, 4, 5, and 6, all which will be separately described hereafter.

3. DOLLOND'S STAND —When the telescope is from four to six feet long, and has not a diagonal eye piece, it becomes very inconvenient to stoop to the eye-end when the altitude is considerable, and the centre of motion at the middle of the tube; to remedy which inconvenience, Mr G Dollond lately contrived the stand which is represented by fig. 1 of Plate V. The frame work of this stand is sufficiently intelligible from inspection of the figure, it is composed of bars of mahogany, and rests on three castors, two of which are made fast to their respective legs in the usual way, and the third stands under the middle of the lower horizontal bar, that connects the two opposite legs, so that the frame has all the advantages of a tripod. As before, *a* and *b* denote the two ends of the telescope, which carries a finder, and has its centre of motion under the eye-end of the main tube, which therefore remains at the same height from the ground in all degrees of elevation of the object-end. At some distance beyond the centre, a clamping piece of tube surrounds the main tube, and forms the superior support, while it regulates the altitude by its sliding motion along the tube which it embraces, but does not pinch too closely to create much friction, though it may be set fast by a clamping-screw in any required degree of elevation. The principal contrivance is, to give motion to this sliding piece without shaking the stand, or putting the body of the observer out of a standing or sitting position, while the requisite motions are given to the telescope thus elevated, and apparently in an unmanageable situation. This object however is accomplished by simple means in the following manner the strong bar *c f*, having an horizontal axis *d e*, going across the top of the frame, and resting in two brass perforated supports at their extremities, or pivots, ascends from nearly the middle of the frame, and reaches to the sliding piece above described, with which it is connected by two joints, one allowing vertical, and the other horizontal motion, but at present we will attend only to the mechanism that produces the former. A lever or tail-piece *d g* has a hole in its upper end, through which the horizontal axis *d e* passes freely, and when it has taken a vertical position, the horizontal screw *h g* takes hold of it, and passes

several turns of the screw through its inferior end, which is tapped for a corresponding thread; then a small screw, near the milled nut *k*, enters the frame, and passes into a groove turned on the axis of the said screw *k g*, thereby preventing this axis from quitting its situation either inwards or outwards. The consequence is, that the tail-piece *d g* is acted upon by the screw *k g*, either backwards or forwards, accordingly as the nut *k* is turned in a direct or retrograde direction, but no effect is produced on the elevation of the telescope while the tail-piece remains detached from the axis of the large bar or support *c f*, this being intended only to regulate the slow motion. The approximate elevation is first gained by a push given to the lower extremity of the support *c f*, which immediately raises the opposite or superior end, and with it the telescope, the sliding piece adapting itself to its true position on the tube, then, while the elevation remains unaltered, by means of a slight pressure by the left hand on the support at *f*, a clamping screw, of which the horizontal axis is visible at *h*, is moved by its ring at the exterior end, till it has clamped the tail-piece *d g*, to the axis *d e* of the support, after which any further slow motion in altitude may be given to the telescope by turning the milled head *k* by the left hand, in the requisite direction. With respect to the horizontal motion before referred to, a horizontal screw *i*, every way similar to the screw *k g*, lies at right angles to this, across the frame, and carries a milled nut at each end, which appear in the figure; the axis of this screw passes through the metallic piece that forms the lower portion of the joint or centre of vertical motion, under the eye-end, and when this piece is acted on by the screw, the telescope has a slow horizontal motion round the second joint, at the upper end of the support *c f* Thus, if the right hand manage the screw at *i*, and the left that at *k*, while the eye is applied near *a*, a compound motion in any direction may be given to the telescope, while the heavenly object is passing through the field of view, and an observation may be completed with the greatest ease, as well as comfort, notwithstanding the great length of the telescope

4. **Smeaton's Support**.—When a telescope exceeds six or seven feet in length, it may be used by placing the object end on some elevated peg, post, or tree, and guiding the eye end by a two-legged adjustable support, that has the means of giving slow motion, both in altitude and azimuth. We have seen such a support at Greenwich, but in the present improved state of achromatic telescopes it can very seldom be wanted. The representation of a contrivance for this purpose, supposed to have been invented by Smeaton, is given in fig 8 of Plate XXX. of Astronomical Instruments, in the new Cyclopædia edited by the late Dr Rees, and is described under the article Telescope.

5. **Tulley's improvement of Varley's stand**.—On a reference to the figure of Dollond's stand, which has been above described, it will be obvious, that the frame is not adapted for easy conveyance from one place to another, all the parts of the frame being permanently fixed, and the frame itself too bulky to be admitted into a convenient packing-box. Varley's stand is not liable to this inconvenience, and as improved and now constructed by the Tulleys, is perhaps one of the best stands for a moderately long telescope, of any that have been contrived without the equatorial motion This stand is represented by fig 1 of Plate VI, in such a position as is calculated to exhibit all its most essential parts, as much as is possible in one figure As before, the tube *a b*, forms the telescope of more than an ordinary length, with its eye piece and finder, it rests on two mahogany *Y*s attached to a frame of the same wood, immediately contiguous to the main tube, which is buckled to the said frame by two leathern

straps, seen between the letters *c* and *d*, which method of fixing renders the stand useful for a telescope of any dimensions. This frame is partly concealed in its elevated position, by a second frame under it, on which the upper frame is incumbent; these two frames are united by a bolt and tapped nut near their elevated ends, but so that the bolt forms a centre of motion, and allows the upper frame to glide over the other at its lower end, either to the right or left; this motion is produced and regulated by a piece of circular rack-work, the racked portion of which is made fast to the lower frame, while the pinion and its axis are attached to the upper one; this axis is seen with its milled head, and squared end, at *e*, under the eye-end of the telescope. The lower frame has its axis of motion in elevation across the framed tripod, or rather is hinged thereto at its lower end, so as to be capable of any elevation. The frame of the tripod is formed of two pieces, namely the trapezium *f g h i*, and the triangle *k l m*, both of mahogany, and when the bracing bars are unscrewed, these two portions will lie flat upon one another, by means of the hinges at *k* and *l*. The two upper frames that form the bed of the telescope when detached from the racked bar *n*, that supports them, will also lie down upon the flat tripod, when the telescope is dismounted, and the stand, being rendered thus compact, may be suspended on a pin against a wall, or packed for carriage in a long shallow box. Across the triangular leg of the tripod is an horizontal axis, turning freely on its end in brass caps attached to the mahogany at *o* and *p*, and a handle on the axis of a pinion made fast to the horizontal axis above described, acts with the teeth of the racked bar *n*, and holds it in any given situation, with respect to its angle of altitude. At the superior end of this racked bar is a hinge of brass, the upper part of which is fixed to a sliding piece, that fits into a dove-tailed parallelogram forming the under part of the lower frame, and the quick motion in elevation is given to the telescope by the sliding motion of the dove-tailed piece, produced by turning the handle of the rack-work, which rack-work may be clamped by the milled head of a screw, seen near the handle at the lower part of the racked bar. But to produce the slow motion in elevation, a handle *q* is inserted on the squared end of a brass rod, that passes under the lower frame, till it reaches the sliding dove-tailed piece, the tapped part of which it enters, and the screw formed on its upper end gives the slow motion as the handle *q*, which has a Hooke's joint, is turned round. Then, when another handle with a Hooke's joint is applied to the axis of the pinion at *e*, attached to the upper frame or bed of the telescope, and acting with a horizontal rack fixed to the under frame, the right hand will manage the former, and the left the latter, in regulating the compound motion, which may be necessary in keeping a moving object in the field of view of the telescope. The mahogany board seen at *r*, serves for a table to hold the different eye pieces, and also as a stretching brace to the frame-work. This stand, like Dollond's, which is a more recent contrivance, has its centre of motion near the eye end of the telescope, and the body of the observer is therefore kept in an erect posture, while a heavenly body is observed even at a considerable altitude.

What adds importance to stands for long telescopes is, that the magnifying power of any telescope is considered the best, when it is derived more from the object-glass than from the eye-piece, because as the image of any object to be viewed is entirely formed by the object-glass, and is enlarged in proportion to its focal length, the eye-piece is only employed as a microscope to view that image, such as it is, and no additional power that can be applied at the eye will rectify an imperfect image, but on the contrary, will render the imperfection more visible.

§ X EQUATORIAL OR PARALLACTIC STANDS (Plates I II V)

1. In all the preceding stands for achromatic telescopes the motions produced are in altitude and azimuth, which are sufficient when a single view of any heavenly body is all that is required, but when the body is to be kept within the field of view for some time, to afford the opportunity of examining it minutely, as in the case of measuring the diameter of a planet, or of the distance between two contiguous stars, it becomes necessary that the telescope should have such a motion as will enable the observer to follow the body through its circle of declination, as it alters its right ascension, without the trouble of making repeated adjustments for new altitudes The most perfect stand that a telescope can have, for effecting this purpose, is a large equatorial instrument furnished with graduated circles and having the nicer adjustments, such as we shall have occasion to describe in the sequel of our volume, but as these instruments are very expensive, more simple stands have been contrived to perform a like office in a satisfactory manner.

2 The first stand that was contrived to give a motion parallel to the equator was by Smeaton, who fixed a cylindrical block of durable wood on three legs with hinges at the upper ends, that allowed them to extend themselves into the form of a high wooden tripod The principal parts of this stand with its brass appendages are shown in figure 7 of Plate I., in perspective, where all the essential parts are presented to view, and may be thus explained. The cylindrical portions *a* and *b* are so divided by an inclined section, that in their first position they form a perfect cylinder, but when the upper portion *b* is turned half round on a round bolt of steel, made fast to it, and passing down through the lower portion *a* under which it receives a fastening nut, the upper face of the part *b* is inclined towards the horizon in an angle equal to the co latitude of the observatory where it is intended to be used A strong but small plate of brass is attached to the lower half of the divided cylinder near *d*, and the end of a spring catch appears lying in a notch of this plate, which catch is the end of a lever made fast to the upper half of the said cylinder and concealed from view the use of this catch is to keep the two halves of the cylinder in the position given them in the drawing, when the half *b* is turned half-way round, to give the requisite inclination to its superior end, which in the first position is horizontal, and may be kept so by a notch in another small plate at the opposite side, which the same catch acts with, but which is not seen in the figure When the two halves *a* and *b* are so placed on each other as to form an exact cylinder, which we call the first position, the motion of a telescope carried by the brass plate *c* lying upon and made fast to the superior face of the portion *b* of the cylinder, has an horizontal motion, and in this position the two halves form the head of a plain stand, and together are called Smeaton's Block, but in the second or present position the circular plate *c* takes an equatorial position when its plane is made to lie in the plane of the equator of the heavens, by making it face the exact north point.

This plate *c* has a strong tube of brass fixed at right angles to its plane exactly at its centre, which, descending into the body of the upper half of the block, forms a socket to hold the lower end of the polar axis *e*, which is detained in its place by a collar and screw at its lower extremity, but so as to be capable of revolving without shake. The strong radial bar *f* is made fast to the polar axis *e* at the face of plate *c*, and carries a steel tangent screw at its remote end acting with the circular rack made at the periphery of the fixed plate *c*, from which it may be detached by the milled head of another screw that points to the centre of the plate, and draws out or pushes in the bed of the tangent screw at the end of bar *f*, as occasion may require. When the tangent screw is detached, the bar *f* and polar axis *e* will move round with a quick unrestrained motion, but when attached to a handle with a Hooke's joint, seen near *b*, it produces a slow motion The brass part *g h* is formed into a square frame by two parallel sides, and two hinge bolts, the bolt at *g* forming a hinge with a portion of bar *f*, round which the frame turns, when the lower end of screw *i*, which passes through the remote bolt, presses on the face of the semicircular end of the bar *f*, by being turned round in a forward direction. Another tangent screw passes through the middle of this brass frame *g h*, and acts with the circular rack cut on the periphery of the semicircle of declination, and a second Hooke's joint with a handle turns this screw when necessary, to give a slow motion in this direction. The screw *i* brings the tangent screw into action when turned forwards, but when turned backwards detaches it, and leaves the semicircle free to take a quick motion in elevation. When the screw *i* is turned round, the bolt into which it screws turns on its pivots in the side bars of the small quadrangular frame, and allows the screw to keep its perpendicular position as it regards the face of the bar *f* on which it rests. The vernier of the semicircle is attached to the side of the polar axis *e*, by an intervening bar on which it is adjustable for zero, and the semicircle is divided to shew degrees and minutes of altitude or declination, accordingly as the block is fixed in its first or second position. The upper part of the polar axis is divided down its middle to admit the semicircle to pass through the slit, and the centre of motion in altitude is a bolt passing through the upper end of the polar axis, and kept to its place by a tapped nut, which creates friction enough to keep the semicircle steady at any elevation. To the upper part of the semicircle, above its centre of motion, a metallic bed is fixed for the tube of the telescope to lie in, to which it is fixed by two milled nuts taking hold of the screws attached to the main tube in the usual way. In our figure a short piece of large tube is placed on the bed to show the situation of the telescope when mounted; and it is easy to perceive that the two handles will guide the motion, the right hand one in declination, and the other in right ascension.

3. SECOND CONSTRUCTION.—The reader will have perceived from our description of Smeaton's Block, that it is adapted for only one latitude when it has no foot screws to alter the elevation of the polar axis, in fig 7. of Plate II. is the representation of a second construction, in which the whole instrument is seen in a state ready for use, and which allows of any elevation that may be required. The different parts and their uses will be easily apprehended from a short description, after our explanation of Smeaton's block has been read and considered. The cylindrical piece of mahogany *a* has the three feet attached to it by as many strong brass hinges, which allow the feet to approach one another till they come in contact, or to separate in the form of a tripod till the three-armed piece *d* prevents their further separation, by being brought into a horizontal situation, which their respective hinges at each end will allow. This contrivance keeps the feet steady at the extreme limit of their extension, and yet when pushed upwards at the central junction will ascend and permit the feet to approach to any distance, till

they come in contact for packing in a box. Instead of the upper half of an inclined block, the letter *b* here represents a quadrantal arc of brass perforated with a circular opening and graduated to 90°, and a screw with a milled head passing through this circular opening, will fix the quadrantal arc in any given degree of elevation by passing through the vertical frame made fast to the cylindrical block *a*. The circular metallic plate *c* is here, as in Smeaton's construction, the equatorial circle of right ascension in time, and has its axis of motion passing through the horizontal axis of the quadrantal arc at *e*, and descending to the end of the arc to which it is made fast, so that altering the degree of this arc, as indicated by a vernier not seen in the figure, will alter also the elevation of the axis of plate *c*, which, though descending, is here the polar axis, being as before placed at right angles to the plane of the equatorial circle *e*, hence this axis may have any inclination given that the quadrantal arc *b* will allow. The axis *e* of the quadrantal arc has more length than that of a common joint, it being composed of a strong perforated plate with pivots at the ends, and admitting a cylindrical hole large enough to contain the upper end of the polar axis. To the face of the equatorial circle *c* a frame is fixed at right angles, which therefore ascends in the direction of the polar axis, and carries the semicircle of declination on its upper end, the diametrical bar of which is attached by a pair of screws to the tube of the telescope near its centre of gravity, and the handles with the Hooke's joints act with the racked edges of the circle *c*, and semi-circle of declination, in the same manner as was described above with Smeaton's block, and are detached and attached for quick and slow motion in a similar manner The edge of a ring lying under plate *c* has the graduations for right ascension, and the vernier is fixed to the circular plate, but as the scale of this drawing is small, the parts could not be all distinctly represented in the figure We have placed an entire telescope on this stand, to show the manner in which it is kept steady in use, how the object may be readily found, and how the lines of a micrometer applied to it may be illuminated A system of tubes nicely fitted into one another are made fast to the eye-end of the main tube at *f*, and drawing out of one another descend to the legs, to which two holding pieces of brass are screwed that receive each a strong pin turning on a joint at the lower extremity of each of two sets of tubes, thereby giving steadiness to a second point of the main tube, which is too long to be firmly supported at the centre of gravity only, and while these tubes prevent tremors in the telescope, they do not impede its motion in either declination or right ascension, when the handles are used for giving slow motion by means of their respective screws When the telescope is dismounted, the system of tubes being pushed into one another occupy a small space, and their pins are inserted into holding-pieces at *h* made fast to the principal tube At *i* is a small achromatic telescope, which enlarges the object linearly about six or eight times, made fast to the main tube, and adjusted to such a parallel situation, that it will always view the same object to which the large telescope is directed at any time, and as its field of view is much more extensive than the field of the greater telescope, it is used as a *finder* only, and has therefore two cross lines of silver wire placed at right angles to each other across its field of view, so as to intersect one another at the centre, where the object is brought to before it will be visible in the large telescope. The milled nut at *k* has a pinion on its axis which moves the racked tube *m* out of the fixed surrounding tube *l*, and the tube *n* is usually called the *drawer*, the outer end of which holds an eye-piece *o* of either the celestial or terrestrial construction, as may be required, it will also receive any of the micrometers which are adapted to it, such as will be described in some of our subsequent sections. At *p* is a circular aperture made in the side of the main tube, which may be covered by a piece of larger tube that surrounds it, and which, in being turned round will either cover it or suffer it to remain open, accordingly as it is turned more or less, a corresponding hole being made in this revolving piece of tube for the purpose of either admitting or excluding the light of a lamp, that is necessary when the wire micrometer or any divided scale is used at the eye-end of the telescope for an oval piece of gilt metal placed diagonally in the tube opposite the hole *p*, and being itself perforated, reflects the mitigated light of the lamp towards the eye, while its central aperture admits the converging rays coming from the object-glass to pass also in the same direction, till they form the required image at the lines or strokes of the micrometer. When the object is very small, or emits but little light, it will disappear in the superior illumination of the lamp, which in such case may be darkened by the interposition of ground glass, oiled paper, or coloured glass, as the observation may require. A good telescope mounted in this manner and having micrometrical appendages, is capable of performing much service in the hands of a skilful observer.

4. THIRD CONSTRUCTION.—When an achromatic telescope has a large aperture and a corresponding length of tube, its weight will be too great to be well supported by either of the preceding stands, and Dollond has constructed a more substantial one, which we will now describe, as being convenient for an observatory, and quite competent to its purpose. This stand is exhibited with a telescope mounted upon it so clearly in figure 2, of Plate V., that the reader, who has perused our account of the two other stands, will require but little additional information respecting that, beyond what an inspection of the drawing will give him. The leading features of this stand are, that the tripod remains with its feet always extended, and is firmly braced by cross bars to keep it steady, that it has feet-screws of adjustment for position, and that its polar axis is made long enough to ensure an undeviating motion in right ascension, it has a level suspended under and parallel to the main tube, for adjusting the semicircle of declination; and a prop of sliding tubes is applied at the object-end of the telescope, as well as a pair at the eye end, which give the instrument the advantage of support at three equidistant parts of the tube. The upper end of the polar axis bears a strong square plate of brass to which a pair of bearing pieces are attached, that supply the centre of motion in declination, or in altitude, when the vernier at the lower end of the polar axis points to zero on the large quadrantal arc, that is graduated on one of its faces. This quadrantal arc is made fast at its lower end to a cross bar, that connects the two single legs, and passing between the two wooden bars that form the double leg, is fixed at the upper end to another cross bar that connects a pair of horizontal bars, supported in that position by the legs to all which it is fast, so that this quadrantal arc is so firmly fixed, that it is strong enough to keep the polar axis in any angle of elevation, when clamped to it by the usual apparatus at the vernier. The centre of motion of the polar axis is round a steel bolt, connecting the tops of the two single legs, and the two screws for the respective slow motions work with circular rack-work, from which they may be discharged at pleasure, as in the other constructions. The small circle between the racked circle made fast to the upper end of the polar axis, and the square plate above, is the graduated circle of right ascension, which in the Plate is made small, that the other parts may not be concealed, but it may and ought to be much larger, to correspond with the scale on which the other parts are constructed, particularly as the si-

dereal time of making an observation by the instrument is read on this circle. As the polar axis has its elevation measured on the large quadrantal arc, it is necessary that this should be properly adjusted, when this axis stands at zero, which adjustment is effected partly by the level and partly by the feet screws; and when it is once placed in the meridian, with its hour circle adjusted, and on a pillar, or other firm basis, the whole should be made fast, or at least have metallic holes in which the feet-screws may rest permanently.

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§ XI. THE CONSTRUCTION AND USE OF THE DYNAMETER. [Plate XI.]

1. In many of the most delicate observations made in practical astronomy, it is of the utmost importance to know the exact magnifying power of the telescope used, particularly when micrometrical measurements are the means employed, to obtain the desired results; for several of the micrometers, hereafter described, depend entirely on the knowledge that the observer has acquired of this power, which is made one of the data of computation. Various practical methods have been devised of determining the magnifying power of a telescope, but we shall confine ourselves principally to the description of a small instrument, originally contrived by Ramsden, for the purpose of measuring the image that the object-glass, or large speculum, of a telescope forms at its solar focus. It has been already stated, that, if the solar focal distance of the object-glass of any telescope be divided by the focal distance of its eye-piece, considered as a single lens, the quotient will be the magnifying power of the said telescope. But when the aperture of the telescope is exactly known, its diameter, on the principle of similar triangles, will be to the diameter of its disc, formed at the solar focus, and seen through the eye-piece, in the same ratio that the focal length of the object-glass is, to the focal length of the lens representing the eye-piece; hence the diametrical measure of the aperture divided by the diametrical measure of its image, or disc, will also be the magnifying power.

2. An instrument therefore, that will give the true diameter of the image of any given aperture, will afford the means of obtaining the magnifying power. This operation can only be performed by optical means, because the disc to be measured is without substance, and is also formed within the small tube holding the eye piece. The most simple method of getting the measure in question is by means of a second eye-piece of the positive construction, holding a slip of mother-of-pearl, nicely divided into the hundredths of an inch, in the common focus of the two lenses; for if this be held before the eye-piece of the telescope, at a short distance from the eye-lens, as a simple microscope is used for viewing any object, the disc of the object-glass will be seen lying on the scale of divisions, and a little dexterity in making the proper adjustment for good vision, will enable the observer to count how many divisions are covered by the said disc, which will be so many hundredth parts of an inch. An eye piece of this kind is represented by fig. 13, of Plate XI., where the line $a\ b$ shows the place of the divided scale, before the two lenses that form the eye piece, and if the fractional part of a division could be estimated with perfect accuracy, such an eye-piece would form a good dynameter, or instrument for obtaining the *measure* on which the *power* depends, as the name imports.

3 But Ramsden considered this method of procuring a measure of the luminous disc, only as approximating to the accuracy, that a good determination of the magnifying power demands, and he therefore had recourse to an ingenious contrivance for getting a more correct measure by the aid of double images, which plan is highly advantageous, in a case where a vernier cannot be applied, to subdivide a graduated scale. It is also of equal importance, that the effective part of the aperture be truly measured, and therefore care must be taken to diminish it, by a perforated cap, till it is known that none of the light admitted is intercepted by an internal diaphragm, which may be ascertained by sticking a triangular piece of paper on the external face of the object-glass, pointing to the circumference, and by examining its image in the disc through the dynameter, or through a positive eye-piece, or even a single lens, for it will appear there, if any, and what portion of the whole aperture is cut off, from the portion of the triangle that is intercepted.

4 In constructing the double image dynameter, it is necessary to divide the eye lens of a positive eye-piece into two equal halves, and to separate their centres, while the two semi-lenses continue in contact at the line of junction, for then the quantity of separation is made the measure of the distance of the two images, formed by the respective semi-lenses. This might have been effected by a single screw, with a micrometer head, moving one of the two semi lenses, and leaving the other stationary, but in that case, which would have afforded the simplest construction, one of the images would have moved across the field of view of the dynameter, while the other remained fixed; the consequence of which would have been, that one of the two discs of an object would have been viewed too obliquely, and one half of the field only would have been disposable, in getting the visual angle, which would thus have been confined to a narrow limit. It became indispensable therefore, that both the semi-lenses should move in contrary directions, to make the whole field available, and as the sum of the two motions must be the amount of the separation of the centres, it was judged proper to make the contrary motions as nearly alike as mechanical means could effect, though it is not absolutely necessary that they should be exactly so. This plan of a dynameter has been executed by a double screw on the same axis in two different ways, which we will now successively describe.

5. The original construction of Ramsden is represented in section by figures 14 and 15 of Plate XI., in which the same letters indicate the same parts, *a b* is the outer tube containing the mechanism of the dynameter within, and *c d* is an inner tube sliding into the outer one, and forming a circular frame for the interior parts, that are attached to it, *e* is the milled nut, and *f* the divided head of the double screw *h i*, of which the portion *h* has the fine, and the portion *i* the coarse threads, of one half the number per inch. the end of the coarse screw *i* rests against a piece of metal, attached to a half tube, that holds the semi-lens *k*, and acts with a tapped nut *r*, attached to the half tube bearing the other semi lens *l*, but the finer screw *h* acts with a nut *s* fixed to the outer tube *a b*, which gives the screw a slow motion inwards, when the motion is direct, but the contrary when retrograde, this slow motion pushes the semi lenses *k* and *l* from the centre of the field, in consequence of the half tubes that hold them having a common motion, like that of a lever, round the pivot *o*, at the remote end of the tube *c d*, which carries a species of gimbal for this purpose, but at the same time the coarse screw *i*, acting with nut *r*, attached to the semi-lens *l*, draws this semi-lens in the opposite direction by double the quantity of motion, and the difference of the two contrary motions is the actual quantity, that removes

it from the centre, in an opposite direction. Thus the two semi lenses have each the same quantity of contrary motions, produced by a turn of the milled nut *e*, and the divided head is indicated by a cranked index *g*, which is carried by a tubed piece *n*, attached by a screw to the cock *m*, screwed fast to the surrounding tube *a b*, and as this index is adjustable by friction, its edge shows the division on the micrometer's head, while at the same time the edge of the divided head, or ring, shows, on the scale of the index, the number of revolutions at any time made by the screw, in either direction. The zero of the divisions on the scale of the index *g* is at the middle, and as the divisions are just equal to the spaces between the threads of the fine screw *h*, every revolution makes the head *f* advance or recede a quantity equal to one of those divisions, at each revolution. By this mode of graduation the measure is obtained at each side of zero, and the sum of the two measures will correct for the index error, if any. In order to keep the screw always in close action, a long spring presses against the back part of the semi-tube, carrying the semi-lens *l*, which will however yield to the power of the screw, when its motion is direct, and on the good action of this spring, in opposition to the screw, will the uniformness of the indication depend, provided the screw be perfect, as to the inclination of its threads, which is an indispensable requisite in a micrometer of this construction. The second lens may be seen between *c* and *d*, at the remote end of the inner tube, and the common focus of this and of the two equal semi-lenses lies beyond both, where a sliding case or tubular cover *t u* holds a piece of mother of-pearl exactly one tenth of an inch in diameter, for showing the appearance of a luminous disc, and also for proving that five or ten revolutions of the screw will separate the two images of it, in the way exhibited in the field represented by figure 16. and if these images are seen in exact contact at the whole number of revolutions on each side of zero, the scale is considered correct, but not otherwise. The better practice however is, to procure a good ivory or glass scale, divided into perfect hundredths of an inch, and while viewing them, as through a simple microscope, to measure their spaces, by making the image of the first stroke cover the second, third, &c. in succession at each revolution, or half of a revolution, as the screw may require. In this way not only will the value of the screw and its goodness be ascertained, but frequently an error in the scale examined will be detected to the $\frac{1}{10000}$th of an inch, where the fine screw has 100 threads in the inch, and its head is divided into 100 parts, as is usually the case with dynameters of this construction, for though the separation produced by the two contrary motions is equal to the motion that would be produced by the coarse screw of 50 threads acting alone in one direction, and viewed through the semi lenses alone, yet when the second lens is put on, the magnifying power is doubled, and the tenth of an inch, instead of being measured by five revolutions, will require ten, as though there had been only the single motion produced by the fine screw. This circumstance gives the instrument-maker the advantage of adjusting the distance between the whole lens and the two semi-lenses so, that, though the screws may not have the exact number of threads per inch that theory requires, yet the final adjustment of their joint power, as an eye-piece, will compensate the defect, and make the number of revolutions commensurate with any good scale of equal parts. As the truth of the dynameter's measure is the most important consideration, we were induced to take a dynameter to pieces and to examine all its parts, both optical and mechanical, and in doing so we discovered the extraordinary property, above noticed, of the arrangement of the lenses, as an adjustment for the otherwise imperfect value of the screw, when read off as decimal numbers of an inch, and we have since learned from Mr. Dollond, that his practice is, to make the final adjustment of the scale in this manner.

6 SECOND CONSTRUCTION.—The second construction of the dynameter, and that which Dollond now adopts, is represented by figures 17 and 18 of the same Plate, the former of which shows the external appearance of the instrument seen edgewise, and the other exhibits the internal parts, when the cover of the small frame is removed, by taking out the four corner screws. We have put the same letters to the same parts that were used in describing the other construction, as far as they go, but as this is by much the more simple construction, and on that account we think preferable, we shall not have occasion to dwell so long on the account we have to give of it The interior mechanism of this dynameter is contained in the oblong box *a b c d*, from which the lid is removed, *e* as before is the milled nut, and *f* the divided head containing one hundred divisions, both made fast to the thick screw *h*, which is what is called a right-handed screw, containing about fifty threads in the inch, this screw is perforated and tapped by a left-handed screw through its tubular part, which the second screw *i* enters; so that here there are two separate screws with each its own axis, one right and the other left-handed, but having each the same number of threads in the inch. The inner screw *i* enters the fang of the small flat plate *m*, which bears the semi-lens *k*; but, though it cannot be withdrawn from its connexion, it has liberty to turn round the external threads of the larger screw, cut with a female screw tapped in the fang of the small plate *n*, which carries the semi-lens *l*, these plates *m* and *n* are of like breadths, and are kept parallel to each other by the bed formed by parallel interior sides of the frame or box *a b c d*, and the horse-shoe spring *o*, confined by the end and sides of the frame, presses these two plates against their respective screws, to prevent shake, or loss of motion, when the change of its direction takes place. The *modus operandi* may be thus explained; when the milled nut is turned in a forward direction, the thick screw *h*, from its exterior connexion with the fang of plate *n*, draws the semi-lens *l* towards it, but by its interior connexion with the left-handed screw *i*, pushes this screw from it, and also the plate *m*, which carries the semi-lens *k*, the spring *o* in the mean time pushing both the plates against their respective screws; thus the plates move in contrary directions, not, as in the case of screw *i* in the former plan, by the difference of two contrary motions, but by the sum of two equal absolute motions in contrary directions. The cover *t u*, containing the one-tenth of an inch of mother-of-pearl, is here seen in its place in the form of a tube, adjustable for distance for distinct vision, when the mother of pearl is turned back from covering the central hole, and when the end *t* is applied to the eye-piece of a telescope, the adjustable tube is found useful in keeping the dynameter to its true position for obtaining measures, which are shown on the scale *g*, carried by the plate *n*, where a division corresponds to a revolution of the screw. When the second lens of this dynameter was removed, the semi-lenses measured the tenth of the inch, or disc of pearl, by 2.66 revolutions, but when it was returned to its place, the measure was exactly five revolutions, as it ought to be.

7. Having described the two common constructions of the double image dynameter, we may now proceed to explain the method of using this beautiful little instrument for the purpose of ascertaining the magnifying powers of any telescope.

The vast advantage attending the use of the dynameter is, that it does not require any knowledge of the thickness and focal lengths of any of the lenses employed in a telescope, nor yet of their number or relative positions; neither does it make any difference whether the construction be refracting or reflecting, direct or inverting, one operation includes the result arising from the most complicated construction, and sets theory at defiance, with respect to calculations that must take into consideration the previous determination of all the preceding requisites, the obtaining of which is attended with practical difficulties almost insurmountable.

When a telescope has a particular eye-piece applied to it, for the purpose of having its magnifying power ascertained by the dynameter, it must first have a cap put over its object-end, with a circular hole of well-known dimensions smaller than the whole aperture, it must then be directed to some heavenly body for adjustment to the solar focus, particularly if the power to be determined is to be used in celestial observations, and lastly the dynameter must be held in the hand, before the eye-piece, at such a distance, and in such a direction in the axis of vision, that the image of the aperture facing the sky may be a perfect colourless and well defined disc, in which situation the screw of the dynameter must be turned forwards until one of the two discs, that will soon appear, just comes in contact with the other, like two planets, as seen in fig 16 When this is the case, the number of revolutions, as read on the divisions of the scale *g* to the right of zero, and the hundredth parts of a revolution, read on the micrometer head, must be marked down as the first part of the measure, the same operation must then be performed by carrying the images first into one, and afterwards by the backward motion of the screw into a similar contact at the other side, and the divisions of the scale *g*, now read to the left of zero, will give entire revolutions, and the divisions on the micrometer read backwards, or subtracted from one hundred, will give the parts of a revolution to be put down with the revolutions under the former measure, when half the sum of the two will give a mean, that will include the correction for any index error that may exist at the time. For instance, when an eye-piece of small power was screwed into an achromatic telescope of about sixty-three inches focal length, and a cap was put on, with an aperture of just three inches, a dynameter by Ramsden, which made ten revolutions in the tenth of an inch, or one hundred in the inch, was used to get the dimensions of the image of this diminished aperture at the solar focus, and the measures were found $\left.\begin{array}{l}\text{to the right } 6\ 00\\ \text{to the left } 6\ 10\end{array}\right\}$ average $= 6\ 05$, or according to the notation belonging to this dynameter $\frac{605}{10000} = .0605$ when decimally expressed, then $\frac{3.0000}{.0605} = 49\ 58$ was the power by this experiment But when the telescope has several limitations of aperture, it will contribute to accuracy to use more than one, thus, with an aperture of only two inches, the measures were found to the right 4.025, to the left 4.035, giving 4.03 as the average, then $\frac{403}{10000} = .0403$ was the quotient, and we had $\frac{2.0000}{.0403} = 49.63$ for the power by this experiment, and as the two powers thus obtained are very nearly the same, the mean (49.6) may be considered as the power required to be found.

8 Hence, when a dynameter of this description is used, it will only be necessary to put down the numbers as they are read, without reference to the integer, and to prefix a cipher with the decimal point before it, in order to reduce the revolutions and parts to the decimal denomination of an inch, on a supposition that the inch is divided into 10000 parts, which is

the product of one hundred revolutions into one hundred divisions on the divided head, that were previously found to measure an exact inch.

9. But the dynameter is frequently constructed with a screw that makes only fifty revolutions in the inch, or five revolutions in one tenth; when this is the case, the numbers read on the scale and micrometer head jointly, must be *doubled*, before the cipher is prefixed, and then the expression will be the decimal part of an inch as before.

10 As a second example, a dynameter, having only five revolutions in the tenth of an inch, made agreeably to the second construction by Dollond, was applied to a telescope of 43 .2 inches solar focus, when adjusted for vision by a planet, the diminished aperture being only an inch and a half, when the average of the measures taken to the right and left of zero was found to be 0 825; then the double of this reading, with a cipher and decimal point prefixed, will give the measure of the disc 0165 as the proper divisor, and $\frac{1 .5000}{0165} = 90 .90$ gave the magnifying power with a certain eye-piece that was then in use. Also with a six-feet Newtonian telescope the aperture was diminished to 4 02 inches, and the measure of its disc, taken on the average at 2 .365, gave the power $\frac{4 .0200}{.0173} = 84$, when a wire micrometer formed the eye-piece. In the same manner the magnifying power of any Gregorian or Cassegrainian telescope may be taken with both facility and accuracy.

11. The dynameter is of the greatest utility when a table of powers is wanted in micrometrical measurements, taken with any telescope by means of an eye-piece with variable powers, composed of either two or four lenses, as being suitable to be used with the prismatic solids of double refraction at the ocular end of the tube; for as the measures depend as much on the power of the telescope as on the refracting angle of the prism, it is requisite that they should both be accurately known. The scale of the variable eye piece, both of the celestial and terrestrial construction, is a scale of equal parts, and therefore if the power be known at each extreme limit, the powers at all the intermediate points may be readily computed, by the constant addition or subtraction of determined differences.

12. When a practical astronomer has different telescopes in his possession, it will be desirable, on account both of convenience and accuracy, to adapt his variable eye-pieces to them all, and to make a scale of powers for each telescope; for the ascertained focal length of each telescope will operate as a check on the powers determined solely by the dynameter, when the analogy existing between the determined powers and the corresponding focal lengths is examined; and this mode of treating the powers will give to any individual telescope all the correction that arises from the analogous examination of the different telescopes.

13. The following operations will exemplify this plan of tabulating the successive powers of five telescopes to which a terrestrial pancratic eye piece (fig. 2. of Plate II.) has been adapted. The solar focal distances are these, viz.

Tel. 1 30 .50 inches, achromatic refractor.
2. 43 .20 ditto.
3. 67 .50 ditto.
4. 71 .75 ditto, Newtonian reflector.
5. 76 .25 ditto, achromatic refractor.

The sliding tube of the eye-piece, that carries the pair of eye-glasses, will draw out so far, as to admit a scale of six inches, divided into sixty equal parts to be marked on it with a graver, which divisions are indicated by the exterior end of the outer tube In the annexed little table the numerators of the respective fractions denote the apertures used, and the denominators show the decimal numbers derived from the dynameter's screw, in each of the two positions, at 0 and 60 of the scale, and the integers, or quotients resulting from the divisions, are the respective magnifying powers, as determined by a Dollond's dynameter of fifty threads in the inch

	POSITION 0 P	POSITION 60 P
Tel. 1	$\frac{1\ .5000}{.0418}=35\ .90$	$\frac{1\ .5000}{.0214}=70\ .09$
2	$\frac{1\ .5000}{.0295}=50\ .85$	$\frac{1\ .5000}{.0151}=99\ 33$
3	$\frac{2\ 0000}{.0254}=78\ .80$	$\frac{2\ 0000}{.0129}=155\ .03$
4.	$\frac{4\ .0200}{.0476}=84\ 45$	$\frac{4\ .0200}{.0244}=164\ 75$
5.	$\frac{3\ 2400}{0361}=89\ 75$	$\frac{3\ 2400}{01852}=174\ 40$

Now as telescope 1 has the largest disc in proportion to the aperture, we may conclude that its power is obtained with greater accuracy than the powers of the other telescopes, and also that all the powers due to the position 0, are more correct than the powers obtained at the position 60, for the same reason, we therefore make telescope 1 the standard of analogical comparison, as in the following Table

	POSITION 0			POSITION 60		
For Telescope 2.	As 30. 5 . 35. 9	43. 2	50. 52	As 30. 5 . 70. 10 ::	43. 2 :	99. 28
3	As 30. 5 : 35. 9	67. 5	79 45	As 30. 5 70. 10 :.	67. 5	155. 14
4.	As 30 5 : 35. 9	71. 75	84. 45	As 30. 5 70. 10	71 75	164 90
5	As 30. 5 35 9 ::	76. 25	89. 73	As 30. 5 . 70. 10 ·	76. 25	175 25

14. These two determinations by the dynameter, and by analogy, are so nearly the same, that either of them might be depended upon, as sufficiently exact for the nicest purposes, the average of the two will be,

P P
for Position 0—Tel 2,.......50. 67,..... for Position 60. .. ...99. 30.
for Ditto. .... 3,. .....79. 12,......for Ditto . ...155. 08.
for Ditto.......... .... 4,.. ... 84. 45, ...for Ditto . ...164. 82.
for Ditto................5,.......89. 74,......for Ditto ....174. 80.

If these averages are adopted as the best powers, it will make very little difference which telescope is used in micrometrical measurements, where the *power* is one of the elements for determining them, their respective values being rational.

15. But in constructing a Table there is another consideration that must influence the choice we make of the extreme powers, which is, that their differences must be so taken, that they may be divisible by the number of divisions on the scale of the interior tube of the eye-piece, which in this instance is 60, otherwise the tabular quantities will accumulate improperly by constant addition, and will require periodical coaxing to bring the conclusion right. If we take, for the first telescope, the numbers determined by the dynameter, we shall have 70. 09—35. 90 = 34. 19 for the whole difference, which is not divisible by 60, but if we substitute 70. 1 for 70. 09 the difference 34. 2 is divisible by 60, and 0. 57 will be the tabular multiple or difference from unit to unit, to be applied to 35. 9, and to every succeeding number, till at the lowest line of the table (60), the number 70. 1 is the exact concluding number. In like manner, if we take 50. 85 at the position 0, and substitute 99. 45 for 99. 3, the average number, we shall have 99. 45 – 50. 85 = 48. 6, which number divided by 60, will make 0. 81 the tabular multiple for the second telescope. Likewise by taking 155. 00, 164. 85; and 174. 35, instead of the exact numbers above determined, from which they differ imperceptibly, for the powers of the other three telescopes at position 60, and by retaining those determined by the dynameter at position 0, the continual tabular differences will be respectively 1. 27, 1. 34, and 1. 41, with which the subjoined table was completed without a remainder. The utility of the table will require scarcely any explanation, for if the position of the sliding tube, as indicated by the divisions on the scale inserted on its surface, be made the argument in the first column, the corresponding power of any of the five telescopes will be seen by inspection, in the same horizontal line, under the proper column. For instance, when the tube is drawn out to 40 on the scale, the powers of the five telescopes will be 58. 70, 83. 25, 129. 60, 138. 05, and 146. 15, and in like manner the powers are given for any other relative position of the lenses. The fifth telescope will have the same power at the position 39, that the fourth will have at 45, and the third at a little space short of 52, when the same eye-piece is used with them all in succession.

TABLE

OF THE VARIABLE MAGNIFYING POWERS OF FIVE TELESCOPES

Scale	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5	Scale	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5
0	35 90	50 85	78 80	84 45	89 75	30	53 00	75 15	116 90	124 65	132 05
1	36 47	51 66	80 07	85 79	91 16	31	53 57	75 96	118 17	125 99	133 46
2	37 04	52 47	81 34	87 13	92 57	32	54 14	76 77	119 41	127 33	134 87
3	37 61	53 28	82 61	88 47	93 98	33	54 71	77 58	120 71	128 67	136 28
4	38 ,18	54 09	83 88	89 81	95 39	34	55 28	78 39	121 98	130 01	137 69
5	38 75	54 90	85 15	91 15	96 80	35	55 85	79 20	123 25	131 35	139 10
6	39 32	55 71	86 42	92 49	98 21	36	56 42	80 01	124 52	132 69	140 51
7	39 89	56 52	87 69	93 83	99 62	37	56 99	80 82	125 79	134 03	141 92
8	40 ,46	57 33	88 96	95 17	101 03	38	57 56	81 63	127 06	135 37	143 33
9	41 03	58 14	90 23	96 51	102 44	39	58 13	82 44	128 33	136 71	144 74
10	41 60	58 95	91 50	97 85	103 85	40	58 70	83 25	129 60	138 05	146 15
11	42 17	59 76	92 77	99 19	105 26	41	59 27	84 06	130 87	139 39	147 56
12	42 74	60 57	94 04	100 53	106 67	42	59 84	84 87	132 14	140 73	148 97
13	43 31	61 38	95 31	101 87	108 08	43	60 41	85 68	133 41	142 07	150 38
14	43 88	62 19	96 58	103 21	109 49	44	60 98	86 49	134 68	143 41	151 79
15	44 45	63 00	97 85	104 55	110 90	45	61 55	87 30	135 95	144 75	153 20
16	45 02	63 81	99 12	105 89	112 31	46	62 12	88 11	137 22	146 09	154 61
17	45 59	64 62	100 39	107 23	113 72	47	62 69	88 92	138 49	147 43	156 02
18	46 16	65 43	101 66	108 57	115 13	48	63 26	89 73	139 76	148 77	157 43
19	46 73	66 24	102 93	109 91	116 54	49	63 83	90 54	141 03	150 11	158 84
20	47 30	67 05	104 20	111 25	117 95	50	64 40	91 35	142 30	151 45	160 25
21	47 87	67 86	105 47	112 59	119 36	51	64 97	92 16	143 57	152 79	161 66
22	48 44	68 67	106 74	113 93	120 77	52	65 54	92 97	144 84	154 13	163 07
23	49 01	69 48	108 01	115 ,27	122 18	53	66 11	93 78	146 11	155 47	164 48
24	49 58	70 29	109 28	116 61	123 59	54	66 68	94 59	147 38	156 81	165 89
25	50 15	71 10	110 55	117 95	125 00	55	67 25	95 40	148 65	158 15	167 30
26	50 72	71 79	111 82	119 29	126 41	56	67 82	96 21	149 92	159 49	168 71
27	51 29	72 72	113 09	120 63	127 82	57	68 39	97 02	151 19	160 83	170 12
28	51 86	73 53	114 36	121 97	129 23	58	68 96	97 83	152 46	162 17	171 53
29	52 43	74 34	115 63	123 31	130 64	59	69 53	98 64	153 73	163 51	172 94
30	53 00	75 15	116 90	124 65	132 05	60	70 10	99 45	155 00	164 85	174 35

16. Double Vision.—When a good dynameter is not at hand, the magnifying power of a telescope may be ascertained by double, or what is called *false vision* with considerable accuracy, when due care is taken in taking the measure, and in reducing the power thus gained to the power that belongs to the solar focus. If a slip of writing paper one inch long, or a disc of the same material of one inch diameter, be placed on a black ground at from 30 to 50 yards distance from the object end of the telescope, and a staff painted white, and divided into inches and parts by strong black lines, be placed vertically near the said paper or disc, the eye that is directed through the telescope when adjusted for vision, will see the magnified disc, and the other eye, looking along the outside of the telescope, will observe the number of inches and parts that the disc projected on it will just cover, and as many inches as are thus covered will indicate the magnifying power of the telescope, at the distance for which it is adjusted to distinct vision. If we call F the measure of the solar focal length in inches and parts, and F' the terrestrial focal length, at any given distance, which will always be longer than the solar focal length, we shall have the following analogy for obtaining the solar power P, from the

terrestrial or measured power P' thus, as $F' : F :: P' : P$. For instance a disc of white paper, one inch in diameter, was placed on a black board, and suspended on a wall contiguous to a vertical black staff, that was graduated into inches by strong white lines at a distance of 33 yards, 2½ feet, and when the adjustment for vision was made with a 42-inch telescope, the left eye of an observer viewed the disc projected on the staff, while the right eye observed that the enlarged image of the disc covered just 58½ inches on the staff, which number was the measure of the magnifying power P', at the distance answering to the focal distance F', which in this case exceeded F by an inch and a half, then, agreeably to the analogy above given, we have, as $43.5 : 42 :: 58.5 : 56.5$ nearly, hence the magnifying power due to the solar focal length of the telescope in question is 56.5, and the distance 33 yards 2½ feet, is that which corresponds to an elongation of the solar focal distance of an inch and a half, if we compute by the formula (given at § IV. 22.) $\frac{F'F}{F'-F} = D = \frac{43.5 \times 42}{1.5} = 1218$ inches $= 101.5$ feet, or 33 yards 2½ feet, and by a like process the magnifying power of any other telescope may be taken, and reduced to the solar power.

17. In a manner similar to the preceding, the whole field of view may be projected on a distant wall, and the space thus covered by the luminous circle may be considered as the subtense of an angle, to be measured by a sextant or circular instrument, and when the value of the field of view in minutes and seconds is previously known, by the passage of an equatorial star, or otherwise, the large measured angle divided thereby, will give the power of magnifying, depending on the distance of the wall, which may then be reduced to the solar power. When the telescope is large, it will however intercept a large portion of the projected luminous field, and render this method difficult to practise, but if the disc proposed above, be reduced from an inch in diameter to one half, or one quarter when the telescope is long, the method of double vision may be applied with considerable accuracy, in obtaining even large powers, when the observer has accustomed himself to adjust the pupils of his eyes to two different distances at the same time, to enable him to judge correctly of the measure, that the projected image of a disc occupies on a graduated scale, which faculty will be obtained by a little practice; and is absolutely necessary when the lamp micrometer, or other binocular instrument is proposed to be used. The practice of the Tulleys is, to take the focal distance of a known eye-piece, and to divide the solar focal length of any proposed object-glass thereby, to gain the magnifying power of the telescope used with such eye-piece and object glass.

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§ XII THE DORPAT REFRACTING TELESCOPE [PLATE VII.]

1. THE most expensive refracting telescope that ever was constructed, is that which was made by the celebrated Fraunhofer of Munich, for the observatory of the Imperial University at Dorpat, and received by Professor Struve in the year 1825. The aperture of this telescope is nine French inches or about 9.43 English, and its solar focal length about 14 English feet, the main tube being 13 French feet, exclusively of the small tube that holds the eye-pieces. The

smallest of the four magnifying powers is stated to be 175, and the largest 700, which it is said, "in favourable weather, presents the object with the utmost precision." Professor Struve transmitted a short notice of the arrival and erection of this gigantic instrument, to the council of the Astronomical Society through their President, which is published in Part I of Volume II of their Memoirs, accompanied by a plate giving the perspective view of the telescope, mounted on its equatorial stand, which we have copied in our Plate VII. "This masterpiece" says the indefatigable Professor, "was sold to us by Privy counsellor VON UTZCHNEIDER, the chief of the optical establishment at Munich, for 10500 florins (about £950 sterling), a price which only covers the expenses which the establishment incurred in making it. This generosity, this sacrifice to science, deserves every praise, especially as the Professor and Academician Chevalier FRAUNHOFER has offered to contribute also in future towards perfecting this splendid masterpiece of art." Accordingly we learn, that a perfect micrometrical apparatus has been ordered as an appendage, to render the instrument serviceable for measuring the small angles by which double stars are separated, and also the angles of position; this apparatus, which is now probably completed, will consist of four annular micrometers, two of them to be double; a lamp circular micrometer with four eye-pieces; a repeating lamp net micrometer with position circles and four eye-pieces; one of the circles to be graduated on silver, and to read to minutes by a pair of opposite verniers.

2. The frame work of this stand is of oak, inlaid with pieces of mahogany in an ornamental manner, and the tube is of deal, veneered with mahogany and highly polished. The whole weight of the telescope, and of its counterpoises, is supported at one point at the common centre of gravity of all the ponderous parts; and though these weigh 3000 Russian pounds, of which the frame-work constitutes 1000, yet we are told, that the remaining 2000 are balanced in every situation, and that "this enormous telescope may be turned in every direction towards the heavens with more ease and certainty, than any other hitherto in use!" "It may be turned" continues the Professor, "in declination with the finger, and round the polar axis with still less force, a weight of three pounds being fixed at some distance from the eye-end, by which the friction is overcome."

3. The basis of the frame is formed of two cross beams, each nine feet seven inches long, seven inches wide, and seven and a quarter deep, the ends of which are seen in figure 1. at *A*, *B*, *C*, and *D*; these are braced by four smaller bars, forming a square, of which one is seen at *E*; this braced cross is fastened down to the floor by eight strong screws, four near the outer ends, and four near the junction, six of which are visible in their places; in the centre is firmly fixed a perpendicular post, six feet and one inch high, and seven inches square, which is propped at the north, east, and west sides, by three posts of an elliptical form, denoted by *G*, *G'*, and *G''*, which are made fast at their lower ends to the beams of the cross, and at their upper ends to the vertical post. An inclined beam *H*, of the same thickness, rests on the southern end of the meridianal beam of the cross, and is attached to the vertical beam in an angle of inclination equal to the altitude of the polar axis, to which therefore it lies parallel; but all the remaining parts not yet described are metallic.

4. When the object-end of the telescope is elevated to the zenith, it is 16 feet 4 inches, Paris measure, above the floor, and as 13 feet 7 inches of this height belong to the telescope, its eye end in this position will be 2 feet 9 inches high. The polar axis of the instrument de-

noted by the letter *I*, is a cylinder of steel, 39 inches long, and proportionably thick; it revolves in two cylindrical collars, and its lower end, being rounded and highly polished, rests on a steel plate, attached to the strong bearing piece *K*, secured to the inclined beam *H*, and has therefore very little friction, the weight being supported by a pair of friction-rollers, near the common centre of gravity; and a counterpoise *L* applied to support the axis in any given position. To the lower extremity of this axis is fixed a circle of 13 inches diameter, which is so graduated as to indicate single minutes of time, a pair of opposite verniers read to two seconds, and a smaller quantity may be had by estimation*.

5 The axis of the telescope's motion in altitude, which stands at right angles to the polar axis, and is nearly of the same dimensions, passes through a long brass tube *M*, forming a part of the frame, which is screwed to the upper end of the polar axis by twelve strong screws, this axis carries the circle of declination, which is 19 inches in diameter, divided from 10′ to 10′, and has a vernier reading 10″, or 5″ by estimation. The tube of the telescope being fixed to the frame-work nearer to the eye-end than the middle, has two counterpoises attached to a pair of levers which serve, at the same time, to balance the two ends, and to prevent the natural tendency of the longer end to bend, which is certainly a very ingenious contrivance, for as the levers have their fulcra at the common centres of motion and of gravity, and have each two axes in the form of gimbals, they act on the tube in all directions that different positions may require, while their remote ends are inserted into holes in the strong ring *N*, that surrounds the tube, at the part most liable to bend. The brass frame holding the two axes, appears to be clamped to the tube by two other strong rings, one at each side of the centre of motion. A bent lever carrying the weight *O*, has a double ring that embraces the near end of the axis of the declination circle, and the axis itself is said to carry another weight, which two weights together form the counterpoise of this axis, and though it does not appear how the counterpoise *L* is attached to the polar axis, it is probably made with a similar ring, to lay hold of the upper end of this axis, so as to be adjustable. The slow motion in altitude is given to the telescope by a Hooke's joint applied to the screw of the clamp, which has a spring urging it against a strong iron bar *P*, attached to the end of the cylinder *M*, that forms a stop to the circle; and a slow equatorial motion is given by a second Hooke's joint, taking hold of an endless screw, acting with the racked edge of the hour circle; while a spring presses it into action uniformly, and a lever is employed to raise it out of the rack when necessary. The handles taking hold of these screws extend to the reach of the observer, who can thus, we are assured, point his telescope in right ascension and declination with as much certainty as when using the best meridian instruments.

6. Another peculiarity of this extraordinary instrument is, that clock work applied to the equatorial axis, gives it a smooth and regular sidereal motion, which, it is affirmed, keeps a star in the exact centre of the field of view, and produces the appearance of a state of rest in the starry regions, which motion can yet be made solar, or even lunar, by a little change given to the place of a pointer, that is placed as an index on the dial plate. A weight placed on a projecting piece, coming from the tangent screw, overcomes all the friction of the machine; and a balance, vibrating in circular arcs, regulates the velocity of the motions; which balance must

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* While the Author was writing this Section, and had proceeded so far, he received information that the large telescope of the Astronomical Society had fallen to his lot, his tender having been accepted.

consequently be adjustable, from its connexion with the aforesaid pointer, that occasionally changes the rate of going of the clock. Another use of this momentary regulation is, that thereby a star that is out of the centre of the field, either in advance or arrear, can be soon brought to its required place in the field, and kept there by putting the pointer again to its original place We do not however find, that there is any provision made for counteracting the effect of varying refraction in the different changes of altitude, which must necessarily interfere with the intended operations of the clock, when its motion is continued for several minutes on the same star A provision is made for keeping the clock in motion during the act of winding it up The different weights that are introduced for the purposes of overcoming friction and of maintaining the clock's motion, appear in their places in the plate Professor Struve considers the optical powers of his telescope superior to those of Schroeter's 25 feet reflector, from having observed σ Orionis with fifteen companions, though Schroeter observed only twelve, that he could count with certainty. Nay, he seems disposed to place it in competition with the late Sir William Herschel's 40 feet reflector, but his reasons are grounded entirely on supposition.

7 By way of preparing himself for the task of re-measuring the double stars, the professor took his net-micrometer from a five feet refractor, and adapted it to the large instrument, and gave several specimens of its power in measuring small angles, with a magnifying power of 540 His first object was to ascertain the diameter of a spider's line, two of which nearly alike were placed in the focus of his eye-piece, and he found their diameters to be each 0″.5, as deduced from the following measures,

viz..................1 .06
0 .98
1 .06
1 05
0 99
0 .99
1 .00
1 .02
1 .00
1 .05
1 .00
1 .08
The mean 1. 023

These observations were made by reducing the line of light to the *minimum visibile*, and from the twelve measurements of the double diameters it appears that the probable error on a single measurement is only .024 or one-fortieth of a second, but as the lines of visible light between the spider's lines in both positions were included in the measure, the thickness is given too great at each repetition. A black board with white dots placed at different intervals, was then erected at the distance of about 900 toises, and the measures of the lines joining their centres were found as follow viz.

Distance.	Diff from mean	Distance	Diff from mean	Distance.	Diff from mean
7″.90	0″.06	5″.10	0″.08	1″.66	0″.01
8 .05	0 .09	5 .13	0 .05	1 .63	0 .02
7 .99	0 .03	5 20	0 .02	1 .55	0 .10
7 .95	0 .01	5 .20	0 .02	1 .66	0 .01
7 .81	0 .15	5 .02	0 16	1 .72	0 .07
8 .08	0 .12	5 .26	0 .08	Mean 1 .65	
7 .88	0 08	5 .08	0 .10		
8 .02	0 .06	5 .26	0 .08		
Mean 7 .96		Mean 5 .18			

From these 21 observations the author deduces the probable error of an individual measurement, from every double observation, to be equal to $0''.055 = \frac{1}{18}$, when the objects are perfectly quiescent. The nearest dot had a diameter of $0''.8$, and the circumferences stood at $0''.8$ from each other he remarks, however, that such an exactness of measurement will certainly be impossible in the heavens, owing to the glimmering of the stars, and the apparent motion of the sphere. The latter difficulty, he further remarks, would be insuperable with the greatest magnifier, were it not for the clock-work, which, by turning the telescope, makes the heavens appear immoveable. He then gives a list of observations made on double stars on December 24 and 26, 1824, which we propose to notice when we have described the different micrometers, and shown the methods of applying them to the measurement of small angles.

8. The finder of this telescope has a focal distance of 30 French inches, and 2.42 aperture

9. The principles on which we have been accustomed to form our opinion of the steadiness in the position of a long telescope, at the first sight of the plate, induced us to entertain a doubt of the good performance of an instrument of such large dimensions, when supported only at the centre of motion, but persons of credit, who have witnessed the excellence of the whole machine, have controverted our judgment by matter of fact, to which we must therefore yield our assent. It is much to be regretted that the ingenious contriver and maker of this grand telescope is now no more, the continental astronomers will mourn his loss.

§ XIII ON REFLECTING TELESCOPES

1. Though reflecting telescopes have not been made subservient to astronomical observations, where measurements have been used, except in Short's equatorial, and in cases where micrometers are applied, yet their great powers have rendered them highly useful in viewing and examining the different parts of the heavens for the purpose of making discoveries. There has been no considerable alteration in the qualities and proportions of the metals, proposed and used for specula by Sir Isaac Newton, but the skill and care with which their curves are now formed, have brought these instruments into high estimation, and no astronomer, who prides himself on the value of his instruments, will be content to remain without a good reflector. There are four varieties of construction, the Newtonian, Cassegrainian, Gregorian, and Herschelian, so called from their respective inventors It will perhaps be trifling with the reader to say, that the large speculum in all the constructions is concave, and that the four varieties depend on the small specula, or on having no small speculum at all.

2. In the Newtonian telescope, which is the most convenient for ordinary purposes, the small speculum is quite plane, or ought to be so if such desideratum could be completely effected, in this construction the converging rays that have been reflected by the large speculum, are intercepted by the small one before they converge to a focus, but being turned at right angles, they form an erect image of the object before, or in the eye piece, that is inserted into the side of the main tube, near its aperture. When the focal length of the large speculum is from 6 to 7 feet, and the aperture from 6½ to 7 inches, the instrument has sufficient power and light for giving an interesting view of any of the heavenly bodies, and, being used with the

head of the observer in an erect position, is peculiarly convenient; particularly when it is mounted on a good stand, and has a small parallel telescope as a finder, to assist the observer in detecting the object required.

3. The Cassegrainian telescope has a convex face in its small speculum, which returns the converging rays coming from the large speculum, back through a hole made in its centre, and the image is viewed inverted by the Huygenian eye-piece at the depressed end of the tube, when this is directed to the heavens.

4. The Gregorian, on the contrary, has a concave face on the small speculum, which makes the reflected rays return after they have crossed one another, so as to form a second image in the front of it, and the picture of this image is viewed through a central aperture made in the large speculum in an erect position, by means of an Huygenian eye-piece, applied in the same manner as in the Cassegrainian telescope.

5. The Herschelian telescope is the most simple of any, inasmuch as it has only one speculum, placed a little obliquely, as it regards the length of the tube, it produces an image at one edge of the mouth of the tube, which is viewed there by the eye-piece, while the back of the observer is turned towards the object viewed. The great diameter of the speculum of this instrument, together with the length of its focus, give it powers that far exceed those of any other instrument, but the difficulty and expense attending the casting and polishing of a metal four feet in diameter, and 40 feet in focal length, will ever deter an ordinary mechanic from undertaking the Herculean labour of making one of the largest size. Indeed a telescope of 15 or 16 inches in diameter, and 20 or 25 feet focal length, has been found not only more manageable, but quite adequate to the important purpose of penetrating into space, and examining minute objects, which is the main purpose of the Herschelian instrument

6. The observations which we have to offer to the practical astronomer, by way of guide, in his choice of a Cassegrainian or Gregorian instrument, will apply with equal propriety to them both. An inspection of the reflecting surfaces of the metals, will show their colour and fineness of polish, but will not afford any criterion of the *figure*, on which their principal excellence depends to judge of this requires a previous adjustment of all the parts that constitute the instrument. In most of these reflecting telescopes there are screws of adjustment at the back surface of the specula, by means of which the reflected rays, coming from the large one, are made to fall concentrically on the small one before the second reflection, and again to come towards the eye in the direction of the line of vision, after the second reflection This adjustment is not easily effected even by the maker, and is very troublesome to an amateur astronomer, who has not been previously instructed how to proceed The usual method of placing the two metals in their proper positions is, by screwing the eye-piece into its place without its lenses, and by looking repeatedly through the diminished aperture at the eye-end, as often as any screw is altered of either of the metals, until, after many slight alterations of position, the images of the two specula are seen through the eye-hole exactly concentric, but when there is any parallax arising from the size of the eye-hole, some uncertainty will remain as to the final accuracy of this adjustment. We have accidentally discovered a method that removes this doubt, by which the accuracy of the adjustment may be instantaneously discovered, and by means of which the relative positions of the two specula may be easily ascertained and secured let the two lenses of the eye-piece be restored to their place, and procure a Ramsden's eye-

piece from some transit instrument, or spider's line micrometer, that will render an object visible in the compound focus of the two lenses, of which it is composed, then hold this second eye-piece in front of the one already in use, and, by varying the distance a little, find the position in which the image of the large speculum is seen, well defined through both eye-pieces, as when a dynameter is used, and if the image of the small speculum is seen precisely on the centre of the large one, now that there is no parallax, the metals may be considered as rightly placed; but if not, the proper screws must be used in succession, till the required position is determined, when the face of the large metal stands at right angles to the length of the tube, the adjustment may generally be finished without disturbing it; and when the bed that receives it has once been properly finished, it will be advisable not to alter it, unless some accident should render such alteration indispensable.

7. When the metals have their parallelism and concentricity insured, the instrument may be directed to some distant luminous point, as a white disc on a black ground; or a star will do still better for this purpose, then in adjusting for distinct vision, observe if the figures of the metals are well adapted to each other. this may be done partly by noticing if the disc, or star, is well defined, and free from irradiations; and partly by carrying the small speculum a short distance beyond and short of distinct vision, by the proper screw, and by examining if the disc or star enlarges alike in similar changes of position, if the result be satisfactory, the metals may be considered as well placed, and as well adapted to each other.

8. To try whether the large speculum is formed to a curve that partakes of the parabolic quality, its aperture must be partially covered, first at the central part, and then round the circumference, by tin, pasteboard, or stiff paper, and if, on trial, the same adjustment for distinct vision of a distant object is good in both these cases, and also when the speculum is all exposed, the *figure* may be considered good, and in this case the least alteration, in the adjustment for good vision, will produce a confusion of images, and prove that the large speculum has a good figure; but if these effects are not produced, which it is the maker's business to insure, the instrument will be incompetent to perform several of the nicer observations in astronomy, though it may give a satisfactory view of the surrounding country, or even of the sun and moon. When a mistiness appears in the field, it is a proof that the aberrations are not corrected, and that the figure of at least one of the specula is not perfect. We have learned from an astronomical friend, intimately acquainted with reflecting telescopes, that if a cover be put on the mouth of a large reflecting telescope, with a circular aperture, of about one-half the diameter that the tube has, in such way that the diminished aperture may fall entirely at one side of the opening of the tube, the effect thus produced on the telescopic appearance of a star is wonderful, a star of the first magnitude will frequently, by this contrivance, be seen quite free from irradiations, and will have its circumference clearly delineated; though the same effect will not be produced by a similar diminution of aperture, made concentric with the aperture of the tube. This manœuvre in the management of a reflecting telescope is founded on a supposition that in most large metals, if not in all, *one side* of the surface, used by itself, will give a more distinct image, than will be produced by rays coming from both sides, and mixing together, want of homogeneity, or want of uniform temperature in both sides of the large speculum, may occasion the phenomenon in question; and this is rendered probable by the circumstance, that, on turning round the excentric cover, there will be found some positions that give better images than others. The secret, however, is worth knowing, particularly if the telescope

is not good with its full aperture; and in all measurements of diameters of the planets, and of distances between double stars, we have known the expedient tried with different telescopes, and are quite satisfied with the result.

9. It has been affirmed by a gentleman eminent in the practical departments of science, that much light is lost, or absorbed, in the *crossing* of the rays in a Gregorian telescope, which is not the case in the Cassegrainian construction, but we do not find that this assertion has been confirmed by the experiments that have been subsequently made, on the comparative appearances of the same object by twilight, when instruments of the two different constructions, but with precisely the same magnifying powers and apertures, as well as the same composition of the metals, have been tried against each other. In an experiment of this kind, which we witnessed, the print on a card became illegible after sunset by both telescopes at the same time, as nearly as could be ascertained When a Gregorian telescope has a large aperture in proportion to its focal length, it has obtained the appellation of *dumpy*; we have before us one of this description by Watson, who was an eminent maker, which has an aperture of four inches, with a focal length of only fourteen, which defines a planet beautifully, and shows the companion of Polaris. It will bear a power of nearly 200, and is packed in a box with a tripod that renders it an useful travelling instrument for observing occultations

10. The magnifying power (P) of the Newtonian and Herschelian telescopes, like the achromatic refractor with a celestial eye-piece, is known by comparing the distance of the image of any remote object from the large speculum with the focal length of the lens or eye-piece used; for as often as the latter is contained in the former, so often is the telescope said to magnify, or augment the linear dimensions. but the Gregorian telescope has its power ascertained by the same method as the refracting telescope with a terrestrial or four-glassed erect eye-piece, on account of its repeating the image in an enlarged state, but in a contrary position (§ VII.), if we call the solar focal distance of the large speculum F, and the distance of its focus, or of the place of the first image, from the small speculum f, the distance of the conjugate focus, or of the second image, from the small speculum D, and the focal distance of the eye-piece, or of its equivalent lens d, then the magnifying power will be obtained from the following formula,

$$\frac{FD}{fd} = P.$$

11. The magnifying power of the Cassegrainian telescope is obtained in nearly the same way, but as the first image, which would be formed at the *virtual* focus, behind the small speculum, if this speculum did not intervene, is prevented by its interposition, the image that is formed at the conjugate focus, before the eye piece, must be considered as the *second* image, if, then, we call the focal distance of the large speculum, which must be considered as extending beyond the small speculum, F, the distance from the small speculum, where the first image would be formed, if the rays were not intercepted, f, the distance from the small speculum to the second or actual image, D; and the focal distance of the eye-piece, or equivalent lens, d; the magnifying power of this telescope will be had also by the same formula,

$$\frac{FD}{fd} = P.$$

12. But as the dynameter will give the magnifying powers of all the four constructions, when the limits of their effective apertures are known, in the same manner that it gives the

power of a refracting telescope, it will always be both the easiest and most correct operation to apply this elegant little instrument in the way that has been described in our eleventh section.

13. The Newtonian and Herschelian telescopes may have any of the different eye-pieces or micrometers applied to them, since they form but one image of the object viewed, which is an advantage they possess over the other two constructions; but the Cassegrainian and Gregorian instruments require eye-pieces of a description that have diaphragms placed where the image is formed, as well as diminished eye-holes, just large enough to admit the principal pencil of rays, that emerge from the eye-lens, to enter the pupil, without admitting any of the extraneous light of the sky, which would render the image invisible. The circular hole in the central part of the large speculum, in these two constructions, must be equal to the dimensions of the small speculum, lest any false light should be admitted, or any reflected rays lost.

14. The diameter of the large speculum must bear the same proportion to the diameter of the small one, that the focal distance of the former does to the distance of the first image from the latter, which image is not situated exactly in its focus, but is made the radiant for the formation of the second image, and is therefore a little out of the focal place.

For instance, if the solar focal distance of a large speculum be taken at 26 inches, and its aperture at 4.5; then, if we assume the distance of the image, formed at the focus, to be 2.5 inches before the concave small speculum of a Gregorian telescope, we shall have the proportions thus, as 26 : 2.5 :: 4.5 : 0.43, or $\frac{2.5 \times 4.5}{26} = 0.43$ will give the diameter of the small speculum, but if we suppose the distance of the image 3.5, then the diameter will be required to be upwards of 0.6 of an inch.

§ XIV. STANDS FOR REFLECTING TELESCOPES. [PLATE VI.]

1. TULLEY'S STAND FOR A CASSEGRAINIAN OR GREGORIAN TELESCOPE.—The best stand that has yet been contrived for a telescope of Cassegrain's or Gregory's construction, as it regards both steadiness and facility in the management of the motions, is that which is represented in perspective in fig. 2. of Plate VI. which stand was the invention of the senior Tulley, on which he mounted a Gregorian of ten feet focal length, and nine inches aperture, that could be managed with as much ease as any instrument of ordinary dimensions. The peculiarity of this construction consists in the methods by which quick and slow motions may be given, both in altitude and azimuth, while the tube has two places of support, that insure its firmness in any altitude between the horizontal and zenith positions. The body of the frame is composed of four legs, with as many crossed braces of cast iron, stretching outwards as they descend, for the sake of obtaining an enlarged base in all azimuthal directions, which is requisite for the due support of a heavy metal resting on the lower end of the tube; which metal would overturn a tripod of ordinary dimensions, by extending beyond its side, when turned round into any of the three situations. The upper end of the frame is contracted, so as to be covered by an iron circular plate, *a*, which is screwed fast to it; and these together form a

metallic table, for bearing the superincumbent weight of the moving parts of the instrument. Figures 3, 4, and 5 are detached parts of the instrument, bearing the same letters of reference that are applied to the corresponding parts of the drawing in fig. 2, and were deemed necessary to render the explanation of the two motions sufficiently intelligible. In figure 3, the part *a* is a section of the circular head of the above-mentioned table, *b* is a section of a strong brass ring, formed of two edge bars, one seen with its racked teeth in front, lying between the circular plate *a* and a similar circular iron plate *c*, and the other concealed between the said plates, which it separates, by being interposed without any fastening The second round plate *c* carries the metallic frame that holds the tube of the telescope, and is therefore pressed by the weight of the speculum metal and other heavy parts of the instrument, which pressure also applies to the interposed horizontal edge bar of the brass ring *b*, while an endless screw, cut on the axis of the handle *d*, takes hold of the racked vertical edge-bar of the same ring; when, therefore, the hand pushes the plate *c*, and the parts resting on it, round in azimuth by a quick motion, the interposed brass ring, being connected with it by the cranked piece that presses the screw into action near *a*, has also a quick motion in azimuth, together with the handle *d* and its Hooke's joint; but when the hand ceases to push in giving the quick motion, this handle, being turned, will move the racked ring *c* by a slow motion, and with it all the superior parts of the instrument, including the telescope, which is uppermost; for when the quick motion has ceased, the weight pressing on the ring *b* keeps it quiescent, while the handle travels round its racked edge, on being turned, and carries with it the plate *c* by a slow motion. Thus the telescope may have either a quick or a slow motion in azimuth communicated to it, without detaching the screw of the handle from the rack of the brass ring, which will either move round, or remain fixed, accordingly as the motion is required to be quick or slow, that is, produced by a push, or by the revolution of the handle The effect of this contrivance is equally surprising and convenient. The centre of these motions is a cylinder of steel, made fast to the central part of the plate *c*, and descending through the middle of plate *a*, under which a tapped nut keeps it down to its place. The quick and slow motions in altitude are produced by a different but equally beautiful contrivance

2. The telescope is mounted on the brass-work, carried by the iron circular plate *c*, by means of the double lever *e*, in the way already described (§ IX 2.), and has the usual eye-pieces of the Huygenian construction, as well as a finder attached to the main tube, as represented in the principal figure. Two squared brass rods, *f* and *f*, are jointed at their lower extremities to the bottoms of the vertical portions of the small bearing frame, near the face of the plate *c*, and will move into any angular position between the horizontal and vertical lines; the upper ends of these rods are connected with the remote end of a metallic sliding-piece, *g h*, by a thick pin or bolt, which passes through all the parts, and forms a hinge the piece *g h* is seen enlarged in figures 4 and 5, the former of which shews the appearance to an eye placed under the telescope and looking upwards, and the latter exhibits a lateral view; it has a dovetail on the concealed face, acting with a grooved bar, *k l*, made fast to the lower part of the main tube, seen in fig. 5, and two thumb-screws at *g* and *h* fix it occasionally to a long round rod, *m n*, that lies parallel to the tube nearly its whole length, and passes through the two ends of the sliding piece *g h*. The relative positions of this long round rod, *m n*, and of the sliding-piece *g h*, determine the angle of inclination that the square rods *ff* shall assume; and these

rods give the angle of altitude at which the tube of the telescope is obliged to stand. To effect these purposes, the eye-end of the long round rod has an oblong sliding frame of brass, $o\,p$, attached to it, which may be moved forwards or backwards by a long screw, q, within reach of the hand, near the eye-piece, and when this brass frame is moved the rod moves with it, and being clamped by the two thumb-screws to the frame $g\ h$, this frame also moves in its concealed groove, and raises or depresses the supporting square levers $f\!f$, thereby altering the elevation of the telescope. When the sliding piece $g\ h$ is not clamped to the long rod $m\ n$, the telescope may be elevated by a quick motion, occasioned by a push of the hand, while the said sliding piece approaches the eye-end of the telescope, and raises the levers $f\!f$ towards a vertical position, and, on the contrary, a depression of the telescope produces a corresponding depression of the supporting levers, and a backward motion in the sliding-piece $g\ h$, which may be clamped by the thumb-screws at any given situation.

3. The method of producing the slow motion in altitude may be thus explained at the end p of the oblong brass frame $o\,p$, is a joint made by a thick but short cylinder of brass, turning on its pivots to accommodate itself to the screw q, which it is tapped to receive; the piece r, which limits the inner end of the screw, is made fast to the main tube of the telescope, as is also the piece s, which limits its outer end, by a shoulder on its axis; hence, when the screw is turned by the holding piece at q, it is prevented either advancing or receding by the fixed and limiting pieces r and s; but as the long side bars of the frame $o\ p$ pass through square beds made in the fixed piece r, and are kept in those beds by a thin plate passing across, from o to r, on the under face, and screwed fast to the piece r, the tapped joint piece p is obliged to move, by the turning of the screw q, and with it the oblong frame, and affixed long rod $m\ n$; and as this rod is occasionally clamped to the sliding piece $g\ h$, to which the upper ends of the levers $f\!f$ are attached by a joint, the whole apparatus will move together, with such easy motion as is given by turning the screw, whenever the rod $m\ n$ is clamped to the piece $g\ h$; and as the telescope is connected with the square levers $f\!f$, by the intervention of the sliding-piece $g\ h$, it will also partake of the motion thus imparted by the screw q, which motion will be in elevation or depression, according to the direction in which the screw is turned, and will be more or less slow, according to the velocity with which it is moved.

4. The aperture of the main tube is at the remote end, and the large speculum, perforated at the centre, rests at the eye-end on slender springs, that keep it to its bed without shake, and free from such pressure as might alter the figure of its curve. The Huygenian eye piece is at the end of the small tube, and is adjusted for distinct vision by rack work that is not seen. The finder, fixed parallel to the main tube, is a small achromatic refracting telescope, with cross wires in its common focus, by means of which the object is readily found; and a long rod passing within and parallel to the concealed side of the main tube, not visible in the figure, regulates the place of the small speculum with respect to its distance from the large one, by making it approach to or recede from the remote end of the main tube, accordingly as the distance of the object viewed demands, for getting distinct vision.

5. The telescope thus mounted is first directed towards the celestial object to be viewed, partly by a push given to its eye-end in the direction of the required azimuth, and partly by the quick motion in altitude, while the long round rod $m\ n$ is unclamped, and then the finder will point out the exact place wanted, when the two screws of the handles produce the slow

compound motion, that is required for getting the object into the field of view; after which the final adjustment of the vision must be obtained by the proper screw. A reflecting telescope of either of the constructions for which this stand is adapted, is seldom supplied with a micrometer, except sometimes with a divided slip of mother-of-pearl, on account of the eye-piece not only being of the negative kind, but having a very small eye-hole to exclude the superfluous light that would spoil the vision, which is not the case with the other constructions.

6. Sir W. Herschel's Stand for a Newtonian Telescope.—The stand represented by Fig. 6 was contrived by Sir William Herschel so long ago as the year 1778, before he had conceived the grand scheme of constructing the gigantic instruments, that afterwards proved what effects may be produced in science by an union of ingenuity and perseverance. In the early part of his catoptric labours Herschel had noticed the bad effects of tremors produced by imperfect stands, and employed his mind in contriving methods of obviating such effects, by supporting the tube, that holds the metals, at two distant points, and by suspending the end that contains the large speculum by a cord and system of pulleys, that might also be subservient to the elevation of the object-end, or, which is the same thing in effect, the depression of the large speculum. The stand now under our notice was the result of such contrivance, and is represented as holding a seven-feet Newtonian telescope in a state of elevation, such as is proper for enabling an observer to examine a heavenly body.

7. All the essential parts of this instrument are so clearly exhibited in the perspective drawing, that a short description will suffice to render them intelligible. The principal frame is formed in the shape of a large chair, of strong but light bars of mahogany, which need no other description than a reference to the engraved figure, where the four legs are seen terminated by as many castors, the shorter two of which legs constitute the front portion, and the longer two the back part of the stand, while the sides are braced by each an inclined bar, made fast to the vertical bars, that constitute the back and the two longer legs. These four vertical and inclined bars have each a groove made longitudinally on their interior faces, two of which are visible in the figure, and a second smaller frame of an oblong shape, denoted by the letters *a b c d*, has four pins of metal inserted at the four corners, near the said letters respectively, which pins are just large enough to enter the grooves, and to slide without friction when a motion is given to the small frame. The mahogany tube of the telescope rests, at some distance from the lower end that contains the large speculum, on the lower cross-bar *c d* of the small frame, and a system of pulleys, made fast to the upper bar of the said small frame, has a cord that supports the weight of the frame and a portion of the telescope, this cord being made fast to the upper end *a b* of the small frame, embraces the pulleys, and then passes over the fixed rollers at *g* and *f*, which change the direction of motion, and is finally attached to a cylinder, carrying the handle *e*, round which it is coiled in drawing up the small frame. When the lower pair of pins, near *c* and *d*, have descended down the grooves in the long vertical bars of the back, they meet with the ends of the grooves made in the inclined bars, and continue to descend down them till the second pair of pins at *a* and *b* are in the same situation, when all the four begin to slide in the grooves formed in the inclined bars, as far as the turns given to the handle *e* will permit. A ratchet wheel on the axis of this handle, holds the small frame and telescope resting on it at any given position. A long square tube of mahogany, *k k*, is fixed at the middle of the front part of the main frame, which contains the square *h*,

that may be raised, either by a cord wound round the axis of the handle having a ratchet wheel *i*, or by hand; in the latter case a straight rack, acting with a spring-piece, detached by pressure on the button at the upper *k*, will detain it in a given position. This bar *h* is simply for giving quick motion in elevating the upper end of the tube, and when it has brought the telescope nearly to the elevation required, the handle *m*, made fast to its superior end, carries a pinion that acts with a wheel, having a second pinion, that lifts the vertical rack of a bar of brass, *l*, which descends down the wooden bar *h*, and at its upper end supports a small horizontal metallic frame *n*, on a joint of which the main tube is made fast at *o*. The slow motion in altitude is produced by this third handle *m*, which, by means of the wheel-work seen detached, gradually elevates the racked bar *l*, and frame *n*, together with the upper end of the telescope, and keeps its situation without a clamp. By these means the tube of the telescope may be carried into a situation nearly vertical. The lower end of the tube merely rests on the cross-bar *c d* of the small frame, and therefore is at liberty to ascend or descend with the bars *h* and *l* when they are elevated or depressed; and as the whole stand is light, and moves on castors, the quick motion in azimuth is given by turning it round on its feet, till the heavenly body is within the adjustment of the horizontal screw *p*, which gives the slow motion in azimuth. The small plane speculum is fitted into the central part of the tube at a short distance from the aperture *q*, *r* is the finder constructed in the usual way, and *s* the eye piece screwing into the brass bar, that slides between two dove-tailed cheeks, fixed on one of the octagonal faces of the wooden tube, and as the tube has an oblong perforation under this brass bar, the small speculum is carried by it, and is therefore adjustable for distance from the principal concave speculum, which faces it at the opposite end of the tube, by means of a pinion and racked bar of brass. From this description our readers will perceive that the observer, who uses this instrument, may stand in an erect position with his eye at *s*, while he makes his observations, his right hand being employed at one of the handles, while his left may be used as occasion shall require. The stand is easily moved about on a level floor, and is quite manageable without an assistant, but is not calculated to retain a meridian or any other stationary position.

8. VARLEY'S STAND FOR A NEWTONIAN TELESCOPE.—The ingenious mechanic employed by the late Lord Stanhope, made such an alteration in the construction of the stand for a Newtonian telescope, as may be considered an original contrivance; he left out the cord and pulleys, made the principal frame to stand on three legs only, and while he retained the property of supporting the telescope at two distant points, gave to one of these points an axis of motion, which gives the advantage of an elevation in a true vertical circle, as well as the capability of being fixed permanently in the meridian, or in any given azimuth, and yet it retains all the required both quick and slow motions. By an additional improvement that has lately been made in the construction by Tulley, the telescope will also admit of being poised in any situation, even in its zenith position, into which it is capable of being placed with as much ease as in any degree of ordinary elevation. This construction is exhibited in perspective by Fig 7., where the telescope is placed in the same position nearly as its predecessor, and the stand may be thus described. The base *a b c d* is a four-sided horizontal frame of wood, supported by three rollers fixed fast to the lower side, one at the middle of the narrow end *a b*, and the two others at the corners *c* and *d*, so as to form the angular points of a triangle; this frame is a detached platform to place the stand upon, when it requires to be moved about in any direction;

but when the stand is required to be made permanently stationary, it may be put aside, as not forming a requisite part of the telescope's support. At the points *a b c* and *d*, the four feet of the principal frame ascend in directions inclining towards one another, the feet at *a* and *b* being so near together, that they may be considered as one, and on this account we shall denominate the frame triangular, with respect to the horizontal plane terminating its superior portion. The leg *e*, the strong edge-bar *f*, and the leg *g*, all of mahogany, constitute one side of the main frame, which is braced by the two bars *h h'*, at the opposite side, over the bar *a d* of the platform, is a similar side of the main frame, the legs over *c* and *d* are tied together by the cross horizontal wooden bar *i*, which is all that the remote end of the frame, which appears foreshortened in the drawing, requires, the upper part being left open to admit the lower end of the tube between those legs, when the object end is elevated. A pair of bracing bars, similar to *h h'*, of which a part of one is seen joined to the leg above *d*, are applied also to the concealed side of the frame; and a pair of short edge-bars connect the legs over *a* and *b* into one, in the form of a narrow parallelogram. The two black marks on the strong edge-bar *f*, to the right of this letter, show the end of a strong cross bar, that unites the two principal sides of the frame a little beyond the middle, and the two similar black marks, to the left of the letter *f*, mark the end of a short edge bar, that fixes the said two sides together, near the tops of the two contiguous legs, which we have considered as one, these two cross edge-bars form, with the bar *f*, and its opposite corresponding one out of sight, a four-sided figure, which holds a board of mahogany, that forms a table of the shape of a trapezium, which is useful for containing the eye pieces and other appendages, that are not in use at any time, while the largest cross-bar serves as a stop for the tube, when erected into the vertical position. To the edge bar *f* of the principal frame above described, a strong upright bearing-piece *k* is firmly attached by a dovetail, and also a similar one *k* to the opposite bar, which is concealed in the figure by the tube of the telescope, to the upper ends of these pieces beds of brass are attached, which receive the horizontal pivots of a second frame, the upper end of which frame is seen at *l* and *m*, where a racked edge-bar is made fast, of which the use will be explained presently This second frame is of an oblong figure, braced by cross-bars, and made as light as the required strength will admit, and a strong piece of mahogany *n*, similar to *k*, made fast by a dovetail to the edge *m n*, bears a strong metallic pivot, and a similar piece *n'*, bearing a corresponding pivot at the opposite side of this second frame is hid from the sight, these two pivots removed from each other by the whole breadth of the frame, which is about eighteen inches, and resting on the brass beds, attached to the upper ends of the bearing-pieces *K* and *K'*, support the whole superincumbent weight of the telescope and of the appendages, nearly at its centre of gravity, when the large speculum is in the tube, and afford a good horizontal axis of motion in altitude. Between *l* and *m*, under the elevated end of the second frame, a plate of brass is screwed fast, which is connected by a hinge to the top of a fine rack, consisting of a racked edge-bar pinned to a sliding brass tube, which descends into the central part of the coarse rack, bearing the handle *o* and its concealed wheelwork that actuate this fine rack, the second handle *p*, which has a strong pinion on its axis impelling the coarse-rack, is attached to the hollow mahogany bar *q*, which allows the coarse rack to pass through it, and may be clamped in any position; the surrounding piece of squared brass on which the letter *r* is placed, turns on strong pivots in two brass cocks *s s'*, attached to the legs over *a* and *b*, and adapts itself to the position of the wooden bar *q*, as it departs from a true vertical position, in its ascent or descent, the strong screw on the interior end of the milled head *t*, passes through one of the pivots of the clamping-piece *r*, and fixes the bar *q* to it in any given elevation. The quick motion in altitude is given by the sliding of this bar *q* up or down, when the fixing screw *t* is released; but when it is made fast, either a very slow motion may be given by the uppermost handle *o*, acting with the fine rack, or a motion a little more quick may be given by the handle *p*, by means of the coarse rack, which is made fast to a second wooden bar, the lower end of which is concealed in the hollow bar *q*. When these bars and racks are drawn out of one another to their utmost extent, they admit of the second frame taking a vertical position, by a motion round its horizontal pivots. After the second motion has been given by the handle *p*, a thumb screw fixes the bar of the coarse rack by clamping the socket, through which its axis passes, and thus the frame will be sustained in any angle of elevation, and yet will be under the command of the small handle *o* and concealed wheelwork, without a second clamping screw. The bed of the telescope is a solid board of mahogany, holding large Ys to receive it, the upper end of which bed is seen at *u v* above the racked edge bar, and a pinion borne by it at *v* gives it a motion round a strong pin, passing through the second frame at its lower end, so that while the second frame is at rest in any angle of elevation, the bed of the telescope can have an easy motion parallel to the horizon, by means of the milled head of the pinion's axis at *v*. The screw seen at *w* fastens the bed to one of the octagonal sides of the mahogany tube. The finder and sliding bar of brass, for adjusting the eye-piece and small speculum to distinct vision, are similar to those already described in *fig.* 6., and need not further notice, but the little quadrant at *x*, which carries a level on its vernier bar, and which is an addition of our own, is exceedingly useful in placing the telescope to any required degree and minute of elevation, when properly adjusted; or for giving the approximate altitude of a heavenly body, that may at any time have been the object of observation. We have had a six feet Newtonian telescope by Tulley mounted on the stand which has been here described, and though its aperture is only seven inches, its performance is excellent, and its steady and manageable motions render it a most convenient instrument for the accomplishment of measurements, by micrometers of every description, as far as the optical powers are competent.

ADJUSTMENTS OF THE NEWTONIAN TELESCOPE.

9. *To prove if the small speculum be a perfect plane.*—Place a small achromatic telescope on a stand, and direct it to a printed card at about thirty yards distance, and adjust for distinct vision; if the reading is not magnified enough, bring the telescope nearer, till the print can be read. Then place the small speculum in its box, at a like distance from the card, in such way, with respect to height and angle of reflection, that the same card may be seen by an eye looking at right angles across the axis of the telescope; then having turned the telescope round a quarter of a circle, place it so that its object-glass may point towards, and also approach near to the reflecting face of the small speculum, and in this situation read the card as reflected; and if it be as distinctly visible, as when viewed before directly, without altering the former adjustment of the eye-piece for distinct vision, the speculum is not only well polished, but its surface is flat and suitable for its purpose. If the telescope requires to be elongated for dis-

tinct vision of the reflected card, it shows that the focus of the object-glass is lengthened by diverging rays, and that consequently the reflecting face of the small speculum is a little *convex;* but on the contrary, if the eye-piece requires to be pushed in, to gain distinct vision in the speculum, it shows that the focus of the telescope's object-glass is altered by converging rays, and that therefore the face of the speculum is *concave.*

10. *To place the small speculum in the centre of the tube.*—Let a terrestrial eye-piece, with its lenses taken out, be screwed into the place of the celestial eye piece of the telescope, so as to guide the eye down it without much parallax, then the oval section of the piece of tube that holds the small speculum will appear circular, and must be so adjusted, by sliding the holder in its groove of brass, and by bending the arm that carries the said speculum, that the circle bounding the field of view, within the empty tube of the terrestrial eye-piece, and the circular ring, that surrounds the small speculum transformed by position from an ellipse, shall be truly concentric. When this purpose is effected, which ought always to be done by the instrument-maker, the sliding interior bar of the speculum-holder may be screwed fast to the exterior bar, that carries the eye-piece, so that when the speculum is taken out, it may always be properly replaced by the limitation ensured by the screw, which may be a thumb screw, applying at the outside of the tube.

11. *To adjust the face of the small speculum to an angle of forty-five degrees with the axis of the telescope.*—Though the slope of the small speculum's surface is by construction as nearly forty-five degrees as mechanical means can effect, yet it will seldom be found to make an exact reflection of the large speculum up the centre of the terrestrial eye-tube, without an adjustment of its position, this adjustment is made by means of three small screws placed in a triangle behind the small speculum, any one or more of which may be turned until the illuminated sides of the main tube be seen apparently in the same direct line with the line of sight down the terrestrial eye-tube.

12. *To adjust the large speculum to suit the verified position of the small one.*—Hitherto the position of the large speculum has been disregarded, the next and last adjustment is, to place this speculum at right angles to the axis of the main tube, by means of the three screws of adjustment that act on the brass box containing it, for it would be injurious to the shape of the curve given to the speculum's surface, if the screws were to act on the metal itself, or even if the metal were placed too tight in its box of brass, the variable temperature would not affect the speculum metal and brass alike, and an injury would be occasioned in the figure of the reflecting surface by a change of temperature. The adjustment of the principal speculum's position is known to be correct, when any gauging piece of wood, or metal, applied to the edge of the large tube's aperture, at opposite sides successively, are seen reflected up the main tube of equal lengths as seen at the mouth, and when the eye looking down the terrestrial tube, without glasses, also sees the small circular speculum in the exact centre of the large speculum reflected up the empty tube of the terrestrial eye-piece If now either the celestial or terrestrial eye-piece, having their lenses in, be adjusted for distinct vision of any object, and a positive eye-piece be applied, after the manner of a dynameter, in the place of the eye, the diminished images of both the specula will appear perfectly concentric if not, the screws of the large, or of the small speculum must make them so, and then the telescope is prepared for use.

13. *To judge of the goodness of a large Newtonian Speculum.*—When the Newtonian telescope is properly adjusted, and directed to view a card, or other object, placed at a moderate distance, let it be set to distinct vision of the said object as nearly as the eye can judge, then turn the screw for distinct vision so that the object becomes gradually indistinct, or until a round luminous point becomes enlarged into a disc of a certain diameter, when the small speculum is pushed in, or made to approach the large one, do the same, by means of a contrary motion given to the adjusting screw, and stop when the disc appears of the same size as the first disc, then, having previously marked with a pencil the place of distinct vision on the external sliding bar, take notice whether the spaces moved over by the bar inwards and outwards, for the formation of the two respective similar discs, be equal, if they are, the figure of the reflecting face of the large speculum is properly *parabolic*; but if the spaces moved over are not equal, the figure is not good, when the *inward* motion is *less* than the outward motion, to produce the same disc, the figure of the reflector is *hyperbolic*, but if greater, it will be *spherical*. In a true parabolic figure the least motion produced by the screw, in the position of the small speculum, will begin to produce indistinctness, or to form a disc; but when the figure is imperfect, the change of distinctness will be gradual, and will require more motion to be given to produce the same effect.

§ XV THE HERSCHELIAN FORTY-FEET TELESCOPE [PLATE VIII]

1. AFTER the ingenious and persevering contriver of the Newtonian stand which we have described, had cast and polished an immense variety of specula for telescopes of different sizes, he finished a twenty-feet reflector with a large aperture in the year 1782, which was mounted on a stand that admitted of being used without a small speculum, in making *front observations*, and many of his discoveries and measurements of double stars were made with this instrument, and with some of the Newtonian construction, before in the year 1785 he had put the finishing hand to the gigantic structure, which soon became the object of universal astonishment, and which forms the subject of our present section. The late Sir William Herschel had succeeded so well in constructing reflecting telescopes of small aperture, that they would bear higher magnifying powers than had been ever applied, but he found that a deficiency of light could only be remedied by an increased diameter of the large speculum, which therefore was his main object, when he undertook to accomplish a work, which to a man less enterprising would have appeared impracticable. The difficulties he had to overcome were numerous, particularly in the operative department of preparing, melting, annealing, grinding, and polishing a mass of metal, that was too unwieldy to be moved without the aid of mechanical powers. The various contrivances and processes are detailed in the 85th volume of the Philosophical Transactions of London (1795), and the figures contained in eighteen engraved plates, are referred to in explaining the different parts of the machinery. It will not however be expected, that we should enter minutely into all the details of the construction of this monument of skill and industry united, but that we endeavour to give our readers a general idea of the structure and

method of using the largest telescope that ever was pointed to the starry regions. As we had occasion to give a description of the forty-feet telescope in another work, which we previously submitted to the perusal and approval of the inventor himself, we shall have no hesitation in availing ourselves of our own labours, in giving the same account in this place.

2. Our Plate VIII contains a view of this magnificent telescope suspended in its stand, copied, with some unimportant omissions, from Plate XXIV of the volume of the Philosophical Transactions above referred to. This view, taken from a station to the south-west of the erection, represents the telescope elevated in the meridian line, and affords the means of seeing the anterior portion of the instrument and of its numerous appendages; but does not allow the mechanism that supports the inferior end of the tube, and that gives motion to some of the adjustments, to be explained by a reference to their parts, and therefore must be comprehended from a verbal description. The foundation, on which the frame work is erected, consists of two concentric circles of brick-work, one 42 and the other 21 feet in diameter, both sunk 2½ feet into the ground, and battered from the breadth of two feet three inches to one foot two at the top, where they are capped with paving stones of twelve inches and a quarter wide, and three thick. In the centre of these circles is fixed fast into the ground, by brickwork and opposite braces of wood, a vertical beam as a centre of motion, round which the whole of the frame-work may have a circular motion in azimuth, the plane of the larger circle being made perfectly level. The platform, that connects the different parts of the frame-work near the ground, has three principal horizontal beams lying parallel to each other, and three others also parallel to each other, crossing the former ones at right angles, besides various bracing beams, that tie the whole compactly together, by iron bolts passing through the places of crossing. In our drawing, the outer circle of brick-work and masonry is denoted by the letters A B, and the circumference of the platform of wood by C D. Under each opposite end of the six main beams is fixed a roller of six inches in diameter, and eight long, having each a strong iron frame bolted into the end of its respective beam, so that the outer circle has twelve rollers; but these were not sufficient to bear the whole at the distance of twenty-one feet from the centre of motion; therefore eight more rollers, nearly equidistant, were fixed to strong parts of the platform, so as to be borne by the inner circle of twenty-one feet diameter, and thus the whole platform, with its superstructure, is capable of making a revolution, when sufficient force is applied, in a direction tending round the central vertical beam, that enters a hole at the junction of the two central main beams, and that ascends but a little way above the ground. Six out of the twelve rollers of the outer circle are seen between the brick-work A B, and C D, the circular edge of the platform; the rest may be imagined, not only on the remainder of this circle, but also in the inner circle which is concealed. The axis of motion of these rollers all point towards the central beam, and also their diameters and frames are precisely similar in dimensions, by which means they bear alike on the basis of masonry. At twelve feet distance from, and all round this moveable platform, are fixed fast into the ground eight equidistant posts, to an opposite pair of which the ends of a long pliable rope are hooked, that produce the motion in azimuth, which rope, being conducted over two separate pulleys, fixed upon the platform, at opposite sides of the centre, has its ends turned in the direction of tangents, that point in opposite directions to their respective posts. The middle part of the rope is made to pass round one of the spokes of a large wheel, carried by the platform, before

it winds round the axle, so as to coil up both ends of the rope equally, which rope therefore pulls by both tangental ends alike, so as to apply an equal force at each opposite pulley, while the resistance of the posts produces the requisite motion, without a strain on the centre. This mechanism gives the operator a great mechanical advantage. That part of the platform *C*, which connects the extreme ends of the three longitudinal beams, above the rollers at *A*, is made strong, and is the support for a pair of double ladders, that are seen ascending to the summit of the whole frame-work, one on each side of the large tube *E*; and at *D* is another similar support for two other double ladders, which, ascending in like manner, meet the former ones, and cross into them in such way as to admit of being bolted together at the points of crossing. These ladders are propped by other short ladders, as seen in the figure, and some upright masts, of which one is seen erected over the roller at *B*, ascend in like manner and afford the means of obtaining horizontal braces at different heights, all round the frame, except where the elevated end *E* of the telescope requires an open space to be left between the front ladders, for its different degrees of elevation.

3. The transverse beam *F G*, which lies horizontally over the crossings of the double ladders, and is bolted to them, receives the hooks of the different pulleys, that will presently be described, at the same time that it connects and braces together all the ladders at their upper extremities. These ladders are each forty-nine feet two inches long, and the height of the transverse beam *FG* about forty-five, which will therefore admit the long tube of forty feet to be raised under it into a vertical position. Below the mouth of the large tube, a gallery, *H I*, with its attached brackets *K* and *L*, rests upon the slopes of the interior halves of the double ladders at *K* and *L* respectively, and may be made to slide up or down into any state of elevation, by two systems of pulleys, and ropes going round the blocks hooked at the junction of each pair of ladders, to the transverse beam *F G*, as appears in the Plate; and when this gallery is lowered to the landing of the pair of steps M, a party of persons may be admitted into it to gratify their curiosity, the floor being thirteen feet and a half by six feet one inch and a half, and palisaded at the front, as well as in part at both ends. The bases, or sliding parts of the brackets, are prevented from slipping aside by lateral rollers of brass, acting against the straight sides of the middle pole of each double ladder, while other rollers of the same metal, acting under them, diminish their friction when drawn up or let down by the respective pulleys. In the framing of these brackets it was necessary to introduce contrivances for allowing some deviation of the gallery from an exact level, in case one of the brackets should be elevated by its pulley faster than the other; which contrivances are not easily described without drawings of the several parts, or without inspection.

4. The tube of the telescope, which is thirty-nine feet four inches in length, and four feet ten inches in diameter, is composed entirely of iron; it having been ascertained that a wooden tube of proper dimensions would have exceeded an iron one in weight by at least three thousand pounds. The sheets were first put together by a kind of seaming that requires no rivets; and when the sides of the iron platform were cut straight, it was lifted by proper tackle into a hollow gutter, and then brought gradually, by various tools, into a cylindrical form and united. Various hoops are fixed within the tube, and longitudinal bars of iron connecting some of them, are attached to the two ends of the tube, by way of bracing the sheets and preserving the shape perfect, when the pulleys are applied to give the necessary elevation at the upper

end, and that the speculum may be kept secure at the lower end. The hoop, by which the upper end of the tube is suspended, is eight inches broad, and thicker than the rest, and the system of three pulleys, seen at *N*, with each a double block, has a corresponding set at *O*, hooked to the transverse pole *G F*, the bars to which the blocks are hooked being so bent, that the moving ropes will not come in contact, nor will the elevated tube have its vertical motion disturbed by the tackle, either in ascending or descending, which was an important precaution. The lower end of the tube is firmly supported on rollers, that are capable of being moved forwards or backwards by a double rack, connected with a set of wheels and pinions seen at *R*, which we will not attempt to describe more minutely, but the use of which every mechanic will comprehend without particular explanation. The handles and barrels round which the respective ropes are coiled are not given in the drawing, but are situated on the platform near the corners of the small building in the present position of the frame. Originally there were several appendages, near the mouth of the tube, sliding by pulleys, or fixed to the tube, for the purpose of regulating the *sweeps* taken by this instrument, but as the twenty-feet reflector was afterwards used for this purpose, they were taken off, and have been omitted in our drawing. By an adjustment at the lower extremity of the tube, the speculum is turned to a small inclination, so that the line of collimation may not be coincident with the longitudinal axis of the tube, but may cross the tube diagonally, and meet the eye in the air at about two inches from the edge of the tube, which is the peculiarity of the construction, that supersedes the necessity of applying a second reflector Hence no part of the head of an observer intercepts the incident rays, and the observation is taken with the face looking at the speculum, or by what the inventor has called, by way of distinction, the *front view*, the back being turned towards the object to be observed. Besides the pulleys of elevation and of azimuthal motion, there are others for the purpose of communication, as well as speaking-pipes, repeating-bells, and signals by clock-work, which cannot be fully described without appropriate drawings, but the dexterity of the celebrated astronomer rendered some of them superfluous.

5. The large speculum is enclosed in a strong iron ring, braced across with bars of iron, and an enclosure of iron and tin sheets makes a case for it, it is lifted by three handles of iron attached to the sides of the ring, and is put into and taken out of its proper place in the tube by the help of a moveable crane, running on a carriage, which operation requires great care. Three small vanes attached to the edge of the tube at the mouth assist to place the line of collimation right at the eye piece on the edge of the tube, when they are seen reflected from the speculum in the proper oblique direction. The speculum metal for the twenty-feet telescope was made of a composition of tin and copper in the proportion of 7 75 . 20, but the metal of the large speculum would have been too brittle for its weight, which was 1050 pounds, its diameter being four feet; it had therefore a larger portion of copper which made it liable to tarnish, and less brilliant than a better proportioned metal, but a second speculum of the same dimensions was kept as a spare one, ready to supply its place, when required to be re polished. The metals were procured for the large telescope at a warehouse in Thames Street, London, where they kept ingots of two kinds, ready made, one of white, and the other of bell-metal, and the large specula were composed of two ingots of bell-metal for one of white. It was not to be expected that a speculum of such large dimensions could have a per-

feet figure imparted to its surface, nor that the curve, whatever it might be, would remain identically the same in changes of temperature, therefore we are not surprised when we are told, that the magnifying powers used with this telescope seldom exceeded 200, the quantity of light collected by so large a surface being the principal aim of the maker.

6. The raising of the balcony, and sliding of the lower end of the tube are effected by separate tackles, and require only occasional motions; but the elevation of the telescope requires the main tackle to be employed, and the motion usually given in altitude at once was two degrees, the breadth of the zone in which the observations were made, as the motion of the sphere in right ascension brought the objects into view. A star however could be followed for about a quarter of an hour. Three persons were employed in using this telescope, one to work the tackle, another to observe, and a third to mark down the observations.

7. The elevation was pointed out by a small quadrant fixed to the main tube, near the lower end, but the polar distance was indicated by a piece of machinery, worked by a string, which continually indicated the degree and minute on a dial in the small house adjoining, while the time was shown by a clock in the same place; Miss Herschel performing the office of registrar. The range in altitude was limited by the striking of a bell, and sheets of large paper, ruled into squares, were used by way of a register, each square being equal to a quarter of a degree, and when a cross was inserted, it showed that the corresponding portion of the sky had been observed; but when a single stroke only was inserted, it indicated that such place had been only partially examined. The degree of approximate accuracy, with which the place of a double star or nebula was thus laid down, was most extraordinary, though still wanting rectification.

8. The stand, pulleys, and other appendages which we have here described, when made of smaller dimensions, are equally suitable for the twenty-feet telescope, which is not only more manageable, but more generally serviceable, as it may be used in states of the weather, when the large instrument would not perform so well.

9. Mr. Herschel, who inherits all the skill and zeal of his father, has lately ground and polished a new speculum for the twenty-feet tube, which shows the double stars well defined, and with which he has observed an additional number, that, in conjunction with Mr. South's second list, will greatly augment the catalogue that was lately published jointly by those astronomers in the Philosophical Transactions of London; the former gentleman has already given a paper on the subject to the Astronomical Society of London, and the latter has presented his second series of observations to the Royal Society. The public have also been promised, or have reason to expect, a large increase to the present list of double stars from Professor Struve, who is now using Fraunhofer's large refracting telescope for this purpose.

10. During his developement of various celestial arcana, the late Sir William Herschel introduced the application of a new power called the *space-penetrating* power of his instruments, which consisted in a certain union of light with magnifying power, which property may be said to belong exclusively to instruments of large dimensions, as will be seen hereafter.

11. The discovery of the satellites of the Georgium Sidus was made by the twenty-feet reflector, after it had been converted from the Newtonian to the Herschelian construction; affording a proof of the superiority of the latter over the former when the same speculum was used. But it was reserved for the forty-feet telescope to discover the sixth and seventh satel-

lites of Saturn, only one of which is within the reach of the twenty-feet instrument, or even of a twenty-five-feet reflector, with which the Georgian planet was afterwards viewed. "In beautiful nights, when the outside of our telescopes is dropping with moisture discharged from the atmosphere," says this eminent astronomer, "there are now and then favourable *hours*, in which it is hardly possible to put a limit to the magnifying powers. But such valuable opportunities are extremely scarce, and with large instruments it will always be lost labour to observe at other times "* And in the same paper he adds, "in order, therefore, to calculate how long a time it must take to sweep the heavens, as far as they are within the reach of my forty-feet telescope, charged with a magnifying power of 1000, I have had recourse to my journals, to find how many favourable hours we may annually hope for in this climate. It is to be noticed, that the nights must be very clear, the moon absent, no twilight, no haziness, no violent wind, and no sudden change of temperature, then also, short intervals for filling up broken sweeps will occasion delays, and, under all these circumstances, it appears, that a year which will afford ninety, or at most one hundred *hours*, is to be called very productive." In the equator, with my twenty-feet telescope, I have swept over zones of two degrees, with a power of 157, but an allowance of ten minutes in polar distance must be made for lapping the sweeps over one another where they join As the breadth of the zones may be increased towards the poles, the northern hemisphere may be swept in about forty zones; to these we must add nineteen southern zones, then fifty-nine zones, which, on account of the sweeps lapping over one another, about five minutes of time in right ascension, we must reckon of twenty-five hours each, will give 1475 hours And, allowing 100 hours per year, we find, that with the twenty-feet telescope the heavens may be swept in about fourteen years and three quarters. Now the time of sweeping with different magnifying powers will be as the squares of the powers, and putting p and t for the power and time in the twenty-feet telescope, and P=1000 for the power in the forty-feet instrument, we shall have

$$p^2 \quad t \quad P^2 \quad \frac{t P^2}{p^2} = 59840.$$

Then, making the same allowance for 100 hours per year, it appears that it will require not less than 598 years, to look with the forty-feet reflector, charged with the above-mentioned power, only one single moment into each part of space, and even then, so much of the southern hemisphere will remain unexplored, as will take up 213 years more to examine.

12. When stars of the first class, such as ε Bootis, γ Leonis, α Lyræ, &c and clusters of small stars were examined, the magnifying powers used were as follow, viz. 460, 625, 932, 1159, 1504, 2010, 2398, 3168, 4294, 5489, 6450, and 6652 but in general the space-penetrating power was greatest with smaller magnifying powers. As we shall have occasion to treat of the space-penetrating power in a distinct section, we cannot conclude our present article better, than by subjoining a list of the valuable papers communicated, from time to time, by our indefatigable observer, to the Royal Society of London, together with their titles, and the volumes in which they are contained, as well as the years in which they were printed, in order that our readers may the more readily refer to them.

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* Phil Trans Vol. XC p. 80 1800.

13. A LIST OF SIR WILLIAM HERSCHEL'S PAPERS CONTAINED IN THE PHILOSOPHICAL TRANSACTIONS OF LONDON.

	Vol	Page	Year
Astronomical Observations on the Periodical Star in Collo Ceti.	70	338	1780
Ditto relating to the Mountains of the Moon.	70	507	1780
Ditto on the Rotation of the Planets round their Axes.	71	115	1781
Account of a Comet, and Description of a Position-micrometer.	71	492	1781
On the Parallax of the Fixed Stars.	72	82	1782
Catalogue of Double Stars.	72	112	1782
Description of a Lamp Micrometer.	72	163	1782
A Paper to obviate Doubts concerning the Great Magnifying Powers.	72	173	1782
Discovery of the Georgium Sidus.	73	1	1783
The Diameter of the Georgium Sidus from 3″.5 to 5″, by Disc-micrometers.	73	4	1783
On the Proper Motion of the Sun and Solar System.	73	247	1783
On the Remarkable Appearances of the Planet Mars.	74	233	1784
Account of some Observations tending to investigate the Construction of the Heavens.	74	437	1784
Catalogue of Double Stars.	75	40	1785
Construction of the Heavens.	75	213	1785
Catalogue of New Nebulæ and Clusters of Stars (1000).	76	457	1786
The Cause of Indistinctness of Vision from the Smallness of the Optic Pencil.	76	500	1786
Remarks on Miss Herschel's New Comet.	77	4	1787
Account of the Discovery of Two Georgian Satellites.	77	125	1787
Account of Three Volcanos in the Moon.	77	229	1787
On the Georgian Planet and its Satellites.	78	364	1788
Observations of a Comet.	79	151	1789
A Second Thousand New Nebulæ and Clusters of Stars.	79	212	1789
Discovery of the Sixth and Seventh Satellites of Saturn, the Ring, &c.	80	1	1790
Satellites of Saturn and Rotation of his Ring; Tables, &c.	80	427	1790
On Nebulous Stars, properly so called.	81	71	1791
Saturn's Ring, and Rotation of his Fifth Satellite.	82	1	1792
Observations on the Planet Venus.	83	201	1793
Observations of a Quintuple Belt on the Planet Saturn.	84	28	1794
On the Solar Eclipse of Sept. 8, 1793 ($8^h\ 40^m\ 3^s$).	84	39	1794
On the Rotation of Saturn on his Axis ($10^h\ 16^m\ 0^s.4$).	84	48	1794
On the Nature and Construction of the Sun and Stars.	85	46	1795
Description of the Forty-feet Telescope.	85	347	1795

	Vol	Page	Year
Changes among the Stars, Comparative Brightness, &c. 1st Catalogue.	86	166	1796
Periodical Star *α* Herculis, Rotation on its Axis, &c 2d Catalogue.	86	452	1796
Third Catalogue of Comparative Brightness of the Stars.	87	293	1797
Changeable Brightness of Jupiter's Satellites.	87	332	1797
Discovery of Four Additional Satellites of the Georgium Sidus, &c.	88	47	1798
Fourth Catalogue of Comparative Brightness.	89	121	1799
Space-penetrating Power of Telescopes	90	49	1800
Investigation of the Powers of Prismatic Colours, &c.	90	255	1800
Experiments of the Refrangibility of the Invisible Rays of the Sun.	90	284	1800
Experiment on Solar and Terrestrial Rays, &c.	90	293	1800
Observations tending to investigate the Nature of the Sun, &c.	91	265	1801
Additional Observations on the same subject.	91	354	1801
Observations on the Two Lately Discovered Bodies (Ceres and Pallas).	92	213	1802
500 New Nebulæ and Clusters of Stars.	92	477	1802
Observations of Mercury over the Sun Nov. 9, 1802.	93	214	1803
Changes in the Relative Situations of Double Stars.	93	339	1803
The Changes in the Relative Situations of Double Stars continued.	94	353	1804
Experiments on Telescopes for Measuring Small Angles.	95	31	1805
On the Direction and Velocity of the Motion of the Sun and Solar System.	95	233	1805
On the Singular Figure of Saturn.	95	272	1805
On the Quantity and Velocity of the Solar Motion.	96	205	1806
Observations and Remarks on the Figure, &c of Saturn and his Ring.	96	455	1806
On Coloured Rings between the Lenses of an Object-glass.	97	180	1807
On Olber's Planet, and on a Comet	97	260	1807
On a Comet and on Saturn.	98	145	1808
Cause of the Concentric Coloured Rings.	99	259	1809
Supplement to Ditto.	100	149	1810
On the Construction of the Heavens	101	269	1811
On a Comet, its Construction, &c	102	115	1812
On a Second Ditto.	102	229	1812
On the Sidereal Part of the Heavens.	104	248	1814
On the Satellites of the Georgium Sidus, and Telescopic Apparatus.	105	293	1815
On the Milky Way.	107	302	1817
Astronomical Observations, &c. on the Relative Distances of Clusters of Stars, and on the Powers of Telescopes, &c.	108	429	1818

On the Places of 145 New Double Stars, June 8, 1821. Memoirs of the Astronomical Society, Vol. I. p. 166.

§ XVI HERSCHELIAN TELESCOPE AS CONSTRUCTED BY RAMAGE [PLATES IX X]

1 The construction of the stand for supporting and regulating the motions of the Herschelian telescope has been considerably simplified by an ingenious tradesman of Aberdeen, Mr. Ramage, who has succeeded in mounting two large specula of fifteen inches diameter, and twenty-five feet focal length, in such way as to be manageable by an observer without an assistant. His experience in casting and polishing metals of various sizes, during a period of about a dozen years, has qualified him to prepare specula of great lustre, and with an unusually high polish; and if he proceed with his labours with equal skill and assiduity, it is hoped he will succeed in giving the whole surface of his metal such a perfect curve as may free it from the effect of remaining aberrations. An instrument of the construction and dimensions above noticed has been put up on a platform at Greenwich, which at first astonished the observers, who had not been accustomed to examine a heavenly body with an instrument possessing so much light; and its performance was deemed quite extraordinary, but when the first impression had subsided, and different trials had been made in different states of the atmosphere, it was discovered that the central portion of the speculum was more perfectly figured, than the ring bordering on the extreme edges. When the aperture was limited to ten or eleven inches, the performance, as to distinctness in the definition, was greatly improved, and the light was so brilliant, that the astronomer royal was disposed to entertain an opinion, that it might equal that of a good refractor of the same dimensions. When however very small and obscure objects are to be observed, the whole light of the entire aperture may be used with advantage on favourable evenings. Much is to be imputed to the existing state of the atmosphere, as well as to the powers of the telescope, when large instruments are used of either the refracting or reflecting construction, but more particularly of the latter, when the aperture is great. A description of Ramage's twenty-five feet telescope was read before the Astronomical Society of London on the 11th of November, and on the 9th of December 1825, which has been published in the second volume of their Memoirs, illustrated by two engraved plates, which were drawn and engraved at the joint expense of the Society and of the author of this volume, and which therefore serve the purposes of both the works. A perspective drawing of the telescope mounted in its stand, as erected at Greenwich, is represented in Plate IX, which exhibits almost all the most essential parts of the mechanism united in their places, and Plate X. gives detached parts, for the sake of explaining more in detail the dimensions and structure of the separate portions, before they are put together. As Plate IX. contains a well executed picture of the instrument, that speaks for itself, we will not spoil its appearance by adding letters of reference to it, but shall describe the constituent parts given separately in Plate X, by the assistance of the same letters that were added by the author himself, from whose description it will not be necessary materially to deviate.

2. The platform, upon which the frame of the telescope revolves in azimuth, is shown in fig. 1. of Plate X., in which the circle denoted by the letters *A A A*, is a railway composed of cast iron, twenty-seven feet and a half in diameter, and four inches in breadth; seven equal segments, of nearly four feet each, are united by dove tails, and laid on the ends of eight piles

in a horizontal position, to complete the rail-way. The base of the upright stand consists of four straight beams *B B B* and *B*, each twenty-five feet long, the two middle ones being parallel to each other, at the distance of two feet, but the two outer ones are placed obliquely, with respect to each other, and to the middle beams, being separated at one end by fourteen feet, and at the other by only six, and are intended to prevent all lateral bending or yielding of the incumbent frame-work. These four beams are connected at their extreme ends by a pair of cross beams *E E* and *D D*, the former of which is fifteen feet and a half long, and forms the basis for the two long ladders, *M N*, and *O P*, seen in fig. 2, but the latter one, which is only seven feet in length, supports the parallel braced frame shown in fig. 3. Under the points at *E*, *E*, *D*, and *D*, are fixed four metallic rollers, each six inches in diameter, set in frames of iron, that are directed so as to carry the whole frame round the central strong iron pivot, in a post fixed fast in the ground at C, where the cross beam connects the parallel beams, and holds the pivot steady. The front part of the upright frame consists of the two triangular ladders seen in fig 2, the inner sides of which are formed parallel to each other, at the distance of five feet, that the tube of the telescope, which swings between them, may have a little horizontal sweep. The height of these ladders when erected is thirty-one feet, and the width of each at the lower end five feet, which gradually diminishes to the point of junction at the top. The other end of the frame is that which is seen in fig. 3, forming a parallelogram of thirty-one feet by six, meeting the two ladders at their upper extremities, and being united to them by strong screw bolts at the points of meeting These parts are braced together, as seen endwise in fig. 4.

3. The gallery, in which the observer stands when he makes his observations, is made to slide upwards or downwards between the two parallel sides of the ladders, having friction rollers to facilitate the motion, and ratchet stops, which slide over the steps of the ladder in the ascent of the gallery, but which, falling by their own weight into the way of the steps, prevent its descent for more than six inches at a time, which contrivance is therefore a security against any accident that might occur in the event of a rope giving way. The tackle used for raising the gallery and lowering it, are a winch *F*, and system of pulleys, such as are seen in fig. 5, represented by the letters *A A B B* and *C*, the two first pulleys are attached to the gallery at equal distances from its centre, and the next two, *B B*, are made fast to the ladders near their summits, and the fifth pulley *C*, fixed to the strong cross bar connecting the tops of the ladders, serves to change the direction of motion of the cord, to guide it to the barrel of the winch to which one end is made fast, while the other end is fixed near the right hand pulley *B*. The winch *F* may be seen in Plate IX, as well as in fig 1 of Plate X. lying across the main beams *B* and *B*, near the ground

4. The tube of the telescope is twenty-five feet in length, and eighteen inches in diameter, being composed of deal boards five eighths of an inch thick, in the form of a twelve-sided prism, and strengthened internally by rings of hard wood, which allow a space of fifteen and a half inches for the aperture. This tube is elevated to any height by a quick motion given to it by a winch and tackle, similar to what have been above described as connected with the gallery. Two pulleys *A*, *P*, (fig. 6) are made fast to the upper side of the tube, the pulley *A* at about twelve inches from its open end, and the pulley *P* at seven feet below, the rope, after passing over these and three other pulleys *C*, *C*, *C*, fixed to the upper cross-beam, is wound round the barrel having a winch below, at *L* in figs. 1 and 6, and seen also in Plate IX,

occupying a situation across the main beams, opposite the barrel *F* before described. When the quick motion in elevation has been given by the winch *L* to the approximate altitude, the remainder is given by the handle placed at the end of the double barrel *D*, which the author has not fully explained in his original description, as producing the slow motion of adjustment. When one end of a cord has been made fast to the barrel *L* above described, and wound a few times round it, it passes over the right hand pulley *C*, then over the pulley *A* fixed to the tube, and then, taking an oblique direction, passes over the left-hand pulley *C*, and also the middle pulley *C*, before it descends and passes under the pulley *P*, the second pulley fixed to the tube, where, if made fast, it would elevate the tube by the quick motion, without further appendage, this effect indeed is actually produced while the pulley *P* is at rest, and while the block seen at *E* is stationary, which is always the case while the double barrel *D* remains at rest, notwithstanding the barrel at *L* may be turned; for in this case the opposite end of the rope, made fast to the block *E*, may be considered as made fast to the tube, but there is a second cord *e e*, which surrounds the block *E*, and has one of its ends fast to the thicker portion of the double barrel *D*, and the other to the smaller portion, after they have been coiled several times, in opposite directions, round their respective ends of the said barrel; hence, as the handle *F* of this double barrel is turned, the cord, that is unwound from one end, is wound round the other, which operation does not allow the block *E* to remain stationary, but causes it to move with a slow motion, depending on the difference of the diameters of the two ends of the barrel, which may have any proportion given them, that the nature of the slow motion requires. When, therefore, the winch *L* remains quiet, and the end of the cord surrounding it is stationary, the block *E* pulls the opposite end very slowly, and the cord acting at the pulley *P* gives a slow elevation to the telescope, or otherwise a depression, accordingly as the handle *F* is turned forwards or backwards, that is, accordingly as the larger end of the barrel is gaining or losing its cord. In this manner the slow motion in altitude is effected by the observer himself, as he stands in the gallery, to the extent of three or four degrees. Near the lower end of the main tube a finder is fixed with cross wires, and also a small quadrant with a level, by means of which the approximate altitude can be given by the observer before he ascends into the gallery, and a divided arc of three degrees, with a radius equal to the length of the whole tube, is pointed to by an index made fast to the gallery, from which the additional altitude or depression can be known in the gallery, as it is altered by the slow motion thus occasioned by the double barrel.

5 The tube itself is supported at its depressed end by two metallic rollers *A*, *A*, seen in fig 7, of each four inches in diameter, which are fixed at the opposite ends of an axle tree fourteen inches long, and move on the base *G H*, shown in fig. 1, as the telescope is elevated, in order that the elevated end may just reach the eye of the observer The backward and forward motion is produced by a rope doubled like the rein of a carriage-harness, by pulling one rein or the other, as the motion requires, and is thus effected when the observer has hold of the middle or doubled part of the rope, both the ends are laid down the under side of the tube, and the one surrounding the pulley *B* (fig 6.) and made fast to the tube is fixed at *H*, (figs. 1. and 6.) and being pulled, makes the tube approach, while the other end, surrounding the pulley *I*, on the cross-beam *D D*, being pulled, makes the tube recede, and thus an alternate mo-

tion can be produced by simply pulling the reins by turns, which may hang on a peg, near the hands of the observer, when not wanted.

6. From the construction of the tackle by which the tube is suspended a free motion may be obtained in any state of elevation, except when the tube is directed to, or nearly to, the zenith; in which situation, if the horizontal range be given, the pressure will be upon one of the two bottom rollers, *A*, *A*, only, to obviate which inconvenience, a pivot is placed at right angles to the axle of the rollers, which, turning in a frame attached to the end of the tube, places the weight on both the rollers equally in every position of the telescope.

7. The quick motion in azimuth is thus produced: at the narrow end of the frame a barrel is fixed, seen at *K* in fig. 1, to which a coiled rope is made fast, and after passing round two fixed pulleys to change the direction of motion, has its remote end fastened to one of the five piles driven into the ground at the equidistant points *a*, *a*, *a*, *a*, and *a*, to any of which it will hook as occasion may require; and when the winch is turned, the whole frame will turn in azimuth as far as the pile will draw it, after which the rope is hooked to another pile to complete the motion, that is necessary for pointing the tube nearly to the azimuth of the object; after which the observer can give an horizontal range of about eight degrees, by swinging the elevated end of the tube to the right or left, as his back is turned towards the object. The speculum, like Sir William Herschel's, is placed at the depressed end of the tube, and has adjusting screws to turn it into the angle of obliquity necessary for placing the image in the field of the eye piece, which is adjustable at *F* (fig. 6.), at the left-hand side of the mouth of the tube, and has screws for turning its axis of vision in a line pointing to the centre of the speculum, as well as rack-work for giving distinct vision. The reins of rope which regulate the distance of the lower end of the tube from the circular railway, serve also to give the horizontal range, and to hold the tube in its place.

8. The eye-pieces adapted to this telescope have powers which magnify the object linearly from 100 to 1500 times, which are competent to fulfil all the purposes of vision when cleared of aberration. When the telescope is placed in the plane of the meridian and elevated, together with the gallery, into any required altitude, the *meridianal sweeps*, formerly practised by Sir William Herschel, and continued by his son Mr. Herschel with great success, in the examination of double stars and nebulæ, may be managed with great ease. We do not learn, however, that any micrometer has hitherto been applied to the telescope under our notice; but there can be no doubt of its utility being greatly enhanced by such appendage, and by a corresponding table of measurements.

9. When the telescope is not in use, the speculum is removed, the gallery and tube are lowered, and the latter is laid horizontally in a long case, formed between the parallel cross-beams from *H* to *G*, fig. 1, and is protected from the weather by a covering of oil-cloth. It has been asserted, that a fifty-feet telescope by Ramage, of twenty one inches aperture, is intended to be substituted for the twenty-five feet instrument at present erected at Greenwich, and the speculum we know is prepared, though not yet fully tried; but whether the plan will be carried into execution at the expense of the honorable Navy Board or not, is perhaps not yet determined.

§ XVII ON THE SPACE-PENETRATING POWERS OF TELESCOPES

1. Sir William Herschel was the first observer of the heavenly bodies, who made a distinction between the *magnifying* power and *space-penetrating* power of a telescope, and has given us examples that prove clearly the necessity of distinguishing one species of power from the other, particularly when large instruments are used. It is well known that a small star, for instance, the companion of Polaris, is seen through a telescope of large aperture with a smaller magnifying power, than it can be rendered visible with a small aperture even by a high power if the magnifying power is sufficient to separate the small star from the rays surrounding the edge of the large one, it is all that is required, but the quantity of light admitted into the pupil must be competent to draw the star out of space, if such an expression be admissible, and to render it distinguishable by the assisted eye. This joint effect is produced by a certain union of magnifying power with quantity of light, so as to produce a ratio between the values of the two properties, that shall be favourable to the observer. If the whole quantity of light emitted by the surface of a luminous body might be denominated by the letter L, the mean copiousness of light emitted from all the physical points of the said body by C, and the number of such points of emission by N, then Sir William assumed that CN, the copiousness multiplied by the number, is always equal to L. But as the quantity of light that enters the telescope, or, more strictly speaking, that enters the pupil applied to a telescope, is what is concerned in telescopic vision, it may be distinguished by the small letter l, and then the equation will be $CN = l$ The density of light however decreases in the ratio of the square of the distance of the eye from the luminous body, and if that distance is represented by D, then the expression becomes $\frac{l}{D^2}$. In natural vision the quantity l depends on the size of the pupil, which is different in different persons, and even in the same eye in different states of contraction or of dilatation. The author assumes the mean diameter of the pupil to be $\frac{2}{10}$ of an inch, for the sake of illustrating his mode of computing the space-penetrating power of a telescope. If the diameter of the aperture, or of the object glass, be denominated by A, and that of the pupil by a, then the quantity of light admitted by the telescope at any given distance from the luminous object will be expressed thus $\frac{A^2 l}{D^2}$, and the quantity admitted by the pupil thus $\frac{a^2 l}{D^2}$. But, in the use of optical instruments, the pencil of light transmitted towards the eye, may be greater than the pupil can receive at once, in which case the expression $A^2 l$ will not be applicable; if therefore the magnifying power be denoted by m, the aperture and this power must be so modified, that $\frac{A}{m}$ ought not to exceed a; and then the brightness of a luminous object seen by the eye of an observer at a distance will be expressed by $\frac{a^2 l}{D^2}$.

2 The author is aware that mathematicians may object to the use of this expression, on the ground that an illuminated object is equally bright at all distances. which objection he obviates by shewing that there are different states of brightness that are distinguishable, but

which in common language are confounded with each other; since the quantity of light emitted by any luminous body is expressed by $C\,N$, as above stated, this definition implies that there may be a modification of the brightness thus expressed, it may arise from the greater value of C compared with the value of N, or from the greater value of N compared with the value of C, in the former case the brightness is called by our author *intrinsic*, and in the latter *aggregate*, but in all cases the *absolute* brightness will be defined by $C\,N$.

3. Now the mathematical demonstrations of opticians, which have regard to what is here called *intrinsic* brightness, or to the illumination of the picture of objects on the retina of the eye, will not be at variance with what the author affirms of *absolute* brightness. To show the distinction more clearly, when it is said that the sun must be as bright to an observer on the planet Saturn, as to an observer on the Earth, Sir William admits that the picture of the Sun on the retina of the Saturnian observer may be as *intensely* illuminated, as that on the retina of the terrestrial observer, the apparent diameter being smaller in the first case than in the second, but he cannot admit, that the former *absolutely* receives as much light from the Sun as the latter, since this would be denying the well known decrease of light with the increase of distance. For, if we consider that the Sun must appear a hundred times less to an eye on Saturn than on the Earth, in consequence of the increased distance, while it is said to be *intrinsically* as bright there, it must be here *absolutely* an hundred times more bright. And what is true of the Sun will be equally so of the stars, though their distances exceed all calculation the light we receive from each of them will be properly expressed by $\frac{a^2 l}{D^2}$, and therefore their absolute brightness will vary in the inverse ratio of the squares of their distances. Hence it is concluded, that stars cannot be seen by the naked eye when they are situated more than seven or eight times farther from us than the star Sirius, that they become very soon, comparatively speaking, invisible to the powers of our best telescopes, and that their *visibility* depends on the *space-penetrating* power of the instrument used There are however other considerations that must affect the space penetrating power of the telescope, such as its good definition of an object viewed, the state of the atmosphere, the nature of the luminous body, whether self-luminous, or only performing the office of a reflector, and the powers of the observer's eye so that the subject cannot be fully reduced into any general terms, that will comprehend all cases in which the power may be employed, which has the property of penetrating into space, or of rendering very distant objects visible.

4. If we could, from the doctrine of parallaxes, or otherwise, assign the true distances to stars which, from our sensations alone, we now call stars of the first, second, third, &c. magnitudes, the expression $\frac{a^2 l}{D^2}$ would afford us the means of rendering stellar observations most interesting, in developing the structure of the starry regions, but our sensations cannot be halved or quartered, and as we have no means of determining the actual distances of the stars, as we have of the sun, moon, and planets, we cannot avail ourselves of the term D, in the general expression of brightness in the former case, as we can in the latter, but we may substitute the *name* of the object for its *distance*, and make a scale of comparative brightness thus, $\frac{a^2 l}{\odot^2}$, $\frac{a^2 l}{Sirius^2}$, $\frac{a^2 l}{\beta\ Tauri^2}$, $\frac{a^2 l}{Polaris^2}$, &c. which is all that the unassisted eye can pretend to.

If we were to assume, that the distance of the nearest star, say of Sirius, or Arcturus, is 412530 times farther from us than the Sun, as has been imperfectly calculated from the doctrine of parallaxes, and also that stars of the second magnitude are placed at double their distance, and so on, the diminution of brightness of the stars of the fourth, fifth, sixth, and seventh magnitudes, our author has computed, would answer pretty well to the mathematical expression of their brightness comparatively taken, and the differences would diminish with their distance, for instance, the calculated ratio of the brightness of a star of the sixth magnitude, to that of the seventh, is but little more than $1\frac{1}{3}$: 1, but still the eye can discriminate this small difference in actual observations, though different estimations of magnitude, from comparative brightness, will be formed by different observers, even under the same circumstances of light and magnifying power, according to the degrees of their experience, and faculty of making comparisons. In a favourable state of the atmosphere the utmost that the unassisted eye can reach, is a star of the seventh magnitude, and hence it is concluded, that no star more than eight, nine, or at most ten times farther from us than Sirius, can be perceived by the naked eye.

5. Now since the brightness of luminous objects is inversely as the squares of the distances, it follows that the space-penetrating power must be as the square roots of the light received by the eye of the observer, and this power in natural vision is expressed by $\sqrt{a^2 l}$, for if the brightness of light be truly expressed in natural vision by $\frac{a^2 l}{D^2}$, and in telescopic vision by $\frac{A^2 l}{D^2}$, the artificial power of penetrating into space, will be to the natural power, as $A : a$, which is as the diameter of the speculum, or of the object glass, to the diameter of the pupil of the eye, when D^2, the term common to both expressions, is omitted. But by pursuing the method of M. Bouguer, given in his *Traité d'Optique* (p. 16 fig 3.), our author determined by a rough experiment, that out of 100000 incident rays, 67262 were reflected by a well polished plane mirror at a single reflection, and that therefore two reflections preserved only 45242 of the said rays. Before this reflected light reaches the eye, there will be a still further diminution of the rays; and it was found by Bouguer's method, that when a piece of well polished glass, about the thickness of an eye lens, was the transmitting substance, about 94825 rays out of 100000 passed through, so that, when two lenses are used, 89918 only will be transmitted, and with three lenses 85265. Then by compounding these amounts of lost rays, in the Herschelian construction 63796 rays of the 100000 will reach the eye; but in the Newtonian, only 42901, with a single lens, or, with an eye-piece containing two lenses, 40681 will be the effective number, if we admit that there is no uncertainty in the quantities thus determined. For the correction to be derived from this loss of light in reflection and transmission, we may put x; and as there will be some further loss in the Newtonian telescope, from the second reflection from the small speculum, if we put b for its diameter, we shall have (A^2-b^2) for the real incident light; which being corrected for loss in reflection as above by x, will give the general expression $\sqrt{x\,l.(A^2-b^2)}$ for the space penetrating power of a telescope, where there are two reflections, or $\sqrt{x\,l.A^2}$ where there is only one, or where $b=0$ Then if we put natural light $l=1$, and divide by a, the diameter of the pupil of the eye, the general formula for telescopic

vision becomes $\frac{\sqrt{x(A^2-b^2)}}{a}$, whatever we assume or determine the aperture of the iris to be

6. When a Newtonian telescope of twenty feet focal length was used with an aperture of twelve inches, on an evening when a distant church steeple could be no longer distinguished by the naked eye, the dial of a clock placed against its wall was visible, when the space-penetrating power, agreeably to the general formula, was $\frac{429 \times (120^2-15^2)}{2} = 38, 99$, where the diameter of the pupil is taken at $\frac{2}{10}$, and where A, a, and b are expressed in tenths of an inch. From this experiment it appears that telescopic vision is performed by the *absolute*, and not by the *intrinsic* brightness of objects For though the intrinsic illumination of the picture on the retina, which is made by the telescope, does not exceed that which is made by natural vision, yet the *absolute* brightness of the magnified picture exceeds that of the natural picture in the same ratio that the area of the first, or component quantity N, in the first, exceeds that in the natural picture; and $N C$ in the telescopic picture exceeds $N C$, or the absolute brightness, in the natural picture, in this instance, about 1500 times

7 It is in this way that the large aperture of night glasses and of comet-finders, with comparatively small magnifying powers, perform their office better than telescopes of the ordinary construction, though they may not magnify the object more than seven or eight times, in these constructions, if we suppose the diameter of the object-glass to be not more than two inches and a half, the diameter of the optic pencil will be from $3\frac{1}{8}$ to $3\frac{1}{7}$ of an inch, which exceeds the natural diameter of the pupil in day light, taken at two-tenths only, therefore, to give such telescope its full effect, the value of a must be increased, but must not be assumed, even in the dark, to exceed $\frac{A}{m}$, nor in this instance to be less, we may then put $a = \frac{2.5}{7}$ or $= \frac{2.5}{8}$, and $x = .853$ for three refractions, and we shall have $\sqrt{\frac{.853 \times 2.5^2}{a}} = 6.46$ or 7.39 for the space-penetrating power, which are respectively but little short of the magnifying powers.

8. From the various examples given by Sir William Herschel for illustrating the distinction between magnifying power, and that species of power which we are now considering, we will take his observations on the nebula between η and ϱ Ophiuchi, discovered by Messier in 1764, with a ten-feet reflector, having a magnifying power of 250, and a space-penetrating power $= \sqrt{\frac{.43 \times (89^2-16^2)}{2}} = 28.67$ His note is, " May 3, 1783 I see several in it, and make no doubt a higher power and more light will resolve it all into stars. This seems to me a good nebula for the purpose of establishing the connection between nebulæ and clusters of stars in general "

" June 18, 1784. The same nebula viewed with a large Newtonian twenty-feet reflector, penetrating power $\sqrt{\frac{43 \times (188^2-21^2)}{2}} = 61.18$, and a magnifying power of 157, a very large and very bright cluster of excessively compressed stars The stars are but just visible, and are of unequal magnitudes the large stars are red, and the cluster is a miniature of that near Flamsteed's forty second Comæ Berenices, right ascension $17^h\ 6^m\ 32^s$, polar distance 108° 18′."

Here a penetrating power of about twenty-nine, with a magnifying power of 250, would barely show a few stars, when in the second instrument, the first power of sixty, with the second of only 157, showed them completely.

9. Subsequently to the date of the latter observation the twenty-feet Newtonian telescope was converted into an Herschelian instrument by taking away the small speculum, and giving the large one the proper inclination for obtaining the front view, by which alteration the power in question was improved from sixty-one to seventy-five, and the advantage derived from the alteration was evident, in the discovery of the satellites of the Georgian planetb y the altered telescope, which before was incompetent on the score of penetration.

"March 14, 1798. I viewed the Georgian planet with a new twenty-five-feet reflector. Its penetrating power is $\sqrt{\frac{.64 \times 240^2}{2}} = 95.85$, and having just before also viewed it with my twenty feet instrument, I found, that with an equal magnifying power of 300, the twenty five-feet telescope had considerably the advantage of the former." From this account we might be led to infer, that the difference of the performance might depend on the ratio of the lengths of the respective telescopes, but when we examine the formulæ, we shall find that the apertures differed in the proportion of 18.8 24 Again, the following quotations will show the superior power of the forty-feet telescope as compared with the twenty-feet instrument.

"Feb. 24. 1786. I viewed the nebula near Flamsteed's fifth Serpentis, with my twenty-feet reflector, magnifying power 157. The most beautiful extremely compressed cluster of small stars; the greatest part of them gathered together into one brilliant nucleus, evidently consisting of stars, surrounded with many detached gathering stars of the same size and colour. R A. $15^h\ 7^m\ 12^s$, P. D. 87° 8′."

"May 27, 1791. I viewed the same object with my forty-feet telescope, penetrating power $\sqrt{\frac{64 \times 480^2}{2}} = 191.69$, magnifying power 370. A beautiful cluster of stars. I counted about 200 of them. The middle of it is so compressed, that it is impossible to distinguish the stars."

"Nov. 5, 1791. I viewed Saturn with the twenty and forty-feet telescopes.

"Twenty-feet. The fifth satellite of Saturn is very small. The first, second, third, fourth, and fifth, and the new sixth satellites are in their calculated places.

"Forty feet. I see the new sixth satellite much better with this instrument than with the twenty-feet The fifth is also much larger here than in the twenty-feet, in which it was nearly the same size as a small fixed star, but here it is considerably larger than that star."

10. These examples, and various others that are contained in the 90th Volume of the Philosophical Transactions of London, 1800, to which our readers may turn, explain sufficiently the nature and extent of that species of power that one telescope possesses over another, by virtue of its enlarged aperture, but the exact quantity of this power must always be in a certain degree uncertain, while some of the elements of computation, that enter into the general formula, remain imperfectly determined.

11. The author's distinction between *intrinsic* and *absolute* brightness, will probably be more intelligible to general readers, if we substitute the terms *intensity* (or density) and *quantity* of light, the latter of which is that which affects the visibility of a luminous object seen at a distance. The intensity of light on the retina will be the same in the unassisted eye, as when a telescope is used, if allowance be made for the loss in reflection or transmission, as has been proved by La Grange in one of the volumes of the Memoirs of the Berlin Academy, for the whole quantity of light admitted into the eye will be expressed by $\frac{A^2}{a^2}$ agreeably to what we before said on this subject (§ IV 20.), and the principle on which the linear magnifying powers is obtained by the dynameter (§ XI 1.) is $\frac{A}{a}$, and consequently the power of magnifying the *area* of a picture will be also $\frac{A^2}{a^2}$, but the *intensity* of telescopic light is the whole quantity divided by the superficial magnifying power, which therefore is $= 1$ Sir William Herschel has a note in p 79. of his paper, that explains his meaning thus

12. " The mean intrinsic brightness, or rather illumination of a point of the picture on the retina, will be *all the light that falls on the picture, divided by the number of its points;* or $C = \frac{l}{N}$ Now, since with a greater magnifying power m, the number of points N increases as the squares of the power, the expression for the intrinsic brightness $\frac{l}{N}$ will decrease in the same ratio, and it will consequently be in general $N \propto m^2$, and $\frac{l}{N}$ or $l \propto \frac{1}{m^2}$, that is, by compounding $C\,N \propto \frac{m^2}{m^2} = l = 1$, or absolute brightness a given quantity "

13. We may now sum up the doctrines of Sir William Herschel on this subject in the following epitome He maintained that the visibility of an object is proportioned to the quantity of light that paints its image on the retina, which he has called *absolute brightness.* And since the quantity of light received by an eye or telescope from a distant object, is in the inverse ratio of the square of that distance, when the same visibility is required at a greater distance by means of a telescope, it is necessary that the quantity of light should be increased in the direct ratio of the square of that distance for then a telescope will have the power of showing an object equally clear at a great distance, as the eye will see it at a short one, provided that the square root of the quantity of light received by the telescope, be in the same proportion to that received by the eye, as the greater distance is to the less This power is called the *space-penetrating* power, as compared with that of the naked eye, and when put into an algebraic expression, is represented by the formula above given, viz. $\frac{\sqrt{x}\,(A^2-b^2)}{a}$, when there are two reflections, as in the Newtonian, Cassegrainian, and Gregorian constructions, but in the Herschelian, where $b = 0$, the formula becomes $\frac{\sqrt{x}.A^2}{a}$ or $\frac{A\sqrt{x}}{a}$.

14. The most proper objects for proving the powers of a telescope, are Saturn's ring and satellites, the Georgian satellites, the double stars of the first and second classes of Sir W. Herschel, and the various nebulæ.

§ XVIII AN HISTORICAL ACCOUNT OF THE DIFFERENT METHODS OF MEASURING SMALL CELESTIAL ARCS

1. The division of the circle, or of a quadrant, into such minute parts as will indicate the smallest denomination of the sexagesimal notation, has never been attempted on the limbs even of the largest instruments; but a great variety of contrivances have been devised for subdividing the fractional part of a degree into its subordinate parts, which it will be the purpose of this section to enumerate, in order that each kind of micrometrical measurement may afterwards be particularly described and explained

2. The first person who divided the sphere into zones was the father of the Greek philosophy, Thales, who about six hundred years before the Christian era, introduced the five imaginary circles which still remain on our globes, namely, the arctic and antarctic circles, the two tropics, and the equator, so that he must have observed the greatest and least meridian altitudes of the Sun, as well as his diameter. These circles were more fully explained by his disciple Anaximander, and about a century afterwards Anaxagoras wrote his book "On the Quadrature of the Circle." From these circumstances, and from the consideration that Thales had determined the apparent diameter of the Sun to be *half a degree*, it is more than probable that the circle was divided into quadrants, and the quadrant into 90°, long before the astrolabe was invented.

3. The astrolabe of Hipparchus was an armillary sphere better calculated for explaining than observing the places of the heavenly bodies, but that of Ptolemy was a planisphere more adapted for the purpose of observing the heavenly bodies, and his observations made therewith afforded data for the composition of his *Almagest*, or *Great Syntax* but the small radius of the astrolabes did not admit of very small subdivisions, and it does not appear that any plan for lessening the spaces of the subdivisions was attempted in the early ages of astronomy.

4. Delambre (*L'Astronomie Ancienne*, Tome I. p. 290.) was of opinion, that Hipparchus composed his catalogue of the stars in longitudes and latitudes, which could only be derived from the use of the sphere, and Ptolemy found, from a comparison of his own catalogue with that of Hipparchus, that the longitudes had varied in the interval about 2° $\frac{2}{3}$, while, as he supposed, he found the latitudes remaining constant. Hipparchus however, in his Commentary on Aratus, speaks only of right ascensions, whence Delambre concludes, that he changed his mode of observing and of registering his stars, after a movement in longitude had been discovered. This discovery in itself shows that the *degree* was a determined quantity, and also that *one third* of it was appreciated, either by estimation or division, at that period.

5 We have scarcely any records transmitted to us during the fourteen centuries of darkness in which the sciences were obscured, that throw any light on the subject of astronomical observations, except that the Arabic numerals found their way into England about 1133, where they were first observed engraved on a chimney piece at Helmdon in the county of Northampton. This occurrence, trifling as it may seem, afforded notwithstanding the means of deciphering what had been done and recorded by the Eastern nations, and the tables composed in these numerals have providentially become the common language of astronomy, as well as of commerce, in all nations.

6 In the year 1436, John Muller, better known by the appellation of Regiomontanus, from his birth-place in France, having by some means obtained a copy of Ptolemy's Almagest, and having been instructed in the doctrine of the sphere at Leipsic and Vienna, was determined to acquire a knowledge of the Greek language, that he might understand the contents of this astronomical work, which study eventually led to his contriving astronomical instruments that he might tread in the path of his remote predecessor in this branch of the sciences This learned and ingenious cultivator of science, assisted by Bernard Walther, and other mechanical geniuses, constructed different instruments which are described in a posthumous treatise of his, preserved in the British Museum, and entitled "*Scripta clarissimi Mathematici M Joannis Regiomontani de Torqueto, Astrolabio armillari, Regulâ magnâ Ptolemaicâ, Baculoque Astronomico*," &c. (fol. Nurembergh, 1544). The regula we understand was made of tin, for the purpose of taking the altitudes of the Sun and Moon, but it is difficult to say what was the mode of applying the graduations, or indeed what was the shape or size of this instrument, the Baculum Astronomicum was a rectangular or astronomical radius, probably the prototype of the cross-staff, by means of which the angular distances between the planets were measured. The Torquetum was a species of portable equatorial, described by Appian in his "*Introductio Geographica*," etc and also by Bailly in his "*Astronomie Moderne*", but the accounts fall short of satisfying our wishes, as to the precise nature of the measures afforded by those instruments.

7. We have passed over the Chinese and Egyptian methods of observing the heavenly bodies, as having more of antiquity than accuracy to recommend them, and as being therefore not within our purpose.

8 We find the astrolabe still used by Copernicus two centuries and a half after the time of Regiomontanus, but not finding it competent to the correction of the tables of Ptolemy and Alphonsus, with which he had reason to be dissatisfied, this Prussian astronomer determined to erect a quadrant, and to fix it in the meridian line at Thorn, where we may consequently fix the first regular observatory. We regret that we cannot give the dimensions of this quadrant, but the altitude of the Sun was taken by it with considerable comparative accuracy, by the shadow of a pin inserted into the centre of the graduated limb, affording a proof, that the subdivisions of the degree were yet in a rude state. But the other heavenly bodies, except the moon, afford no shadow, and therefore their altitudes could not be taken with the quadrant without some sort of apparatus for affording *sights* to be taken This defect, and also the limited radius of the quadrant, were supplied by the construction of a parallactic quadrant of fir, the radius of which was extended, according to Ben Martin, to four cubits, the quadrantal arc of which was divided into 1414 equal parts, so that each division must have contained $\frac{1414}{90^\circ}$, or 3 49″ 137 &c of sexagesimal measure. This is probably the first approximation towards a real micrometrical measurement of a small celestial arc, and the huge dimensions of this mechanical contrivance, contrasted with the diminutive size of any of our pocket micrometers, as now applied to a telescope to determine single seconds, afford a striking proof of the vast improvement that has taken place in the art of constructing astronomical instruments

9 The next step towards attaining the grand desideratum of measuring very small arcs on the limb of an instrument was attempted, and in a certain degree accomplished, by the celebrated Danish astronomer, Tycho Brahe, who was born 73 years after Copernicus, and if he

had confined himself to making astronomical observations, his name would have been placed high in the annals of astronomy. That he might not be interrupted in his favourite pursuit, he availed himself of the munificence of his sovereign, and built himself a princely observatory on the Isle of Huen, in the Sound, which he called Uranienburgh, or Uranibourg (Castle of the Heavens), his arrival on the island was on the 8th of August, 1576, and he expended 200000 rix dollars in the construction and fitting up of his building, exclusive of his detached house, called Stiernberg, or Mountain of the Stars, one half of which sum was supplied by Frederic II In this sumptuous retreat Brahe continued his observations for twenty years, until a pension, which the king had granted him, was withdrawn, after which he retired to Prague, where he dedicated a work on astronomy to Rodolph II. He was the author of several works, but his Rodolphine Tables, his Catalogue of 1000 Stars, both published by his friend Kepler, his *Astronomiæ Instauratæ Mechanica*, &c, and his *Historia Cœlestis*, are chiefly those in which astronomers can now have an interest. The author has described in his *Mechanica* four principal instruments by the names of *Armillæ*, *Zodiacales*, and *Æquatoriæ*, which seem to have been complete circles, the diameters of which varied from four and a half to ten feet in diameter The graduation of these circles, which is our present object of inquiry, is said to have been into degrees and 10′ spaces, like most of our larger modern instruments; and the subdivision of the 10′ spaces was effected by triangular diagonals to the accuracy of fifteen, and even of ten seconds in the largest instrument. Here then we have a rapid advance in the progress towards accuracy, from 3′ 49″.137 to 10″, and it might be supposed that little was then left to be accomplished by modern improvements; but though little, what yet remained to be done was important, as we shall see when we come to examine the recent contests among practical astronomers about splitting the second, and the various means employed to ensure the scarcely-discernible differences that modern refinement in the construction and use of instruments has effected, together with the minute quantities now introduced into the formulæ for computing the corrections. Tycho, as might be expected from his instruments, discovered the refraction of the rays of light in passing through the atmosphere.

10. As Peter Nunez, or Nonius, an eminent Portuguese mathematician and physician, died at 80 years of age in the year 1577, he must have been contemporary with Tycho Brahe, and was the contriver of a certain method of subdividing the arc of a circle which must have been known to him. Nonius in his book *De Crepusculis*, explains his method as consisting of 45 concentric circles described on the limb of his quadrant, the outermost of which was divided into 90, the next into 89 divisions, and so on till the innermost was divided into 46 only, each arc having one division less than the next larger; then when the edge of the index rested on any particular half of a degree on the limb, it would necessarily lie over some dividing stroke in one of the concentric circles, and thus point out what fractional part of the half degree ought to be added to the number of degrees and halves shown on the graduated limb, viz. whether $\frac{46}{90}$, $\frac{47}{90}$, or $\frac{48}{90}$, &c. of 30′ would be the additional quantity But as there are nine *prime* numbers among the 45 circles to be divided, it was found difficult to divide them correctly, and in 1573 Thomas Digges published his Treatise in London, called *Alæ seu Scalæ Mathematicæ*, which was considered as the invention of one Richard Chanseler In this treatise the *diagonal* scale was recommended in preference to that of Nonius, which from

that time it superseded, and it is more than probable that Tycho Brahe availed himself of Digges's scale, as its description was published three years before the observatory at Huen was erected.

11. As yet the use of a telescope had not been practised in making astronomical observations, and we need not therefore point out to the reader how superfluous nice divisions on the limb of an instrument must have been, when the observations in altitude were made by the unassisted eye if Tycho Brahe had been master of a good telescope of modern construction, his observations would have been a treasure, as constituting a remote standard of comparison with observations of recent date, and in this respect would have had the advantage over Bradley's. The first authentic notice that we have of the use of a telescope in astronomy, is the account that is given of the solar spots having been discovered by Galileo and Harriot in the year 1610, and the *Rosa Ursina* of the Jesuit Scheiner contains a minute description, with engravings of a great variety of the solar spots taken by him in 1611 and subsequent years at Rome, by the aid of a telescope mounted on a polar axis, which observations, in the opinion of Riccioli, Des Cartes, and Hevelius, were as accurate as could be desired, and were published in one volume folio, in the year 1630. It does not however appear that Scheiner had any method of measuring the sizes and relative situations of the spots on the sun's face; for had this been the case, his labours would have been more permanently useful, as objects of reference in subsequent ages But when the telescope had been once successfully pointed to the heavens, it was not long before a micrometer was invented and applied to the eye-piece, to subdivide the field of view into proportional parts. This invention introduced a new mode of measuring small arcs, which exceeded every other that had preceded it, both in facility of application and in accuracy, though the refracting telescope was at first made of a troublesome length.

12 Monsieur Auzout, a Frenchman, has generally been asserted to have been the first inventor of a micrometer, in the year 1666, but Costard has shewn that the honour of the invention is due to an Englishman of the name of Gascoigne, who fell in the civil war near York, in the year 1644 a letter on the subject written by the inventor himself is said to be preserved in the library of the Earl of Macclesfield.

13. Costard has not given any other notice than of the existence of Gascoigne's micrometer, which we have said was the first we have on record, but Richard Townley has given an account of it, in one of the early parts of the Philosophical Transactions of London (No. 25, p 457 May 1667, and Vol. I. Abrid. p. 225), in which he says he had it improved by an ingenious watchmaker. Gascoigne, says the writer, not only devised an instrument, before the civil wars, of as great a power as M. Auzout's, "but had also for some years made use of it; not only for taking the diameters of the planets, and distances upon land, but had further endeavoured, out of its preciseness, to gather many certainties in the heavens; amongst which I shall only mention one, viz the *finding the moon's distance*, from two observations of her horizontal and meridianal diameters; which I the rather mention, because the *French astronomer* [Picard?] esteems himself the first that took any such notice, as thereby to settle the moon's parallax." This instrument, continues the writer, "is small, not exceeding in weight, nor much in bigness, an ordinary pocket watch [of that period], exactly marking above 40000 divisions in a foot, by the help of two indexes; the one shewing hundreds of divisions, the other the divisions of the hundred, every last division of my small one containing $\frac{1}{10}$ of an inch, and

that so precisely, that as I use it, there goes [go] about 2½ divisions to a second." Dr. Hooke has given a description of this micrometer with an engraving, in the work above referred to (No 29 p 540. Nov 1667, and Lowthorp's Trans Abrid Vol I. p. 226), which is in substance this a small cylinder lies across the eye-tube of a telescope, and has one third of its length cut into a fine screw, and the remaining two thirds into a coarser screw of half the number of threads per inch, then, as this cylinder is confined to its place at both ends, and has one bar connected with the fine screw and another with the coarse screw, which bars are grooved into each other like the manner of a sliding rule, it is easy to conceive that, if two sights or pieces of metal, with strait edges adjusted for parallelism, are respectively made fast to the separate bars, a turn given to the common arbor of the screws will move the said bars with different velocities, so as to separate the edges of the sights by this difference, gradually as the screws revolve*, and the index and face connected with the arbor will indicate the parts of a revolution, while the graduated bar, actuated by the coarse screw, shows the number of revolutions at any time performed. Another office of the fine screw is, to keep the middle point of the opening, between the separating sights, in the line of collimation of the telescope at all times, whatever that opening may be, which it does very correctly, for while the coarse screw moves one sight from the other considered as stationary, the fine screw moves them both, together with the whole frame, in a contrary direction with one half of the velocity, in consequence of the bar to which its female screw is attached being fixed fast to the tube of the telescope. Dr. Hooke proposed however to substitute hairs for the strait edges of two plates, and no doubt adopted this construction in his zenith sector, with which he professed to measure *single seconds*, when disputing with Hevelius. This micrometer of Gascoigne seems to possess all the qualities necessary for producing good measures, and may be considered as the *prototype* of our best spider's-line micrometers, which have the advantage of delicacy of structure and good workmanship, but not of self adjustment for collimation. With respect to its priority, Dr. Bevis refers to an original letter of Gascoigne, addressed to Mr. Oughtred, written in 1640-1, (Phil. Trans. Vol. XLVIII. p. 190.) in which are contained several curious observations made by him on the diameters of the moon and planets. and Flamsteed informs us in his *Prolegomena* (p. 95 of Vol. III of his Historia Cœlestis,) that the inventor measured the diameters of the Sun and Moon, and the intermutual distances of the stars in Pleiades, by his micrometer in August 1640; and also that the improvement which Townley made, was the substitution of one screw for the two which at first were used. The micrometer was afterwards given to Flamsteed himself and used at Derby, with 15 and 7 feet telescopes, in the years 1671, 1672, 1673, and 1674, and was brought with him on his appointment to Greenwich. A more complex micrometer, with two separate screws, is described in Dr. Rees's Cyclopædia, and represented in Fig. 7. of Plate X. of Optics, which measures in a similar manner. Auzout's micrometer was used by himself and Picard, and is stated to have had the property of dividing a foot into 24000 or 30000 parts (Phil. Trans. No. 21. p 373, Jan. 1666.), and in this respect must have been inferior to Gascoigne's, as described by Dr. Hooke.

14. De la Hire has asserted (Royal Academy of Sciences, 1717), that the world is indebted to Huygens, who certainly was competent to any invention of this sort, for the first

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* It is on this principle that Mr. Barton's machine performs its beautiful divisions.

micrometrical measures of the planets, and that the instrument he used was a *virgula*, or tapering piece of metal, interposed in the common focus of the eye-lens and object-glass, which would consequently cover the diameter of the planet, and, by sliding forwards across the tube, might be made an exact measure. His observations on Saturn's Ring were thus made in 1659, and consequently before Auzout had published his account of a micrometer, but not before Gascoigne had constructed and used his. Dr. Hooke had two or three other ways of measuring the diameters of the planets to the accuracy of a second, mentioned in the Philosophical Transactions in May 1667, and described in his posthumous works, p. 497, &c. which we have not at present before us.

15. With respect to the tapering plates of Huygens, Sir Isaac Newton has observed, that the measures taken by them are somewhat bigger than they ought to be, for that all lucid bodies seen upon dark ones appear larger than they really are, and on the contrary, all opaque bodies seen upon bright ones appear smaller, which illusion has been confirmed by the measure of Mercury's diameter, observed upon and at a distance from the sun. But, as Dr. Smith has observed, this error may be avoided by using long tapering openings cut in brass plates, or by forming long triangles of hairs in a frame that will slide through oblong openings made in the sides of the eye-tube. Hence we find that Cassini used the square divided into four smaller squares, and twice bisected diagonally by a pair of hairs, and his method of applying this figure to the observation of eclipses is seen in No 236 of the Philosophical Transactions. Dr. Smith observes, that this is the best method of observing the places of Mercury and Venus, or of any of the planets and comets when near the Sun. And Dr. Halley (Phil. Trans. No. 363.), speaking of the accuracy of this way of observing, says, "that he himself was present when Dr. Pound and his nephew Mr. Bradley did this way demonstrate the extreme minuteness of the Sun's parallax so exactly, that upon many repeated trials it was not more than 12″, nor less than 9″." It may be worthy of remark here, that as the modern wire micrometers were derived from Gascoigne's model, with but slight alterations, so the modern Continental method of observing the paths of the new comets has been borrowed from Cassini's plan, in some form or other, that makes the declination depend on the time of passage through the limited field of view, by the assistance of computation.

16. When additional optical power had been supplied to the human eye by the use of the telescope, the subdivisions on the limb of an instrument were no longer too minute to exceed in accuracy the errors of observation, which was the case with Tycho Brahe's instruments, hence it was natural to expect that some further improvement should take place in the known methods of subdividing and reading the quantities measured by the graduated arc of a circle, for though the micrometer would measure a small arc very correctly, such for instance as that subtended by the diameter of a planet, or by a line connecting two stars seen both at the same time in the field of the telescope, yet this instrument could not determine the point in the heavens where the measured arc, or line substituted for a small arc, is situated, without the aid of a larger graduated arc, that would refer the said small arc to some known point, as the pole, the horizon, or zenith, or at least to some parallel indicated by the passage of a known star. Hence in 1631, the next year after the publication of the *Rosa Ursina*, Peter Vernier, of Franche Comté, published his account of a new quadrant, to which was applied the scale still designated by the inventor's name, and still used with small instruments, and even with the

larger modern instruments of the German construction. Hence we see that the scale bearing the name of Nonius, which is often improperly confounded with the Vernier, preceded it by about half a century, and is quite a different contrivance. We have at present before us an old reflecting octant, and also a fore-staff, both graduated according to the method of Digges, the intersections or subdivisions of which are indicated by the *fiducial* edge, as it has been called, of the index they read to single minutes only. In the year 1634, an ingenious foreign workman, Joannes Ferrerius, (John Ferrers?) contrived a method of indicating the subdivisions of the degree of a circular arc, by the intersection of curved lines, tending towards the central part of the instrument, with straight lines drawn from the centre of the divided arc, by means of which single minutes might be read, but this contrivance never gained the preference over the diagonal scale of Digges, much less over the Vernier. The description of this mode of subdividing has been given by John Baptist Morin, Regius Professor of Mathematics at Paris, who died in November 1656

17. We have now arrived at a period which constitutes a remarkable epoch in the history of science, and particularly of practical astronomy In the year 1660 King Charles II. founded the Royal Society, and in the same year Huygens introduced into England his method of grinding and polishing lenses. In 1670 the Royal Observatory at Paris was begun, and Cassini appointed to superintend it, and on the 10th of August 1675 the first stone was laid of the Royal Observatory at Greenwich, the appointment to which was fortunately given to Flamsteed in the following year. The commencement of these noble institutions demanded a supply of instruments, suitable for effecting a systematic survey of the heavens by men of the first-rate talents. Before this time (in 1668) Hevelius of Dantzic had published his *Cometographia*, that led to his contest with Dr. Hooke, respecting the comparative merits of plain and telescopic sights, which contest remains a standing proof of the rooted prejudice, which a partial view of a subject may excite in the human mind, even in matters which admit of positive proof Dr. Hooke contended that Hevelius could not measure a small arc in the heavens by means of plain sights to a greater accuracy than one *minute*, whereas he could determine, by means of a telescope furnished with a micrometer, a similar arc within a single *second*, and yet the dispute terminated with the public opinion in favour of plain sights! Even Dr. Jurin has spoken favourably of Hevelius's pretensions, in his Essay on Distinct and Indistinct Vision. Dr. Hooke, it is well known, had a zenith sector at Gresham college, of thirty-six feet focal length, which had a micrometer constituting a part of its eye-piece, that would no doubt be capable of performing the work above attributed to it

18. The first instruments that were fixed up at Greenwich, were a large sector or sextant, constructed by Flamsteed himself, in conjunction with his assistant Abraham Sharp, and an arch firmly attached to a wall, which had its face in the plane of the meridian, whence it was denominated a *mural arch*. This instrument was constructed for taking both right ascensions and declinations, after the models of Tycho Brahe and Hevelius; but the art of dividing was then in a rude state, and the work performed by those instruments is not now considered accurate

19. As if Providence had taken especial care to furnish Greenwich with the means of accomplishing the grand object of its new institution, George Graham came into the world in an obscure village in Cumberland (Kirk-linton), in the very year that produced the fabric for a

national observatory in England; and the mural quadrant, which was the produce of his ingenuity and labour, together with his sector, pendulum, and dead-beat-escapement, may be said to have contributed more to the advancement of practical astronomy, than all that had been done before him. Dr. Smith has given a particular description of Graham's quadrant, made for the use of Dr. Halley, and the reader who has not seen it may form an opinion of its powers, by being informed that two concentric arcs are graduated, one struck with a radius of 96.85 inches, and the other of 95.8; the inner one is divided into 90 degrees, and each degree into 12 parts, or into 5′ spaces, by a method that exhibited great ingenuity. (See GRADUATION in Dr. Rees's Cyclopædia, and in Dr. Brewster's Encyclopædia) The outer arc is divided into 96 equal parts, and each part into 16 subdivisions, so that each subdivision contains the measure of 3′ 30″ .9375, or $\frac{90°}{1536}$. The vernier of the arc of 90° reads $\frac{1}{10}$ of 5′ or 30″, and the vernier of the second arc of 96 parts reads $\frac{1}{16}$ of 3′ 30″ 9375, or 13″ 1835975, when the coincidence of some stroke on the vernier plate is not exact with any stroke on the limb, the remainder of the seconds are taken by estimation This quadrant is fixed on the eastern side of the meridian wall at Greenwich, and was adjusted by Dr. Halley, by the aid of a plumb line alone, but in Dr. Bradley's time the zenith sector, which will be described in its place, afforded the better means of obtaining the zero of the divisions, and the catalogues of Doctors Bradley and Maskelyne will bear testimony to the skill of the ingenious artist, so far as the southern stars are concerned.

20. When the English Government fixed on the celebrated Bird to make a second quadrant, to be placed on the opposite side of the same wall, and to look towards the north, that the northern stars might also be observed, he constructed it of brass, to avoid the effects of unequal expansion, to which Graham's quadrant was liable, from its limb being composed of two metals, iron and brass, pinned together; and also made the tangent screw of slow motion the measure of the additional quantity of seconds, not exactly indicated by the vernier, as Jeremiah Sisson had done previously, in his large mural arch, now remaining at Richmond observatory. Bird's portable quadrants were afterwards all furnished with micrometer heads to his measuring screw, and if the clamping apparatus, of which it formed a part, had not been liable to get too much play from wear, the plan of measuring by a compound reading microscope, first suggested by the Duke de Chaulne, and adopted by Ramsden, might not have become necessary.

21 This new method of reading by a compound microscope rendered the errors of dividing very sensible to the eye, and at the same time afforded the means of carrying the improvements in the art of dividing, in the hands of Ramsden, and subsequently of E. Troughton, to the *ne plus ultra* of perfection. The Duke de Chaulne published two pamphlets in French, folio, the one explaining his new method of dividing, and the other giving an account of his microscopic micrometer, the very next year after Bird had published the account of his method of dividing, which was rewarded by the English Board of Longitude, and from the publications above mentioned, the date of micrometrical readings, now generally applied to English instruments, may be fixed, though improvements in their construction and mode of application, have been gradually made since their first introduction. Though a circle of two feet diameter may now be graduated to show single seconds, by the micrometrical contrivance for reading its subdivisions, yet our competitors in Germany still give the preference to the more simple mode

of subdividing by the vernier, which notwithstanding is much more difficult to read than the graduated head of a micrometer, and is by no means so accurate, since something must always be left to estimation.

22 The screw micrometer has been used in various forms, the simplest of which was the ring contrived by Kirchius, with two opposite screws meeting one another and measuring the space between by the number of threads and parts of a revolution that separated their ends from a state of contact to the extent of the arc measured. But in every micrometer where the screw is employed, the distances and inclination of its threads are supposed to be perfectly formed, which is not always the case, a perfect screw being one of the most difficult achievements in mechanical operations.

23. In Dr. Bradley's time the screw micrometer had been simplified by omitting one of the two screws, and by measuring from a line fixed out of the centre of the field of view, so that one motion only was necessary; but it had been adapted only for measuring either a horizontal or vertical arc, and not an oblique one, the telescopes being at that time very long and the eye-tubes incapable of turning round, that they might hold the weighty apparatus of the micrometer in a given position; he therefore introduced a piece of circular rack-work to give the micrometer plate a circular motion to adapt it to any obliquity of the arc to be measured, and Dr. Smith has given an engraved delineation thereof, which is copied into the same plate of Rees's Cyclopædia which we have before referred to. An instrument of Bradley's construction, at present in our possession, is much too heavy to be used with an ordinary achromatic telescope, though Dr. Maskelyne has thought proper to give a description of its use, extracted from Bradley's papers, in Vol. LXII. of the Philosophical Transactions of London.

24. The micrometers that measure small angles by the aid of a simple telescope, have undergone great improvements since their original invention. The different telescopic micrometers that have been constructed, may be divided into *single image, double image,* and *binocular micrometers;* and the varieties of each would admit of still further classification, if we thought it necessary to describe very minutely all the contrivances that have been partially used, in conjunction with the simple telescope. We proposed merely to give a brief historical account in this section of the inventions as such, and shall give the particular descriptions, in the following sections, of such micrometers as have recommended themselves to actual service in the hands of modern practical astronomers

25. We have hitherto omitted mentioning the *reticulum* or net of Roemer, the Danish astronomer, who assisted Picard and Cassini in their observations at Paris, but shall have occasion hereafter to speak of its use, when fixed in the focus of the eye-piece of a telescope This ingenious astronomer is said to have been the first who suggested the construction of a double image micrometer about the year 1675, but it does not appear certain, that he ever matured any plan himself that was followed by others; but in science it is frequently of great use merely to *suggest* an idea that may give rise to an invention.

26 In 1743 Savery communicated to the Royal Society his construction of a double image micrometer, which Short extracted from the minutes, and published in Vol. XLVIII. of the Transactions (1753), and Bouguer proposed a similar construction in 1748 to be used as an *heliometer*. The method consisted of the application of two object-lenses, placed side by side,

to the same tube of a telescope, which consequently formed two images of the object viewed, and a scale was used at the common focus to measure the distance from exact contact of the limbs of the two suns or moons thus formed, which measure depended on the distance of the centres of the two contiguous object-lenses.

27. This instrument, no doubt, gave origin to Dollond's object-glass micrometer, in which the object-glass was divided into two halves, each giving a distinct image, and the distance between the centres of the semi-lenses being measured by a scale of equal parts, subdivided by a vernier at the object end. The account of this instrument was published as a communication also from Short, in the same volume of the Transactions that has just been referred to; and its construction will be described more particularly hereafter. Dr. Maskelyne found that the object-glass micrometer was adapted for measuring diameters, or distances between two visible points, better than for obtaining differences of right ascension or of declination, and he contrived a ring to hold two wires placed at right angles in the focus of the eye-piece, capable of being turned round by a button, to which visible lines any kind of measures may be referred, as to a zero, his description of which may be found in Vol. LXI. of the said Transactions, Part II p. 799.

28. The astronomer royal, however, was not altogether satisfied with the performance of the object-glass micrometer, and in 1776 contrived a plan of producing double images by two wedges or prisms of glass rendered achromatic, which were proposed to slide along the inside of the tube, so as to make its whole length a scale of the measure. This scheme, ingenious as it was, does not appear to have been effectually carried into execution, having been applied only to a small telescope.

29 In all probability Dr. Maskelyne's contrivance gave the hint to the Abbot Rochon, who applied a solid piece of rock crystal, formed into a double achromatic wedge, one of which wedges possessed the property of producing two images by double refraction, while the other rendered them nearly achromatic. This mode of measuring small angles was precisely similar to the one proposed by the astronomer royal, though the account of it was published seventeen years afterwards

30. Dr. Brewster's patent telescope is also made to measure on a similar principle, where a second object-glass, divided into two halves, with their centres a little separated from each other, is made to slide more or less from the principal object-glass, thereby altering the magnifying power, and at the same time producing two images.

31. But in our historical sketch we must not pass over the labours of Ramsden, the contriver of the dividing engines, the accuracy and utility of whose instruments of every description have borne the test of practical experience among the first-rate astronomers of different countries, and have only been excelled by the superior inventions and workmanship of Troughton, the specimens of both which are too numerous to be particularized in this section

32. Neither shall we pass in silence over the micrometrical contrivances of the late eminent searcher of the heavens, Sir William Herschel, and of other ingenious men, who have enriched our catalogue of micrometers by their happy inventions. Indeed the circles that have lately been constructed by instrument makers of our own times, may be said to be micrometrical all round their circumferences, and to leave nothing more to be hoped for on the score of

accuracy both of construction and division; and fortunately the veteran Troughton, with a liberality peculiar to himself, has put his junior artists in full possession of his original methods of constructing and dividing instruments, that have enabled them to perpetuate his skill, and to partake of his well earned fame.

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§ XIX SPIDER'S-LINE MICROMETER

1. Having explained the various constructions of different telescopes, and given a brief historical account of the methods of measuring celestial arcs in the successive stages of progressive improvement in practical astronomy, we proceed now to describe those useful appendages to the telescope, which modern artists have ingeniously contrived, for measuring small arcs with the utmost accuracy, and which have greatly contributed to the importance of comparative observations. Astronomers have agreed to give the appellation of *micrometer* to every mechanical or optical contrivance, that enables the observer to subdivide a small arc into minutes and seconds, or into seconds only, without reference to the local situation where that arc is situated in the celestial sphere. It would be an infringement of our plan, to enter into a detailed description of the rude micrometers which are now no longer in use, of which we could enumerate several, exclusively of those that have been incidentally mentioned in our preceding section; but to give our readers satisfactory information on the subject, as it has reference to existing observers, it seems necessary to divide micrometers into three classes, namely, SINGLE-IMAGE, DOUBLE-IMAGE, and BINOCULAR contrivances, any of which may, or may not, have the further property of measuring angular positions, accordingly as a graduated circle is added or omitted in the construction.

2. In most of the earliest micrometers the *screw* seems to have been fixed upon as the best mechanical mode of subdividing a small space, first by separating the parallel edges of two metallic plates, for which fine wires were afterwards substituted; but recently spiders' lines have been introduced by Troughton, and what was formerly called the *wire micrometer*, is now denominated the *spider's line micrometer*, and sometimes *Troughton's* micrometer, though these lines are now generally used by instrument-makers, who vary the construction to suit particular purposes, and who have introduced contrivances of their own, that produce varieties more nominal than real. The construction which we propose to describe first is Troughton's, which is represented under different aspects in figures 1, 2, and 3, of our Plate XI, and constitutes a positive eye-piece for a refracting, Newtonian, or Herschelian telescope. fig. 1 exhibits a horizontal section of this beautiful little instrument, taken in the direction of the axis of vision, fig. 2 shows the internal structure, when the lid of the oblong brass box is displaced; and fig. 3 gives the external appearance to an eye viewing it in the direction of the length of the telescope into which it is screwed in a vertical position. In figure 1, *a b* is a positive eye-piece containing the lenses *a* and *b*, constructed as we have already explained [§ IV. 16.]; *c d* is a piece of short tube, into which the eye-piece slides by friction, so as to be adjustable for distinct vision, which piece screws into a brass cylindrical box *e f*, having two openings cut at the opposite sides, to admit the oblong box, shown in fig. 2, to pass through it, and to keep its place by the friction of a dove tail, made along each side of it, which fits two pair of notches

made in the open part of the cylindrical box *g h* is a circle of brass racked at the periphery, and having one of its faces graduated into four quadrants by single degrees, to be indicated by a single stroke made on the side of the cylindrical box, it acts with an endless screw *w*, seen in fig 3, this graduated circle is connected with the telescope by means of the screw *t*, fig. 1, which is attached to it, but as the screw is held fast in its place by the two potences *x x*, that are screwed to the cylindrical box, turning the screw makes it travel round the racked circle, and thus carries the boxes and eye-piece into any given situation that may be required, while the stroke of the cylindrical box points out the number of degrees, that at any time have been passed over in turning round The small circle at *y*, in fig 3, is the eye-hole in the cap of the eye-piece, seen also in fig. 1 From this short description of the optical part, we pass now to the mechanical or micrometrical part of the instrument, which may be explained by a reference to fig 2 the internal forks *k* and *l* are so nicely fitted into each other, and into the parallel sides of the oblong box, that when they are displaced by their respective screws *o* and *p*, which are turned by the milled nuts *m* and *n*, seen in all the figures, there is not the least lateral shake, and as the pins *q* and *r*, that pass into suitable holes in the metallic ends of the said forks, have each a spiral spring surrounding them, which press the forks back in a direction opposite to the action of their screws, there is no sensible loss of motion, whichever way the screws may respectively turn, which is an important condition in any construction, where the measure depends on a screw. There are two spiders' lines laid across the forks respectively, one attached to the prongs of the inner fork *k*, and the other to the prongs of the outer fork *l*, and the scratches, in which the lines are imbedded, at the same time insure the parallelism of the two lines, and prevent their touching as they pass over one another, or lie in apparent contact When these lines are well placed, they appear as one line at the position of Zero. Some of the opticians make an adjustment for the parallelism of position, by making one of the beds or scratches on the head of a screw, which entering the prong admits of being turned a little. The long line running at right angles to the small ones may be a wire, as its only use is to place the micrometer in a direction that shall take in both the objects viewed, between which the distance is required to be measured, and to compare the position of that line with a true horizontal or vertical line, as the case may be, and it is convenient to have this line well marked in the field of view. In Troughton's micrometrical screw there are about 103.6 threads in the inch, and the inclination of the threads has been so often proved to be uniform all round, that it has been copied by several of the other London instrument-makers, and, in ordinary temperatures, it may be expected that the same values will be derived from the same object-glass, or speculum, whichever of these micrometers be used, provided that the object measured be situated in the solar focus of the telescope. We have compared the measures taken by the micrometers of Troughton, T. Jones, and Robinson of Devonshire Street, and have always found them identical, when a slight allowance is made for the unavoidable errors of observation. Dollond's screw is different, but the value of the screw is of no importance, when the screw itself is perfect Sometimes the line-micrometer is constructed with only one screw, and has one, two, or three fixed lines of departure, according to the inclination of the maker, to render it capable of measuring either large or small angles, but whatever may be the deviations from the instrument we have described, the mode of applying the screw is the same in them all.

3. In using this micrometer, the notched scale, bounding one side of the field of view, will always show how many entire revolutions of either screw have been made, in obtaining a measure, provided that the two measuring lines have been previously laid one exactly over the other, so as to bisect the small circular hole, made near the middle of the scale, that represents Zero, and also provided that the point *o* on the divided ring or head of the screw which is adjustable by friction, is then pointed to by the index *r*, made fast to the oblong box by two screws. If the angle to be measured be very small, one line may measure it, first to the right and then to the left of zero, and one half the sum of the two measures will be the mean measure, including the index error; but if the angle is considerable, such as the diameter of the Sun, or of the Moon, one of the lines may be drawn 15 or 20 notches to the left of zero, and then the line moving to the right will complete the measure, when the sum of the two quantities will give the whole number of revolutions and parts indicated, the tabular value of which sum may be had from a computed table by simple inspection. When the angle is only a fractional portion of the whole field, the two screws, starting from zero, may move in the same direction, and pass one another alternately in taking separate measures, so as to form a repeating instrument, and the last measure divided by the whole number of repetitions will give a correct result. When this micrometer is used as a celestial eye-piece, to view the primary image of an object in the solar focus, it will make no difference in the number of revolutions of the screw measuring such object, whether the magnifying power be large or small; for the object and the scale are enlarged by the eye-glasses in the same proportion, the only limit of power being, that the whole object shall be included in the field of view; but that the micrometer may be advantageously applied in measuring angles of different magnitudes, a set of different positive eye-pieces, some of them diagonal, are usually supplied with each micrometer, to suit the telescopes with which they are intended to be used.

4. The easiest method of giving a value to a revolution of the screw, when used with any individual telescope of a moderate size, is to try how many of them will exactly measure the vertical diameter of the sun in summer, when his altitude is so high, as to have nearly the same refraction at the upper and lower limbs; for the whole diameter reduced into seconds, and divided by the number of revolutions, and decimal parts observed on the head of the screw, will give the value of a single revolution, provided allowance be first made for the diminution of the sun's diameter by the difference of the two refractions at the superior and inferior limbs, which diminution will depend on the sun's altitude at the time of taking the measure. For example, on the morning of the 25th of June, 1826, when the sun's altitude was about 40° 30, his vertical diameter was measured with a Troughton's micrometer, by 40.98 revolutions of the screw, when applied to a refracting telescope of 43 2 inches solar focal length, in order to ascertain the value of a single revolution; on that day the sun's semi diameter, as given in the Nautical Almanac, is 15′ 45″.6, and consequently the whole diameter was then, 31′ 31″.2; from Bradley's table of refractions we have at app. zen. dis. 49° refrac. 1′ 5″.4, and at 50° we find 1′ 7′ .8, the difference in one degree being 2″.4 or 1″.2 for half a degree, the diameter of the sun nearly, the correct apparent diameter therefore may be taken at 1890″, and $\frac{1890''}{40.98} = 46''.12$ is the value of a single revolution of the screw which separates the two lines of the micrometer. In like manner, on the 20th of July 1826, when the sun's altitude required

the correction $-1''.1$ for its apparent vertical diameter, the measure taken by a telescope of 30.5 solar focal length, with the same micrometer, was 28.985 revolutions of the screw, and, the semi-diameter on that day being 15′ 46″.2, the corrected diameter was 1891″.3, which divided by 28.985 gave the value of a revolution in this case 65″.25.

5. Now as the values of the micrometer used with any two telescopes, are to each other in the inverse ratio of their solar focal lengths, we can examine the accuracy with which the measures of the sun's diameter were taken in these two examples; when compared together, they stand thus $\frac{43.2\times46''.12}{30.5}=65''.3$, and $\frac{65''.25\times30.5}{43.2}=46''.1$ nearly the two determinations were therefore made with almost the same accuracy, the product of each solar focal length, by the determined value of the screw's revolution, being in all cases a *constant quantity*, when allowance is made for the errors of observation When a telescope is so long that the image of the sun occupies more than the field of view, this method cannot be applied; but when the *constant product* due to a shorter telescope, or to an average of several telescopes, is determined, this constant, divided by the solar focal length of the large telescope, will give the value of a revolution of the micrometer's screw applied to it, as well as if the sun's diameter could have been actually measured thereby By these means the values of the micrometer's screw have been determined when successively used with the five telescopes, which we had occasion to introduce to the notice of our readers, when explaining the use of the Dynameter (§ XI. 13.). The analogical method of correcting the magnifying powers we there explained, will equally apply in correcting the comparative values of the micrometer's screw. Thus if we take for telescope 1 the numbers $30.5\times65''.25=1990''.125$, and for telescope 2, $43.2\times46''.12=1992''.384$, as determined by the sun, the mean constant will be 1991″.254, and $\frac{1991.254}{43.2}=46''.09$ will be the rectified value derived from the mean of the measures taken by the two telescopes, when the micrometer is adapted to telescope 2 But when there are several telescopes, the mean derived from the whole number will be better, and in this way we have rectified the comparative values of the screw with all the five telescopes, without departing sensibly from the solar values, and the following tables contain the results, derived from multiples of the respective values of each single revolution.

6 THE VALUES OF TROUGHTON'S MICROMETER WITH EACH OF FIVE TELESCOPES

TABLE I.

ENTIRE REVOLUTIONS.						ENTIRE REVOLUTIONS.					
No	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5	No.	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5
1	1' 5"25	0' 46"12	0' 29"48	0' 27"73	0' 26" 1	31	33' 42"75	23' 49".72	15' 13"88	14' 19"63	13' 29".1
2	2 10 50	1 32 24	0 58 96	0 55 46	0 52 2	32	34 48 00	24 35 84	15 43.36	14 47 36	13 55 2
3	3 15.75	2 18 36	1 28 44	1 23 19	1 18 3	33	35 53 25	25 21.96	16 12 84	15 15 09	14 21 3
4	4 21 00	3 4 48	1 57 92	1 50 92	1 44 4	34	36 58 50	26 8 08	16 42 32	15 42 82	14 47 4
5	5 26 25	3 50 60	2 27 40	2 18 65	2 10 5	35	38 3.75	26 54 20	17 11 80	16 10 55	15 13 5
6	6 31 50	4 36 72	2 56 88	2 46 38	2 36 6	36	39 9 00	27 40 32	17 41.28	16 38 28	15 39.6
7	7 36 75	5 22 84	3 26 36	3 14 11	3 2.7	37	40 14 25	28 26.44	18 10 76	17 6 01	16 5.7
8	8 42 00	6 8 96	3 55 84	3 41 84	3 28 8	38	41 19 50	29 12 56	18 40 24	17 33 74	16 31.8
9	9 47 25	6 55 08	4 25 32	4 9 57	3 54 9	39	42 24 75	29 58.68	19 9 72	18 1 47	16 57 9
10	10 52 50	7 41 20	4 54 80	4 37 30	4 21 0	40	43 30 00	30 44 80	19 39 20	18 29 20	17 24 0
11	11 57 75	8 27 32	5 24 28	5 5 03	4 47 1	41	44 35 25	31 30 92	20 8 68	18 56 93	17 50 1
12	13 3 00	9 13 44	5 53.76	5 32.76	5 13 2	42	45 40 50	32 17 04	20 38 16	19 24 66	18 16.2
13	14 8 25	9 59 56	6 23 24	6 0 49	5 39 3	43	46 45 75	33 3 16	21 7 64	19 52 39	18 42.3
14	15 13 50	10 45 68	6 52.72	6 28 22	6 5.4	44	47 51 00	33 49 28	21 37 12	20 20 12	19 8 4
15	16 18 75	11 31.80	7 22 20	6 55 95	6 31 5	45	48 56 25	34 35 40	22 6 60	20 47.85	19 34.5
16	17 21 00	12 17 92	7 51 68	7 23 68	6 57 6	46	50 1 50	35 21 52	22 36 08	21 15 58	20 0 6
17	18 29 25	13 4 04	8 21 16	7 51 41	7 23 7	47	51 6 75	36 7 64	23 5.56	21 43 31	20 26 7
18	19 34 50	13 50 16	8 50 64	8 19 14	7 49 8	48	52 12 00	36 53 76	23 35 04	22 11 04	20 52 8
19	20 39 75	14 36 28	9 20 12	8 46 87	8 15 9	49	53 17 25	37 39 88	24 4 52	22 38 77	21 18 9
20	21 45 00	15 22 40	9 49 60	9 14 60	8 42 0	50	54 22 50	38 26 00	24 34 00	23 6 50	21 45 0
21	22 50 25	16 8 52	10 19 08	9 42 33	9 8 1	51	55 27.75	39 12.12	25 3 48	23 34 23	22 11 1
22	23 55.50	16 54 64	10 48.56	10 10 06	9 34 2	52	56 33 00	39 58 24	25 32 96	24 1 96	22 37.2
23	25 0.75	17 40 76	11 18.04	10 37 79	10 0 3	53	57 38 25	40 44 36	26 2 44	24 29 69	23 3 3
24	26 6 00	18 26 88	11 47.52	11 5 52	10 26 4	54	58 43.50	41 30 48	26 31 92	24 57 42	23 29 4
25	27 11.25	19 13 00	12 17 00	11 33 25	10 52.5	55	59 48.75	42 16 60	27 1 40	25 25.15	23 55.5
26	28 16.50	19 59 12	12 46.48	12 0 98	11 18 6	56	60 54 00	43 2 72	27 30 88	25 52 88	24 21 6
27	29 21 75	20 45 24	13 15 96	12 28 71	11 44.7	57	61 59.25	43 48 84	28 0 36	26 20 61	24 47.7
28	30 27 00	21 31 36	13 45 44	12 56 44	12 10 8	58	63 4 50	44 34 96	28 29 84	26 48 34	25 13 8
29	31 32 25	22 17 48	14 14 92	13 24 17	12 36 9	59	64 9 75	45 21 08	28 59 32	27 16 07	25 39 9
30	32 37 50	23 3 60	14 44 40	13 51 90	13 3.0	60	65 15 00	46 7.20	29 28 80	27 43 80	26 6 0

The VALUES of TROUGHTON'S MICROMETER with EACH of FIVE TELESCOPES.

Table II.

Hundredths of a Revolution						Hundredths of a Revolution					
Parts	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5	Parts	Tel 1	Tel 2	Tel 3.	Tel 4.	Tel 5
1	0″ 65	0″ 46	0″ 29	0″ 28	0′ 26	51	33′ 27	23″ 51	15″ 04	14 14	13′ 31
2	1 30	0 92	0 59	0 56	0 52	52	33 92	23 97	15 33	14 42	13 57
3	1 95	1 38	0 88	0 84	0 78	53	34 57	24 43	15 63	11 70	13 83
4	2 60	1 84	1 18	1 11	1 04	54	35 22	24 89	15 92	14 97	14 09
5	3 26	2 30	1 47	1 39	1 30	55	35 88	25 35	16 22	15 25	14 35
6	3 91	2 76	1 77	1 67	1 56	56	36 53	25 81	16 51	15 53	14 61
7	4 56	3 22	2 06	1 95	1 82	57	37 18	26 27	16 81	15 80	14 87
8	5 .21	3 68	2 36	2 22	2 08	58	37 84	26 73	17 10	16 08	15 13
9	5 86	4 14	2 65	2 50	2 34	59	38 49	27 19	17 40	16 36	15 39
10	6 52	4 61	2 95	2 77	2 61	60	39 15	27 66	17 69	16 64	15 66
11	7 .17	5 07	3 24	3 05	2 87	61	39 80	28 12	17 99	16 92	15 92
12	7 82	5 53	3 54	3 33	3 13	62	40 45	28 58	18 28	17 19	16 18
13	8 47	5 99	3 83	3 60	3 39	63	41 10	29 04	18 58	17 47	16 41
14	9 12	6 45	4 13	3 88	3 65	64	41 75	29 50	18 87	17 75	16 70
15	9 .78	6 91	4 42	4 16	3 91	65	42 41	29 96	19 17	18 02	16 .96
16	10 43	7 37	4 72	4 44	4 17	66	43 06	30 42	19 46	18 30	17 .22
17	11 09	7 83	5 01	4 72	4 43	67	43 71	30 88	19 76	18 58	17 48
18	11 74	8 29	5 31	5 00	4 69	68	44 36	31 34	20 05	18 85	17 71
19	12 39	8 75	5 60	5 28	4 95	69	45 01	31 80	20 35	19 13	18 00
20	13 05	9 22	5 90	5 55	5 22	70	45 67	32 27	20 64	19 41	18 27
21	13 70	9 68	6 19	5 83	5 48	71	46 32	32 73	20 94	19 69	18 53
22	14 35	10 14	6 49	6 11	5 74	72	46 97	33 19	21 23	19 96	18 79
23	15 00	10 60	6 78	6 38	6 00	73	47 62	33 65	21 53	20 24	19 05
24	15 65	11 06	7 07	6 66	6 26	74	48 27	34 11	21 .82	20 52	19 31
25	16 31	11 52	7 37	6 94	6 52	75	48 93	34 57	22 12	20 79	19 57
26	16 96	11 98	7 66	7 22	6 78	76	49 58	35 03	22 41	21 07	19 83
27	17 61	12 44	7 96	7 49	7 04	77	50 23	35 49	22 70	21 35	20 09
28	18 26	12 90	8 25	7 77	7 30	78	50 89	35 95	23 00	21 62	20 35
29	18 91	13 36	8 55	8 05	7 56	79	51 54	36 41	23 29	21 90	20 61
30	19 57	13 83	8 84	8 32	7 83	80	52 20	36 88	23 58	22 18	20 88
31	20 22	14 29	9 14	8 60	8 09	81	52 85	37 34	23 88	22 46	21 14
32	20 87	14 75	9 43	8 88	8 35	82	53 50	37 80	24 17	22 73	21 40
33	21 52	15 21	9 73	9 16	8 61	83	54 15	38 26	24 47	23 01	21 66
34	22 17	15 67	10 02	9 43	8 87	84	54 80	38 72	21 76	23 29	21 92
35	22 83	16 13	10 32	9 71	9 13	85	55 46	39 18	25 06	23 56	22 18
36	23 48	16 59	10 61	9 98	9 39	86	56 11	39 64	25 35	23 84	22 44
37	24 13	17 05	10 91	10 26	9 65	87	56 76	40 10	25 65	24 12	22 70
38	24 79	17 51	11 20	10 54	9 91	88	57 41	40 56	25 94	24 40	22 96
39	25 44	17 97	11 50	10 82	10 17	89	58 06	41 02	26 24	24 68	23 22
40	26 10	18 44	11 79	11 09	10 44	90	58 72	41 49	26 53	24 96	23 40
41	26 75	18 90	12 09	11 37	10 70	91	59 37	41 95	26 83	25 24	23 75
42	27 40	19 36	12 38	11 65	10 96	92	60 02	42 41	27 12	25 51	24 01
43	28 05	19 82	12 68	11 92	11 22	93	60 67	42 87	27 42	25 79	21 27
44	28 70	20 28	12 97	12 20	11 48	94	61 32	43 33	27 71	26 07	24 53
45	29 36	20 74	13 27	12 48	11 74	95	61 98	43 79	28 01	26 34	21 79
46	30 01	21 20	13 56	12 95	12 00	96	62 63	44 25	28 30	26 62	25 05
47	30 66	21 66	13 86	13 03	12 26	97	63 28	44 71	28 60	26 90	25 .31
48	31 31	22 12	14 15	13 31	12 52	98	63 94	45 17	28 89	27 17	25 57
49	31 96	22 58	14 45	13 59	12 78	99	64 59	45 64	29 19	27 45	25 83
50	32 62	23 05	14 74	13 87	13 05	100	65 25	46 12	29 48	27 73	26 10

7. The manner of using these tables can hardly be misapprehended: Table I. belonging to the notched scale, gives the measures of an angle due to any number of *entire* revolutions that the micrometer can take in, and many more, when the magnifying power is great; and Table II. contains the values of the fractional portions indicated by the micrometer's divided head, the sum of which quantities gives the whole measure. Thus if the whole number of revolutions on the scale of notches be 19, and the parts on the head of the screw be 67, out of the 100 there inserted we shall have for the five telescopes under our notice the following values, viz.

	Tel. 1	Tel. 2	Tel. 3	Tel. 4	Tel. 5
Table I....... 19	20′ 39″.75	14′ 36″.28	9′ 20″.12	8′ 46″ 87	8′ 15″. 9
Table II .... .67	43 .71	30 .88	19 .76	18 .58	17 .48
Respective Sums	21 23 .46	15 7 .16	9 39 .88	9 5 .45	8 32 .57

8. The micrometer under our consideration may also be applied as part of a terrestrial eye piece with four lenses, by means of an adapter, in which situation it supplies the place of the two eye-lenses, which must be previously withdrawn. We have already stated (§ VII. 10.) that the terrestrial eye-piece has a variation of magnifying power depending on the distance between the two separate pairs of lenses, as is the case with the compound microscope, and that a new value is given to the micrometer's screw at every new distance. When an interior tube, that draws out, contains the pair of eye-lenses in the micrometer, a scale of equal divisions will indicate the magnifying powers; but not the variable values of the micrometer's screw due to each position.

9. These values will be to the tabular values belonging to the celestial telescope, inversely as the length of the secondary or direct images in the terrestrial telescope, are to the lengths of the primary or inverted image in the celestial eye-piece. The ratio of $V : v$, or $\frac{V}{v}$ can always be most correctly determined by experiment, by means of the micrometer itself, where the value v denotes the number of seconds in a revolution of the screw, when the enlarged secondary image is measured, and V the value, when the smaller or primary image is the object. In all cases the number of revolutions multiplied by the value of the screw will be the whole measure of the object observed; and as there are as many values in the terrestrial construction, as there are distances between the primary and secondary images, on which the magnitudes of the latter depend, it is obvious, that as many different measures of an object may be taken, as there are variations in the magnitudes, or, which is the same thing in effect, in the distances marked on the interior sliding tube. In general the increase of the secondary image, as compared with the primary one, made at the solar focus of the object-glass of a refracting telescope, is nearly as two to one in the ordinary terrestrial eye piece, which may be increased till the ratio is about 3 : 1, by the separation of the two pairs of lenses, in drawing out the sliding tube that holds the eye-piece or micrometer, but this ratio depends on the relative focal distances of the object and amplifying lenses, and also upon the distance between them, a small variation in any of which distances will greatly affect the place of the secondary image. In

our telescope numbered 1, of which the solar focal distance is 30.5 inches, the focal distance of the object lens *a* is 3.00 inches, that of the amplifying lens *b* 3.25, as nearly as can be ascertained, and the distance between them 4.7 When the inner tube is pushed home with the micrometer attached to it, the distance between the primary and secondary images is by measurement 12.75, and in this position *v* is found by experiment to be $32''\frac{1}{3}$.

10. In obtaining the distance between the two images in the home position, the tube itself is 11.5, and the distance of the solar focus 1 25 from the end of the tube, at the face of lens *a*, the sum of which is 12.75, as has been stated. Now as the inner tube is drawn out, the solar focus approaches lens *a*, till it has been withdrawn about five inches, in which position the solar focal image falls on the flat face of the said lens, and on being withdrawn still further, this primary image falls between the lenses *a* and *b*, where its distance from the secondary image can no longer be measured, but we may determine by experiment, as well as by computation, the points on the sliding graduated tube where the required values of the screw shall fall. When a distant object has been measured by the micrometer with the value 65″ 25, say by 20 revolutions, making the whole measure 1305″, the same object may be measured by $\frac{1305}{32}$, $\frac{1305}{31}$, $\frac{1305}{30}$, &c. as far as the scale will allow, the quotients arising from which fractions will be so many revolutions of the screw to which the micrometer's lines must be separated respectively, then drawing out the tube that holds the micrometer, till the same object is exactly included between the separated lines, at each of the different positions, will give the several points of the scale, where 32″, 31″, 30″, &c. must be marked The separations necessary for this purpose, counted in revolutions and parts, will be $\frac{1305}{32} = 40.78$, $\frac{1305}{31} = 42.10$, $\frac{1305}{30} =$ 43 50, &c.; and in this way 13 points were inserted, beginning with 32″, and ending with 20″, in the space of 6.5 inches, which is the whole length of the scale, the first space being about $\frac{1}{10}$, and the last $\frac{9}{10}$ of an inch, the spaces gradually increasing from the outer end, as the values of a revolution decrease. When the scale is thus graduated for 13 positions of the micrometer, as indicated by the end of the fixed exterior tube, as many measures may be successively taken of any object, forming a second image, as there are points marked on the inner tube, for the number of revolutions and parts of the screw, multiplied by the number of seconds engraved at the proper points of the scale, will give the whole measure, and an average of all the measures, taken at the different points, may be expected to give a result more accurate than will be obtained from a single measure taken at any one of the positions, hence we may denominate this method of using the micrometer *polymetric*, to denote the number of different new measures that may be successively taken by the same telescope

11. If we could determine with precision the solar focal lengths of the lenses *a* and *b* and also the exact distance between them, we could determine the whole scale theoretically by the following formula,

$$\frac{1 - \frac{d}{a} + \frac{V}{v}}{\frac{1}{a} + \frac{1}{b} \cdot \left(1 + \frac{d}{a}\right)}, \text{ in which}$$

$a =$ focal length of the object-lens,

$b =$ focal length of the amplifying lens,

d = the distance between them,
D = the distance from b to the second image,
V = the value in seconds of the screw's revolution at the first image,
v = the value in seconds of the screw's revolution at the second image;

but as the smallest error in any of the measurements would greatly affect the values of D, on which the scale depends, it is better to determine practically the values v at the two extreme points of the intended scale, and then to compute the distances by the formula in the following manner. For the sake of brevity let us put $A = \dfrac{1-\frac{d}{a}}{\frac{1}{a}+\frac{1}{b}.\left(1-\frac{d}{a}\right)}$, and $B = \dfrac{1}{\frac{1}{a}+\frac{1}{b}.\left(1-\frac{d}{a}\right)}$, and then the formula will become $D = A + B\dfrac{V}{v}$. Now we have already determined the extreme ends of the scale to be at the points 32″ and 20″, and by measurement we find, that the distance of the second image from lens b was 6.8 inches in the first case, and 13.3 in the second, by substituting these quantities for v and D in the respective positions, and 65″.25 for V, we shall have $6.8 = A + B\dfrac{65.25}{32}$, and $13.3 = A + B\dfrac{65.25}{20}$, by which equations we can thus determine A and B, subtract the first equation from the second, and the difference will be $6.5 = B\left\{\dfrac{65.25}{20}-\dfrac{65.25}{32}\right\} = B\left\{3.26-2.04\right\} = B.1.22$, which gives $B = \dfrac{6.5}{1.22} = 5.328$; this value of B being substituted in the first equation, affords $6.8 = A + 10.87$, from which we deduce $A = -4.07$. From these values of A and B the former equation, that gives the value of D, will be $D = -4.07 + 5.328.\dfrac{65.25}{v}$; and from this formula, taking the quantities 32″, 31″, 30″, 29″, &c. for v successively, we may compute a table of the corresponding values of D in inches and parts, as follows.

TABLE I.

v	D	v	D
32″	10.87	25″	13.91
31	11.22	24	14.49
30	11.59	23	15.12
29	11.99	22	15.80
28	12.42	21	16.55
27	12.88	20	17.37
26	13.37		

The differences between the first and each of the successive numbers in column D, will give the successive distances on the scale measured from the first point 32″, the differences of which distances will give the intervals as marked on the scale. The following table contains these respective numbers corresponding to the factors 32″, 31″, 30″, &c. which may be useful in graduating the scale, or in correcting it, if not accurately determined by the practical method already explained; but in this instance the practical determination accords exactly with the theory

TABLE II.

v	Diff	Intervals.
32″	0 .00	0 .35
31	0 35	0 .37
30	0 .72	0 .40
29	1 .12	0 .43
28	1 55	0 .46
27	2 .01	0 .49
26	2 .50	0 .54
25	3 04	0 .58
24	3 62	0 .63
23	4 .25	0 .68
22	4 93	0 .75
21	5 68	0 82
20	6 50	

12 The only variable quantity in our formula is v, which varies inversely as the second image, which image varies directly as the magnifying power with the same eye-piece; therefore if a scale of equal parts were inserted also on the sliding tube, it might have the corresponding magnifying powers also engraved, or otherwise tabulated, suitable for the use of the micrometer's eye-piece.

13. As a second example, we determined the points in the sliding eye-tube of the telescope numbered 5, which has a solar focal length of 76 25 inches, where the two extreme values of a revolution of the same micrometer were 8″ and 5″, and the whole interval 5 .6, at the first of which points, when the telescope was adjusted for distinct vision of a distant object, the distance from the lens b to the second image, situate at the micrometer's lines, was 8 .2 inches, and when drawn out to the point 5″, the distance was 13 8. Then by our formula, without regarding the focal lengths of lenses a and b, or the distance between them, we have $8.2 = A + B\frac{26''\ 1}{8}$, and $13.8 = A + B\frac{26''\ 1}{5}$ to find the points of the scale, which will be the true positions of the microscope, for giving 8″, 7″, 6″, and 5″, for each revolution of the screw, when used with this telescope, which points may be determined by the following steps, for gaining first the values of A and B, and from them the distances of the respective points from lens b, and then the differences of those distances, as well as the intervals between them; thus.

$$5.6 = B.\ \left\{\frac{26.1}{5} - \frac{26\ 1}{8} = B\ \right\{ = 5\ 22 - 3\ 26 = B \times 1.96$$

$$B = \frac{5.6}{1\ 96} = 2\ 857, \text{ and } 2\ 857\ \frac{26\ 1}{8} = 9\ 321$$

$$8.2 = A + 9.321, \text{ and therefore } -9.321 + 8.2 = -1.121 = A.$$

Hence $D=2.857.\frac{26.1}{v}-1.121$ will give the place of each point, as in the following table, which contains all the quantities.

v	D	Diff	Intervals
8″	8 .20	0 .00	1 .33
7	9 .53	1 .33	1 .78
6	11 .31	3 .11	2 .48
5	13 .79	5 .59	

In this computation it happens that the whole interval, between the points 8″ and 5″ per revolution, is just $\frac{1}{100}$ of an inch less than was determined by experimental vision and measurement; but where the interval from point 6″ to point 5″ is as much as 2 .48 inches, this discrepancy in the position of the micrometer will produce a very small error, if it could be even discernible in the observation.

14. From a comparison of these two scales we perceive that a telescope of short focal length will have the advantage of more polymetric points, than a telescope of a long focus, provided the pair of inner lenses are made proportional to the focal length of the object-glass, as they regard the due magnifying power and proper quantity of light; for the lenses *a* and *b* may have such short focal distances, and be placed so near together, that the high power thus gained will enlarge the second image too much, and there will be a great deficiency of light.

15 When the micrometer is set to the positions denoted by the figures 32″, 31″, 30″, &c. or 8″, 7″, 6″, 5″ engraved on the respective scales, these numbers become the factors suitable for multiplying the corresponding number of revolutions and parts of the screw, and a mean of all the products with either telescope, will give the measure with extreme accuracy.—As an example, we directed the larger telescope to a distant window, when the sun was concealed by a cloud, and took the following measures of its breadth when situated at the distance of more than a mile; viz.

1st position 8″ × 14 .15 revolutions = 113″.20
2nd do..... 7 × 16 .20 do. .... = 113 .40
3rd do.. . 6 × 18 .75 do. .... = 112 .50
4th do..... 5 × 22 .55 do. .... = 112 .75

Mean of the four = 112 .96

16. In this experiment none of the single measures differs from the mean of the four so much as half a second. When the image formed at the solar focus was measured by these two telescopes without the terrestrial tube, they gave 4 .30 × 26 .1 = 112″.23, and 1 .72 × 65″ .25 = 112″.23 also, but with much smaller magnifying powers.

17. When the micrometer is applied to determine the angle of position, that a line uniting two stars makes with the horizontal line, it must be attached to a telescope that will move in a vertical line when placed in the meridian, or otherwise that is mounted on a good parallactic stand, if an equatorial instrument is not at the observer's command. In some of the best micrometers there is an adjustment for putting the index stroke to zero on the divided

circle, which must be done while the horizontal wire is in its proper position, at right angles to a plumb-line seen at a distance in the meridian of the place, otherwise the distance from zero must be applied as an index error in each observation, accordingly as it is positive or negative. If the micrometer has no adjustment of this sort when screwed home into the telescope, an adapter that will turn stiffly round will supply the defect. Some of the micrometers have moreover an horizontal rack, which becomes vertical or inclined, according to the position given it by the screw acting on the racked edge of the divided circle, but unless the pinion of the rack is tight enough to keep its place in every position, it may be the cause of some inaccuracy in the measures, particularly in taking differences of declination. Indeed it is desirable that the sliding long box holding the lines should be made fast, for then the lines put to zero will measure the same zenith distance to the right and left, while the instrument is reversed in a zenith position. In Troughton's construction there are fixing screws in the interior part of the cylindrical box, that will produce sufficient friction on the cheeks of the dove-tail, when turned quite home, and when the tube that holds the eye piece is unscrewed and taken away, a small screw-driver may be admitted to effect the tightening of those cheeks. When the table of measures is made for the spaces included between the two measuring lines, the measures must be taken accordingly, but if the thickness of the spider's line should be included in the measure, an allowance must be made for it, either in the observation or in the construction of the table, for when the eye-piece has a short focal distance, the thickness of the line, though very small, will introduce an error into the measure, if not in some way allowed for.

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§ XX. OTHER METHODS OF DETERMINING THE VALUE OF A MICROMETER'S SCREW.

1. In the preceding section we have explained and exemplified the method of obtaining the value of a single revolution of a micrometer's screw, by means of the sun's diameter. But as there have been different determinations of his mean diameter, and as large telescopes give a greater image of his disc than can be included in the ordinary field of view of an eye-piece, we have thought it advisable to give other original methods of appreciating the revolution of a screw, when applied micrometrically to any given telescope, such as have no reference to the sun's diameter, and as may be applicable to telescopes of any length.

2. The method that has been most practised, though probably not the most accurate, and which we will call the second method, is that which derives the screw's value from the equatorial passage of a star without declination, or from a star of known declination reduced to the equator, and the only objection to this method is, that the arc corresponding to the time of the passage through any interval contained between two lines, when separated by a given number of the screw's revolutions, cannot be ascertained with the same degree of precision, that an arc in declination can be measured. It supposes a second of *time* to be measurable into fifteen parts, when a very small arc is determined in this way, which is impracticable; when however the arc is large, such as the whole field of view of a telescope, the entire number of seconds will make the difficulty merge, by rendering the residuary fractional

portion of the last second of little importance, when distributed among the whole number of seconds repetitions also of the same passage, or of the passages of different stars, in or near the equator, will afford an average which will contribute to diminish the error of an *arc* thus derived from its corresponding *time* The tables containing the values of Troughton's screws given by Messrs. Herschel and South, in the 22d and 23d pages of their valuable volume of *Observations of the Apparent Distances and Positions of* 380 *double and Triple Stars*, were constructed from values of the respective screws thus deduced, for telescopes of five and seven feet focal lengths, which are respectively 0′ 31″.582 and 0′ 24″.044. The exact focal length of the smaller telescope, we have been informed by Mr. South, is 63.2 inches, and we shall presently have occasion to show, by another method, that the determination with respect to it is good, but with regard to the larger, we have not yet been able to ascertain its exact focal length, though we can entertain no doubt of its scale being equally good. Indeed the coincidence of the resulting measures, taken by the two separate micrometers, is of itself a sufficient proof.

3. The third method of giving the screw its value that we have to offer, is a comparative operation, when two stars have been found that pass the field of view at the same position of the telescope, and at a short interval of time after one another, the difference of their altitudes may be determined by a good circular instrument placed in the meridian, and this difference divided by the number and parts of the revolutions of the micrometer's screw, applied to the given telescope, will show the value of one of its revolutions without farther trouble or computation. As it is necessary that the telescope in question should preserve its elevation unaltered during the interval of the two passages, it would be better to select two stars that are seen in the field at the same time, and then the first of them may be made to pass along one of the lines, while the second is directed along the other by making a proper separation of the lines before the first has disappeared: but this must be done while the stars are passing the meridian, and if they have small polar distance, they will continue the longer in the field to give time for the requisite operation.

4. The fourth method of gaining a knowledge of the screw's value, is not dependent on celestial observations any farther than that the solar focal distance, of the telescope to be used, is required to be previously measured in a state of adjustment to distinct vision of any heavenly body, and that the number of threads in the inch, which compose the screw, should be correctly known. In Troughton's micrometer we have had occasion to conclude, from various experiments, that there are 103.6 threads in the inch, but somewhat more or less according to the structure of the metal and temperature in which the screw was originally made and afterwards examined, for it is not to be expected that two screws, even made from the same die, will be precisely alike through their whole length, particularly if examined under different degrees of temperature both of the scale and screw, which are generally of different metals; but the results arising from our assumption, will prove, that it is not far from the truth. If we consider the telescope's focal length to be the radius of an imaginary circle, with the object glass at the centre, and the solar focus, with the screw in it, at the circumference, we can easily compute the small arc that is equal to $\frac{1}{103.6}$ of an inch, from the

known proportion of the circumference to the diameter of the circle, thus, let us put

$1 : 3.1416$ as the ratio between the diameter and circumference,
$f =$ the solar focus, or radius of the circle,
$n =$ the number of revolutions in the inch,
and $1296000'' =$ the seconds in the whole circle;

then we shall have v, the value of the scale, from the following formula.

$$v = \frac{1296000''}{f \times 2 \times 3.1416 . n};$$

In this expression f is the only variable term, and therefore we have $\frac{1296000}{2 \times 3.1416 \times 103.6} =$ $1990''.97$, a *constant* quantity, and $\frac{1990''.97}{f} = v$ for any telescope, whatever may be the length of f. If we take our five telescopes, and substitute the respective focal lengths already given (§ XI. 13.), we shall have the numbers and corresponding values of the micrometer's scale, which are here subjoined,

Tel. 1. $\frac{1990''.97}{30.5} = 65''.27$ for the value of a revolution,
Tel. 2. $\frac{1990.97}{43.2} = 46.09$ for do.
Tel. 3. $\frac{1990.97}{67.5} = 29.49$ for do.
Tel. 4. $\frac{1990.97}{71.75} = 27.74$ for do.
Tel. 5. $\frac{1990.97}{76.25} = 26.11$ for do.

Four out of five of these determinations differ only $\frac{1}{100}$ of a second, and the fifth only $\frac{3}{100}$ from the values respectively determined by the sun, and forming the basis of our tables in the last section, which confirmation is highly satisfactory, as it regards both the methods of assigning the tabular values We are now in a situation to ascertain the true value of Mr. South's micrometer attached to his equatorial instrument, on a supposition that its telescope's solar focal length is just 63.2 inches, which by our formula will be, $\frac{1990''.97}{63.2} = 31''.503$ in each revolution. This value is smaller than Mr South's tabular number by $0''.077$ in each revolution, and will amount to a second in about 13 revolutions, but in a very small arc the difference will not be perceptible, and therefore cannot sensibly affect the measures that he took of the double stars.

5. The fifth and last method that we propose to describe, of appreciating a revolution of the micrometer's screw, is by means of terrestrial measurement alone. By a case in plane trigonometry it may be easily computed, that, at the distance of 100 yards, a single yard placed either horizontally or vertically at right angles to the line of sight, will subtend an angle of 34′ 22″.4, and that it will subtend 36′, or a minute for each inch, at the distance of 95.49 yards, and if the rays of light issuing from a graduated scale of inches, placed at the latter dis-

tance from the object-glass of a telescope, could be distinctly seen with the micrometer adjusted for celestial vision, a value might be given to one revolution of its screw, by substituting as many inches as the telescope will measure at once for the sun, and then proceeding as in our first method described in the last section. But it is well known, that to view a near object with a telescope of considerable length, the eye-piece requires to be drawn out, and that the telescope thus elongated has an increased magnifying power, with a corresponding decreased value of its micrometer's revolution, therefore a correction for a want of parallelism of the rays issuing from a near object, must necessarily be applied to the angle measured by the telescope so situated, which we may call the *apparent*, by way of distinction from the *true*, angle subtended at the object-glass. When the distance of the scale is known, that correction may be conveniently obtained by the subjoined formula,

$$e = \frac{f^2}{d-f}$$

in which e = the elongation of the telescope,
f = the solar focal distance of the object-glass,
d = the distance of the scale from the object-glass,

then, if we put f' for $(f+e)$, we shall have $f' : f$: apparent angle : true angle.

6 Some years ago we borrowed one of Troughton's new chains with 5 feet steel links, to enable us to prove how far this method of obtaining a value for his micrometer's screw could be depended upon, with an achromatic telescope of 45.75 solar focal length, to which it was then applied, and the following detail will explain both the method, and the dependence that may be placed on it A staff painted white, and divided into inches by black strokes, was erected perpendicularly on a level plane, at an unknown distance from the object-end of the telescope, where a yard was measured by 18.19 revolutions of the micrometer's screw, after distinct vision had been obtained by the proper screw of the rack. The point under the object-glass was then determined by a plumb line, and the distance therefrom to the staff carefully measured by the new chain, which was found to be 261.9 yards, the true angle subtended by the yard at that distance being by computation 13′ 7″.57 When all the quantities concerned in computing the elongation e, are turned into yards and parts, we shall have

$$\left.\begin{aligned} f &= 1.2708 \\ f^2 &= 1.61493264 \\ d &= 261.9 \end{aligned}\right\} \text{ and } \frac{f^2}{d-f} = \frac{1.61493264}{261.9-1.2708} = 0.0062 = e$$

Then we have 1.2708 : 1.277 :: 13′ 7″.57 : 13′ 11″.41, hence $\frac{791.41}{18.19} = 43''.508$ was found to be the value of one revolution, which was very nearly the same that had been previously determined from measuring the sun's diameter, and also scarcely differs from what will result from our fourth method, which gives $\frac{1990''.97}{45.75} = 43''.51$.

7. When the telescope is a long one, and has the place of the solar focal adjustment marked on the sliding tube, or on the drawer of the rack, the place of the terrestrial adjustment may be marked with the fine point of a pencil, and the distance between the marks will give the elongation, e, sufficiently near without computation For instance, in the experiment just related the

interval was apparently a quarter of an inch, and as 0 .0062 of a yard is 0 .2232 of an inch, the value of the screw's revolution would have been very nearly the same, if a quarter of an inch had been substituted for the true value of e, obtained by computation.

8. In determining the true angle subtended by a yard at a considerable distance, considered as the base of a small triangle, the perpendicular and hypothenuse may be taken as equal to each other; and on this supposition the complement of the logarithm of the distance in yards and parts, will always be the logarithmic sine of the true angle subtended.

9. To facilitate the practice of this method of giving a value to a scale, and also to enable the practical astronomer to determine the distance of his meridian mark, or of any other object that may be required at one station, by means of a micrometer and a graduated staff, we computed the following table, which will give either the true angle, or corresponding distance by inspection, accordingly as one is known and the other required, it gives the distances corresponding to every minute and second subtended by a yard, from 1′ to 30′ 59″ inclusive, and was originally computed for another work, in which it has been before published. We also gave tables for showing the corrections for converting the apparent into the true angles, and *vice versâ*, but as these corrections vary with the focal length of the telescope, it is better to compute the true angle as we have directed by the ratio $\frac{f}{f'}$. For instance, if we reverse the data of our last experiment, we shall have the apparent angle subtended by a yard at an unknown distance = 13′ 11″.41, as obtained by the micrometer applied to the telescope (No. 5), and the elongation $(e) = 0 .2232$ of an inch, to determine the true angle and corresponding distance; which operation will stand thus; as $1.277 (= f') : 1.2708 (= f)$ 13′ 11″.41 : 13′ 7″.57 for the true angle, then in the first page of the table we look for 13′ at the side, and 7″ at the top, and at the junction of the horizontal line with the vertical column we find 262 .08, and in column 8″, we find 261 .75; the difference between which is 0 .33, which multiplied by the fractional part .57 will give 0 .1881, to be added to 261 .75, making together the true distance in yards 261 .9381.

In the same way the true distance may be determined by any other telescope, having a spider's-line micrometer, with a table of its values, when its solar focal length is known, and also the elongation of its terrestrial focal length.

A TABLE

OF

DISTANCES IN YARDS,

CORRESPONDING TO THE ANGLES SUBTENDED BY ONE YARD

True angle	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
1′	3437.70	3381.34	3326.80	3274.00	3222.80	3173.26	3125.15	3078.54	3033.25	2989.30	2946.60	2905.10
2	1718.85	1704.64	1690.67	1676.93	1663.40	1650.10	1637.00	1624.11	1611.40	1598.93	1586.63	1574.52
3	1145.90	1139.56	1133.30	1127.11	1120.99	1114.92	1108.94	1103.00	1097.15	1091.33	1085.60	1079.90
4	859.43	855.86	852.32	848.81	845.33	841.89	838.46	835.06	831.70	828.36	825.05	821.76
5	687.54	685.25	682.99	680.73	678.45	676.27	674.05	671.86	669.68	667.51	665.36	663.22
6	572.95	571.36	569.78	568.21	566.65	565.10	563.55	562.02	560.49	558.98	557.46	555.96
7	491.10	489.93	488.77	487.62	486.46	485.32	484.18	483.05	481.92	480.80	479.68	478.56
8	429.71	428.81	427.91	427.03	426.16	425.28	424.40	423.53	422.66	421.80	420.94	420.08
9	381.96	381.25	380.55	379.85	379.15	378.46	377.77	377.07	376.39	375.70	375.02	374.34
10	343.77	343.19	342.62	342.05	341.49	340.93	340.36	339.79	339.23	338.69	338.13	337.57
11	312.52	312.02	311.52	311.09	310.63	310.17	309.70	309.23	308.77	308.31	307.85	307.39
12	286.47	286.07	285.68	285.28	284.89	284.49	284.10	283.71	283.32	282.93	282.55	282.16
13	264.41	264.10	263.76	263.42	263.09	262.76	262.42	262.08	261.75	261.42	261.09	260.76
14	245.55	245.26	244.97	244.67	244.38	244.09	243.81	243.52	243.23	242.94	242.66	242.37
15	229.18	228.92	228.67	228.41	228.16	227.91	227.66	227.41	227.16	226.91	226.66	226.41
16	214.85	214.62	214.40	214.17	213.95	213.72	213.51	213.29	213.08	212.86	212.64	212.42
17	202.22	202.02	201.82	201.62	201.42	201.22	201.03	200.83	200.64	200.44	200.25	200.05
18	190.98	190.80	190.62	190.35	190.27	190.09	189.92	189.74	189.57	189.40	189.23	189.05
19	180.93	180.77	180.61	180.45	180.29	180.13	179.98	179.82	179.67	179.51	179.35	179.19
20	171.88	171.73	171.59	171.44	171.31	171.17	171.02	170.88	170.74	170.60	170.46	170.32
21	163.70	163.57	163.44	163.31	163.18	163.05	162.92	162.79	162.66	162.53	162.41	162.28
22	156.26	156.13	156.01	155.89	155.78	155.66	155.54	155.42	155.31	155.19	155.08	154.96
23	149.46	149.35	149.24	149.13	149.03	148.93	148.82	148.71	148.60	148.49	148.38	148.27
24	143.23	143.13	143.03	142.94	142.84	142.75	142.64	142.54	142.44	142.34	142.24	142.14
25	137.51	137.41	137.32	137.23	137.14	137.05	136.96	136.87	136.78	136.68	136.59	136.50
26	132.22	132.13	132.05	131.96	131.88	131.80	131.71	131.62	131.54	131.46	131.38	131.29
27	127.32	127.24	127.16	127.08	127.00	126.93	126.85	126.77	126.69	126.61	126.54	126.45
28	122.78	122.70	122.63	122.55	122.48	122.41	122.33	122.26	122.19	122.11	122.04	121.97
29	118.54	118.47	118.40	118.33	118.26	118.19	118.13	118.07	118.00	117.93	117.86	117.79
30	114.59	114.52	114.46	114.39	114.33	114.27	114.20	114.14	114.08	114.01	113.95	113.89
31	110.89	110.83	110.77	110.71	110.65	110.59	110.53	110.47	110.42	110.36	110.30	110.24
32	107.42	107.36	107.31	107.25	107.20	107.14	107.08	107.02	106.97	106.92	106.86	106.81
33	104.17	104.12	104.06	104.01	103.95	103.89	103.84	103.79	103.74	103.69	103.64	103.59
34	101.11	101.06	101.01	100.96	100.91	100.86	100.81	100.76	100.71	100.66	100.61	100.56
35	98.22	98.18	98.13	98.08	98.04	97.99	97.94	97.90	97.85	97.80	97.76	97.70
	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″

A TABLE

OF

DISTANCES IN YARDS,

CORRESPONDING TO THE ANGLES SUBTENDED BY ONE YARD, CONTINUED

True angle.	12″	13″	14″	15″	16″	17″	18″	19″	20″	21′	22″	23″
1′	2867 75	2825 60	2787 ,25	2750 16	2713 07	2678 73	2644 38	2610 91	2578 27	2546 11	2515 10	2485 10
2	1567 57	1550 84	1539 27	1527 06	1516 02	1505 ,50	1494 05	1483 90	1473 30	1462 86	1452 56	1442 38
3	1074 27	1068 71	1063 25	1057 75	1052 35	1047 01	1041 74	1036 19	1031 31	1026 17	1021 10	1016 07
4	818 50	815 22	812 05	808 86	805 70	802 57	799 46	796 37	793 32	790 21	787 20	784 20
5	661 00	658 98	656 88	654 80	652 73	650 67	648 62	646 59	644 56	642 56	640 56	638 56
6	551 47	552 98	551 50	550 03	548 58	547 12	545 66	544 23	542 80	541 37	539 95	538 56
7	477 66	476 ,30	475 26	474 16	473 08	472 00	470 92	469 84	468 77	467 70	466 63	465 59
8	419 23	418 38	417 53	416 69	415 85	415 01	414 18	413 35	412 53	411 70	410 88	410 06
9	373 66	372 98	372 31	371 64	370 97	370 31	369 65	368 99	368 33	367 68	367 02	366 36
10	337 62	336 47	335 93	335 38	334 82	334 29	333 76	333 22	332 68	332 14	331 61	331 08
11	306 93	306 47	306 02	305 57	305 12	304 67	304 22	303 77	303 32	302 87	302 43	301 99
12	281 ,77	281 39	281 01	280 63	280 24	279 86	279 49	279 11	278 73	278 36	277 98	277 60
13	260 13	260 11	259 78	259 45	259 12	258 80	258 47	258 15	257 82	257 49	257 16	256 83
14	242 09	241 80	241 52	241 24	240 96	240 68	240 40	240 12	239 84	239 56	239 29	239 01
15	226 ,60	225 91	225 67	225 42	225 ,18	224 ,93	224 69	224 44	224 20	223 95	223 71	223 46
16	212 20	211 98	211 ,76	211 54	211 33	211 11	210 90	210 68	210 ,47	210 25	210 04	209 82
17	199 86	199 66	199 47	199 28	199 09	198 90	198 71	198 52	198 33	198 13	197 94	197 75
18	188 88	188 70	188 53	188 36	188 19	188 02	187 85	187 68	187 51	187 34	187 17	187 ,00
19	179 04	178 88	178 73	178 57	178 42	178 ,26	178 11	177 96	177 81	177 65	177 50	177 35
20	170 80	170 04	169 90	169 76	169 62	169 48	169 31	169 20	169 ,06	168 92	168 ,78	168 61
21	162 16	162 03	161 91	161 77	161 65	161 51	161 39	161 26	161 14	161 01	160 89	160 76
22	154 85	154 73	154 62	154 50	154 38	154 27	154 15	154 03	153 92	153 80	153 69	153 ,57
23	148 17	148 06	147 96	147 85	147 75	147 64	147 54	147 43	147 33	147 22	147 12	147 ,01
24	142 05	141 95	141 85	141 75	141 66	141 ,56	141 ,46	141 36	141 27	141 17	141 08	140 98
25	136 41	136 32	136 25	136 14	136 ,05	135 ,96	135 87	135 79	135 70	135 61	135 52	135 43
26	131 21	131 ,12	131 04	130 ,95	130 87	130 79	130 ,71	130 ,62	130 54	130 46	130 38	130 29
27	126 38	126 30	126 23	126 14	126 07	125 99	125 92	125 84	125 77	125 69	125 62	125 51
28	121 90	121 83	121 76	121 ,69	121 62	121 54	121 47	121 40	121 33	121 26	121 18	121 11
29	117 73	117 66	117 59	117 ,52	117 46	117 39	117 32	117 ,26	117 19	117 12	117 05	116 98
30	113 83	113 76	113 ,70	113 64	113 58	113 51	113 45	113 39	113 33	113 26	113 20	113 ,14
31	110 ,18	110 13	110 07	110 ,01	109 95	109 89	109 83	109 77	109 71	109 65	109 59	109 53
32	106 75	106 70	106 64	106 59	106 54	106 49	106 43	106 38	106 32	106 27	106 21	106 16
33	103 54	103 49	103 44	103 39	103 33	103 28	103 23	103 18	103 13	103 08	103 02	102 97
34	100 ,51	100 46	100 41	100 37	100 ,32	100 27	100 22	100 17	100 12	100 ,07	100 02	99 97
35	97 ,66	97 62	97 57	97 52	97 48	97 43	97 38	97 34	97 29	97 24	97 19	97 15
	12″	13″	14″	15″	16″	17″	18″	19″	20″	21″	22	23′

A TABLE

OF

DISTANCES IN YARDS,

CORRESPONDING TO THE ANGLES SUBTENDED BY ONE YARD, CONTINUED.

True angle	24′	25′	26′	27″	28′	29″	30″	31	32′	33′	34″	35″
1′	2455 50	2420 00	2398 40	2370 82	2343 87	2317 55	2291 .80	2266 61	2241 97	2217 87	2191 30	2171 20
2	1432 37	1422 40	1412 75	1403 11	1393 62	1381 31	1375 08	1365 97	1356 93	1348 10	1339 30	1330 72
3	1011 05	1006 16	1001 27	996 43	991 61	986 90	982 20	977 51	972 93	968 36	963 81	959 36
4	781 28	778 31	775 42	772 51	769 61	766 78	763 .98	761 11	758 31	755 51	752 78	750 01
5	636 61	634 66	632 70	630 77	628 85	626 91	625 .03	623 15	621 23	619 40	617 .55	615 71
6	537 18	535 75	534 30	532 99	531 .62	530 25	528 88	527 53	526 .18	524 .84	523 50	522 18
7	464 55	463 51	462 47	461 40	460 .45	459 41	458 36	457 34	456 .33	455 .33	454 32	453 32
8	409 25	408 42	407 61	406 81	406 03	405 22	404 .43	403 61	402 .85	402 06	401 .29	400 51
9	365 71	365 06	364 12	363 78	363 14	362 49	361 86	361 22	360 59	359 96	359 35	358 71
10	330 55	330 02	329 49	328 96	328 41	327 92	327 40	326 .88	326 36	325 .84	325 33	324 82
11	301 55	301 11	300 67	300 23	299 80	299 36	298 93	298 .49	298 06	297 65	297 .20	296 77
12	277 23	276 86	276 49	276 12	275 75	275 38	275 01	274 65	274 29	273 92	273 56	273 18
13	256 51	256 20	255 90	255 58	255 27	254 96	254 64	254 33	254 02	253 71	253 39	253 08
14	238 73	238 45	238 18	237 90	237 63	237 35	237 08	236 81	236 54	236 27	236 00	235 73
15	223 22	222 98	222 74	222 50	222 26	222 02	221 78	221 54	221 31	221 07	220 83	220 59
16	209 61	209 40	209 19	208 97	208 76	208 55	208 34	208 13	207 92	207 .71	207 50	207 29
17	197 56	197 37	197 18	196 99	196 81	196 62	196 44	196 24	196 06	195 .87	195 69	195 50
18	186 83	186 66	186 49	186 32	186 15	185 98	185 82	185 64	185 .48	185 31	185 15	184 98
19	177 20	177 04	176 89	176 74	176 59	176 44	176 29	176 14	175 .99	175 84	175 69	175 54
20	168 .51	168 37	168 24	168 10	167 .97	167 .82	167 69	167 54	167 .41	167 27	167 14	167 00
21	160 61	160 51	160 39	160 26	160 14	160 01	159 89	159 76	159 64	159 51	159 39	159 27
22	153 46	153 34	153 23	153 12	153 01	152 89	152 78	152 67	152 56	152 46	152 33	152 22
23	146 91	146 80	146 70	146 59	146 49	146 38	146 28	146 17	146 07	145 97	145 86	145 76
24	140 88	140 78	140 69	140 59	140 50	140 40	140 31	140 21	140 12	140 02	139 93	139 83
25	135 .34	135 25	135 16	135 07	134 98	134 89	134 81	134 73	134 64	134 55	134 46	134 .37
26	130 21	130 13	130 05	129 97	129 89	129 80	129 .72	129 .64	129 56	129 48	129 40	129 .31
27	125 47	125 38	125 31	125 23	125 16	125 .08	125 01	124 93	124 86	124 78	124 71	124 63
28	121 04	120 97	120 90	120 83	120 76	120 .69	120 62	120 55	120 48	120 41	120 34	120 ,27
29	116 92	116 85	116 78	116 71	116 65	116 58	116 .52	116 45	116 39	116 32	116 26	116 .19
30	113 08	113 01	112 95	112 89	112 83	112 77	112 71	112 65	112 59	112 .52	112 46	112 .40
31	109 48	109 42	109 36	109 31	109 25	109 19	109 13	109 08	109 02	108 .96	108 91	108 85
32	106 10	106 05	105 99	105 94	105 88	105 83	105 77	105 72	105 66	105 61	105 .55	105 50
33	102 92	102 87	102 82	102 77	102 72	102 67	102 62	102 56	102 51	102 46	102 41	102 36
34	99 93	99 88	99 83	99 78	99 73	99 69	99 64	99 59	99 54	99 50	99 45	99 40
35	97 11	97 06	97 02	96 98	96 93	96 .89	96 84	96 79	96 75	96 70	96 66	96 61
	24′	25″	26″	27″	28″	29″	30	31″	32″	33″	34″	35″

A TABLE

OF

DISTANCES IN YARDS,

CORRESPONDING TO THE ANGLES SUBTENDED BY ONE YARD, CONTINUED

True angle	36″	37″	38″	39″	40″	41″	42″	43″	44″	45″	46″	47′
1′	2148 65	2120 41	2104.70	2083 47	2062 62	2042 20	2022 10	2002 64	1983 28	1964 40	1945 85	1927 00
2	1322 19	1313 77	1305 45	1297 23	1289 13	1281 13	1273 22	1265 41	1257 70	1250 06	1242 55	1235 10
3	954 91	950 51	946 15	941 83	937 55	933 27	929 15	924 94	920 81	916 72	912 66	908 64
4	747 33	744 62	741 95	739 29	736 65	734 03	731 43	728.83	726 27	723 72	721 19	718 69
5	613 87	612 05	610 21	608 44	606 65	604 87	603 10	601 35	599 60	597 86	596 12	594 41
6	520 87	519 55	518 21	516 95	515 65	514 36	513 06	511 81	510 55	509 29	508 04	506 78
7	452 33	451 31	450 36	449 37	448 40	447 42	446 45	445 49	444 53	443 57	442 62	441 67
8	399 73	398 95	398 19	397 41	396 66	395 88	395 12	394 37	393 63	392 88	392 13	391 39
9	358 09	357 47	356 85	356 23	355 62	355 01	354 40	353 79	353 18	352 58	351 98	351 35
10	324 32	323 81	323 29	322 78	322 28	321 78	321 28	320.78	320 28	319 79	319 29	318 79
11	296 36	295 92	295 50	295 08	294 66	294 24	293 82	293 40	292 98	292 56	292 15	291 73
12	272 83	272 47	272 11	271 75	271 40	271 04	270 68	270 32	269 97	269 62	269 28	268 93
13	252 76	252 46	252.15	251 85	251 54	251 24	250 93	250 62	250 32	250 02	249 71	249 41
14	235 46	235 19	234 92	234 64	234 38	234 11	233 84	233 57	233 31	233 05	232 79	232 53
15	220 36	220.12	219 89	219 66	219 43	219 19	218 96	218 72	218.49	218 26	218 03	217 80
16	207 09	206.88	206 67	206 46	206 26	206 05	205 85	205 64	205 44	205 23	205 03	204 82
17	195 32	195 13	194 95	194 76	194 58	194 40	194 22	194 03	193 85	193.67	193.49	193 31
18	184 82	184.65	184 49	184 32	184 16	183 99	183 83	183 67	183 51	183 34	183 18	183 01
19	175 39	175 24	175 09	174 94	174 79	174 64	174 49	174 34	174 20	174 05	173.91	173 76
20	166 87	166.73	166 60	166 47	166 34	166 20	166 07	165 94	165 81	165 67	165 54	165 41
21	159 15	159 02	158 90	158 78	158 66	158 54	158 42	158 29	158 17	158 05	157 93	157 81
22	152 11	151 99	151 88	151 77	151 66	151 54	151 43	151 32	151.21	151 10	150 99	150 88
23	145 66	145 56	145 46	145 35	145 25	145 14	145 04	144 94	144 84	144 74	144 64	144 54
24	139 74	139 64	139 55	139 45	139 36	139 26	139 17	139 08	138 99	138 89	138 80	138 70
25	134 29	134 20	134 11	134 02	133 93	133 84	133 76	133 67	133 59	133 50	133.42	133 33
26	129 23	129 15	129 07	128 99	128 91	128 82	128 74	128 66	128 58	128.49	128 41	128 33
27	124 55	124 47	124 40	124 32	124 25	124 17	124 10	124 02	123 95	123 88	123 80	123 72
28	120 20	120 13	120 06	119 99	119 92	119 85	119 78	119 71	119 64	119 57	119.50	119 43
29	116 13	116 06	116 00	115 93	115 87	115 80	115 74	115 67	115 61	115 54	115.48	115 42
30	112 34	112 28	112 22	112 16	112 10	112 04	111 98	111 91	111 85	111 79	111 73	111 67
31	108 79	108 73	108 67	108 62	108 56	108 50	108 44	108 38	108 32	108 27	108 21	108 15
32	105 45	105 40	105 34	105 29	105 23	105 18	105 12	105 07	105 02	104 97	104 91	104 86
33	102 31	102 26	102 21	102.16	102.11	102 06	102 01	101 96	101 91	101 86	101 81	101 76
34	99 35	99 31	99 26	99 21	99 16	99 11	99 06	99 02	98 97	98.92	98 87	98 83
35	96.57	96 52	96 48	96 43	96 38	96 34	96 29	96 25	96 21	96 16	96 12	96.07
	36″	37″	38′	39″	40″	41″	42″	43″	44″	45″	46″	47′

A TABLE

OF

DISTANCES IN YARDS,

CORRESPONDING TO THE ANGLES SUBTENDED BY ONE YARD, CONCLUDED.

True angle	48"	49"	50"	51"	52"	53"	54"	55"	56"	57"	58"	59"
1'	1909 82	1892 31	1875 11	1858 21	1841 62	1825 32	1809 30	1793 60	1778 12	1762 92	1747 98	1733 29
2	1227 75	1220 47	1213 30	1206 20	1199 20	1192 24	1185 42	1178 61	1171 93	1165 30	1158 77	1152 30
3	904 65	900 72	896 80	892 90	889 06	885.21	881 46	877 72	873 97	870 30	866 65	863 02
4	716 18	713 71	711 25	708 80	706 37	703.96	701 57	699 18	696 81	694 49	692 15	689 83
5	592 71	591 01	589 32	587 64	585 96	584 30	582 65	581 02	579 38	577 76	576 15	574 55
6	505 52	504 30	503 08	501 85	500 63	499 42	498 22	497 02	495 82	494.63	493 46	492.27
7	440 73	439 79	438 86	437 92	436 98	436 07	435 15	434 24	433 32	432.42	431.51	430 61
8	390 64	389 90	389 17	388 44	387 71	386 98	386 25	385 51	384 81	384 10	383 39	382 67
9	350 78	350 18	349 59	348 99	348 41	347 82	347 25	346.66	346 08	345 49	344 92	344 34
10	318 30	317 81	317 32	316 83	316 35	315 85	315 38	314 90	314.42	313 94	313 47	312 99
11	291 32	290 92	290 51	290 07	289 68	289 28	288 88	288 47	288 07	287 67	287 27	286 87
12	268 56	268 23	267 87	267 52	267 18	266 83	266 49	266 15	265 81	265 46	265 12	264.78
13	249 11	248 81	248 51	248 21	247 91	247 61	247 31	247 02	246 72	246 42	246 13	245 83
14	232 27	232 01	231 75	231 49	231 23	230 97	230 72	230 47	230 22	229 96	229 70	229 44
15	217 57	217 34	217 12	216 89	216 66	216 43	216 21	215 98	215 75	215 52	215 30	215 08
16	204 62	204 41	204 21	204 00	203 80	203 60	203 40	203 21	203 01	202 81	202 61	202 41
17	193 13	192 95	192 77	192 59	192 41	192 23	192 05	191 87	191 69	191 51	191 33	191 15
18	182 85	182 69	182 53	182 37	182 21	182 05	181 89	181 73	181 57	181 41	181 25	181 09
19	173 62	173 47	173 33	173 18	173 04	172 89	172 75	172 60	172 46	172 31	172 17	172 03
20	165.28	165.14	165 01	164 88	164 75	164 61	164 48	164 35	164 22	164 09	163 96	163 83
21	157 69	157 57	157 45	157 33	157 21	157 09	156 97	156 85	156 73	156 61	156 49	156 37
22	150 77	150 66	150 55	150 44	150 33	150 22	150 11	150 01	149 90	149 79	149 68	149 57
23	144 44	144 34	144 24	144 14	144 04	143 94	143 84	143 74	143 64	143 54	143 44	143 34
24	138 61	138 52	138 43	138 33	138 24	138 15	138 06	137 96	137 87	137 78	137 69	137 60
25	133.25	133 16	133 07	132 98	132 90	132 81	132.73	132 64	132 56	132 47	132 39	132 31
26	128 25	128 18	128 10	128 03	127 95	127 87	127 79	127 71	127 63	127 56	127 48	127 41
27	123 65	123 58	123 51	123 43	123 36	123 28	123 21	123 13	123 06	122 99	122 91	122 84
28	119 36	119 29	119 22	119 16	119 09	119 02	118 95	118 88	118 81	118 74	118 67	118 60
29	115 36	115 29	115 23	115 17	115 11	115 04	114 98	114 91	114 85	114 78	114 72	114 66
30	111 61	111 55	111 49	111 43	111 37	111.31	111 25	111 19	111 13	111 07	111 01	110 95
31	108 10	108 05	107 99	107 93	107 88	107 82	107 76	107 70	107 64	107 59	107 53	107 48
32	104 80	104 75	104 70	104 65	104 59	104 54	104 48	104 43	104 38	104 33	104 27	104 22
33	101 71	101 66	101 61	101 56	101 51	101 46	101 41	101 36	101 31	101 26	101 21	101 16
34	98 78	98 73	98 68	98 64	98 59	98 54	98 49	98 45	98 40	98 36	98 31	98 27
35	96 03	95 98	95 94	95 89	95 86	95 81	95.77	95 72	95 68	95 64	95.60	95 56
	48"	49"	50"	51"	52"	53"	54"	55"	56"	57"	58'	59"

§ XXI MICROMETRICAL SCALE WITH A CONSTANT MAGNIFYING POWER

1. The most simple, as well as the cheapest micrometer in existence, is a plain divided scale of mother-of pearl, glass, or other transparent substance, that is capable of receiving the dividing strokes. Such a scale, divided into the hundredths or two-hundredths of an inch, and placed in the common focus of the object and eye-glasses of a good celestial telescope, affords the means of obtaining the measure of a terrestrial angle, within the accuracy of a few seconds, even under the unfavourable circumstance of our being obliged to estimate the fractional portion of a division. In astronomy, also, it may sometimes be used with advantage, when the body emits light enough to render the dividing strokes visible by night, or when adventitious light is introduced into the telescope for this purpose. We do not confine our remarks to the simple scale of two parallel lines, divided and subdivided into spaces by long and short strokes, that may be easily counted, but we include all reticulated divisions made in the form of squares, right angled or otherwise, where the divisions are equal among themselves, and invariable in their values, which such divisions will always be with the same telescope, used with the same magnifying power, and taken as the measure of a distant object. The value of such a scale may be ascertained from either celestial or terrestrial objects, when the angle is previously known, that is measured by a certain number of its divisions. When the sun is used, the telescope will of course be adjusted to the solar focus, at the moment of taking the measure, and the value of a division obtained from a comparison of his vertical diameter, corrected for the effect of refraction, with the divisions of the scale, will be the true value, and as his semidiameter is given in the Almanacs for every day in the year, the number of seconds covered by one division will be readily had from the whole number of seconds, composing the diameter at the time, divided by the whole number of divisions and fractional parts, that will just measure this diameter. The horizontal diameter of the sun, which would require no reduction for refraction, is not easily measured, as we before noticed, by reason of his motion in right ascension through the telescope.

2. When a terrestrial subtense of an angle is used, the angle will vary inversely as the distance, and the telescope will require to be elongated, by means of its drawer, to have distinct vision of a near object, hence the angle, measured by the solar scale, will be too great by a quantity depending on the elongation of the telescope, as is the case with the spider's-line micrometer (§ XX. 5. 6), therefore the value of the scale obtained from a terrestrial object, placed at a near distance, will not be correct for astronomical purposes. If the solar focus of the object-glass be denoted, as before, by f, and the elongation of the telescope, when distinct vision is obtained, be called e, then the elongated focus will be $f + e$, for which we may, as before, substitute f', and then the magnifying powers of the telescope, for the sun and for the terrestrial object, will be to each other directly as $f : f'$, when the same object glass and eye-piece are used. The quantity e may be measured on the drawer of the telescope, when successively adjusted for vision of the sun, and of the terrestrial object as we have already stated, but may be more accurately obtained by the well known equation before exemplified, $\frac{f^2}{d-f} = e$.

3. Now the image of an object placed at any distance will increase with the increase of the magnifying power of the telescope, which, with the same eye-piece, varies with the focal length, and more divisions will be required to measure the larger than the smaller image, but we have seen that the tabular value of a single division on any scale, is found by dividing the whole known angle by the number of divisions that will just measure it. If, therefore, we put a for the angle to be measured, which is previously known, n' for the number of divisions that will measure an angle subtended by the larger image, and n for the number that will measure the smaller image, formed in the solar focus, we shall have $f' : f :: n' : n$, and $f'n = fn'$, also $\frac{fn'}{f'} = n$, and $\frac{f'n}{f} = n'$. We shall have also $\frac{a}{n'} = v'$, and $\frac{a}{n} = v$, the two respective values of a division of the scale corresponding to the focal lengths f' and f.

4. As an example; when an exact yard, painted white and divided by black strokes, is erected vertically at the distance of 98.25 yards from the object-glass of our refracting telescope (No. 5.) of 76.25 inches solar focal length, and when distinct vision is obtained by drawing out the small tube, let it be required to determine, what is the quantity of the elongation (e); what the error in the measurement of an angle subtended by nine inches, as then measured by the solar scale, and what the proper values of the same scale, corresponding to f' and f respectively, when 40.4 divisions of the scale just cover nine inches?

By our Table given in the last section, a single yard, at the distance of 98.25 yards, will subtend an angle of 34′ 59″.4, or 34′.99; and 76.25 inches are equal to 2.118 yards; then $\frac{2.118 \times 2.118}{98.25 - 2.118} = \frac{4.485924}{96.132} = 0.04666 = e$; and $\frac{2.118 + 0.04666 \times 34.99}{2.118} =$ 35′.7618, or 35′ 45″.7080 will be the angular measure of the yard, belonging to the elongated focal length f', one-fourth of which is 8′ 56″.425, the measure due to nine inches at the said focal length; but the measure due to the solar focal length f, is $\frac{34' \; 59''.4}{4} = 8' \; 44''.85$, so that the difference, or error, belonging to the measurement at 98.25 yards, is 11″.575 In the next place we have $\frac{a}{n'} = \frac{524''.85}{40.4} = 12''.99 = v'$,

$$\text{and } \frac{fn'}{f'} = \frac{2.118 \times 40.4}{2.16466} = 39.53 \text{ divisions} = n,$$

hence we have $\frac{a}{n} = \frac{524''.85}{39.53} = 13''.276 = v$ If we suppose that, by shortening the distance, the image of the object increases in the same proportion as the divisions of the scale are required to be more numerous, and call the increased angle 8′ 56″.425 $= a'$, then $\frac{a'}{n'}$ will be $= \frac{a}{n}$, and $\frac{536''.425}{40.4} = 13''.277$ will also be the solar scale.

5. The focal length, multiplied by the value of a division of its proper scale, will always be a constant quantity, whatever that focal length may be, provided that an invariable celestial eye piece be used. Thus when the quantity e is reduced into inches, it will be 1.68, which added to 76.25, the solar focus f, will make the elongated focus $f' = 77.93$ inches, which will give 77.93 × 12″.99 = 1012″.31, as the constant product; while 76.25 × 13″.277 will also pro-

duce 1012.37; and with respect to the measures, we have 40.4 × 12".99 = 524".796, and 39.53 × 13".277 = 524".739. In this instance the value of the scale could not have been derived directly from the sun's diameter, because, as the magnifying power was 162, the field of view was much too small to contain the whole of it.

When we say that the product of the focal length into the value of a division of the scale is a *constant quantity*, we mean not to confine this quantity as belonging exclusively to one telescope, but to take it as a quantity by which a value may be given to the same scale when applied to any other telescope, whatever may be its focal length; for this constant quantity 1012".3, divided by the focal length of any other telescope, will give the value of a division on the said scale. To show that this is the case, in practice as well as theory, we may subjoin the following comparative measures, taken by a scale of 203.7 divisions in the inch, with the telescopes No. 5. and No. 2 successively with the former instrument the constant product is 76.25 × 13".277 = 1012".37, and with the latter the value of one division on the scale will be $v = \frac{1012.37}{43.2} = 23''.43$. The larger telescope, having the divided scale in front of a positive eye piece, was first directed to a small newly-painted window, at the distance of more than a mile, the breadth of which was measured by exactly eight squares of the vitreous disc, and 8 × 13".277 = 106".216 was the measure thus obtained; the same eye-piece was then applied to the smaller telescope, and when the adjustment for good vision was made, the same window, viewed from the same station, was measured by 4.5 divisions, as nearly as the eye could estimate, and 4.5 × 23".43 = 105".43 was the resulting measure, differing about three quarters of a second from the former measure. The spider's-line micrometer was then applied to No. 5. telescope, and the same object was measured by 4.075 revolutions of the screw, which, at 26".1 per revolution, gave the measure 106".3575, or about a quarter of a second more than the divided scale produced, when applied to the same telescope.

6. A value may also be given to a divided scale by any of the methods we explained in our last section, if we substitute a division on the scale for a revolution of the screw thus by our fourth method we have the following computation abridged. viz.

$$\frac{1296000''}{76.25 \times 2 \times 3.1416 \times 203.7} = \frac{1296000''}{97591.45} = 13''.279.$$

If we want the number of divisions in the inch, from the determined value of the single division, taken at 13".277, by reversing the operation we shall have

$$\frac{1296000''}{76.25 \times 2 \times 13.277 \times 3.1416} = \frac{1296000''}{6360.931} = 203''.74.$$

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§ XXII. ON THE DIFFERENT METHODS OF ILLUMINATING THE LINES IN THE EYE-PIECE OF A TELESCOPE [Plates IV VII XIV]

1. In our two preceding sections it has been presumed, that the spider's lines and scales in the micrometers will be visible in all celestial observations, made by night as well as by day, whereas the fact is, that they are hid in darkness, except when the moon or some of the larger planets

are observed, in all nocturnal observations, until some adventitious light is introduced into the small tube that contains the micrometer, and the different modes, by which the illumination has been effected, will form the subject of this section. The first method of introducing light into the telescope was by means of a reflector of white paper, held by a small frame attached to the object-end of the main tube, in which it turned on pivots, like a small swing-glass, to any angle that the situation of a candle or lamp might demand, to reflect the rays up the tube, and that the object to be viewed might not be excluded from the telescope, an oval hole was made at the centre of the reflector, such as would appear circular, when placed in a proper degree of inclination. This contrivance was in use in Bird's time, and the quadrants, which we have seen of his construction, were supplied with such an appendage. When achromatic object-glasses became general, their enlarged apertures would not admit of being thus closed, without suffering great loss of light, and a less objectionable mode of introducing additional light became a desideratum.

2. When Dr. Usher, of the Dublin Observatory, had a new transit-instrument made by Ramsden, he proposed to have the reflector removed from the object-end of the telescope to the middle, where the cone of light converging from the object-glass would have the diameter of its section diminished one half, and to make the axis of the telescope hollow with tubular pivots, to admit of the light of a lamp passing through to the reflector, when placed at an angle of 45°, and perforated as before. This plan was approved and adopted with success, and has continued to be the usual mode of illuminating a transit-instrument under certain limitations.

3. Soon after telescopes had been shortened, by the achromatic construction of the object-glass, the wire micrometer began to be applied to them, and it became necessary to illuminate their lines in the way the transit-instrument had been made to enlighten its lines in the eye-piece. If we mistake not, Troughton first opened a circular hole in the side of the main tube, above its centre of motion in altitude, and placed an oval ring of gilt metal, deadened so as to reflect a mitigated light up the telescope, in a proper angle of inclination within the tube, which plan is still in use, and is found much more convenient than when the reflector was situated before the object-glass, because in every degree of elevation of the telescope, the lamp, as in the case of the transit-instrument, now remains in a stationary position, and yet gives the same illumination in kind, if not in quantity.

4. In many observations where small stars are the objects, the light, that is necessary for rendering the spider's lines visible, conceals the stars from the view of the eye, and renders the observations extremely difficult, nay often impracticable; in these cases the quantity of light admitted requires to be limited by contracted diaphragms, or by darkening glasses of different shades of colour. Ramsden was accustomed to apply wedges of dark glass between the lamp and the hole into which the light was received, which, by sliding across it, modified the light to suit the observation; and sometimes similar wedges have been applied before the eye-piece to answer the same purpose. Troughton contrived an iris, or variable diaphragm, for the large transit-instrument at Greenwich, which was intended to diminish the quantity of light reflected from the lamp, but which does not seem to have been much used; and Dollond has contrived a method of excluding a portion of the light by applying a variable square opening before the lamp, and sometimes before the hole perforated in the main tube of the telescope. Fraun-

hofer has also invented a lamp that throws light obliquely across the divisions of his glass micrometers, but in a forward direction, which renders the strokes, cut in a peculiar manner on the face of the glass, visible in a dark field, but as this artist's method of making his lines is not well known in England, we have succeeded in effecting the same purpose by giving an oblique direction to rays that are thrown in a backward direction down the tube, so as never to reach the eye. These three last contrivances will be comprehended from the brief descriptions which we propose to give in this section.

5. DOLLOND'S LANTERN.—Fig. 3 of Plate XIV. presents a front view of Dollond's lantern, for limiting the illumination of a transit-instrument, to which it is seen applied in fig. 1 of the same plate, affording a side view, in both of which the Greek characters are made the letters of reference. α shows the body of the lantern, and β its chimney, bent back to take the heat from the telescope; it contains a small Argand's lamp within, which is fed with air by the tube γ, ascending through the bottom, in which are made eight large circular holes, for admitting a still greater supply of air. A milled nut δ, inserted on the end of a horizontal piece of wire, carries a pinion ε, which is seen in fig. 3 acting with two racks at the same time; the projecting part of the lantern carries a solid frame of brass, in which two thin brass plates, lying close one upon the other, will slide up and down in a vertical direction, and having each a rack on the side of a long central opening, admit the pinion ε between them in such way, that when it is turned round, the plates will both move in contrary directions; the ends of these racked plates are seen at ζ and η, and a little below their upper ends they have each a square hole cut of the same dimensions, each side of which is just an inch. When the two plates cover one another their whole length, the two square holes coincide, and allow the light of the lamp to pass through the whole square inch; but as the pinion gives motion to the plates, the diagonal of the square, which lies in a vertical line, gradually diminishes, and the two large squares, moving in contrary directions, still form a diminished square as seen in fig. 3, where the dotted lines show the concealed portion of the interior square while the whole of the exterior one remains visible, though not all open, in this situation as much light only comes from the lamp as can pass through the diminished square, which may be reduced till the whole light is excluded. In a common sized instrument the hand can reach the nut δ, while the eye is viewing the object in the telescope, and can thus regulate the quantity of light that is necessary to be admitted into the axis, for the requisite degree of illumination, as it regards both the spider's line and the object viewed. The lantern carries two small pillars, ι and κ, which enter two holes in the frame of the transit instrument in such way that the centre of the variable square hole is always opposed to the tubular part of the pivot, and therefore the light enters centrally, whatever may be the elevation of the telescope. In addition to this diminution of the quantity of light, the quality is also modified by green or other dark glasses, placed within the projecting part of the lantern, exclusively of a large lens that is intended to make the rays proceed parallel, converging, or diverging, as the adjustment for distance from the lamp may occasion, and when there is a lens moreover in the extreme end of the pivot of the transit's axis, a further modification may be effected, of the nature and extent of which the eye must be the sole judge.

6. From this description it is obvious, that the sliding plates producing the variable square hole may be placed, together with their frame, on the outside of the main tube of a

telescope, and that a long handle, lying parallel to the said tube, may be so placed as to give motion to the pinion, this we understand was the contrivance applied to the telescopes used by Messrs. Herschel and South, when they made their observations on the double stars.

7. Jones, of Charing Cross, has given the sliding plates a cylindrical shape, similar in curvature to the lantern, and has included the whole apparatus within the body of the lantern, so that nothing but the milled nut appears externally, and the whole figure is that of an ordinary lantern; which alteration simplifies the appearance, and retains the property of limiting the quantity and quality of the transmitted light.

8. Fraunhofer's Lamp.—To avoid the inconvenience occasioned by light transmitted directly up the tube of the telescope, the late celebrated optician of Munich contrived to make lines on glass that will become visible on their edges, by means of oblique rays being made to fall on them, while the field of the telescope remains dark, and exhibits very small stars that would become invisible in an illuminated field From the experiments that have been made in London to imitate the strokes made on this artist's micrometers of glass, we have reason to infer, that they are not cut by a diamond, but by etching with the fluoric acid, and by preparing an etching ground of such consistence, as is best suited for the kind of glass that is exposed to the gas, which must contain a large portion of the oxyde of lead. A tube of brass is fixed obliquely to the eye piece of a long telescope, as represented in fig. 2. of Plate VII., and receives a second piece of smaller tube within it, which carries the small lamp, that is contrived to be very light and suitable for containing ardent spirits and a small wick, in such way that whatever may be the elevation of the telescope, the mode of suspension will keep the wick vertical, and a lens in the outer end of the smaller tube, which turns round to any position, condenses the rays, thus obliquely directed, upon the divided face of the glass. The lamp is surrounded by a cylindrical box, one end of which, turned towards the face of the observer, darkens the lamp, and at the same time guards the face from the heat that would otherwise scorch it. The lamp is seen detached in fig. 3. of the plate already referred to, the parts of which may be understood from an inspection of the enlarged figure.

9. Tulley's new Illuminator.—When we had inspected one of Fraunhofer's lamps, put into our hands by a friend in London, who procured one from the Continent, it occurred to us, that the lamp might be removed further from the eye, by applying it to a terrestrial tube instead of a celestial one, and also that the reflected rays might be reflected backwards down the main tube, so as to leave a darker field than could be obtained by throwing the rays obliquely upwards into the interior face of the eye-tube; and on applying to the younger Tulley to execute our plan, he accomplished the construction by placing a concave reflecting ring within the principal tube, which being gilt with dead gold-leaf produces a yellow tint on the lines of a divided disc of glass, cut with a diamond by Turrell, the late pupil of Lowry, to whom we are indebted for the correct execution of most of our engravings. The representation of this illuminated terrestrial eye-piece is given at figs. 5. and 6. of Plate IV., the former of which shows the external appearance of the eye-piece with the lamp attached, and the latter is a longitudinal section of the same, which explains the situation of the different parts. This eye-piece is composed of three tubes, of which *a* is the eye-end of the external one, which has a screw at the internal end attaching it to the telescope, *b* is the visible end of the second tube that holds the pair of field-glasses at each side of *d*, and *c* is the end of the third

tube that holds the pair of eye-glasses, *e* is the diagonal reflecting ring that illuminates the lines on the glass disc *f*, and *g* is a circular hole that admits the light of the lamp *k* that hangs in the small frame or semi-gimbal *i*, that has its motion of adjustment in the small tube *h*, attached to the main tube *a*, by means of which the lamp can be put into its horizontal position in any degree of elevation of the telescope, to which the eye-piece is screwed fast When the tube *c* is drawn out of tube *b*, it separates the two pairs of glasses, and thereby alters the magnifying power of the telescope to what may be required to obtain the measure of an object on the face of the divided disc *f*, and distinct vision of the lines, and if the first image of the object is adjusted by drawing out or pushing in the second tube *b*, when the second image of the object and first image of the disc will be seen inverted between the eye-lenses, and as the divided spaces on the vitreous disc and the first image of the object are alike magnified, there will be no alteration in the value of the divisions, whatever may be the alteration in the magnifying power. This illuminated eye-piece will apply to any refracting, Newtonian, or Herschelian telescope, and will afford the means of measuring the distance between two very small stars in a field that is opaque, except that the strokes made on the glass are rendered visible as a scale of measurement. If the divided disc had a circular motion to put its horizontal lines into any degree of inclination, by means of a graduated adapter, it might be used also as a position-micrometer for determining the relative situations of small stars or satellites that will not bear the light of an illuminated field.

10 If this eye-piece were substituted for the positive eye piece in a spider's-line micrometer, when the disc is displaced, it would illuminate the lines sufficiently to be seen in the opaque field, and would at the same time give the natural position of the stars or other bodies observed. The first hint for adopting the use of a retrograde reflector was given us by Mr. Mossotti, to whom it had been previously suggested by Professor Pictet of Geneva.

11. We have sometimes had occasion to use a reflecting inclined ring in the interior part of a terrestrial tube, where the light of a lamp has been admitted at an aperture in the side of the tube, between the two pairs of lenses, and has been thrown in a small quantity towards the eye, just sufficiently to render the spider's lines visible, when a Troughton's micrometer is substituted for an eye-piece, to measure the second image of an object, according to the plan above explained (§ XIX. 8.), but this method of illuminating is both inconvenient and imperfect.

§ XXIII A NEW POLYMETRIC RETICLE. [Plate III.]

1. When Romer's reticle, Cavallo's divided slip of mother-of-pearl, or the disc of graduated glass described in our twenty-first section, are any of them fixed in the eye-piece of a telescope, for the purpose of measuring small angles, it seldom happens that an entire number of divisions will exactly measure the object of examination, and as the fractional part of a division must necessarily be estimated, the accuracy of the whole measure will be vitiated by such estimation. We have therefore contrived a divided glass micrometer, that will not only render a certain number of divisions exactly commensurate with the linear dimensions of the object to

be examined, but will admit of a succession of such numbers to be made separate measures, which will check one another, and afford an average of several independent measures for a final result, on which account we have denominated it *polymetric*. This desirable property depends on a variation given to the magnifying power of the telescope, while the divisions of the scale remain the same; for it is as easy to vary the apparent diameter of the object to be measured, till it just covers a certain number of divisions on the scale, as it is to open the spider's-lines, till they will include the object between them, except that, in altering the magnifying power of a variable eye-piece, such as we have described in § VI, it becomes necessary to repeat the adjustments for the measure, and for distinct vision, alternately, till the eye is satisfied in both respects. The variable celestial eye-piece represented in figures 15, 16, and 17 of Plate III. is well adapted for receiving a divided disc of glass in the focus of the eye-lens *b*, and the variation of the magnifying power is produced by separating the field lens *a*, by means of the rack and pinion *c*, that move the inner tube; while the single stroke on the sliding index points out the quantity of separation on the divided scale of the outer tube, to which the graduated circle *d d*, of 3½ inches diameter, is attached. When it is not required to ascertain the angle of position, that any given line makes with the horizontal or vertical line, this circle may be disregarded. The eye-lens and a disc of divided glass, which we have named a reticle, are in separate cells of brass, that screw into opposite ends of a short tube, so as to be adjustable for good vision of the lines, and this disc-piece may screw into the place of the eye-lens *b*, when the circle is not wanted, but when the angle of position, as well as the linear distance, is required to be measured, it must be made to slide into a socket, that screws into the vernier piece shown in fig 17, which is detached from its place in fig. 16, for the purpose of affording better explanation. This socket will also receive a set of prisms of rock crystal, instead of the disc-piece, to form a double image micrometer, for which it was originally destined, as will be explained hereafter. We have applied two separate disc-pieces, one, screwing into the centre of the circle, constitutes the smaller of two series of magnifying powers, and the other, sliding into the vernier piece, affords a series of higher powers; but both are used with the same field-lens *a*; the eye-lens *b*, seen in the figure, being that which is used with the prisms for another purpose.

2. When the socket of the vernier, fig. 17, is applied to the ring, fixed to the face of the graduated circle *d d*, which it fits by friction, it may be turned round by the projecting pins *e e*, and the vernier, being in close contact with the limb of the circle, will read to single minutes of a degree. When the vernier is put to zero of the circle, which is divided into four quadrants, by half degrees, the disc-piece is turned in its socket, till the vertical lines coincide with a plumb-line, while the former is viewed in the eye-piece with one eye, and the latter out of the telescope with the other; then turning the vernier round by the pins *e* and *e*, will put the dividing strokes of the reticle into any oblique situation that an observation may require, and the angle moved over will be indicated by the vernier *f*, which will be the angle of position. When the angle formed by a line joining two stars with the horizontal or vertical line is to be measured, the parallel lines, that constitute the 50 squares in each direction, being put into the required angle of position, by gradual adjustment, when the telescope is following the stars on an equatorial stand, are very convenient for giving a succession of opportunities for judging of the final position of the reticle, on which the true measure must depend; for if the stars are near each

other, so as to be seen passing at the same time in pretty close succession, each line will receive the ingress of both stars at the same instant, as many times as there are lines to be passed over, and the observer has time enough to satisfy himself, that the angle of position is truly obtained.

3 The scale on the outer tube, in fig 15, is about an inch and a quarter, divided into 125 parts, though the scale engraved there shows only 110, and a hole made at the end of the tube, where it is joined to the circle, allows the light of a lamp to fall obliquely on the reticle, to illuminate the dividing strokes made by a fine diamond, of which there are 203 .7 in the inch, inserted by Turrell's engine. When linear distances are measured between two stars, or other objects, either a vertical or horizontal line must be turned by the vernier piece, or sliding tube, if the vernier is not used, till it passes through both stars, and if any certain number of spaces on the reticle happen to measure the interval, representing the included arc of a great circle, the position of the index, on the scale of the micrometer, will be the argument with which the table, containing the corresponding factor, must be entered, and the tabular number of seconds and decimal parts given in the proper column, in the same horizontal line, must be multiplied by the number of spaces that just measured the said interval, and the product will be the whole measure of the distance in seconds of a great circle · for when the angle is small, the arc and chord may be considered as the same thing. But it will seldom happen that an exact measure can be had without altering the magnifying power of the eye-piece, and when this is altered by turning the pinion on the axis of the milled nut *c*, there will be different numbers of divided spaces on the reticle, that will be successively commensurate with the small arc to be measured; and each position of the index will have a corresponding tabular factor, so that as many products may be taken as the variations of power made use of will afford. We have computed a table for each disc piece numbered 1 and 2, the former of which applies to the vernier-piece, and the latter screws into the centre of the divided circle, when the vernier is detached; the method of constructing which tables we will now explain.

4. When the variable eye-piece was applied to our refracting telescope, No. 5, having a focal length of 76 .25 inches, and the aperture limited to 3 .24 inches, with a good Dollond's double-image dynameter we took the magnifying powers at the positions 0 and 100 of the scale, as explained in a former section (§ XI), and found them $\frac{3.24}{.023} = 140.87$, and $\frac{3.24}{.0354} = 91.53$ respectively, the difference between which is 49 .34 in 100 spaces, or .4934 in one space, then having prepared the columns of Table I, and inserted the arguments from 0 to 125, we began at argument 100, and put 91 .53 as the power at that part of the table, and by continually adding 0 .4934, that is, by applying 0 .49 twice, and 0 .50 once successively, as we ascended the column, we arrived at the beginning of the table with the proper numerals 140 .87, and then descended from 100, by subtracting, till we arrived at the lower end, where 79 .20 stands opposite argument 125.

5 Having now obtained the column of magnifying powers corresponding to the scale of equal parts on the micrometer, we put their logarithms opposite to them in column 3, and having previously ascertained that the magnifying power of the telescope multiplied by the value of a single division of the reticle's scale (P V), will always be a *constant product*, whatever the argument may be, which product in this case is 1914″.4, the logarithmic values (log. V.) were obtained by subtracting the successive logarithms in column 3 from 3 .28203,

the logarithm of the constant, and inserted in column 4, of which the natural numbers are given in column 5, as the proper factors to be used with their corresponding arguments in every instance.

6 In like manner Table II. was constructed for the disc-piece No. 2. that has squares of similar dimensions, the magnifying powers, at the positions or arguments 0 and 100, were found by the dynameter to be 107.61 and 69.51, and the difference in 100 divisions of the reticle's scale 38.1, or 381 in each unit, from the continual application of which, in the manner that has been explained, the column of powers was completed, and afterwards the other three columns by means of the logarithm 3.16524, which is the logarithm of the constant product 1463″, that had been previously ascertained to be the product due to the lens of smaller power. The tabular quantities thus arising are contained in Table II, and we have given the columns of logarithms in both Tables to show that, though the powers are materially different with the two lenses in the different Tables, yet the *factors* in column 5., corresponding to the different arguments, are very nearly the same, and would perhaps have been precisely so, if the two lenses had occupied exactly the same place in the micrometer, as it has reference to the field lens *a* in the different positions. We were at first surprised to find such similarity in the factors of the two Tables, with such different magnifying powers; but when we recollected that these powers vary as $F+f-d$ (§ VI. 2.) in the eye-piece now used, and that F and d are common to both Tables, while f alone differs, and when we considered that the eye-lens represented by f magnifies the object, or rather its image, and also the reticle in the same proportion, at every position of lens *b*, we perceived that the eye-lens has nothing to do with the factors contained in the fifth column, though it is concerned in the whole magnifying power as determined by the dynameter. The values of the factors are modified by the focal lengths of the object-glass and field-lens, in conjunction with its distance from the eye lens, while the last only assists the natural power of the eye, in rendering the image and scale visible at a short distance, and therefore under a like enlarged visual angle.

7. The subjoined Tables, computed for our reticulated micrometer and telescope 5., to which they are exclusively appropriated, will serve as models for the construction of similar Tables for any other telescope, and their construction and use may be further illustrated by the following examples. When the disc-piece numbered 1. was inserted into the vernier piece, and applied to the telescope, a remote object that subtended an angle of 564″.5, as measured by a spider's-line micrometer, was viewed when the index stood at position 100, and just covered 27 divisions, but when the index was moved to position 0, the same object was measured by 41.54 divisions, as nearly as could be estimated; now $\frac{564.5}{27} = 20''.916$ was the value of one division of the reticle at position 100, and $\frac{564.5}{41.54} = 13''.590$ was its value at zero, therefore we had $140.87 \times 13''.59 = 1914''.4 = P\,V$, and $91\ 53 \times 20''.916 = 1914''.4 = P\,V$ also, hence this constant product was taken as the basis of the first Table; and the constant 1463″ was determined in like manner for the basis of the second Table, when the second disc-piece with smaller powers was applied. After the Tables had been completed from these data, as above explained, we directed the telescope to a small window at more than a mile's distance, and measured four panes, included within the frame that was painted white, with both the

disc-pieces in succession, at three different positions of the index, and obtained the following measures

With Table I. and its disc-piece.

$$\text{At position}\quad \left.\begin{array}{r} 30\ldots\ldots\ldots 7\times 15''.185=106''.295 \\ 66\ldots\ldots\ldots 6\times 17\ .677=106\ .062 \\ 103\ldots\ldots\ldots 5\times 21\ .259=106\ .259 \end{array}\right\} 106''.217.$$

With Table II and its disc-piece.

$$\text{At position}\quad \left.\begin{array}{r} 30\ldots\ldots\ldots 7\times 15''.211=106''.477 \\ 65\ldots\ldots\ldots 6\times 17\ .660=105\ .960 \\ 102\ldots\ldots\ldots 5\times 21\ .283=106\ .415 \end{array}\right\} 106''.284.$$

8. These measures were taken in the morning before the sun had emerged from the clouds near the horizon, when the refraction was steady; and the right-hand edge of both the lines was used in making the contacts, that the thickness of the line might not be included in the measure, which is necessary to be attended to in all cases; but the distance between two stars, or other bodies in apparent motion, cannot be taken very accurately, unless they have the same right ascension, so as to allow them to pass along two parallel lines, when the distance has been magnified so as to produce the exact coincidence of both stars with some pair of lines, including a certain number of divisions. When the object viewed is so near as to require the telescope to be lengthened beyond the solar focal length, the angle obtained from either of the Tables will be too great, and must be diminished by $\frac{f}{f'}$, as we have already explained (§ XX. 5. 6).

9. La Lande has given a short notice of a reticulated micrometer of Romer, contrived for observing eclipses with, in which the twelve divisions are capable of being enlarged or diminished, till they will just include the diameter of the sun or moon, so as to divide it exactly into digits, whatever the number of minutes and seconds may be at the time; this alteration in the digits was effected by applying two object-glasses in the tube of the telescope, one fixed and the other moveable, a separation of which lengthened the compound focal distance, and enlarged the magnifying power, and with it the size of each of the twelve divisions that represented the digits *. The tube which held the micrometrical eye-piece was consequently made long, to accommodate itself to the total length of the telescope in each relative position of the two object-glasses.

10. Whether or not this construction was known to Dr. Brewster when he applied two object-glasses in a similar manner to his patent telescope, we do not presume to say; but his introduction of two parallel lines, and also of two fixed points, to include the measured angle, when a due change has been produced in the magnifying power, depends on precisely the same principle, and may be applied to the same purpose. If a reticle were substituted for the two parallel lines, or fixed points, as is done in our micrometer, the polymetric plan of taking several successive measures with such telescope would be equally feasible, though less convenient, particularly with a powerful instrument, where the total focal length must necessarily be greatly altered.

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* Astronomie, Tome II. Art 2357

TABLE I

OF THE

POWERS OF A TELESCOPE OF 76 25 INCHES FOCAL LENGTH,

WITH THE CORRESPONDING VALUES OF A FIXED SCALE OF $\frac{1}{100}$TH PARTS OF AN INCH, THAT VARY INVERSELY AS THE POWERS, WITH EYE LENS No 1

(P × V = THE CONSTANT NUMBER 1914″ 4)

S	P	Log P	Log V	V	S	P	Log. P	Log V.	V.	S.	P.	Log P	Log. V	V
0	140 87	2 14882	1 .13321	13″ 590	42	120 14	2 07969	1 20234	15″ 935	84	99 42	1 99747	1 28456	19″.256
1	140 .38	2 14730	1 13173	13 637	43	119 65	2 07791	1 20412	16 000	85	98 92	1 99528	1 28675	19 353
2	139 88	2 14576	1 .13627	13 686	44	119 15	2 07609	1 20594	16 067	86	98 43	1 99312	1 28891	19 450
3	139 39	2 14423	1 13780	13 734	45	118 66	2 07430	1 20773	16 134	87	97 94	1 99096	1 29107	19 .547
4	138 90	2 14270	1 13933	13 .783	46	118 17	2 07250	1 20953	16 .201	88	97 44	1 .98874	1 .29329	19 .647
5	138 40	2 .14113	1 14090	13 833	47	117 68	2 07070	1 21133	16 268	89	96 95	1 98655	1 29548	19 746
6	137 91	2 .13959	1 14244	13 882	48	117 18	2 06885	1 21318	16 337	90	96 46	1 98435	1 29768	19 846
7	137 42	2 13805	1 14398	13 931	49	116 69	2 06703	1 21500	16 406	91	95 96	1 .98209	1 29994	19 950
8	136 92	2 13647	1 14556	13 982	50	116 20	2 06520	1 21683	16 475	92	95 47	1 .97987	1 30216	20 052
9	136 43	2 13491	1 .14712	14 005	51	115 70	2 06333	1 21870	16 546	93	94 97	1 .97759	1 30444	20 158
10	135 94	2 13336	1 14866	14 083	52	115 21	2 06149	1 22054	16 617	94	94 48	1 .97534	1 30669	20 262
11	135 44	2 13175	1 15028	14 135	53	114 72	2 05964	1 22239	16 688	95	93 99	1 .97308	1 30895	20 368
12	134 95	2 .13017	1 15186	14 186	54	114 23	2 05778	1 22425	16 759	96	93 .50	1 .97081	1 31122	20 .475
13	134 46	2 12856	1 15347	14 239	55	113 73	2 05587	1 .22616	16 833	97	93 01	1 .96853	1 31350	20 583
14	133 96	2 12697	1 15506	14 291	56	113 24	2 05400	1 22803	16 906	98	92 .51	1 96619	1 31584	20 694
15	133 46	2 12535	1 15668	14 344	57	112 75	2 05212	1 22991	16 979	99	92 .02	1 .96388	1 31815	20 804
16	132 97	2 .12375	1 15828	14 397	58	112 25	2 05019	1 23184	17 .055	100	91 53	1 96156	1 32047	20 .916
17	132 48	2 12216	1 15988	14 451	59	111 76	2 04829	1 .23374	17 129	101	91 03	1 .95918	1 32285	21 031
18	131 98	2 12051	1 .16152	14 505	60	111 27	2 04638	1 .23565	17 205	102	90 .54	1 .95684	1 .32519	21 144
19	131 49	2 11889	1 16314	14 559	61	110 77	2 04442	1 23761	17 283	103	90 .05	1 95448	1 .32755	21 259
20	131 00	2 11727	1 16476	14 614	62	110 28	2 04249	1 23954	17 360	104	89 55	1 95206	1 32997	21 378
21	130 50	2 11561	1 16642	14 670	63	109 78	2 04052	1 24151	17 439	105	89 .06	1 94968	1 33235	21 .496
22	130 01	2 11398	1 .16805	14 725	64	109 29	2 03858	1 24345	17 517	106	88 57	1 94728	1 33475	21 615
23	129 51	2 .11230	1 16973	14 782	65	108 79	2 03659	1 24544	17 .597	107	88 .08	1 94488	1 .33716	21 735
24	129 02	2 11066	1 17137	14 838	66	108 30	2 03463	1 24740	17 677	108	87 58	1 94240	1 33963	21 859
25	128 53	2 10900	1 17303	14 895	67	107 81	2 03266	1 24937	17 757	109	87 09	1 93997	1 34206	21 982
26	128 04	2 10734	1 17469	14 952	68	107 31	2 03064	1 25139	17 840	110	86 59	1 93747	1 34456	22 109
27	127 55	2 10568	1 17635	15 009	69	106 82	2 02865	1 25338	17 922	111	86 10	1 93500	1 34703	22 235
28	127 05	2 10397	1 17806	15 068	70	106 33	2 02665	1 25538	18 005	112	85 60	1 .93247	1 34956	22 365
29	126 56	2 10230	1 17973	15 126	71	105 83	2 02461	1 25742	18 089	113	85 11	1 92998	1 35205	22 493
30	126 07	2 10061	1 18142	15 185	72	105 34	2 02259	1 25944	18 174	114	84 61	1 92742	1 35461	22 626
31	125 57	2 09889	1 18314	15 246	73	104 85	2 02057	1 26146	18 258	115	84 12	1 92490	1 .35713	22 758
32	125 07	2 09715	1 18488	15 307	74	104 36	2 01853	1 26350	18 344	116	83 63	1 92236	1 35967	22 891
33	124 58	2 09545	1 18658	15 367	75	103 86	2 01645	1 26558	18 432	117	83 14	1 91981	1 36222	23 026
34	124 08	2 09370	1 18833	15 429	76	103 37	2 01439	1 26764	18 520	118	82 64	1 91719	1 36484	23 165
35	123 59	2 09198	1 .19005	15 490	77	102 88	2 01233	1 26970	18 608	119	82 15	1 91461	1 36742	23 303
36	123 10	2 09026	1 19177	15 552	78	102 38	2 01021	1 27182	18 699	120	81 66	1 91201	1 37002	23 443
37	122 61	2 08853	1 19350	15 613	79	101 89	2 00813	1 27390	18 789	121	81 17	1 .90939	1 .37264	23 585
38	122 11	2 08675	1 19548	15 685	80	101 40	2 00603	1 .27600	18 880	122	80 67	1 90671	1 .37532	23 731
39	121 62	2 08500	1 19703	15 741	81	100 90	2 00389	1 27814	18 973	123	80 18	1 90406	1 37797	23 877
40	121 13	2 08325	1 19878	15 805	82	100 41	2 00178	1 28025	19 066	124	79 69	1 90140	1 38063	24 023
41	120 63	2 08146	1 20057	15 870	83	99 91	1 99960	1 28243	19 162	125	79 20	1 .89872	1 38329	24 171
S	P.	Log. P	Log V.	V	S	P	Log P	Log V.	V	S	P.	Log P	Log V.	V

TABLE II

OF THE

POWERS OF A TELESCOPE OF 76 25 INCHES FOCAL LENGTH,

WITH THE CORRESPONDING VALUES OF A FIXED SCALE OF $\frac{1}{361}$TH PARTS OF AN INCH, THAT VARY INVERSELY AS THE POWERS, WITH THE EYE-LENS No 2

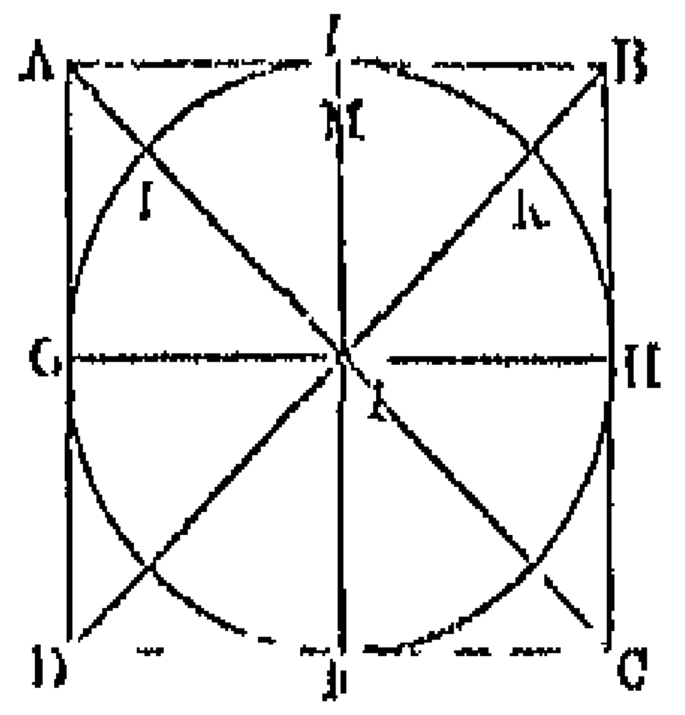
(P×V = THE CONSTANT NUMBER 1463")

S	P	Log P	Log V	V	S	P	Log P	Log V	V	S	P	Log. P	Log V.	V.
0	107 61	2 03185	1 13339	13" 596	42	91 60	1 96190	1 .20334	15" 972	84	75 .60	1 87852	1 28672	19' 352
1	107 22	2 03028	1 13496	13 645	43	91 22	1 96009	1 20515	16 038	85	75 22	1 87633	1 28891	19 .450
2	106 84	2 02874	1 13650	13 693	44	90 84	1 95828	1 20696	16 .105	86	74 84	1 87413	1 29111	19 548
3	106 46	2 02719	1 13805	13 742	45	90 46	1 95646	1 20878	16 173	87	74 46	1 87192	1 29332	19 648
4	106 08	2 02563	1 .13961	13 792	46	90 08	1 95463	1 21061	16 241	88	74 08	1 86970	1 29554	19 749
5	105 70	2 02407	1 14117	13 841	47	89 .70	1 95279	1 .21245	16 310	89	73 70	1 86747	1 29777	19 850
6	105 32	2 02251	1 14273	13 891	48	89 32	1 95095	1 21429	16 379	90	73 32	1 86522	1 .30002	19 953
7	104 94	2 02094	1 14430	13 941	49	88 94	1 94910	1 21614	16 449	91	72 93	1 86291	1 30233	20 059
8	104 56	2 01937	1 14587	13 .992	50	88 56	1 94724	1 .21800	16 520	92	72 .55	1 86064	1 30460	20 .165
9	104 18	2 .01778	1 14746	14 043	51	88 17	1 94532	1 .21992	16 593	93	72 17	1 .85836	1 30688	20 271
10	103 80	2 01620	1 14904	14 094	52	87 79	1 94344	1 22180	16 665	94	71 79	1 85606	1 30918	20 378
11	103 41	2 01456	1 15068	14 147	53	87 41	1 94156	1 22368	16 737	95	71 41	1 85376	1 31148	20 487
12	103 03	2 01296	1 15228	14 200	54	87 03	1 93967	1 22557	16 810	96	71 03	1 .85144	1 31380	20 597
13	102 65	2 01136	1 15388	14 252	55	86 65	1 93777	1 .22747	16 884	97	70 65	1 84911	1 31613	20 708
14	102 27	2 00975	1 15549	14 305	56	86 27	1 93586	1 22938	16 958	98	70 27	1 84677	1 31847	20 .820
15	101 89	2 00813	1 15711	14 358	57	85 89	1 93394	1 23130	17 033	99	69 89	1 .84441	1 32083	20 933
16	101 51	2 00651	1 .15873	14 412	58	85 51	1 93202	1 23322	17 109	100	69 51	1 84205	1 32319	21 047
17	101 13	2 00488	1 16036	14 466	59	85 .13	1 93008	1 23516	17 186	101	69 12	1 83960	1 32564	21 166
18	100 75	2 00325	1 16199	14 521	60	84 75	1 92814	1 23710	17 263	102	68 74	1 83721	1 32803	21 283
19	100 37	2 00160	1 .16364	14 576	61	84 36	1 92614	1 23910	17 342	103	68 36	1 83480	1 33044	21 401
20	99 99	1 99996	1 16528	14 .631	62	83 98	1 92418	1 24106	17 421	104	67 98	1 83238	1 33286	21 521
21	99 60	1 99826	1 16698	14 688	63	83 60	1 92221	1 24303	17 500	105	67 60	1 82995	1 33529	21 642
22	99 22	1 99660	1 16864	14 745	64	83 22	1 92023	1 24501	17 580	106	67 22	1 82750	1 33774	21 764
23	98 84	1 99493	1 17031	14 802	65	82 .84	1 91824	1 24700	17 660	107	66 84	1 82504	1 34020	21 888
24	98 46	1 99326	1 17198	14 859	66	82 46	1 91624	1 24900	17 742	108	66 46	1 82256	1 34268	22 013
25	98 08	1 99158	1 17366	14 916	67	82 08	1 91424	1 25100	17 824	109	66 08	1 82007	1 34517	22 140
26	97 70	1 98989	1 .17535	14 975	68	81 70	1 91222	1 25302	17 907	110	65 70	1 81757	1 34767	22 267
27	97 32	1 98820	1 17704	15 033	69	81 32	1 91020	1 25504	17 990	111	65 31	1 81498	1 35026	22 401
28	96 94	1 98650	1 17874	15 092	70	80 94	1 90816	1 25708	18 075	112	64 93	1 81245	1 35279	22 532
29	96 .56	1 98480	1 18044	15 151	71	80 55	1 90607	1 25917	18 162	113	64 55	1 80990	1 35534	22 664
30	96 18	1 98308	1 18216	15 211	72	80 17	1 90401	1 26123	18 249	114	64 17	1 80733	1 35791	22 797
31	95 79	1 98132	1 18392	15 273	73	79 79	1 90195	1 26329	18 336	115	63 79	1 80475	1 36049	22 935
32	95 41	1 97959	1 18565	15 334	74	79 41	1 89988	1 26536	18 423	116	63 41	1 80216	1 36308	23 072
33	95 03	1 97786	1 18738	15 395	75	79 .03	1 89779	1 26745	18 512	117	63 03	1 79955	1 36569	23 211
34	94 65	1 97612	1 18912	15 457	76	78 65	1 89570	1 26954	18 601	118	62 65	1 79692	1 36832	23 352
35	94 27	1 97437	1 19087	15 519	77	78 .27	1 .89360	1 27164	18 691	119	62 27	1 79428	1 37096	23 494
36	93 89	1 97262	1 19262	15 582	78	77 89	1 89148	1 27376	18 783	120	61 89	1 79162	1 37362	23 639
37	93 51	1 97086	1 19438	15 645	79	77 51	1 88936	1 27588	18 875	121	61 51	1 78895	1 37631	23 .785
38	93 13	1 96909	1 19615	15 709	80	77 13	1 88722	1 27802	18 968	122	61 13	1 78625	1 37899	23 933
39	92 75	1 96731	1 19793	15 774	81	76 74	1 88502	1 28022	19 064	123	60 75	1 78355	1 38169	24 082
40	92 37	1 96553	1 19971	15 839	82	76 36	1 88287	1 28237	19 159	124	60 37	1 78082	1 38442	24 234
41	91 .98	1 .96369	1 20155	15 906	83	75 98	1 88070	1 28454	19 255	125	59 99	1 .77808	1 38716	24 387
S	P	Log P	Log V	V	S	P.	Log P	Log V	V	S	P	Log P	Log V	V

§ XXIV RETICULATED DIAPHRAGMS

1. The ocular diaphragms of Cassini, La Caille, Bradley, and Wollaston cannot properly be called *micrometers* in the sense that we have used the term, but because they afford the data for obtaining an arc of the sphere by differential observations depending on *time*, and convertible into *space*, we shall consider them as belonging to the single image class of micrometers, as the German opticians and astronomers have done, who have only varied the form, but have retained the principle of the contrivance. Indeed La Lande has called reticles (*reticules*) the most simple kind of micrometers. [*Astron.* art. 2350.]

2. Cassini's.—The first reticulated diaphragm that was used in making regular astronomical observations, was by the Parisian astronomer Cassini; but whether the inventor was John Dominic, or his son James, or his grandson Cæsar François (Cassini de Thury), who succeeded one another at the observatory at Paris, is not now quite certain. The field of view, afforded by a thin plate of brass, constituting the ocular diaphragm, was divided by four equidistant diametrical wires crossing one another at so many angles of 45° at the centre, as in the annexed figure.



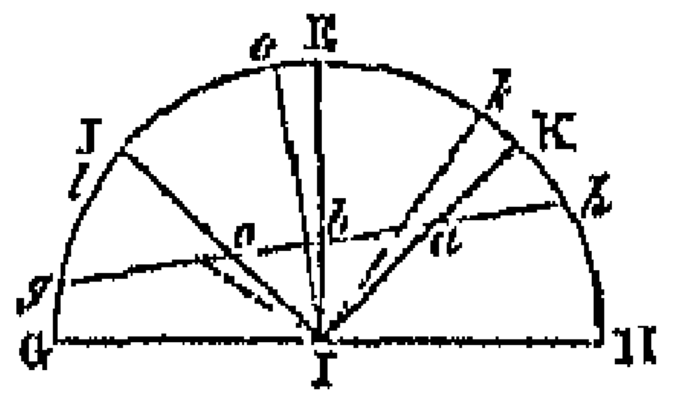
In the square piece of sheet brass *A B C D*, formed with right angles and of equal sides, the lines *A B*, *B C*, *C D*, and *D A*, are bisected by the lines *E F*, and *G H*, and the diagonals *A C*, and *B D*, crossing at the common point of intersection, give the centre from which a circle, equal to the intended diaphragm, is drawn and cut away in a lathe, or otherwise, very exactly; then four diametrical fine wires laid over these lines complete the reticle, which is placed in the common focus of the object-glass and eye-piece of the telescope to be used; which may be either fixed in the meridian, or have an equatorial motion, so that the vertical line, *E F*, may be part of a circle passing through the pole, and the line *G H* parallel to the equator. The tube which holds this diaphragm, must be at liberty to turn round, to adjust for the horizontal position of the equatorial line, as well as to draw out for distinct vision of the lines, that they may have no parallax when the eye moves a little towards the right or left. These adjustments will be known to be properly made when a star will run along the line *G H*, without deviation, viewed in all oblique directions by the observer himself, whose eye must be the judge, and, to save time in making the adjustments, the star may first be brought to the centre of the diaphragm, and before it quite disappears at the edge of the circle, it will be seen whether it ascends or descends, and how much, for in that situation a slight turn of the tube will bring it upon the line, which will then be very nearly parallel to the equator, at the first trial. The principal use of the reticle, in any shape, is to take differences of the right ascensions and declinations of two stars, the place of one of which is known, or of a star and planet, or comet, by the aid of a sidereal clock or other time piece which keeps correct time for a short interval.

3. When the known star precedes the object to be compared with it, and is seen coming into the field, the telescope's screw of slow motion in altitude must bring it upon the equatorial line *GH*, and the exact second must be noted when it passes the vertical line *EF*, while the instrument is firmly placed in this position, or, having an equatorial motion, is not far out of this position; then supposing the second body to be to the south of the known star, and the eye-piece to be of the celestial kind that inverts, it may be expected to come into the upper part of the field when viewing the southern part of the hemisphere, or the contrary; let the minute and second be marked down when it is first seen at *K*, and also when it disappears at *L*, and if the observation be carefully taken, the mean between the two times will be the moment when it passed the line *EF* at *M*, which may also be observed as a check on the other observations, or this last may be taken alone if the second star has passed the line *KI* before it was noticed, then the difference of the times of passage of the two observed bodies will be their difference of right ascension, which added to the known or computed right ascension, either mean or apparent, of the first body, will be the right ascension of the second, in the same denomination. The circumstance of the first star's running along the equatorial line is a proof, that the line at right angles to it is an horary line, even though out of the meridian of the place.

4. With respect to the difference of declination, as the angles *MKL*, and *MLK*, are both angles of 45°, when the angle *KMI* is a right angle, which it will always be when the line *GH* is parallel to the equator, the distances *MK* and *ML*, are equal to one another, and each equal to *MI*, the difference of declination when the time is converted into space and reduced to the equator. The reduction is made by multiplying the small arc, found from the time of semi duration, by the cosine of the known star's declination, which, being nearly the same as the declination of the second body, will give an approximate difference, that applied to the declination of the known star, will give very nearly the declination required, then with the cosine of this approximate declination, the correct difference may be found. When the value of the whole field in minutes of declination is known, the proportion belonging to the interval between the two observed bodies may be assumed near enough by estimation, for finding the cosine of the declination, when used only as an argument, which assumption will save the trouble of a repetition of the computation If the clock indicates true sidereal time, or if the interval is short and the daily rate small, the right ascension and declination thus deduced will be as correct as those of the star from which they are derived, but if the clock is too fast or too slow in consequence of a considerable rate, a proportionable

part of the said rate must be applied to the interval observed, and, if it be solar time, it must moreover be reduced to sidereal time, by the proper table given at page 109 of our first volume.

5. When the planet, comet, or other object to be compared with a known star, follows so closely, that there is not time to have the equatorial line GH properly adjusted to its true position, the observation may notwithstanding be made, and afterwards corrected. Suppose that the visible line of passage be, as in the following figure, along the line hg, which is not



parallel to the line HG, thereby showing that the lines KI, EI, and LI of the diaphragm should have been coincident with the dotted lines kI, eI, and lI, then the first half of the interval observed, ab, will be longer than the second, bc. Now if we call the former interval m, and the latter n, we shall gain the object's distance from the centre of the diaphragm, when it is on the line eI by the formula,

$$\frac{m^2 n + n^2 m}{m^2 + n^2}$$

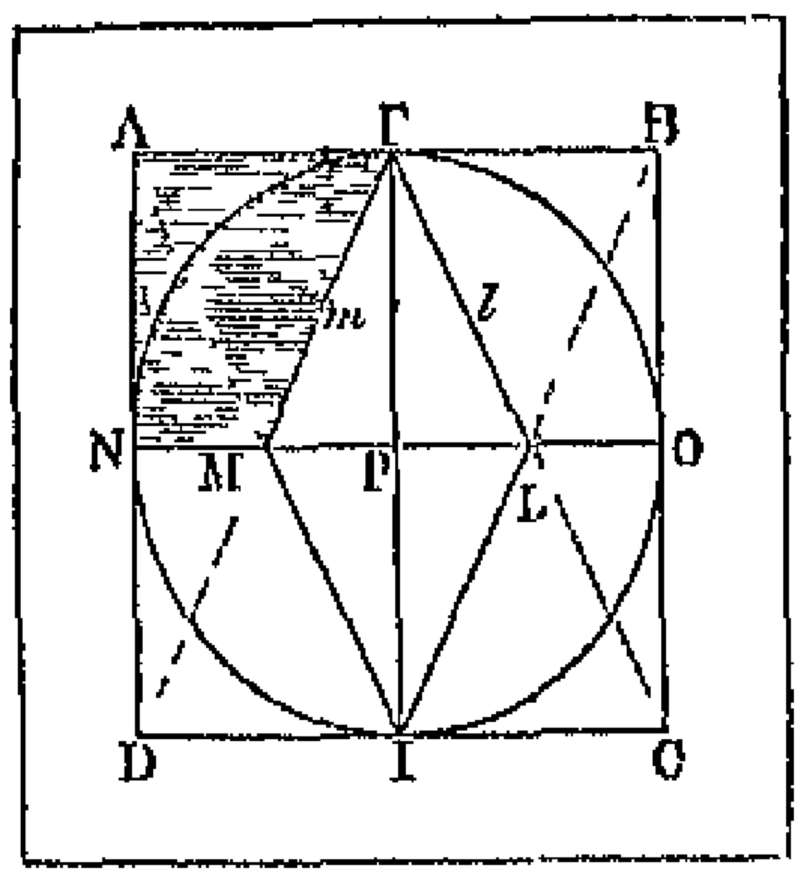
which will be the interval of time to be reduced into a small arc of a great circle, according to the nature and duration of the time indicated, to give the difference of declination. The distance of the point b, from the line eI, on which the correction in right ascension depends, will be had by the following formula.

$$\frac{m^2 n - n^2 m}{m^2 + n^2}$$

and this quantity added to the time of the observed passage at b over the displaced horary line, will give the time of passage over the rectified horary line eI, but if the first interval ab, be smaller than the second interval bc, the displacement of the diaphragm will be on the contrary side, and the correction will be subtractive. When the two intervals ab and bc are passed in like times, it will be a proof that the equatorial line GH of the diaphragm is in adjustment.

6. The objections which Bradley had to the use of Cassini's diaphragm were, first, that the crossing of all the wires at the centre of the field embarrassed the observation, by concealing the known star at the very place where it was most required to be seen, in adjusting the equatorial line; secondly, that two out of the four quadrantal arcs were useless in taking the time, except so far as they showed the line GH, and gave notice of the star's approach towards an ingress into the line KI, when the body did not pass in a small chord above M, and thirdly, illumination was necessary to render the lines visible, which was not admissible when a very small star or a comet was the object.

7. *Rhombus.*—The figure which Bradley preferred, and of which he is said to have been the contriver (though the truth of the assertion has lately been doubted by Valtz of Nîmes), was the rhombus, which is given at the head of the next page;



In the construction of this figure three conditions must be strictly complied with; first, the lines $O\ N$ and $F\ I$, which are the equatorial and horary lines, must be placed truly at right angles to each other; secondly, they must form correct diagonals of the rhombus $F\ L\ I\ M$, and thirdly, the shorter diagonal must be precisely one half the length of the longer, for unless the figure be correctly made, the observations will be charged with errors that will escape correction. We have copied this diaphragm from La Caille's *Cœlum Australe Stelliferum*, which was his third, or *le grand réticule*, and was applied by him to a refracting telescope of 26 inches $3\frac{1}{3}$ lines (French) focal length, made fast to a fixed quadrant each side of the square was 15.4 lines, and the vertical field of view, $F\ I$, was determined to include an arc of 2° 50′ 10″ with this slender apparatus the assiduous French astronomer observed the comparative right ascensions and declinations of the 1942 stars, which are contained in his *Stellarum Australium Catalogus* The observations were begun at the Cape of Good Hope on the 30th of September 1751, and finished on the 25th of June 1752, and were reduced to the epoch 1750. Out of the 25 zones into which the space between the Tropic of Capricorn and the South Pole were divided, each containing 2° 42′ or 2° 43′, the zones 8, 9, &c. to 21 inclusive were observed with this diaphragm.

8. When the equatorial line, $O\ N$, of La Caille's larger reticle was adjusted for the passage of a star, the value of the short diagonal, $M\ L$, was the first thing to be ascertained, which by the passage of several equatorial stars was determined to be 1° 25′ 5″, when the time was reduced into the arc of a great circle. Then as the vertical height of each triangle of the rhombus is equal to the short diagonal, the path of any body moving parallel to the short diagonal, will always cut off a similar triangle, and the vertical distance of such path from the common apex will bear the same proportion to the vertical distance of the large triangle, or to the small diagonal, its equal, that the value of the diagonal in time, is to the observed time of the passage; for instance, from l to m. Therefore when the time of passage of any body, moving along the line $l\,m$ is counted and reduced into a small arc, this arc multiplied by the cosine of the corresponding declination, must be applied to the known declination of the standard star, with the sign + or —, accordingly as it has greater or smaller declination, which may always be known from the passage being in the upper or lower triangle. La Lande has given this general rule, which will apply in all cases, viz.

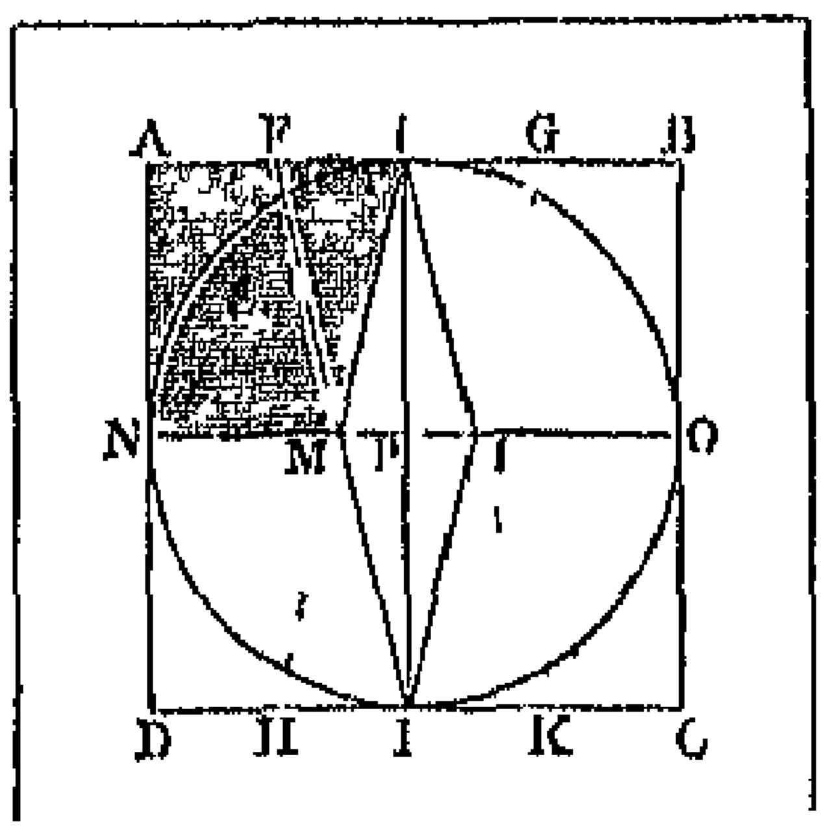
9 "Take either the sum or the difference of the intervals in time employed in the passages, when converted into space and multiplied by the cosine of the mean declination, when it is the difference, the difference of declination will be had without computation, when it is the sum, it must be subtracted from the value of the longer diagonal, or twice the value of the

shorter diagonal, and the remainder will be the difference of declination." Hence the values of the diagonals become of no importance when the two stars or other objects pass in different triangles, above and below the small diagonal, for the difference of declination obtained from the sum of the arcs applied to the known declination of the standard star, will at once give the required declination. The right ascension, as in the use of Cassini's diaphragm, is obtained by applying the time of the object's passing the vertical line *F I*, or which ought to give the same, the middle time between the moments of ingress at *O* and egress at *N*, to the known right ascension of the standard star, this quantity must of course have a positive or negative sign, accordingly as the object follows or precedes the standard star.

10. As an example of the use of this reticle, La Lande wishing to compare the place of Mercury with Spica Virginis, on the morning of the 14th of November 1763, observed this star's ingress on the equatorial wire at $5^h\ 22^m\ 12^s$, and its egress at $5^h\ 25^m\ 24^s$, from which an interval of $3^m\ 12^s$, or an arc of $48'\ 0''$ was deduced, which quantity multiplied by the cosine of $9°\ 55'$, the star's declination at the time, gave $47'\ 17''$, which was the measure of the short diagonal of his reticle, and also of the perpendicular of one of its two triangles, agreeably to the construction of the figure we have referred to. Mercury was seen entering the upper part of the triangle at *l* at $6^h\ 15^m\ 4^s$, and leaving it at $6^h\ 17^m\ 9^s$; then converting the difference of the intervals $1^m\ 7^s$ ($3^m\ 12^s - 2^m\ 5^s$) into arc, we have $16'\ 45''$, which multiplied by the cosine of $9°\ 55'$, the known declination of the star, which passed through the middle of the field, gives $16'\ 30''$ for the difference of declination between Spica Virginis and Mercury, agreeably to the first part of the general rule. With respect to the right ascension it appears, from the recorded times of egress and ingress, that the passages of the two bodies, over the horary line *F I*, must have been at $5^h\ 23^m\ 48^s$, and at $6^h\ 16^m\ 6\frac{1}{2}^s$ respectively, and that the difference $52^m\ 18\frac{1}{2}^s$ converted into arc, at the rate of $23^h\ 55^m\ 50^s$ for 360°, because the clock's daily loss was 14^s per day, will be $13°\ 6'\ 54''$, or $52^m\ 37^s.6$ in time, for the difference of right ascension between Mercury and Spica Virginis, at the moment of Mercury's passage, which was computed to be $18^h\ 16^m\ 6^s$ true time, on the 13th of November 1763.

11. With respect to the three objections which Bradley had to Cassini's reticle, he has avoided two of them; he cleared the central part of the diaphragm from the confusion occasioned by so many crossings, and by making the boundary lines broad, so as to conceal the body for a short time before its ingress, he could mark it and also the egress without illumination; and when the passage was in the upper triangle, the dark solid part concealed the body altogether, when the egress had taken place, and thus gave notice to the observer that the passage took place above the small diagonal.

12. La Caille constructed his second diaphragm with the same longer diagonal very nearly, but with the shorter one diminished one-half: the value of the former was $2°\ 49'\ 29''$ of a great circle, and of the latter $42'\ 22''\frac{1}{2}$. The figure was that which is here subjoined:



This was called *le petit réticule*, but was used in precisely the same way that has been explained, it differs from *le grand réticule* only in the length of the short diagonal, which admits of the passages being performed in less time. It was applied, as well as a pair of parallel plates including an angle of 18′ 20″, in determining the places of the stars included in the first eight zones, beginning at the pole, where the apparent motion was slow The four last zones were observed with the fourth, or *le moyen réticule*, which was constructed like the third, except that the solid dark trapezium was in the opposite corner, $O\ L\ I\ C$, which therefore faced the west when the instrument was turned to the north. Its longer diagonal subtended an angle of 2° 48′ 41″. The star Syrius was generally used for determining the time and going of the clock, as well as the meridional position of the quadrant, by equal circum-meridian altitudes, and the 170 standard stars, marked with an asterisk in the catalogue, had their right ascensions and declinations determined in the same way; and from these the other stars had their places comparatively derived by the simple use of the reticles, which stars were chiefly of the fifth and sixth magnitudes. Besides the catalogue here mentioned, La Caille observed 200 principal southern stars, between the years 1746 and 1752, by the method of equal eastern and western altitudes, which are reduced to 1750 also, and given, together with the observations, in his *Astronomiæ Fundamenta.* We shall probably have occasion to give some examples of these methods hereafter.

13. In the seventy fifth volume of the Philosophical Transactions of London (1785), the Rev. Francis Wollaston has given the formulæ by which the differences of right ascension and of declination may be computed from observations taken with Bradley's rhombus, when its equatorial line is not adjusted parallel to the line of passage of the stars, which may be applied thus:

Let the angle MFL, in either of the preceding figures, which in the larger rhombus is 63° 26′ 6″, be called . . . . a
The small diagonal ML, when the value is known . . . . b
The larger interval, between the oblique and horary wire . . . . m
The smaller interval, between do. in any star's passage . . . . n
The larger interval of a second star . . . . μ
The smaller do. . . . . ν

Then $\frac{2.(m-n)}{m+n}$ = the tangent of the angle which the smaller diagonal makes with a parallel of declination, which call q; and $dD = \frac{2.(n \backsim \nu).\sin(a+q)}{R.\sin a} \times \cos q$; and $d\,AR = \frac{2.(n \backsim \nu).\sin a + q}{R.\sin a} \times \sin. q$; where dD is the difference in declination of the two stars counted in time on the horary line, which must therefore be converted into arc, and multiplied by the cosine of the mean declination; and $d\,AR$ is the difference of right ascension between the same stars. If the two larger intervals only, m and μ, were observed, the same differences may be computed by substituting $(a-q)$ for $(a+q)$, thus

$$dD = \frac{2.(m \backsim \mu).\sin(a-q)}{R.\sin a} \times \cos q = dD$$

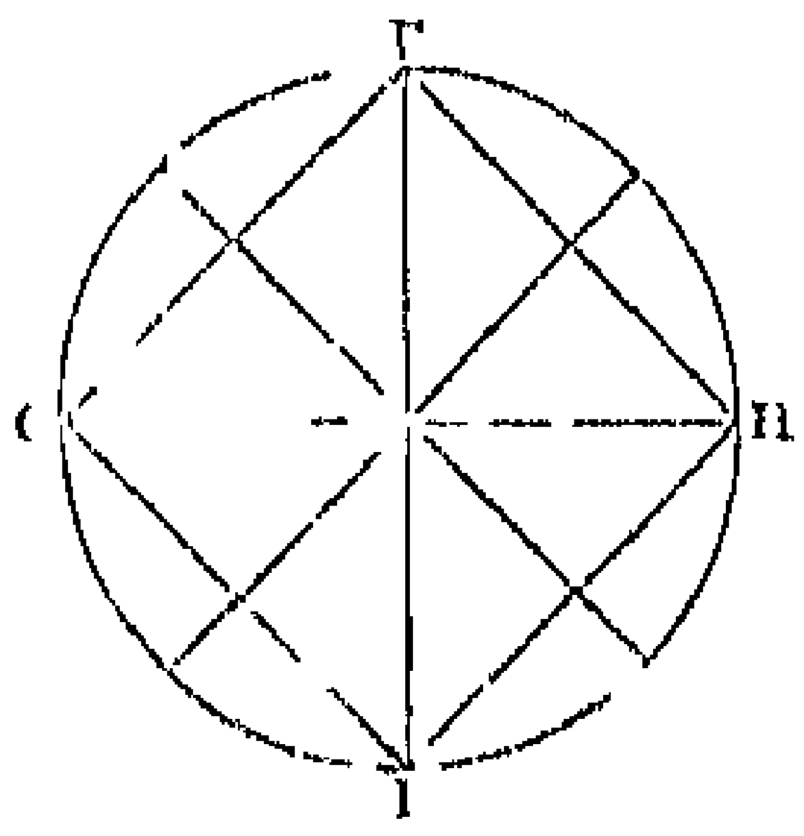
$$d\,AR = \frac{2.(m \backsim \mu)\sin(a-q)}{R\ \sin a} \times \sin q = d\,AR.$$

When the stars pass in different triangles, above and below the small diagonal, the difference of each star from its nearest angular point *F* or *I* may be separately computed by the formulæ

$$\frac{2 . n \ \sin\,(a+q)}{R . \sin a} \times \cos q, \text{ and } \frac{2\,n . \sin\,(a+q)}{R . \sin a} \times \sin q.$$

14. The practical application of these formulæ is very easy; for, when q is found, its log. cosine may be put down in one column, and its log sine in another, then under each the constant log. sine of $(a + q)$, and also the arithmetical complement of sine a, which, being added together, will give two sums for the comparative observations of every star which may pass the field through the same diaphragm, in the same position. When the value of the field is great, and also the difference of declination, the results will be more correct by using the approximate declinations, as before directed, instead of the declination of the centre of the field.

15. Mr. Wollaston proposed an improvement in Cassini's reticle, by inscribing his squares within the circle of the field of view, and placing one of the diagonals, *E F*, in a vertical position for the horary line, and the other, *G H*, for the equatorial line, by which means any of the four included small squares may be used separately, and the observations, made in a pair of them successively, will check one another, as well as the principal observation made in the large right-angled triangle. The annexed figure will explain the improvement,



In this figure the diagonals, being equal to one another, afford a longer passage of a star, which contributes to the accuracy of the observation, as well as leaves a larger portion of the field open for the observations. The formulæ of La Lande above applied to Cassini's reticle, is equally applicable to this figure, when the line *G H* is not placed accurately parallel to a circle of declination.

16. These respective diaphragms may be cut out of a sheet of rolled brass, and placed, when turned circular, in the focus of the first lens of a negative eye piece, or otherwise before the common focus of a positive eye-piece, in the latter of which positions the eye-piece may be changed for one of larger or smaller power, as the observation may require; and if the strokes are nicely cut with a diamond, or eaten by fluoric acid, on a piece of clear glass, with faces perfectly parallel, the reticle, properly illuminated, will be a pretty addition to the eye piece, particularly if the containing cell be made to take out by screwing, when not wanted. But, to use the words of Mr. Wollaston, whose opinion on this subject we may venture to quote, "what is here offered is by no means to be understood as recommending any system of wires in preference to actual measurement with a micrometer, but to render the use

of them as convenient as may be, to such gentlemen as are not provided with better instruments."

17. Dr. Bradley added to the diaphragm a toothed arch with an endless screw, impelling it, as an adjustment for position, before the achromatic principle had shortened the refracting telescopes, but now the adjustment of the equatorial line may be easily made, by simply turning round the loose drawer that holds the eye piece, or, if not loose, an adapter that has got a circular motion.

18. The reticulated diaphragms, in common with all the other species of micrometers, give comparative observations that require no correction for refraction, or for aberration and nutation, since both the bodies, being adjacent, have very nearly the same corrections.

19. M. H. Flaugergues, of Viviers, has shown, in an interesting memoir published in the first volume of Zach's *Correspondance Astronomique* (p. 351), how Bradley's rhombus may be applied to determine the place of any spot on the sun or moon, while the apparent lower limb is passing in contact with the shorter diagonal. He has there proved this general theorem, viz. "The line passing the centre of the sun, in the interval of time between the first exterior contact and the first interior contact, is equal to the line which passes through the same centre, in the interval between the second interior contact and the second exterior contact, and the two lines are each equal to the line which the centre of the sun describes within the [triangle of the] reticle." This theorem supposes the power of the telescope so small, that the whole diameter of the sun may be contained in either of the two triangles of the reticle. When the first limb touches the first oblique line, it is called the first exterior contact, and when it quits the second oblique line, it is called the first interior contact, and the second exterior and interior contacts are when the second limb arrives at the same situations, but the points of contact will not fall in the same parts of the respective lines at each contact. While the passage of the sun's centre is thus twice taken, by means of the four contacts of his limb, with the two oblique lines of the triangle, the spot's passage over the triangle may also be observed, and from the difference of the chord and of the diameter of the sun, obtained from the difference of the times of the two respective passages, the declination of the spot may be determined by computation, while its right ascension will be had from the mean of the times of its ingress and egress, as though it were a star. For instance, if we put T for the time of the interval between the two exterior and interior contacts of either limb of the sun, across Bradley's reticulated rhombus; θ for the time of passage of the spot observed; and $d\,D$ for the difference of the declinations; we shall have $d\,D = (T - \theta) \,.\, (15 \,.\, \cos \text{dec} \odot)$. The time θ must be subtracted from T, or T from θ, accordingly as T is greater or smaller than θ, which ratio will show whether the spot is above or below the sun's centre.

20. The time of passage of the Sun's centre over the longer diagonal may be had by direct observation of the two limbs over this line, for a mean of the two times will be the time of passage of the centre, from which the difference of right ascension of the spot, and of the Sun's centre, may be immediately had without further computation. The author however has shown, how the time of passage of the Sun's centre over the middle of the reticle may be computed from the observed contacts at the oblique lines, by knowing half the angle subtended by the short diagonal; and also how the value of the chord described by the Sun's centre may be determined from the interior contacts alone, in case clouds should intervene; but as the direct

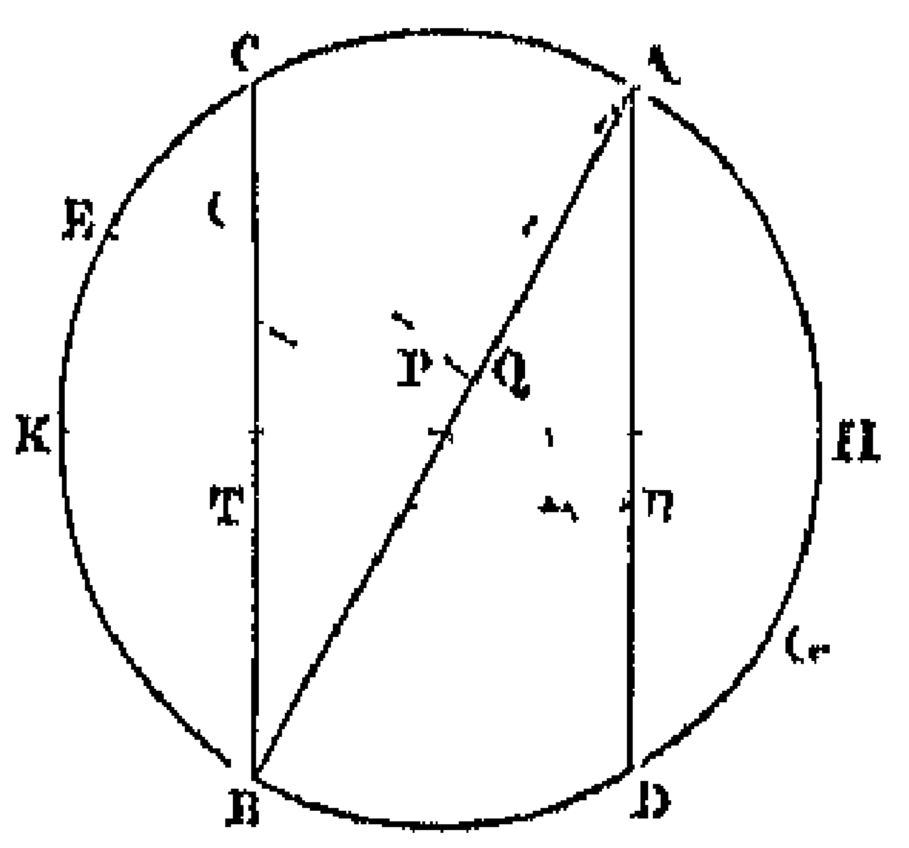
measurement will always be preferable to a deduction of this kind, and as the micrometers which measure arcs of declination, without reference to the *time* of passage, will give more satisfactory results, we will leave our readers to gratify their curiosity by a perusal of the whole memoir.

21. The difficulty of forming the acute angles of Bradley's rhombus with perfect accuracy induced Flaugergues to propose another construction for that which gives the two diagonals as 2 : 1. He observes that the construction of an equilateral triangle is the easiest to be well accomplished of all geometrical right-lined figures; and as the long diagonal of a rhombus, constructed of two equilateral triangles, is to the short diagonal as $\sqrt{3} : 2$, he proposes to multiply the difference of the two chords of passage over the reticle by $\frac{\sqrt{3}}{2}$; in which case instead of the formula above given we shall have

$$d\,D = (T - \theta) \cdot \left(\frac{\sqrt{3}}{2}\, 15.\ \cos \mathrm{dec}\ \odot\right) = (T - \theta) \cdot (12.9904.\ \cos \mathrm{dec}\ \odot);$$

and if the integral number 13 be substituted for 12.9904, the error in the result will be insensible.

22. When the reticulated diaphragm had undergone all the changes of figure that the exigences of practical astronomy seemed to require, M. Benj. Valtz, of Nîmes, from a consideration of the difficulty of perfect construction, and of perfect rectification of the orthogonal position of a rhombus, of any dimensions, invented a new figure, which he conceives to be preferable to any of its predecessors, and which he has described in the third volume of the *Correspondance Astronomique* above referred to (p. 353). He points out the inconveniences arising from the insertion of diagonal lines, as well in Flaugergues's as in Bradley's construction, and also in that of M. Monteiro da Rocca of Coimbra, which makes the acute angles each of 45°, without the use of diagonal lines and then proposes two parallel chord lines to be drawn in a circle at the distance of radius, with their opposite ends united by a single diagonal, like the Greek capital Z, as is represented in the subjoined figure. From the extreme points *A B* of the diameter of the circle *A C B D*, with the same opening of the compass which described the circle, he sets off the two arcs of 60°, *A C* and *B D*; then joins each pair of points by the lines *A D* and *C B*, which will be parallel, and the reticle is constructed as in the subjoined figure.



The author says that a thin plate of metal, thus marked, may be cut out by filing, till the lines delineated form the edges of the narrow flat laminæ left by the file; but as this would be a difficult operation, he proposes to lay flat metallic threads, with straight edges, over the points on the circumference of the circle, by supra-position on one face of the circle, which will be all seen at one adjustment for vision, with a Campani's eye-piece of large field, which we understand to be of the negative construction; after which he proposes a fourth similar flat thread of metal to be laid on the other face of the perforated plate, for the equatorial line, with its edge over $K\,H$, which being used for no other purpose but to adjust for the equatorial position, may remain out of distinct vision when once adjusted. The equatorial line was at first placed at right angles to the diagonal $A\,B$ over $E\,G$, but it was found on trial to be more convenient to lay it on the line $K\,H$, at right angles to the parallel lines, by which means a small portion of the field of view only was rendered ineffective.

23. The observations made with the equatorial line in due adjustment are called orthogonal, as being in right lines parallel to the equator, and when they are so made, the angles formed at the vertex A or B, will be exactly 30°, and its cotangent will be $\sqrt{3} = 1.732$; therefore if we put t for the interval of time taken up in passing from the first to the second line, when reduced to the arc of a great circle, for the first star, and t' for that of the second star, the difference of their declination will be

$$d\,D = 1.732\,(\pm\, t \mp t') \;\ldots.(1);$$

and the difference of right ascension will be had from the times of passing the middle of the space included between the parallel lines, which times will be the means between the exact times of the respective ingresses and egresses of the two bodies.

24. When the equatorial line $K\,H$ is not parallel to a circle of declination, which will be the case when the telescope has not a parallactic stand, and is out of the meridian, it becomes necessary to compute the angle of inclination I from the known interval $T\,R$ between the parallel lines, and the observed oblique line $R\,C'$ reduced to a great circle, which is always known by $\cos I = \frac{a}{b}$, where a represents the arc $T\,R$, and b the arc $C'\,R$. In all the oblique passages the semi arcs $R\,Q$ and $Q\,C'$ will be unequal, but if we designate either of them r for the first star, and r' for the second, and make $d\,P$ the difference of the correction of the passages at one of the parallel lines, and call D the known declination, we shall obtain the triangle $A\,Q\,R$ thus.

$$A\,R = \frac{Q\,R \sin A\,Q\,R}{\sin Q\,A\,R} = 2\,r \sin\,(60^\circ \pm I) \ldots\ldots (2)$$

and in the right angled triangle, $P\,R = A\,R \sin P\,A\,R = 2\,r \sin I.\sin\,(60^\circ \pm I)$

also . . $A\,P = P\,R \operatorname{cotang} P\,A\,R = P\,R \operatorname{cotang} I$

consequently we have . $d\,P = 2\,(\pm\, r \mp r') \sin I.\,\sin\,(60^\circ \pm I) \ldots\ldots(3)$

and likewise . . $d\,D = 15\; d\,P \cos D.\,\operatorname{cotang} I \ldots\ldots(4).$

The correction for the difference of right ascension, $d\,P$, may be computed from the formula marked (3), and the difference of declination from the last formula (4). The observations taken at all the three lines will give two results, and therefore afford a convenient verification.

25. When the stars observed are near the pole, the parallel lines alone will suffice for giving data for the computation of the difference of right ascension, and for the unknown declination D', provided the reticle be adjusted for the diurnal motion; for the former may be

had from the mean times between the respective ingresses and egresses, and the latter from the formula

$$\text{Cos. } D' = \cos. D \, \frac{\sin \frac{15}{2} t}{\sin \frac{15}{2} t'}.$$

The demonstrations of the different formulæ may be seen in the author's own communication.

26. In a long and interesting *note*, which Zach has subjoined to the letter of Valtz published by him, he observes, that this reticular system possesses several advantages, which will probably recommend it to the notice of practical astronomers. In the first place, it requires no illumination, secondly, it does not require the dimensions or values of its lines to be known; and thirdly, it may be used out of exact adjustment to the parallel of declination, because lastly the corrections for the errors arising from want of adjustment, are of easy computation. He recommends slips of tin-foil, or of oiled drawing-paper, to be substituted for wires, in forming the parallel sides of the diaphragm, which would give notice to the observer of the ingress and egress of a luminous body in the absence of light. Such laminæ might be laid on the face of a ring of thin brass, made annular in a lathe, with their straight edges coincident with the points previously marked by the dividers, by means of a gummy substance, or hard spirit varnish, by any dexterous amateur.

27. We shall conclude our account of reticulated diaphragms by describing Fraunhofer's *net*-micrometer, which requires to be used with a lamp of peculiar construction already described (§ XXII. 8.), the principal intention of which is to determine the relative situations of double stars. We are not aware that this micrometer has yet been brought into much use, but as Professor Struve has applied one to his large refracting telescope, astronomers must feel an interest in the nature of its construction. Fig. 4 of Plate VII is an enlarged representation of a disc of (plate?) glass, which the late celebrated German optician divided into two sets of parallel lines, crossing one another in the same acute angle, and placed in the common focus of the object-glass and eye-piece of a refracting telescope, in a cell that admits of a circular motion. The lines that run parallel with the diametrical line *ef*, may be adjusted to stand vertical to a star's circle of declination, and in that position the oblique light of the lamp will illuminate both the vertical and oblique parallel lines, which it would not do so well if the latter were horizontal. The differences of right ascension and of declination may be determined from the observed times of the passages of different stars when the telescope remains fixed in the meridian, or is placed on an equatorial stand out of the meridian. The formula for computation will of course depend on the exact magnitude of the acute angle adopted in the construction, which has not been specified in the account given by Schumacher (*Astronomische Nachrichten*, No. 43.), but may be determined when the angle chosen is convenient for computation The distances from each other of both the vertical and inclined lines, as well as the angle of inclination, are known, and therefore the proportion of the times of transit of a star through the vertical to those of the inclined parallel lines, will afford data for determining the position of those with respect to the parallel of declination. The great number of lines affords the means of making several observations which on an average will give right ascensions and declinations equally exact, whether the differences of declination be great or

small. When the difference of right ascension is small, as in the case of double stars, the transit of both stars cannot well be observed over the same individual line, but one of them may be observed at the first, and the other at the second line alternately, till the observer is satisfied with his observation; and should the net-work experience an alteration of position from any cause, it will in all probability be detected before the computation is commenced. The ingenious artist contrived an engine, by which he could cut straight parallel lines at distances so small as $\frac{1}{10,000}$ of an inch from each other, and to be crossed by other parallel lines at any given angle of declination; and in the *net*-micrometer he formed the parallel lines at such a distance from each other, that the inclined intervals bear the same proportion to the vertical intervals, that the cosine of the angle of inclination bears to radius, so that about as many transits will take place over the inclined as over the vertical lines. To avoid the difficulty of counting a number of lines placed at equal interstices, five lines only are drawn at equal distances from each other, and the sixth line, including the fifth interval, is cut at the distance of one interstice and a half; yet the whole value of any number of intervals may always be known, provided it be noticed how many of the larger kind are included in the whole number. When the cell containing the disc is attached to a revolving graduated circle, the position of a line uniting two stars may also be measured, by first adjusting zero to the equatorial position of the line fg, and then turning the divided disc round, till all its parallels successively receive both the stars at the same instant, as they pass through the field, which contemporary ingresses may be effected by repeated adjustments and subsequent trials, when the telescope is mounted on an equatorial stand properly rectified. We have already given the results of some experimental measurements made with this micrometer by Professor Struve. (§ XII. 7.)

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§ XXV CIRCULAR AND ANNULAR MICROMETERS [PLATE VII]

1. It has frequently happened that astronomers have been obliged to make use of a simple telescope in observing celestial phenomena of rare occurrence, when no micrometer or other graduated instrument has been at hand, in which cases the circular diaphragm of a negative eye-piece has supplied the place of a micrometer. Boscovich appears to have been the first author who proposed to adopt the use of a circular micrometer, in a dissertation published at Rome in the year 1739, which was inserted in the *Actes de Leipzig* of 1740, the title of which was "*De Observationibus Astronomicis et quo pertingat earundem certitudo*." He explains how the ingress and egress of a luminous body may be observed in a dark field, which is not the case with the line-micrometers; and states that the circle of aberration of the rays in the telescope will not alter the figure of the field of view.

2. In the year 1742 La Caille, having occasion to observe a comet, availed himself of Boscovich's suggestion, and gave an account of the manner in which he used a circular field of view with success, in a memoir published by the Royal Academy of Sciences at Paris of the same year. the following is a translation of his own words. "It has happened to me twice, namely, on the 4th of March and the 27th of April, when I could not procure a reticle, that I employed a method [of observing] which may be very commodious on many occasions, and

which is susceptible of great precision when sufficient precaution is taken; it consists in observing only the instants at which two stars enter into and depart from the field of the telescope. for if the field is exactly circular (which it is possible to effect by a diaphragm turned in a lathe), and if its diameter is known by observation, the paths across it may be considered as two parallel chords, which are given in a circle of known diameter. The difference of the times at which the two stars arrive at the middle of their paths, will be their ascensional difference; and the distance between the two chords, which is easily computed, is equal to the difference of their declinations, which will be the more correct the farther the two stars pass from the centre." This paragraph shews that La Caille understood the properties of a circular micrometer, and also the manner of using it with the greatest advantage. M. le Duc de Corigliano observed the passage of Venus over the sun's disc in 1761, with an English Gregorian telescope of 20 inches, by means of its circular field, the value of which he ascertained by observations of both the stars and sun, nearly in the same manner as Olbers and Bessel have done since. But though these instances of the utility of a circular diaphragm were on record, it is remarkable that none of the Continental or English astronomers paid attention to them, till in the year 1798 Doctor Olbers revived the use of the circular micrometer, which he successfully employed in observing the newly discovered planets. The slight notice which La Lande, Kâstner, Koch and others had taken of the circular micrometer, did not prevent the ingenious discoverer of two out of the four minor planets from applying it to a purpose, that furnished elements for the computation of the respective orbits of these wandering bodies, and of various comets that have been since discovered. Indeed, we may attribute the great attention, that has lately been paid to the orbits of comets, to the successful use that Dr. Olbers so judiciously and successfully made of this little contrivance, though applied to a telescope of but small magnifying power.

3. M. Bessel, whose extraordinary talents have fortunately been devoted to the promotion of astronomy, both theoretic and practical, has sanctioned the revival of circular micrometers in Germany, but the French, Italian, and English astronomers have been tardy in following his example. Long computations are troublesome, and instrument-makers are now so expert, that they supply the means of gaining direct measurements of small arcs without the tediousness of arithmetical deductions, or the counting of seconds in observations, which are continually obstructed by the unwelcome obtrusion of clouds. Still however, to observers travelling, and on many interesting occasions when larger apparatus may not be at hand, a portable telescope with a reticulated or with a circular diaphragm, may render important services. The value of the diameter of a circular field will vary inversely as the focal length of the telescope, and may be determined by viewing a graduated staff at any distance, and noticing how many feet and inches can be included at that distance when accurately measured; for the corresponding angle may be computed or taken from our Table in Section XX., and corrected for celestial observations in the ratio of the solar to the terrestrial focal distances $\left(\frac{f}{f'}\right)$, as there explained. But the two eminent astronomers, Olbers and Bessel, have given formulæ by which the value of the diameter of a circle, circumscribing the field of any telescope, may be determined from celestial observations, which formulæ we will present to our readers in succession.

4. Dr. Olbers proposes to observe the passage of the sun's disc across the field of the telescope to be used, and to mark down the times of the exterior and interior contacts of both limbs on the circular circumferences of the field of view, at the moments of entry and departure, he calls the interval of time elapsed between the two exterior contacts m, and the interval between the two interior contacts n, he denominates the time of the sun's semi-diameter's transit for the day in seconds p, and the said semi-diameter in arc d; the two last to be taken from an almanac; then the radius of the circular field of view will be expressed by the formula

$$\frac{(m+n)\,.\,(m-n)\,.\,d}{16\,p^2}.$$

This method differs from Boscovich's only in this respect, that he uses the sun's diameter and declination instead of the diameter and time of passage.

5. Bessel makes use of two adjacent stars that will both pass through the field in succession, while the telescope remains at rest, he calls δ and δ' the declinations of the two stars of which the difference d, is nearly equal to the diameter D of the circle used, and t, T, the respective intervals of time occupied by the stars in passing the whole field or chords of the circle, then he has demonstrated that $D-d=\frac{(15\cos\delta)^2}{4\,d}\,.\,(t^2-T^2)-\frac{(15\cos\delta)^4}{8\,d\,3}\,.\,t^2\,T^2$; but in general the first term of the formula will give the diameter with sufficient accuracy. When the diaphragm is crossed at right angles through the centre by two spider's lines or very fine wires, which is often inserted on the diaphragm of an ordinary negative eye-piece, the passage of a known star along the equatorial diameter will give the value at once, by the interval of its passage turned into arc, and multiplied by the cosine of its declination, which may easily be done when the circle is described on a disc of glass that shows the star on the exterior ring before it enters the circle, provided the telescope has an adjustment for slow motion in altitude.

6 Fig. 5 of Plate VII., represents an enlarged disc of glass divided by Fraunhofer's machine into concentric circles, and applied to his lamp eye-piece, which we have described (§ XXII. 8.), when enclosed in a brass cell fitting the tube of the eye-piece. In this circular micrometer, which has been used by Soldner, the smallest circle appears to the naked eye as a small dot, and can be distinguished only by a magnifier, the number of circles included in the field depends on the magnifying power of the eye-piece · in a telescope of five feet focal length magnifying 110 times, five circular lines only are seen, including the diminutive one · in a middle sized eye-piece magnifying 63 times, eight of these lines are visible, and with the lowest eye piece magnifying 45 times only, eleven lines are within the field. As the passages made over or near the centre of a circle, and also too near the circumference, are both disadvantageous, the first to the computation, and the other to the observation, the concentric rings afford the observer the choice of a circle as the star is passing, that shall be most commodious for his observation without altering the position of his telescope, which must not be disturbed till both bodies have passed and in many cases more rings than one may be used, particularly when the known star has considerable declination, and therefore a slow motion; for then the differences of right ascension and of declination may be separately computed for each

ring. The diameters of the eleven concentric rings were measured with a good microscope by the maker, and determined to be of the following dimensions, when the Parisian inch is taken as unity, viz.

1. = .0038
2. = .0243
3. = .0840
4. = .1678
5. = .2513
6. = .3590
7. = .4426
8. = .5261
9. = .6338
10. = .7178
11. = .8012

One half of the contiguous differences of the diameters will show the breadth of each interstice, which are not equal to one another, but on an average, with the telescope used, the time of passage from one line to another at the equator, was about 10 seconds, and of course more when the declination was considerable. The whole time of passage over one of the larger circles was determined by a known star of small polar distance, when the telescope was made fast, and then reduced to the equator; and from this passage the proportional values of all the other diameters were computed from their tabular dimensions. When the equatorial values of all the concentric circles were thus determined, it was not necessary that two stars following one another should pass through the same circle, the separate values of which were known, but that the passage of each should be a chord to the circle that it was observed in, because the computation gives the distance of each chord from the common centre, and when the declination of that centre is known, the differences applied to it will give the declinations of the observed bodies when the times are reduced into arcs of a great circle, as we shall presently explain.

7. The practical difficulty in using this micrometer with concentric circles, was in obtaining an eye-piece that would give a distinct view of both the small and large circles, at any one point of adjustment; which is a difficulty that must remain insuperable, when the magnifying power is considerable. It was found necessary to place the divided face of the disc of glass towards the eye of the observer, and each eye-piece had a suitable diaphragm for limiting the field properly, as well as a second diaphragm nearer the eye-lens, to intercept that portion of light coming from the lamp, that does not fall on the disc. The largest circle, as might be expected, was not so well illuminated as the rest, but was only used with the smallest power occasionally. If the eye-piece had not the distance between its lenses well adjusted, there would be a distortion and colouration at the extreme circle of the micrometer, which would render it useless at any adjustment for vision. It does not appear that this micrometer has yet been much used, and therefore future experimental trials must prove how far its utility will contribute to the convenience or correctness of differential observations.

8. The most improved construction of the circular micrometer, as last made by the late Fraunhofer, has a circular hole made in the centre of a disc of glass, to the edges of which a ring of steel is made fast, and afterwards truly turned in a lathe, and made thin at both the exterior and interior edges. This ring, when the disc that holds it is in the eye-piece, and applied to a telescope, appears in the field of view as if suspended in the atmosphere; and the instrument is therefore called the *suspended annular* micrometer. Fig. 6 of Plate VII. gives a

representation of this micrometer, of just double the size of one, which we had an opportunity of examining, and was made to apply, like the discs in figures 5 and 6, to the lamp eye-piece shown in figure 2. The advantages of this ring are, that by being fast to the glass, it is not liable to have its shape altered by accidental injury, or by change of temperature, which might affect rings fixed by means of radial arms, connected with the brass work, it has finer edges than could be given to brass, and will admit of double observations being made by immersions and emersions, at the exterior and interior edges of the ring, even when the illumination is imperfect, or when there is no illumination at all, and lastly, in common with other circular micrometers, it requires no rectification for the parallel position of a diametrical line the only requisite necessary to be determined, with a telescope of a given focal length, is the time of passage of an equatorial star over the diameters of the external and internal edges of the ring. If there had been a spider's line or fine wire, bisecting the ring exactly in the centre, and adjustable for the equatorial position, the radius of the circle might have been determined in any degree of elevation, by the formula $r = 15 \cdot \frac{t'-t}{2} \cdot \cos.\ \delta$, in which the path of the star is supposed to be from Q to Q', having its ingress at O and its egress at O', at the corresponding times t and t'; δ being the declination of the centre of the circle, equal to that of the observed star; but as there is no line passing through the centre of the ring, a diametrical line taken by estimation will give an incorrect, or at least an uncertain measure of the radius, by this simple formula; and therefore the method of Olbers, or of Bessel, above explained, must be adopted in practice.

9. When the annular micrometer is applied to a telescope, and has the value of its equatorial radius correctly determined with that telescope, it may be thus made use of in an actual observation that is to be made for the purpose of comparing the place of a small planet, or comet, with that of a known star, in nearly the same parallel of declination; the telescope must be directed towards the body to be observed, and left in such a steady position, that it may describe a chord of the inner circle, which has a better edge than the outer one, from p to p'; then note the times τ and τ' corresponding to the instants of ingress and egress again, keeping the telescope in its place untouched, note the times of ingress and egress of the known star, as at s and s', and call the respective times shown by the same clock θ and θ'; then will the respective times of passing the middle points h and k of the chords $p\,p'$ and $s\,s'$, be denoted by $\frac{\tau'-\tau}{2}$ and $\frac{\theta'-\theta}{2}$, the points of mean passage being both in the horary line $N\ S$, the difference of the right ascensions therefore will be, in solar, or sidereal time, as the case may be, $d\ AR = \frac{\theta'-\theta}{2} - \frac{\tau'-\tau}{2}$, provided the clock has no rate that sensibly affects the interval. The determination of the difference of declinations requires some computation, but is derived from the same data; thus, if we call Δ' the estimated declination of the planet or comet, and Δ the declination of the known star, we have the semi-chords

$$p\,h = \frac{\tau'-\tau}{2} \cdot 15 \cos.\ \Delta',$$

$$s\,k = \frac{\theta'-\theta}{2} \cdot 15 \cos.\ \Delta;$$

then put $\sin x = \frac{s\,k}{r} = \frac{7.5}{r} \cdot (\theta' - \theta) \cdot \cos \Delta$;

$$\sin x' = \frac{p\,h}{r} = \frac{7.5}{r} \cdot (\tau' - \tau) \cdot \cos \Delta',$$

and we shall have $C\,k = r \cos x$ = the distance of the chord $s\,s'$ from C;

$C\,h = r \cos x'$ = the distance of the chord $p\,p'$ from C,

whence we obtain the difference of declination $d\,D = r\,(\cos x - \cos x')$. This formula supposes the two arcs to be both on one side of the centre of the ring, but when they are on opposite sides it becomes $d\,D = r\,(\cos x + \cos x')$. When the observations are made with reference to the outer edge of the ring, the same mode of proceeding must be practised with the arcs $\pi\,\pi'$ and $\sigma\,\sigma'$, and in case the computations are made for both pairs of chords, a mean of both may be taken as a more correct determination than can be expected from either separately, unless there should prove to be a great discrepancy in the two, which, in that case, renders both observations doubtful. A necessary precaution in using this annular micrometer is, that the passages be at a distance from the centre, and also not too near the upper or lower edge of the ring, for reasons that have been above adverted to. For the different formulæ that Bessel has applied to the various cases that may occur, the reader may refer to the *Monatliche Correspondenz* of Baron Zach, Vols. XVII, XXIV, and XXVI.

§ XXVI. LA CAILLE'S METHOD OF REGISTERING AND REDUCING THE OBSERVATIONS MADE WITH HIS RETICLES.

1. THOUGH an experienced astronomer will, we trust, comprehend the descriptions of the different ocular diaphragms, used as micrometers, which we have given in the two preceding sections, and be able to apply the formulæ, respectively annexed, to the computation of the observations made therewith, yet as the young observer, for whose use this work is chiefly intended, may require some direction with respect to registering his observations, that nothing may be left chargeable to his memory, and with respect to reducing them when it may suit his convenience afterwards, we will present him with La Caille's method of proceeding, as a model for his practice, which he may safely adopt as the result of much experience.

2. In order to give the observations, made by a reticulated diaphragm, all the precision they are capable of, La Caille paid particular attention to his clock, on the going of which the accuracy of his results mainly depended; as he had no transit instrument, he took corresponding altitudes of a certain number of principal stars, on each night of observing, to the east and west of the meridian, of which number Sirius was generally one, which could always be observed at the Cape of Good Hope, where he made most of his observations; in this way he could ascertain the rate of his clock within half a second from day to day. The declinations of the same stars were also determined by a great number of meridianal observations, taken by a sextant of six feet radius; they are marked with an * in the catalogue of southern stars, included within the tropic of capricorn, and reduced to the year 1750, as being the *standard* stars from which

the places of the others were derived. Though the observing telescope was made fast to a quadrant well fixed in the meridian, yet if some one at least of the principal stars was not observed on each night, the observations of that night were rejected as uncertain; and the observations of several nights were reduced to the same epoch, while they were going on, to ensure their comparative correctness. This careful and expert observer however confesses, that, notwithstanding he employed all the care and attention that long experience pointed out as necessary, in making his observations in zones by the instruments at his command, yet he cannot pronounce them entitled to confidence, on the score of correctness, within less than *half a minute* of a great circle. His object was, to include in his catalogue all the southern stars of the first to the sixth magnitudes, the differences of which he judged of by the naked eye, from their comparative brightness, and marked them α, β, γ, &c. accordingly on clear evenings, but he observed moreover a great variety of smaller stars, which he put down as being of the seventh magnitude, which he did not reduce. The reasons he has given for marking down his observations on these unreduced stars are, first, that he might obtain a more perfect knowledge of the southern hemisphere; secondly, because it was as easy to observe several stars smaller than those of the sixth magnitude, at their ingresses and egresses in a dark field, as it was to observe large stars, and frequently more easy, by reason of the absence of luminous radiations; thirdly, to avoid falling asleep in the intervals, making up a period of seven or eight hours of watching; and fourthly, because whoever may wish to form a complete catalogue of stars of the sixth magnitude, may be able to admit or reject any portion of those observed stars, which are put down as being of the seventh magnitude, at any time hereafter. The author has given sometimes one and sometimes two days' observations in one page of his *Cœlum Australe Stelliferum*, and at another page, at the same opening of the book, the *tables* that are necessary for the reductions, of both which we shall give exact specimens at the end of this section, as the book is now scarce, after having exemplified the application of them to their respective uses.

3. The purposes for which the small Tables are intended, appear from their titles, but the mode of applying them remains to be illustrated by examples, as well as the method of computing the right ascensions and apparent differences of declination from the times of the ingresses and egresses, as they are tabulated from the observations. In La Lande's example of Mercury compared with Spica Virginis (§ XXIV. 10.), the star was made to pass along the equatorial wire, and the value of the short diagonal of this astronomer's reticle was that arising from its transit, viz. 47′ 17″, which his general rule supposes to be the case in single observations of the same kind; but the same results will accrue if we take the interval of Mercury's passage ($6^h\ 17^m\ 9^s - 6^h\ 15^m\ 4^s =$) $2^m\ 5^s$, for having converted it into arc (1875″) and multiplied it by the cosine of the star's declination 9° 55′ (.985), we shall have 1846″.875, or in round numbers 1847″ = 30′ 47″, which, subtracted from the value of the reticle's perpendicular, or short diagonal 47′ 17″, will leave, as before, 16′ 30″ for the difference. When therefore two bodies that are to be compared together, both pass out of, but parallel to the equatorial wire, their distances from that wire in declination may be separately taken, provided the value of the short diagonal be exactly known, and then the sum or difference of those distances will be their difference of declination accordingly as they passed in the same or in different triangular parts of the reticle; and consequently when the declination of one of the bodies is previously known, that of the other will be obtained by applying this difference with its proper sign. With

respect to the right ascensions, they are derived directly from the clock by mediating between the moments of visible ingress and egress.

4. But when a number of stars are observed in succession, as was La Caille's practice to do, the position of the reticle must first be ascertained by the passage of a known star along the equatorial wire, or by other means, such as the graduation of a good instrument put into due adjustment, when the upper and lower points of the reticle will, from construction, have their places ascertained also, from which points the observed stars will have their declinations more easily computed than from the equatorial wire, because it is the perpendicular distance from the angular point of either triangle of the reticle, that is equal to the line of its passage parallel to the equator, when that line is reduced to a great circle. It was for this method that La Caille constructed his Tables, which we will now explain.

5. Table I. gives the correction of the clock to be applied to the time of the meridian passage of any star, or other body, in the rectified position of the reticle, with its proper sign. Table II. which is sometimes incorporated with Table I., gives singly the correction in *time* arising from an inclination of the vertical diagonal of the reticle, which can best be determined from the observations themselves compared with one another; for which purpose a vertical line bisecting the triangles would be useful. Table III. gives not only the position of the reticle with respect to declination, but will give the declination itself, by inspection, sufficiently near to be used as an argument for the required cosine of the declination in any parallel of the reticle, the star's interval in time being used as the argument of this Table. Hence the first horizontal line gives the declination of each angular point of the reticle, in the upper and lower parts respectively, at the argument $0^m\ 0^s$, or where the star is scarcely visible, and the difference of these tabular numbers will always be found equal to the value of the longer diagonal of the reticle; so that when the exact parallel of the equatorial line, represented by the short diagonal, is ascertained, applying one half of the long diagonal thereto, with opposite signs successively, will give the quantities belonging to the first horizontal line of this Table, and the successive lines corresponding to the different equal intervals are computed, on a supposition that the long diagonal is just double the length of the short one. Thus in the Table for Zone V., for which the small reticle is used, the value of its longer diagonal is 80° 29′ 1″—77° 39′ 11″=2° 49′ 50″, and in that for Zone XIV., the value for the larger reticle is 55° 39′ 1″—52° 49′ 8″=2° 49′ 53″. Table IV., which for the convenience of printing sometimes precedes the one which is used before it, contains the amount of the quantities necessary for reducing the *apparent* into the *true mean* places of the star observed, as they arise from refraction, precession, aberration, and nutation solar and lunar; the mean place of none of the bodies being known, and the reticle being adjusted to the apparent place of a known star. These quantities were not so well ascertained in La Caille's time, as they are at present, by reason of the *constants* not being exactly determined. In copying the tables we have designated the columns of *time* different from columns of *arc*, which the author has confounded with one another, and has thereby rendered his Tables ambiguous, until their construction has been investigated.

6. In exemplifying the use of these Tables, as they have reference to the tabulated observations, it will be convenient to give the examples for the larger reticle used in Zone XIV.

first, because the quantities must be put down as they come from the Tables unaltered, by reason of its short diagonal being just one half the length of the long one. As the stars ζ Aræ and φ Argûs are the *standard* stars, of which the places are determined by other instruments, on May 17 and July 18, 1752, we will first show how they correspond with the observations made of them by the large reticle on the same evening, by giving the operations at full length.

Example 1. For *Right Ascension*, May 17, 1752.

Tabular ingress of ζ Aræ . . .	16^h	37^m	38^s	by clock
Egress	16	38	24	
Difference	0	0	46	= the interval in time.
Half the sum	16	38	1	= the time of meridian. passage.
Table I. and II. for time and inclination .			+18	From Arg. 55° 33′ dec.
Right Ascension	16	38	19	by the reticle.
	16	38	18.5	by the equal altitude instrument.
Reduction from Table IV. .			−15	
Reduced to January 1750	16	38	4	= 249° 31′ 0″

For *Declination.*

Interval 46^s × 15 × .5656 (cos. 55° 33′) = 390″ 264 =		6′	30″.264	subtract
Declination of the upper point of the reticle . .	55°	40	19.0	from Table III.
Apparent declination as observed . . .	55	33	48 736	by the reticle.
	(55	33	49	by the sextant)
Reduction to Jan. 1750 from Table IV. .			−11	with arg 16^h 30^m
Mean declination due to the epoch . . .	55	33	37.736	
Declination given in the catalogue	55	33	31	

Example 2. For *Right Ascension*, May 17, 1752.

Tabular ingress of φ Argûs .	9^h	45^m	53^s	
Egress	9	49	41	
Difference		3	48	= the interval = 228^s.
Half the sum	9	47	47	= time of the merid. passage.
Tables I. and II cor. . . .			+26	with dec. 53° 24'.
Apparent right ascension . .	9	48	13	by the reticle
	(9	48	13	from equal altitudes.)
Reduction from Table IV. about .			− 6	
R. A. reduced to Jan. 1750	9	48	7	= 147° 1' 45"
According to the catalogue . . .				147 1 52

For *Declination*.

Interval 228' × 15 × 5962 (cos 53° 24') = 3039" =		33'	59"	add
Tab. dec. of lower point of reticle	52°	50	19	
Apparent declination	53	24	18	by the reticle.
	(53	24	18	by the sextant.)
Reduction from Table IV. about		−1	12	
Declination reduced to the epoch	53	23	6	
According to the catalogue	53	23	12	

Example 3. For *Right Ascension*, July 18, 1752.

Tabular ingress of ζ Aræ	16^h	38^m	$8^s.5$	
Egress	16	38	42.5	
Difference			34	= the interval.
Half the sum	16	38	25.5	= time of meridian. passage.
Tables I. and II. cor.			−8	
Apparent right ascension	16	38	17.5	by the reticle
	(16	38	18	from equal altitudes)
Reduction from Table IV.			−15	arg. 55° 34'.
Right ascension reduced to Jan. 1750 .	16	38	2.5	= 249° 30' 37".5

For *Declination*

$34^s \times 15 \times 5654 (\cos 55° 34') = 288''.354 =$		4′	48″.354	subtract
Table III upper part	55°	39	1	
Apparent declination	55	34	12 .646	by the reticle.
	(55	34	1	by the sextant)
Reduction from Table IV, about . .			−42	
Reduced declination July 18 . . .	55	33	30	
Do. May 17	55	33	37 .74	
Average of the two	55	33	33 87	by the reticle.
True declination given in the catalogue	(55	33	31	by the sextant)

Right ascension May 17	16ʰ	38ᵐ	4ˢ	= 249°	31′	0″
Do July 18 . . .	16	38	2 .5	= 249	30	37 .5
Average of the two	16	38	3 25	= 249	30	48 .75
R. A. given in the catalogue . . .				249	30	43

Examples with the smaller Reticle in Zone V

Example 4. For *Right Ascension*, Dec 24, 1751.

Tabular ingress of μ Hydri	2ʰ	35ᵐ	9ˢ	= upper part
Egress	2	38	40	
Difference		3	31	= interval = 211ˢ
Half the sum	2	36	54 .5	= the time of meridian passage
Table I +1ᵐ 26ˢ Table II −30ˢ.5 .			+55 5	
Apparent R A	2	37	50	by the reticle
	(2	37	48 .5	by equal altitudes)
Reduction from Table IV .			−1 .4	
	2	37	47 .1	= 39° 26′ 46″.5
R A. according to the catalogue . .				= 39 26 58

For *Declination*

$211^s \times 2 \times 15 \times 1705 (\cos 80° 11') = 1079''.265$		= 17′	59″ 265	subtract
Table III, upper part	80	29	1	
Apparent declination	80	11	1 735	by the reticle
	(80	11	2	by the sextant)
Reduction from Table IV			+41	
Reduced declination (1750) . .	80	11	42 .735	
Declination according to the catalogue .	80	11	42	

Example 5 For *Right Ascension*, Dec 24, 1751.

Tabular ingress β Hydri . .	0^h	6^m	3^s	
Egress	0	16	12	
Difference		10	9	= interval = 609^s
Half the sum	0	11	7 .5	= time of meridian. passage.
Table I +1^m 25^s Table II. −0^m 10^s . .		+1	15	
Apparent R. A.	0	12	22 .5	by the reticle.
	(0	12	22 .5	from equal altitudes.)
Reduction from Table IV.			−9 .1	
Reduced R. A. (1750)	0	12	13 .4	= 3° 3′ 36″
Mean right ascension in the catalogue . .				= 3 3 0

For *Declination* of β Hydri.

$609^s \times 2 \times 15 \times .1968$ (cos 78° 39′) = 3595 536 . .		=59′	55″ 5	add
Declination of lower point of the reticle . . .	77	39	11	
Apparent declination	78	39	6 5	by the reticle.
	(78	39	7	by the sextant)
Reduction from Table IV.			+41 3	
Reduced declination (1750)	78	39	47 .8	
Mean declination in the catalogue	78	39	48	

The reason why the tabular interval in time is doubled, in the fourth and fifth examples, is, that in the smaller reticle the length of the short diagonal is only one half of the height of the two triangles, while the Tables are constructed on the supposition that it is of the same length. These examples, it is presumed, will suffice to render the use of any other reticle with corresponding tables equally easy. The small discrepancies between the computed results and the numbers given in the catalogue, in each example, are owing to the circumstance of the standard stars having their places in the catalogue assigned them, from the mean of various observations taken with the different instruments, whereas our places are derived from the reticles only, to show what dependence may be placed on them.

TABLES FOR REDUCING THE OBSERVATIONS OF ZONE V

(BY LA CAILLE)

SEPTEMBER 24, 1751 APPARENT POSITIONS OF THE PRINCIPAL STARS

	RIGHT ASCENSION.	DECLINATION
Syrii	6^h 34^m 13^s.5	
μ Hydri	2 37 48.5	80° 11′ 2″
β Hydri	0 12 22.5	78 39 7

TABLE I

Correction of the Times indicated by the Clock

Times.		Correction additive	
H	M	M	S
19	0	1	23
21	10	1	24
0	20	1	25
3	0	1	26
5	40	1	27

TABLE II.

Correction of the Errors arising from the Inclination of the Vertical Thread

Declination of the Star		Correction
77°	40′	+ 4″ 5
78	0	− 0 .5
78	20	− 5 0
78	40	−10 .0
79	0	−15 0
79	20	−19 5
79	40	−24 .5
80	0	−29 0
80	20	−34 0

TABLE IV

Reduction to the True Positions, and to the epoch 1750

Hour observed	Right Ascension	Declination
18	−20^s 5	−13″ 5
19	−21 4	− 2 5
20	−21 2	+ 8 5
21	−19 9	+19 5
22	−17 9	+29 0
23	−14 9	+36 0
0	−11 4	+41 0
1	− 5 5	+44 0
2	− 3 7	+43 0
3	+ 0 1	+40 0
4	+ 3 3	+34 0
5	+ 6 0	+25 5
6	+ 7 9	+23 0

TABLE III.

FOR COMPUTING THE APPARENT DECLINATION OF THE STARS.

IN THE UPPER PART.						IN THE LOWER PART			
Intervals of the Transits.		Declination			Diff.	Declination.			Diff
0^m	0^s	80°	29′	1″	502″	77°	39′	11″	634″
1	40	80	20	39	519	77	49	45	615
3	20	80	12	0	534	78	0	0	597
5	0	80	3	6	550	78	9	57	580
6	40	79	53	56	567	78	19	37	568
8	20	79	44	29	583	78	29	5	552
10	0	79	34	46	602	78	38	17	538
11	40	79	24	44	623	78	47	15	524
13	20	79	14	21		78	55	59	

ZONE V. TAKEN BY THE SMALLER RETICLE OF LA CAILLE.

SEPTEMBER 24, 1752. Sirius culminated by the clock at . . 6^h 32^m 55^s.
Revolution of the fixed stars 23 59 51

IN THE UPPER PART

Mag.	H	M	S	Mag	H	M	S
7	19	18	18	6 7	23	56	44
		28	40			57	32
7	19	18	38	7	0	1	5
		20	30			13	10
7	19	22	43	6	0	2	49
		27	46			6	38
7	19	26	26	6 7	0	30	6
		34	35			41	39
7	19	32	57	7	0	53	9
		41	43		1	2	10
7	19	33	4	7	0	54	7
		35	40		1	3	28
6	19	39	21	7	1	22	41
		48	30			34	57
6	19	40	22	7	1	24	51
		42	27			26	36
6	19	41	4	5 6	1	27	26
		49	2			35	37
7	19	44	18	6	1	40	31
		51	48			41	26
7	19	46	18	5 6	1	50	42
		50	12		2	0	58
7	19	47	20	7	1	55	53
		56	16		2	1	41
7	20	7	35	6 7	1	59	53
		15	8		2	4	52
6 7	20	32	21	7	2	1	55
		35	18			8	41
6.7	20	55	58		μ Hydri		
	21	5	48	4 5	2	35	9
7	21	13	48			38	4
		25	57	7	2	36	51
7	21	18	44			40	37
		27	23	5	3	12	42
7	21	58	29			19	17
	22	9	46	7	3	18	17
7	22	6	40			26	47
		13	18	5 6	3	31	33
6	22	7	15			35	47
		12	23	6	3	38	10
7	22	56	45			52	53
	23	11	7	6	3	43	59
5 6	23	25	50			50	56
		30	1	6	4	7	12
7	23	36	31			20	15
		43	11	7	4	17	50
7	23	49	37			27	24
		50	30	6.7	4	31	27
6 7	23	51	0			36	14
		57	41	7	4	36	27
7	23	52	41			50	59
	0	1	36	7	4	49	7
						55	19

IN THE LOWER PART

Mag	H	M	S	Mag	H	M	S.
7	19	20	30		β Hydri		
		22	32	3	0	6	3
7	19	32	36			16	12
		32	58	7	0	9	8
7	19	55	38			13	18
		59	33	6 7	0	26	5
7	19	55	38			26	16
		58	26	6.7	0	48	1
6.7	20	4	4			50	14
		4	25	7	0	49	10
7	20	9	0		1	2	8
		23	8	7	0	52	18
7	20	27	20		1	5	28
		35	12	7	0	58	43
4 5	20	31	2		1	1	58
		33	37	7	0	59	23
4	21	7	31		1	8	12
		15	35	7	1	9	57
4 5	21	7	32			16	37
		15	34	7	1	16	49
7	21	21	59			18	54
		29	55	6 7	1	38	14
6	21	27	35			47	25
		39	43	6	1	47	42
7	21	30	32			58	9
		38	39	5	2	8	48
5 6	21	45	57			9	57
		56	59	7	2	51	43
7	21	55	2		3	4	12
	22	8	42	7	3	6	49
7	21	57	11			11	3
	22	1	39	4 5	3	18	29
7	22	4	47			24	35
		7	12	6 7	3	30	55
5 6	22	23	28			38	45
		30	22	7	3	42	15
7	22	50	5			44	9
		51	21	7	4	32	15
5.6	23	9	54			37	36
		21	10	6.7	4	46	36
5 6	23	18	19			50	51
		24	1				
4 5	23	43	20				
		51	10				
7	23	51	10				
		55	28				
7	0	3	33				
		13	0				
6	0	3	35				
		15	26				

TABLES FOR REDUCING THE OBSERVATIONS OF ZONE XIV

(BY LA CAILLE)

May 17, 1752 APPARENT POSITIONS OF THE PRINCIPAL STARS

	RIGHT ASCENSION		DECLINATION
Syrii . . .	6ʰ 34ᵐ 14ˢ		
ζ Aræ . .	16 38 18.5	. . . .	55° 33′ 49″
φ Argûs . .	9 48 13	. . . .	53 24 18

TABLES I AND II

For the Clock and Inclination of the Vertical Thread

Declination of the Stars	Correction add
52° 30′	29″ 8
53 0	28 0
53 30	26 0
54 0	24 2
54 30	22 2
55 0	20 5
55 30	18 5

TABLE III

For Computing the Apparent Declination of the Stars

IN THE UPPER PART			IN THE LOWER PART	
Interval of Transit	Declination.	Diff	Declination	Diff
0ᵐ 0ˢ	55° 40′ 19″	851″	52° 50′ 19″	901″
1 40	55 26 8	862	53 5 20	891
3 20	55 11 46	872	53 20 11	881
5 0	54 57 14	883	53 34 52	871
6 40	54 42 31	893	53 49 23	860
8 20	54 27 38	904	54 3 43	850
10 0	54 12 34		54 17 53	

TABLE IV

Reduction to the True Positions and epoch 1750

Hour observed	Right Ascension	Declination subtract
12	− 9ˢ 4	1′ 9″5
13	−11 1	1 3 6
14	−12 8	0 54 0
15	−13 9	0 40 5
16	−14 5	0 23 6

July 18, 1752 APPARENT POSITIONS OF THE PRINCIPAL STARS

	RIGHT ASCENSION		DECLINATION
Syrii . . .	6ʰ 34ᵐ 14ˢ.2		
ζ Aræ . .	16 38 18	. . . .	55° 34′ 1″
β Aræ . .	17 4 50.8	. . . .	55 15 34
χ Eridani .	1 46 19.5	. . . .	52 50 39

TABLES I AND II

For the Clock and Inclination of the Vertical Thread

Declination of the Stars	Correction subtract
52° 30′	2″ 5
53 0	3 5
53 30	4 5
54 0	5 2
54 30	6 2
55 0	7.0
55 30	8 0

TABLE III

For Computing the Apparent Declination of the Stars

IN THE UPPER PART			IN THE LOWER PART	
Interval of Transits	Declination.	Diff	Declination	Diff
0ᵐ 0ˢ	55° 39′ 1″	851″	52° 49′ 8″	901″
1 40	55 24 50	862	53 4 9	891
3 20	55 10 28	872	53 19 0	881
5 0	54 55 56	883	53 33 41	871
6 40	54 41 13	893	53 48 12	860
8 20	54 26 20	904	54 2 32	850
10 0	54 11 16		54 16 42	

TABLE IV

Reduction to the True Positions and epoch 1750

Hour observed	Right Ascension	Declination
16	−14ˢ 6	−34″ 5
17	−15 3	−17 0
18	−15 9	+ 1 0
19	−15 6	+19 0
20	−15 0	+36 0
21	−13 9	+51 0

ZONE XIV. TAKEN BY THE LARGER RETICLE OF LA CAILLE.

May 17, 1752	H	M	S
Syrius culminated by the Clock at	6	33	42
Revolution of the Fixed Stars	24	0	1

IN THE UPPER PART								LOWER PART			
Mag	H	M	S	Mag	H	M	S	Mag	H	M	S
6	12	40	40	7	15	20	52	7	13	35	15
		41	57			30	18			41	45
7	12	16	30	7	15	23	50	6 7	13	36	19
		56	0			25	57			39	8
7	12	47	26	6	15	26	57	6	13	37	56
		49	58			36	32			42	11
7	12	50	19	6 7	15	46	52	7	13	38	15
		55	6			48	10			40	41
7	12	51	52	7	15	47	22	5 6	13	39	39
		54	6			51	13			40	7
6 7	12	56	9	7	15	48	10	6	13	46	51
		50	42			50	21			51	9
6 7	13	1	18	7	15	49	51	7	13	48	57
		9	23			51	52			50	2
7	13	8	36	5 6	15	50	45	6 7	13	51	37
		10	44			56	20			58	45
7	13	13	20	6 7	15	56	29	7	14	1	1
		15	19		16	4	31			2	0
6	13	26	8		ζ Aræ			7	14	4	33
		27	18	4	16	37	38			11	31
7	13	30	58			38	21	6 7	14	12	21
		32	1							10	35
7	13	31	5	LOWER PART				7	14	18	55
		42	8		φ Argûs					22	22
7	13	36	16	4	9	15	53	7	14	22	41
		42	47			49	41			28	46
7	13	37	39	7	12	30	21	7	14	22	12
		43	46			33	9			27	4
6	13	44	20	7	12	35	56	7	14	28	51
		48	59			41	9			36	30
7	13	56	33	7	12	36	53	7	14	56	12
	14	3	56			46	25		15	2	10
7	14	0	12	6 7	12	40	55	7	15	0	51
		10	9			43	40			4	23
4 5	14	1	26	7	12	50	31	7	15	16	16
		4	33			52	47			21	19
7	14	9	9	7	12	51	14	6 7	15	38	13
		13	46			52	2			15	3
7	14	10	12	7	12	52	6	6	15	39	3
		14	42			55	7			42	1
7	14	16	40	7	13	10	53	6 7	15	41	27
Neb		18	8			16	13			45	29
6 7	14	20	18	7	13	11	50	7	15	49	30
		22	6			11	55			55	9
6 7	14	21	31	7	13	12	25	5 6	15	49	57
		23	46			12	37			57	35
6	14	23	19	7	13	19	47	6 7	15	53	14
		31	10			26	1			54	17
6 7	14	30	46	6 7	13	23	38	6.7	15	56	2
		31	11			23	56			58	11
7	14	41	44	5	13	21	9	7	15	56	47
		44	39			27	16			58	6
5 6	14	48	17	6.7	13	28	3	7	16	14	41
		57	7			32	4			16	23
7	15	0	12	7	13	31	41				
		3	27			41	33				
7	15	16	43	7	13	34	56				

July 18, 1752	H	M	S
Syrius culminated by the Clock at	6	31	21 5
Revolution of the Fixed Stars	24	0	0 5

IN THE UPPER PART.								LOWER PART			
Mag	H	M	S.	Mag	H	M	S	Mag	H	M	S
7	16	21	32	7	20	3	55	7	18	4	10
		23	11			6	23			10	22
6 7	16	31	43	7	20	33	13	6 7	18	5	53
		35	0			41	7			8	33
	ζ Aræ			7	20	36	16	6 7	18	14	0
4	16	38	8 5			40	1			15	28
		38	12 5	7	20	48	9	7	18	20	27
7	16	49	7			51	30			28	34
		56	56	6 7	20	51	59	5	18	37	22
7	16	56	27			52	13			40	9
	17	2	15	6 7	20	53	20	7	18	52	21
7	17	0	9		21	2	52			58	41
		9	25	7	20	57	27	6 7	18	55	12
	β Aræ				21	1	50		19	1	46
4	17	3	37	5	20	58	8	7	19	0	31
		6	21		21	6	21			2	30
6 7	17	5	52	7	21	1	33	6 7	19	10	20
		10	57			2	5			16	18
7	17	10	36					7	19	14	14
		17	24	LOWER PART						18	53
6	17	13	22	7	16	15	5	7	19	20	42
		22	37			16	56			25	12
7	17	21	49	7	16	30	16	7	19	25	50
		23	4			36	17			27	38
6	17	26	55	6 7	16	35	32	7	19	27	31
		29	31			44	55			31	52
6 7	17	35	33	7	16	36	10	7	19	31	2
		38	2			40	18			36	51
7	17	50	16	7	16	38	16	6	19	45	56
		52	16			47	28			50	56
7	17	53	51	5 6	16	43	32	7	19	46	17
	18	2	31			48	39			48	46
7	17	54	29	7	16	51	51	6.7	19	47	50
		56	17			53	17			50	46
7	18	9	36	7	16	58	26	6.7	19	54	26
		10	11		17	6	32			56	42
7	18	38	55	7	16	58	56	7	20	29	52
		41	17		17	9	32			30	6
7	18	59	32	7	17	5	10	7	20	36	29
	19	8	14			11	40			37	51
5 6	19	0	11	7	17	7	18	6 7	20	46	37
		5	46			8	18			46	47
7	19	0	42	6.7	17	14	24	7	20	52	4
		5	31			16	36			55	28
7	19	4	37	7	17	18	20	7	21	4	30
		5	20	Neb		23	5			5	4
6 7	19	5	0	7	17	20	3	7	21	5	17
		11	2			23	15			6	17
7	19	8	48	6	17	28	13		χ Eridani		
		15	44			32	48	4 5	1	46	18
6 7	19	10	21	7	17	28	19			46	26
		10	51			34	28				
7	19	17	56	6	17	30	39				
		24	15			32	1				
7	19	18	9	7	17	55	33				
		23	4		18	2	5				
6	19	52	42	7	18	2	41				

§ XXVII SMEATON'S METHOD OF USING A WIRE-MICROMETER WITH AN EQUATORIAL STAND

1. When La Lande had announced the utility of accurate observations of the planet Mercury at his two elongations in August and September 1786, the ingenious Smeaton resolved to try his skill in fitting up an achromatic telescope in a way, that would enable him to take some observations before and after the greatest elongation, which happened in August of the year above specified. He first tried to take the meridian passage with a transit instrument, but was disappointed in trying to find the small body in the neighbourhood of the sun at the middle of the day, and therefore resolved to observe it in twilight before the sun rose to effect this he was obliged to mount his telescope in such way, that its motion might be in an equatorial direction, in order the more easily to find his object, and also to keep it in the field of view till he had completed the adjustments of his telescope and attached micrometer. The contrivance which he availed himself of to answer these purposes, was the divided *block* which we have already described (§ X. 2.), on the bed of which he fixed his telescope of 34.66 inches focal length, and placed the stand on the frustum of an hexagonal pyramid of stone, built on a firm foundation, and free from the floor on which he stood. The observatory was at Austhorpe, in the West Riding of York, in latitude 53° 47′ 54″ N. and $5^m 50^s$ W. from Greenwich; the knowledge of which data enabled the observer to adjust for the equatorial position, and to give the telescope the due elevation for Mercury's meridian altitude, as well as to rectify the hour circle to the plane of the equator. The micrometer had two screws for giving differences of declination, in the way Troughton now makes them, and five horary wires crossing the former ones at right angles, which wires he denominated *a*, A, B, C, and D, while he called the declination wires by the italic capitals *A* and *B*, for Australis (southern) and Borealis (northern).

2. That some judgment might be formed of the performance of this apparatus, and of its stability of position, before a trial was made of observing Mercury, Saturn was previously compared with γ Capricorni, and the return of the star to the same place, as to right ascension and declination, after the lapse of two days was so exact, that it raised the observer's confident expectation of being able to get a good observation of Mercury, though the stars with which he was to be compared, could not be observed till the subsequent evening.

3 On the morning of the 23d of September, about three quarters of an hour before sunrise, the air fortunately being clear and perfectly serene, the little planet was readily found with a magnifying power of about 20, in a field of 1° 17′, and when found could easily be distinguished with an opera-glass. Mercury was at that time experiencing very little change of declination, and when the wires were adjusted for his run along one of them, he was suffered to pass along the whole field, and, when he had been brought apparently back again by the proper motion, the telescope was fixed at VI. 34.5 on the horary circle, and at dec. 7° 48′ N. on the vertical semi-circle, at an elevation of about 11° 30′; a Hindley's sidereal clock being referred to for the time, or rather a journeyman half-seconds clock, which was compared with that by Hindley. The observations were then taken, and registered in prepared columns, as in Table I, which follows. In the same evening the astronomer was lucky enough to get ob-

servations of the stars λ Ceti and ο Tauri, but the unfavourable state of the weather prevented Mercury being again observed. On the 26th however, and again on the 30th, the two stars were observed, the telescope remaining *in statu quo*, and were found so nearly in the same parallel of declination, that a resolution was formed of locking the observatory during several days of absence, and of leaving the telescope in its place unaltered. On the 13th of October the positions were found as little altered as could be expected or wished for, after allowance was made for variation of refraction and precesssion, &c. when the places of the stars were again taken. On the 30th of September, and also on the 13th of October, α Orionis was also observed, as being one of Dr. Maskelyne's well determined stars, and therefore more to be depended on as a standard star for a good comparison.

4. *Explanation of the following Tables*.—Columns 1 and 2, in Table I, are sufficiently intelligible from the heading, column 3 contains the minutes, quarters of minutes, and beats or half-seconds, the beats commencing at 15, 30, 45, or 60, as shown by the seconds hand, column 4 gives the observed time reduced into minutes and seconds; and column 5 contains the said times reduced to the middle wire by the help of Table II.; column 6 gives the mean times due to the middle wire on an average of all the observed passages, column 7 contains the number of revolutions and decimal parts of the micrometer's head as read and marked *A* or *B*, accordingly as the southern or northern wire was used; and as the readings had their zeros at 28 .11 turns from the centre for *B*, and at 30 .84 for *A*, the number of revolutions of the micrometer's screws taken from these constant numbers respectively gave the number of revolutions counted from the centre, which are the numbers contained in column 8. When the number of turns read off exceeded the *constant*, the difference being beyond the centre, had an opposite denomination, as was the case in the observation of Mercury, where 28 .85 revolutions made by *B*, exceeded the constant 28 .11 by 0 .74, which therefore became A or S 0 .74.

5. In Table III. the 1st and 2d columns explain themselves; column 3 is the same as column 6 in Table I.; the 4th column contains the corrections to be applied to the preceding column, as determined by comparison with the transit clock, of which the rate was several times taken by actual observations; the fifth column shows the times so corrected; the sixth gives the intervals between the successive observations in time; the seventh the revolutions of the micrometer, as they regard the centre, and the eighth the declination measured from the same centre in minutes and seconds corresponding to the adjoining revolutions.

6. The value of one revolution of the micrometer had been previously assigned from observations, and one revolution was found to be so nearly a second, that dividing the number of revolutions and decimal parts by 1 .08 gave the exact number of seconds without an additional Table

7.

TABLE I.

OBSERVATIONS OF MERCURY, AT HIS ELONGATION IN SEPTEMBER, 1786,

WITH AN EQUATORIAL MICROMETER

Day, Object, and Wires	Hour	Time, as taken by the Clock.	Time reduced to min and sec	Time reduced to the mid wire	Mean of the Wires	Parts of the Micrometer	Micrometer reduced
Sept. 23	A M	M QU BEATS	M S	M S.	M S	REVOL.	REVOL
Merc. to wire *a*	5	24 3 5	24 47 5	26 34 8			
A		25 3 14 5	25 52 3	26 34 8			
Middle wire B		26 2 0 5	26 34 7	26 34 7	26 34 7	B 28 85	S 0.74
C		27 1 6	27 18 0	26 34 7			
D		28 1 15	28 22 5	26 34 6			
λ Ceti to......B	P M 9	15 1 27	15 28 5	15 28 5	15 28.4		
C		16 0 23	16 11 5	15 28.3			
o Tauri to.... B		40 1 25	40 27 5	40 27 5	40 27.4	B 8.39	N 10 72
C		41 0 21	41 10 5	40 27 3			
Sept 26							
λ Ceti to... ...*a*	9	2 2 13	2 36 5	4 23 5			
A		3 2 23	3 41 5	4 23 9	4 23 8	B 16 97	N 11.14
C		5 0 14 5	5 7 3	4 24 1			
o Tauri to ....A		28 2 21	28 40 5	29 22 9			
B		29 1 16	29 23 0	29 23 0	29 23 0	B 8.47	N 19 61
Sept. 30....... *a*	8	47 0 3 5	47 1 8	48 48 8			
λ Ceti to.......A		48 0 13	48 6 5	48 48 9			
B		48 3 8	48 49 0	48 49 0	48 49.0	B 16 97	N 11 14
C		49 2 4 5	49 32.3	48 49 1			
o Tauri to ....A	9	13 0 11	13 5 5	13 47 9			
B		13 3 6	13 48 0	13 48 0	13 48 1	B 8 48	N 19.63
C		14 2 3	14 31 5	13 48 3			
α Orionis to...*a*	11	41 3 12	41 51 0	43 38 0			
A		42 3 20.5	42 55.3	43 37 7			
B		43 2 17	43 38 5	43 38 5	43 37.9	A 15.07	S 15.77
C		44 1 12	44 21 0	43 37.8			
D		45 1 20 5	45 25 3	43 37 7			
Oct 13							
λ Ceti to .... A	7	58 3 0	58 45 0	59 27.4			
B		59 1 25 5	59 27 7	59 27 7	59 27 5	B 16.97	N 11 14
C	8	0 0 21 5	0 10 7	59 27 5			
o Tauri to .. C		25 0 20	25 10 0	24 26 8	24 26 8	B 8 50	N 19 61
α Orionis to...*a*	10	52 2 0.5	52 30 2	54 17 2			
A		53 2 9	53 31 5	54 16 9			
B		54 1 4 5	54 17 2	54 17 2	54 17 1	A 15 07	S 15 77
C		55 0 0.5	55 0 2	54 17 0			
D		56 0 10	56 5 0	54 17 4			
Col. 1	2	3	4	5	6	7	8

8.

TABLE II.

FOR REDUCING THE HORARY WIRES OF THE EQUATORIAL MICROMETER TO THE MIDDLE WIRE, WHEN TAKEN IN MEAN SOLAR TIME

	Wires	Equatorial object		Declination, 7° 48′	
		☉'s run	*'s run	☉'s run	*'s run
The first wire precedes, add	*a*	1ᵐ 46ˢ 2	1ᵐ 46ˢ	1ᵐ 47ˢ 3	1ᵐ 47ˢ
Second wire	A	0 42 1	0 42	0 42 ,5	0 42 4
Middle wire .	B	... ..	... .	... .	... .. .
Fourth wire follows, sub.	C	0 42 9	0 42 8	0 43 3	0 43 2
Fifth wire	D	1 46 8	1 46 6	1 47 9	1 47 6

9

TABLE III.

CONTAINING THE OBSERVATIONS OF TABLE I REDUCED SO AS TO SHOW THE CORRECT DIFFERENCES OF RIGHT ASCENSION AND DECLINATION BETWEEN MERCURY AND THE STARS WITH WHICH HE WAS COMPARED

1786 Date and Object	Hour	Passage over mid hor wire by journ clock	Reduction to mean time	Mean time of the observation	Intervals of mean time of diff obs	Parts of micrometer from the centre	Value of the said parts
Sept 23	A. M	M S	M S	H M S	H M S	REVOL	
☿ to mid wire	5	26 34 7	−3 59 8	5 22 34 9		S 0 ,74	S 1′ 6″
	P M.						
λ Ceti to mid wire	9	15 28 4	−4 0 1	9 11 28 3	15 48 53 4		
ο Tauri, do	9	40 27 4	−4 0 1	9 36 27 3	0 24 59	N 19 72	N 30 26
Sept 26							
λ Ceti to mid wire	9	4 23 8	−4 43 2	8 59 40 6		N 11 14	N 17 11
ο Tauri, do	9	29 23 0	−4 43 2	9 24 39 8	0 24 59 2	N 19 64	N 30 18
Sept 30							
λ Ceti to mid wire	8	48 49 0	−4 50 9	8 43 58 1		N 11 14	N 17 11
ο Tauri, do	9	13 48 1	−4 50 ,9	9 8 57 ,2	0 24 59 1	N 19 68	N 30 17
α Orionis, do	11	43 37 9	−4 50 8	11 38 47 1	2 29 49 9	S 15 ,77	S 24 20
Octob 13							
λ Ceti to mid wire	7	59 27 5	−6 36 4	7 52 51 1		N 11 ,14	N 17 11
ο Tauri, do	8	24 26 8	−6 36 8	8 17 50	0 24 58 9	N 19 ,61	N 30 15
α Orionis, do	10	54 17 1	−6 37 0	10 47 40 1	2 29 50 ,1	S 15 77	S 24 20
Col 1	2	3	4		6	7	8

10. As there was an interval of almost sixteen hours between the times of the passages of Mercury and λ Ceti, it may be satisfactory to show from the observations themselves, that neither the permanent position of the telescope, nor the going of the clock, required any correction for a much longer interval

In *Right Ascension.*

On the 23d Sept. λ Ceti passed at	9^{h}	11^{m}	$28^{s}.3$
26th ditto	8	59	40.6
λ Ceti came sooner after three days by		11	47.7
Tabular Acceleration due to three days		11	47.7
Also ο Tauri Sept. 23 came at	9	36	27.3
26 ditto	9	24	39.8
Acceleration as before within $0^{s}.2$		11	47.5

In *Declination.*

ο Tauri Sept. 23 passed to the north	30′	26″
26 ditto	30	18
It passed more to the south by	0	8

Hence it may be concluded that the variation of position or of rate, would neither of them be sensible in sixteen hours.

11. *Deduction of Mercury's position,* from Table III.

Mercury preceded λ Ceti Sept. 23 by	15^{h}	48^{m}	$53^{s}.4$
λ Ceti preceded ο Tauri by a mean of 4 obs.		24	59
ο Tauri preceded α Orionis by a mean of 2 obs.	2	29	50
Mercury therefore preceded α Orionis by	18	43	42.4
Add correction arising out of difference of refraction			1.1
True difference of right ascension	18	43	43.5

If we take the differences in declination between λ Ceti and ο Tauri on Sept. 23, Sept. 30, and Oct. 13, we shall find them 13″.7, 13″.6, and 13″.4 respectively, whence we may conclude that *ο* Tauri has been well observed, and may be compared with both Mercury and *α* Orionis to connect these two thus,

Sept. 23 A.M. Mercury passed from the centre		1′	8″ S.
ο Tauri ditto		30	26 N
Therefore Mercury was more south than *ο* Tauri by		31	34
Sept. 30, *ο* Tauri passed N. 30′ 17″ *α* Orionis S. 24 20	Sum	54	37
Oct. 13, *ο* Tauri passed N. 30 15 *α* Orionis S. 24 20	Sum	54	35
Hence *α* Orionis passed south of *ο* Tauri		54	36
And therefore Mercury's dec. exceeded that of *α* Orionis, on the 23d Sept. by		23	2 N.
To which add the correction for difference of refraction			6
True difference of declination		23	8

Resulting position of Mercury.

The right ascension of *α* Orionis, Sept. 30, 1786, by Dr. Maskelyne's Tables =				85°	54′	12″
From a revolution in solar time	23ʰ	56ᵐ	4ˢ.1			
Subtract the difference in time	18	43	43 .5			
The time by which *α* Orionis preceded =	5	12	20 .6	= 78	5	9
The *right ascension* of Mercury at the time of observation =				163	59	21
Declination of *α* Orionis, corrected for precession				7°	21′	8″.8 N.
Sum of aberration and nutation					+	8 .4
				7	21	17 .2
Mercury's difference of declination more north, add					23	8
The *north declination* of Mercury at the time of observation				7	44	25 .2

In this determination of Mercury's geocentric place, his parallax has not been considered.

[Phil. Trans. Vol. LXXVII.]

DOUBLE-IMAGE MICROMETERS.

§ XXVIII DOLLOND'S OBJECT-GLASS MICROMETER APPLIED TO A REFLECTING TELESCOPE [PLATE XXIX]

1. HAVING described and explained the different *single-image* micrometers that have fallen within our notice as applicable to astronomical purposes, we may now devote a few sections to the description of such *double-image* micrometers as may be found useful in the hands of a practised observer, which we proposed to consider the *second class* of micrometers (§ XIX. 1.). This class has an important advantage over the single-image micrometers, inasmuch as they require no illumination from a lamp, but as each of the two images that are formed of an object, has usually only one half of the light that a single image of an object possesses in the field of the telescope, the aperture of the telescope, to which a double-image micrometer is intended to be applied, ought to be large enough to compensate the defect of light that bipartition will occasion in each image, which compensation will be effected when the area or square of the diameter is doubled.

2. If we pass in silence over Savery's and Bouguer's plan of obtaining double images, from a pair of complete object-glasses applied to the same telescope, the application of which does not seem to have been adopted in astronomical observations *, we believe that the divided lens placed before the aperture of a telescope, contrived by John Dollond, was the first effectual means that enabled astronomers to apply the principle of double images to celestial measurements. This contrivance was described in the forty-eighth volume of the Philosophical Transactions of London in the year 1753, having been communicated by the excellent optician J. Short, before the aperture of whose reflecting telescopes the divided lens was first used. It is well known that when a perfect lens is divided into two or more portions by lines drawn across its face, these portions, not being in the same plane, will bring the incident rays of light falling at right angles upon their surfaces, to as many focal points, and will form as many images of an object that is luminous and at some distance in those points; but if the pieces have their surfaces put exactly into the same curve, these images will coincide, and one image only, more bright than any of the separate ones, will now appear. If the lens be divided into two equal halves by a straight line passing through the centre, there will be one image while the central parts of the semi-lenses are so kept in contact, that they form a perfect lens; but when the cen-

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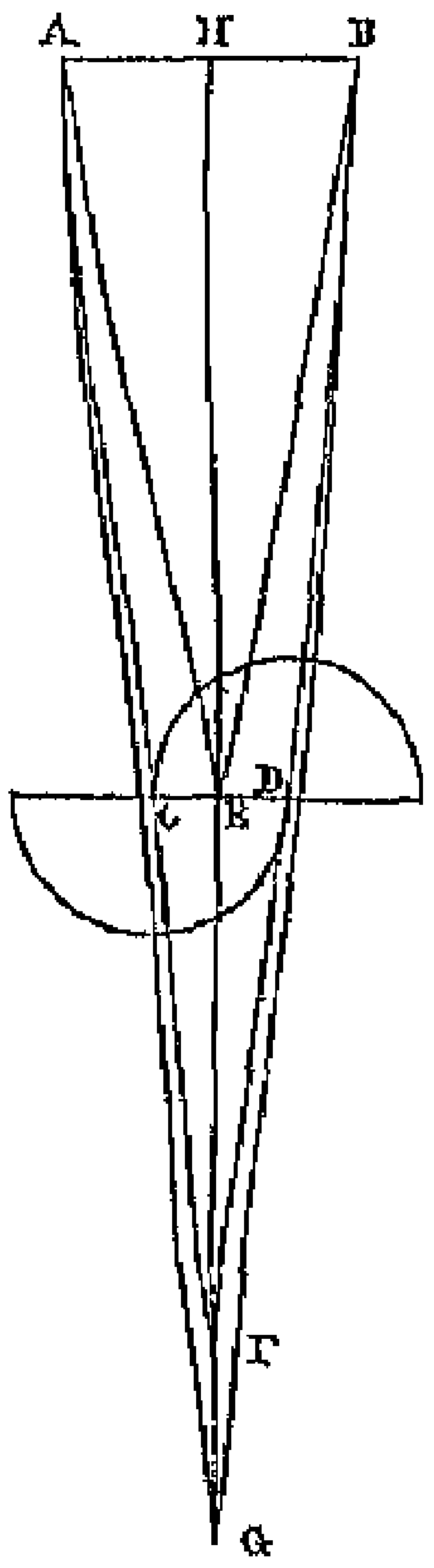
* Mr Benjamin Bevan, of Leighton-Buzzard, makes use of two equal object-glasses in separate tubes, that are so hinged together at the object-ends, as to be capable of being placed parallel, and also inclined in any given quantity that will measure small terrestrial angles, on an arc of large radius, by means of which he measures distances at one station in the operation of levelling

tres of the two halves are a little separated by any mechanical means, two images will appear equally perfect and equally luminous, as they regard each other, but each possessing only half the brilliancy that the single image had, when formed by the perfect lens. The distance between the two images thus formed, will increase as the centres of the semi-lenses are separated, and in the same proportion, so that the quantum of separation of the centres of the two lenses, may be made a scale of measurement of the distances of the central parts of the two images of any object. When the divided lens is fixed in a cell before the aperture of a reflecting telescope, two cones of rays will fall on the large speculum, and will be reflected so as to form two images, according to the construction of the telescope, and the quantity of separation of the semi lenses, which images may both be viewed, or partially viewed, by the eye-piece of the telescope in a magnified state The lens to be divided may be either of the convex or concave kind; and the focal length of the large speculum of the telescope will be either diminished by the former, or enlarged by the latter, so that when the curve of the speculum is not good with parallel rays falling on it, one shape of the lens or the other may be preferred, that will tend to cure the imperfection; but if the telescope be already good with distant objects, an alteration of the parallelism of the incident rays, by either description of lens, will injure the vision of the telescope; the optician therefore, who appropriates a divided lens to a reflecting telescope, ought to adapt the curve of his metal to the nature of the lens, at the time it is polished.

3 When the divided lens had been enclosed in a frame that admitted of gradual separation of its two halves, by a pinion on the axis of a Hooke's joint, acting with a pair of racked bars, and also that allowed a circular motion to take place by means of another pinion impelling a toothed wheel, there immediately became a demand for the instrument on the Continent, and because the telescope would take into its field two images of the sun, when the magnifying power was moderate, it soon obtained the name of an *Heliometer*, which it still retains out of England, and might as well have been called a *Selenometer*, because it will contain two diameters of the moon equally well, and may be applied to measure the distances of the lunar spots from one another, or the breadth of the luminous portion as compared with the whole diameter at any time.

4. There are various ways in which the divided lens may be used as an heliometer, with a refracting telescope, as well as before the tube of any of the reflecting constructions, as the inventor has shown in a second communication made to the Royal Society through the hands of Short, in the year 1754; but the theory will be best explained, by considering it as the chromatic object-glass of a long refracting telescope, divided in the middle, and ground to straight edges that will pass along one another without the admission of false light, agreeably to the construction given in the following figure, which is copied from the author's own delineation, as well as the explanation accompanying it in his first communication, viz:



"The semicircles", says Dollond, "represent the two segments of the object-glass, whose centres C and D are drawn off to the distance CD, and the points A and B are two objects, or different parts of the same object; therefore the lines ACG and BDG represent two rays, that pass through the centres of the segments, and are therefore not at all refracted, but go straight through to G, where they intersect, and G being the respective focus to the distance of the objects from the glass, the two images will coincide at that point It appears from the figure that $AB : CD :: GH : GE$, and from a common proportion in optics $GH : GE :: HE : EF$. Therefore $AB : CD :: HE : EF$, F being the focus of parallel rays; and consequently the angles AEB and CFD are equal, i. e. the angle subtended by the distance of the centres of the segments at the distance of the focus of parallel rays (F), is equal to the angle subtended by the distance between the objects A and B, from (E) the end of the telescope."

5 From this explanation of the principle of measuring by a divided lens, giving two images of an object, it appears, as the author has observed in his second communication, that the angle subtended by the diameter of any object viewed from the object glass, is found without any regard to the distance of that object, or to the distance of the respective focus (G), where the two images are seen in contact, since the measure depends entirely upon the focus of parallel rays and the opening of the segments, and consequently a scale of measurement derived from a terrestrial object placed at any distance would be a good scale for any other distance, when the segments of a single lens are used as the object-glass of a refracting telescope, but according to the present construction of telescopes such an object-glass, producing prismatic colours, is inadmissible.

6. The inventor of the divided lens, not having yet discovered the achromatic principle of the refracting telescope, and perceiving that great inconvenience would arise from the use of an object-lens of very long focal distance, particularly when divided, and requiring an apparatus for producing the separation of the two segments, at length determined to apply it before the tube of a reflecting telescope, where it would be more manageable, and where the different reflections and refractions would not alter the positions and relative magnitudes of the two images of an object, formed by the two separated portions of rays proceeding from that object; consequently he concluded that, notwithstanding the shortening of the instrument, the scale of measurement depending on the separation of the central points of the semilenses, and on the first tendency of the rays to come to a distant focus, would not be altered He therefore preferred this application, and his friend Short promoted his views by his excellent construction of the Gregorian and Cassegrainian telescopes, to which the divided lens was attached to complete the heliometer.

7. The object-end of one of these instruments, together with the handles giving the requisite motions of adjustment for obtaining the measures, are represented by fig 1. of Plate XXIX, in which the lens is of the convex kind, that shortens the focal length of the large speculum, of about two feet, three quarters of an inch. The different parts of this micrometer, as viewed from beyond the object-end of the telescope, are so clearly seen in the figure, that a short description will suffice for rendering their uses intelligible. We must conceive the piece

of tube *A* to be a portion of the remote end of the telescope, into or over which the short piece of tube *B* of the micrometer is applied, and made fast by a thumb-screw or otherwise; this piece *B* carries a concealed wheel formed of a ring, which is indented at the outer edge, and attached to the covered plate of brass *C C′*, in such way that a pinion, on the arbor of which the handle *D* is inserted, will turn it round into any required position, vertical, horizontal, or oblique, which it will retain by means of the friction given to the moving parts. *F* and *G* are two long narrow plates lying upon the under plate *C C′*, to which they are respectively kept close by the rabbeted bars *H H′*, but so as to be at liberty to move in contrary directions, when their indented end pieces are actuated by a concealed pinion placed on the under plate, between the letters *F* and *C*, the arbor of which pinion is embraced by the squared end of the handle *E*, then as the two semi lenses, which appear darkened in the figure, are separately attached to the plates *F* and *G* respectively, it is obvious that their centres may be separated more or less by turning the handle *E*, while its pinion is interposed between and connected with the teeth made on the edges of the said moving plates. At the opposite projecting end of plate *G* is a scale of five inches, each subdivided into the twentieth parts of an inch, and on the contiguous portion of plate *F* is a vernier divided into twenty-five parts, commensurate with twenty-four on the scale of inches, so that the smallest portion that can be measured is $\frac{1}{25}$th of $\frac{1}{20}$th of an inch = $\frac{1}{500}$th part, but it is convenient for the construction of a table, to divide the distance from the centres of the semi-lenses into three denominations, inches, divisions, and subdivisions, as given by the vernier. The two screws that hold the vernier-piece to the plate *F*, pass through oblong holes made in it, which allow of adjustment to zero on the scale of inches, when the centres of the semi lenses, or images of an object seen through the telescope, exactly coincide, and the small screw *I*, which enters the remote end of the vernier-piece, effects the adjustment before the two fixing screws are made fast.

8. From this description it may be perceived that, when the micrometer is attached to the remote end of any telescope, the handles must be long enough to be within reach, while the eye of the observer is applied to the eye-end, and that a piece of strong wire formed into a ring with a couple of hooks at opposite sides embracing the tube near the eye end, will serve to hold the handles in their proper places, parallel to the tube at all times, whatever may be the required position of the micrometer. The principal inconvenience that attends the use of this micrometer is, that its weight deranges the balance of the telescope on its stand, which can only be remedied by a counterpoise, or by additional thumb-screws inserted into the main tube when lying in its bed, at a part nearer to the object-end, than is usual in ordinary telescopes.

9 In giving a value to the scale of the divided object glass micrometer, the sun's disc is usually made the standard measure, and as his motion through the field of view creates no impediment to the accuracy of an horizontal measurement of the diameter by this apparatus, this diameter should always be preferred, because it is not affected by refraction. When the plates of the micrometer have been turned into their horizontal position by the handle *D*, and the sun's disc brought into the field of view with the dark glass to guard the eye, the telescope must first be brought very nicely into distinct vision, by the thumb screw which moves the small speculum, and if the vernier is at *o* on the scale, there should be but one well defined image of the sun, but when the edge appears double, the handle *E* must be used to bring the

two images nicely into one, and the vernier must then be adjusted to zero by means of the screws that regulate and fix its position: when this is done the instrument is fit for use, and the handle *E*, being turned, will produce two images of the sun, which must be gradually separated till their opposite limbs just come into nice contact, when this is effected, the first stroke of the vernier will indicate the inch and parts of an inch on the scale of inches, and the line of the vernier that is coincident with some line of the divisions of the scale, will show the number denoted by the subdivision, as the third denomination to be added to the former quantity and when a Table has been already prepared, the value of this measure may be had by inspection from its corresponding columns, but if not, a Table may be easily constructed from this measure, particularly if it comes out the same, or very nearly so, on two or three observations of the sun on successive days An instance or two will render the method easy to any unpractised observer. On the 18th of May 1825, the sun's diameter, as given in the Nautical Almanac, was 31′ 39″.6 or 1899″.6, and the measure taken by a double-image micrometer, applied to a reflecting telescope by Tulley, was determined to be 3 in. 0 div 21 5 subdiv on an average of several trials, this quantity reduced into its lowest denomination is 1521 5 subdivisions, or so many five hundredth parts of an inch, the value of one such part will therefore be $\frac{1899''.6}{1521.5} = 1''.2485$, or very nearly a second and a quarter, but must not be taken exactly as such for the purpose of constructing a Table, which must be obtained from multiples of the exact quantity the whole scale of the vernier will be $1'' 2485 \times 25 = 31''.2125$, and this being also the value of $\frac{1}{20}$th of an inch on the scale, we shall have $31'' 2125 \times 20 = 624'' 25$ or $10' 24'' 25$ for the value of an inch, and from these values the subjoined Table was constructed

10.

A TABLE

EXHIBITING THE VALUES OF MEASURES TAKEN BY A DOLLOND'S OBJECT-GLASS MICROMETER APPLIED TO A TWO-FEET REFLECTING TELESCOPE

	Inches		Divisions continued		Subdivisions continued
1	10′ 24″ 25	12	6 14.5500	8	9.9880
2	20 48 50	13	6 45.7625	9	11.2365
3	31 12.75	14	7 16 9750	10	12.4850
4	41 37 00	15	7 48 1875	11	13.7335
5	52 1.25	16	8 19 4000	12	14.9820
	Divisions	17	8 50.6125	13	16.2305
		18	9 21 8250	14	17.4790
1	0′ 31″.2125	19	9 53 0375	15	18.7275
2	1 2.4250	20	10 24.2500	16	19 9760
3	1 33.6375		Subdivisions	17	21.2245
4	2 4.8500			18	22.4730
5	2 36.0625	1	1″.2485	19	23.7215
6	3 7 2750	2	2.4970	20	24.9700
7	3 38.4875	3	3.7455	21	26.2185
8	4 9.7000	4	4 9940	22	27 4670
9	4 40.9125	5	6.2425	23	28 7155
10	5 12.1250	6	7.4910	24	29.9640
11	5 43.3375	7	8.7395	25	31 2125

11. As an example for illustrating the use of this Table, and at the same time for showing how the index error may be eliminated when the measure is small, the diameter of Jupiter was taken on both sides of zero on the evening of the same day, which afforded data for the scale, and the following measures were obtained, viz

♃ to the right of zero ....23 subdivisions
Do to the left, read backwards .... 24 do.

Average 23.5 × 1″ 2485 .... = 29″.33975

By the Table, 23 subdivisions give ...28″.71550
$\frac{1'' 2485}{2}$ 0 62425
The value of the measure ....29″.33975

Which may therefore be taken at ....29″ 34.

When the same micrometer was fitted to a good achromatic telescope of 45.75 focal length, the value of a subdivision was by a similar process determined to be 1″.242.

12. The advantages attributed to this micrometer by the inventor, as compared with the wire micrometer that preceded it, may be thus enumerated. 1st, the *motion* of the body observed produces no impediment to either the convenience or accuracy of the observation made by this instrument; 2ndly, no *illumination* is necessary within the telescope, 3rdly, the contact may be made, and the measure ascertained, when the body observed is too large to be all included in the field; 4thly, the scale does not vary with the distance of the object observed*.

13 The principal disadvantages of the construction are, first, a want of perfect vision when the telescope is good with parallel rays without the micrometer; secondly, the measure will be affected more or less by the state of the observer's eye, accordingly as it has a tendency to have distinct vision beyond or short of the focal point, where the rays cross, and where the image is most perfect In the latter case there will be a visible separation of the limbs at the true place of contact, and in the former there will be an over-lapping, either of which errors may amount to a few seconds, and thirdly, the *differences* of declination and of right ascension of two bodies cannot be taken so well, as the absolute distance between them. Hence the wire micrometer and the object glass micrometer were said by Dr. Maskelyne to remedy the defects of each other

14 In the 61st Volume of the Philosophical Transactions above mentioned, this astronomer royal wrote a paper, which was read Dec 12, 1771, in which he proposed an improvement in the use of Dollond's micrometer, by the introduction of cross-wires fixed in a moveable ring at the place of the double image, to one or other of which lines either one or both of two planets, or stars, might be referred, accordingly as the observed difference of right ascension, or of declination, might require. The four annexed figures will explain the Doctor's manner of using his cross-wires in conjunction with a Dollond's micrometer

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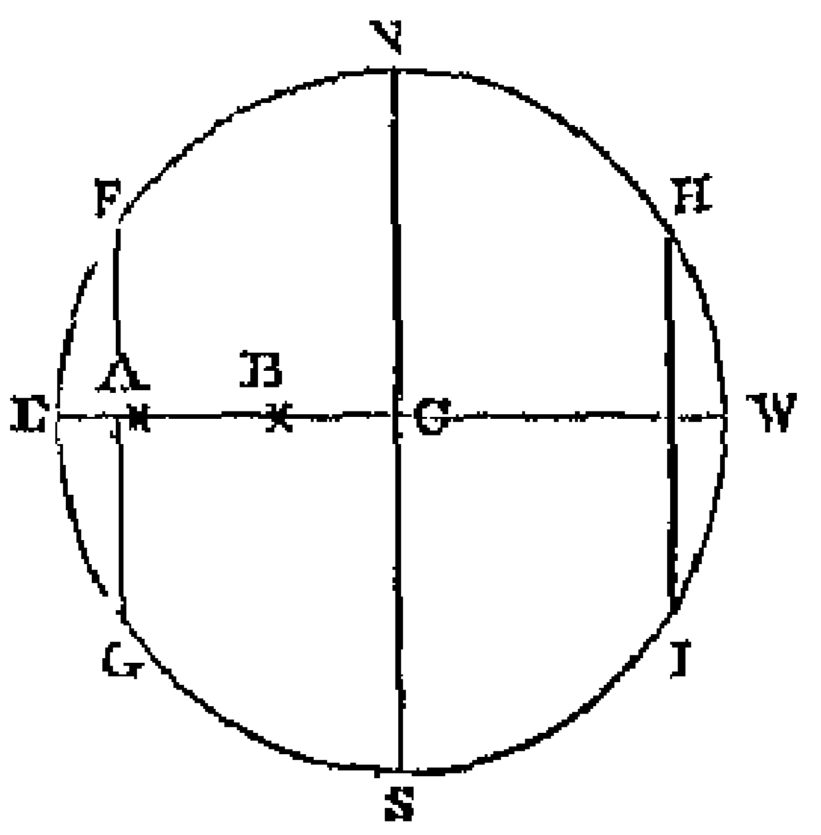
* See Mossotti's experiments and remarks in the following Section.

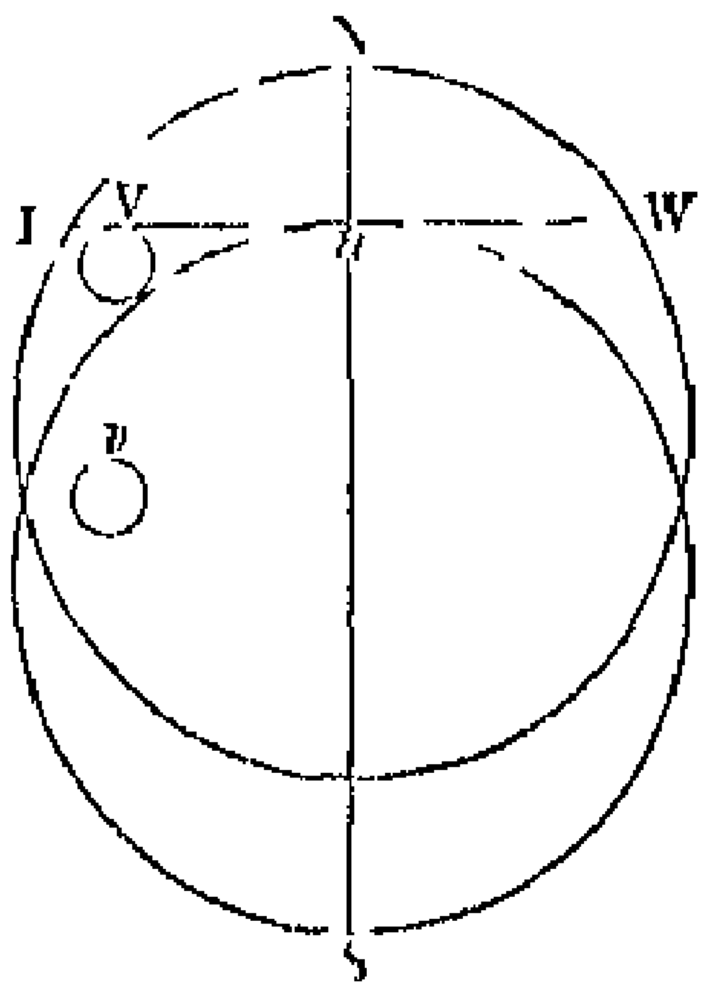


Let the circle represent the field of view, *N S* being the meridian line and *E W* the east and west line of the field, then to find the *difference* of declination and of right ascension of two stars, both appearing in the field, the Doctor opened the semi-lenses to a convenient distance, more or less, which presented two pairs of images, and, turning round the micrometer by the proper handle, he made the two images of the first star to pass over the vertical line *N S* at the same instant, and counted the seconds and parts that intervened, before the passage took place of the second pair of images of the following star over the same line, which interval in time was the difference of their respective right ascensions. Then, partly by the screw of the telescope's elevation, and partly by opening the segments, he contrived to make the north image of one star and the south image of the other run along the horizontal line *E W*, as in the figure, and at that opening the scale indicated the difference of the two declinations. With an equatorial stand this observation might be repeated with very little trouble, and with considerable precision, and an average thus taken would even give a value to the scale, provided the declinations of both stars were correctly known at the time of observation. When the two stars are at some distance from each other in right ascension the additional wires *F G*, and *H I*, are recommended, as affording convenient means of repeating the measures of this denomination.

15. When both the stars are not seen in the field of the telescope at once, it becomes necessary to vary the mode of taking their difference of declination thus; first set the micrometer previously to the supposed or computed difference of declination, and make the first star run along the horizontal wire by the screw of elevation only, and read off, then, when the second star arrives, it must be made also to run along the same wire by only opening the segments by the proper handle, and when this reading is obtained, half the sum of the two readings will be the difference of declination, provided that the two segments recede in contrary directions, by construction, with the same velocity, as Dollond latterly made them, otherwise the result will not be perfectly correct but with this limitation, the object glass micrometer may be made to perform the work of a wire micrometer.

As the planets have sensible diameters when magnified, the mode of observing them will require more minute directions, which therefore we will give in the author's own words:—

"The difference of right ascension and declination between Venus or Mercury and the sun's limb, in their transits over the sun, are to be observed nearly in the same manner as the difference of right ascension and declination of two stars, but the process will perhaps be rendered clearer by the following description,



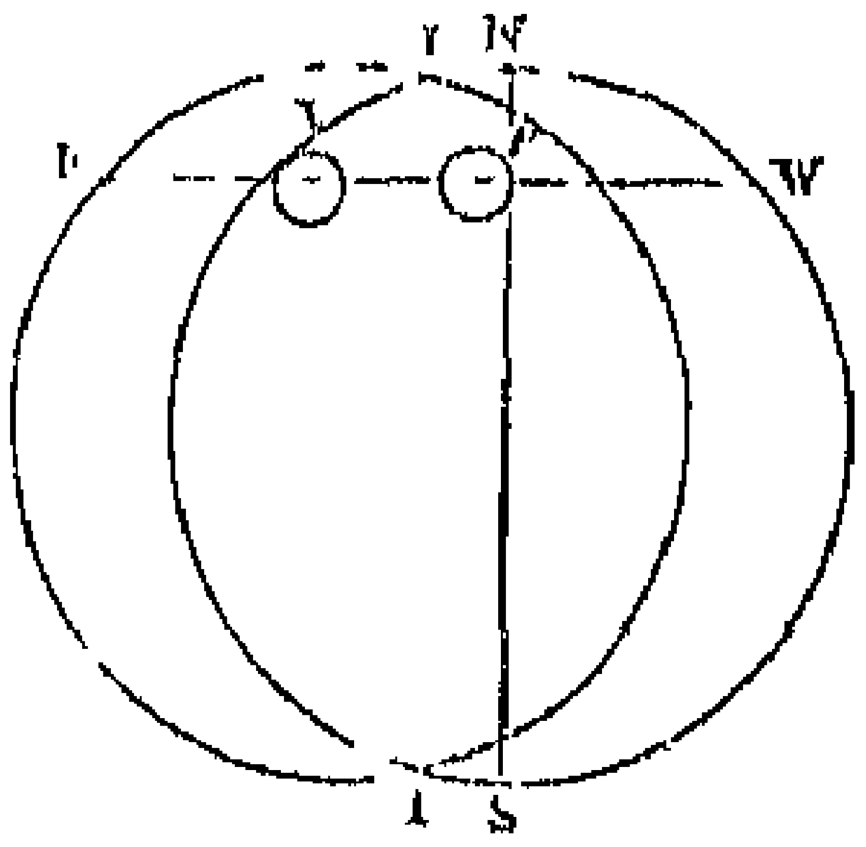
In the first place, turn the moveable wires *E W*, *N S*, into such a position that the sun's north limb *n*, or the planet's north limb *V*, may run along the wire *E W*, which thereby becomes a tangent to the peripheries of their discs.

"Secondly, the semicircular glasses being separated to a convenient distance, turn the micrometer about till the two images of the planet *V v*, pass over the horary wire *N S*, at the same instant.

"Thirdly, separate the glasses of the micrometer to such distance, that the north limb *V*, of the northernmost image of the planet may touch the wire *E W*, at the same time, that the northernmost limb *n*, of the southernmost image of the sun touches the same wire; and the scale of the micrometer will show the difference of declination of the northern limbs of the sun and planet. In like manner, if the glasses of the micrometer be opened to a greater or less distance (according as the planet is nearer the north or south limb of the sun), every thing else remaining unmoved, the difference of declination of the southern limbs of the sun and planet may be observed, by bringing the southernmost limb of the southernmost image of the planet to run along the wire *E W*, at the same time that the southernmost limb of the northernmost image of the sun runs along the same. Half the difference of these two measures, if taken immediately after one another, is equal to the difference of the declination of the centres of the sun and planet at the intermediate time, without any regard to the quantities of the diameters of the sun or planet, or the error of adjustment of the micrometer.

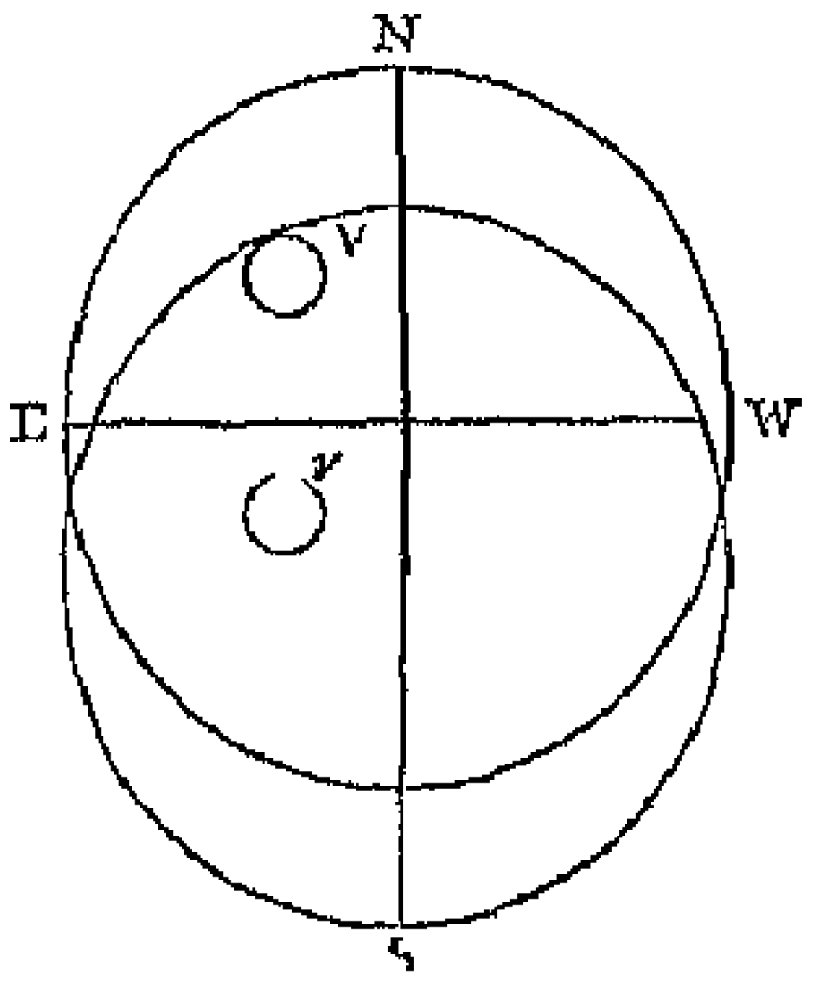
"The difference of the transits of the eastern or western limbs of the sun and planet will give the difference of right ascension, as in the common micrometer.

"Instead of differences of right ascension, distances of the planet from the sun's limb in lines parallel to the equator may be more accurately observed, as follows:



"The glasses being separated to a convenient distance, turn both the wires and micrometer about, so that the two images of the planet may both run along the wire $E\ W$, and separate the glasses so that V, one of the images of the planet, may touch the limb of the sun to the east or west, or rather both alternately. Or perhaps the following method may be preferable: separate the two images of the sun to any convenient distance, so as to produce a considerable angle of intersection of the circumferences at I and T; turn the wires about so that the planet's centre, or its north or south limb may run along the wire $E\ W$; then turn the micrometer about till the two intersections I, T, pass the horary wire $N\ S$ at the same instant, and the micrometer will be in a proper position for measuring distances, in a line parallel to the equator; and the distance of the planet from the sun's limb in a line parallel to the equator, will be obtained by only bringing the glasses nearer together, or separating them farther, till the planet's limb is in contact with the sun's limb. If distances of the planet's near limb from the sun's limb be thus taken to the east and west alternately, and reduced to a given time, by allowing for the motion of the planet by calculation, half the difference of the two reduced measures will be the distance of the planet's centre from the middle of the chord of the sun's disc, passing through the planet's centre parallel to the equator at the given time, without any regard to the quantities of the diameters of the sun or planet, or the error of adjustment of the micrometer. It may be proper to remark, that when the planet is brought to touch the sun's limb, the point of contact will be north or south of the planet's centre, accordingly as the planet itself is north or south of the sun's centre.

"In like manner distances of Venus or Mercury from the sun's limb may be measured in lines perpendicular to the equator.



"The micrometer being brought into the proper position, in the very same manner as for measuring the difference of declination from the sun's north or south limb, as before described, if the planet be brought into contact with the sun's limb to the north and south alternately, half the difference of the two measures reduced to a given time, by allowing for the planet's motion by calculation, will be the difference of declination of the centres of the sun and planet at that time, without any regard to the diameter of the sun or planets, or error of adjustment of the micrometer. And this would be a better observation than measuring the difference of declination of the limbs of the sun and planet by bringing them both in contact with the same wire parallel to the equator, described above as the measuring distances from the sun's east or west limb in lines parallel to the equator, is a better observation than measuring differences of right ascension of the limbs by *time*. By these two observations of distances of an inferior planet from the sun's limb in lines parallel and perpendicular to the equator, its true place with respect to the sun's centre may be accurately ascertained during any part of its transit over the sun's disc, and consequently its nearest approach to the sun's centre, and the time of the ecliptic conjunction may be deduced with great exactness, although the middle of the transit should not be seen, and the sun should be visible only for a small space of time sufficient for taking these observations"

The following order of making the several observations with Dollond's micrometer, in the transit of Venus, was recommended by Dr Maskelyne to the observers who went, on the part of the Royal Society, to the North Cape and to the South Sea, which may serve to elucidate their observations * Instructions to the like effect were also given to other observers, sent by the Royal Society to Hudson's Bay, and the north of Ireland on the same occasion †.

" First, immediately after the first internal contact, you are to observe several diameters of Venus (suppose 12), with 0 of the vernier placed alternately to the right and left hand of the beginning of the divisions of the scale.

" Secondly, You are to observe several differences of declination of the northern limbs of the sun and Venus, and the southern limbs of the sun and Venus alternately.

" Thirdly, If there be considerable time left before the middle of the transit, you are to observe distances of Venus from the sun's limb, to the east and west alternately, in lines parallel to the equator.

" Fourthly, If there still remain considerable time between the middle of the transit, you are to observe several times the horizontal diameter of the sun.

" Fifthly, You are to begin at least half an hour (an hour would be better) before the middle of the transit to measure the nearest distance of Venus from the Sun's limb, and the farthest distance of Venus from the sun's limb, alternately.

" N.B. The same position of the micrometer will serve for both, without turning it about. These observations are to be continued till the very middle of the transit, when the distance will continue the same for a little space of time; but it will be better to continue them for some time longer.

" Sixthly, The same observations which were taken before the middle of the transit, or such as could not, through some impediment, be observed before, may be proper to be observed after the middle of the transit

" Seventhly, It will be adviseable to practise observations similar to these here recommended, previous to the transit of Venus, by means of spots in the sun."

§ XXIX EXPERIMENTAL DETERMINATION OF THE FOCAL LENGTH AND ERRORS OF A DIVIDED OBJECT-LENS, USED WITH A GREGORIAN REFLECTOR

1. In a memoir, *sulla Figura e sul Tempo della Rotazione del Sole*, written by Ottaviano Fabrizio Mossotti of Milan, and published in the *Effemeridi* of Milan, of the year 1821, some experi-

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* See Philosophical Transactions, Vol LIX p. 266, and Vol LXI p 397

† See Ibid, Vol LIX p 480, and Vol LX p 488

ments are detailed by means of which the focal length of a divided object lens was ascertained and its errors pointed out, when applied before the aperture of a Short's reflecting telescope of the Gregorian construction, the substance of which memoir has been kindly put into our hands by the ingenious and learned author himself, and furnishes a suitable addition to the preceding article. The dimensions of the reflecting telescope in question are given in French measures, thus,

	FEET.	INCHES.	LINES.
Distance between the two lenses of the eye piece	0	3	0
Distance from the large mirror to the second eye-lens . . .	0	0	11
Distance of the large mirror to the divided object-glass	2	8	5
Focal distance of the first lens, next the eye	0	1	6
Ditto of the second lens	0	3	0
Ditto of the small mirror . . .	0	5	1
Ditto of the large mirror	2	0	0

2. The focal distance of the divided object-glass being of considerable length, could not be conveniently obtained by direct lineal measurement, and was therefore determined by means of an object of a known length placed at a measured distance, agreeably to the following formula. Let h denote the length of the object, a its distance from the divided object-glass, and b the quantity that measures the separation of the two halves of the object-glass, then the focal distance f of the semi-lenses and great speculum together will be had by the equation $f = \frac{a}{h} \cdot b$.

3. On a vertical wall to the north of the Milan observatory, two white bands were painted, and on the surface of these, two black lines were drawn horizontally, at the distance of 5.8 metres from each other, but in such a way that the ends of the two black lines, both commencing from the same vertical line, were carried one towards the east, and the other towards the west When the heliometer, or micrometrical telescope, was directed to the middle point between the two horizontal and parallel lines, it was depressed 1° 57′ below the true horizontal line, and therefore the distance 5.8 metres was reduced by using the cosine of 1° 57′ as a multiplier, and the resulting distance became 5.7967 metres for the length h of the object, the distance of this middle point from the object-glass was found 650.92 metres $= a$, and when the two separated parallel lines were brought, by separation of the semi-lenses of the instrument in good vision, into one continued straight line, the distance between the centres of these semi-lenses was found $= 4.5586$ inches $= b$. By substituting these values of h a and b according to the foregoing formula, we find $f = 511.3357$ inches

4. After having thus obtained the value of the focal distance, it becomes easy to construct a table that will give every value of b, that is, the angle a corresponding to every distance between the centres of the two semi lenses used as an object glass, and the following equation will be suitable for this purpose, viz.

$$\text{Tang.}\ \tfrac{1}{2} a = \frac{b}{2f}.$$

A TABLE OF VALUES COMPUTED FROM THE FORMULA

b	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
INCHES									
1	6′ 43″.38	0.1	0′ 40″.34	0.6	4 2″ 03	0.01	4″.03	0.06	24″ 20
2	13 26.76	0.2	1 20.68	0.7	4 42.37	0.02	8.07	0.07	28.24
3	20 10.14	0.3	2 1.01	0.8	5 22.70	0.03	12.10	0.08	32.27
4	26 53 52	0.4	2 41.35	0.9	6 3.04	0.04	16.14	0.09	36 30
5	33 36.90	0.5	3 21.69	1.0	6 43.38	0.05	20.17	0.10	40.34

The tabular values here given are to the hundredths of an inch, but should the scale represented by *b*, be subdivided by a vernier, or otherwise into thousandths of an inch, the corresponding values may be readily had by using the column 0.01 instead of 0.001, and carrying the decimal point in the column *a* one figure to the left, thus 0.004 in column *b* will have the value 1″.614 in column *a*.

5. When the object viewed is so near, that the rays of light coming from it cannot be considered as parallel, it will be necessary to reduce the values of *b*, as given by the scale of the divided object-glass, by multiplying them by $1 - \frac{f}{a}$; where *a* denotes the distance of the object from the said object-glass.

6. Some observers have supposed that the numbers given in this table will require a correction, when, from a defect in their sight, they have been obliged to remove the small mirror for the sake of distinct vision, from the position that is necessary for making the rays proceed parallel out of the eye piece, but the theory shows that this supposition is erroneous. When a number of observers, with eyes differently constructed, take the measure of the same angle, it will indeed often happen that their measures will differ from each other by 12″ or 15″, when the whole angle is as much as 30′, and hence the inference arises, respecting the necessity of a correction for the error of the eye; but this difference will occur to the same observer, if, for the sake of distinct vision, the small mirror be a little displaced by the adjusting screw. If the place where the small mirror is situated to give distinct vision were a *point*, it would be easy for any observer to find that point, and the measures taken by different observers, and also by the same observer, as the total angle varies, would be equal to one another, but with the instrument under consideration the small mirror might be moved ten or twelve thousandths of an inch, without producing any sensible difference in the distinctness of a visible image, and consequently different observers adjusting, each for himself, might place the small mirror in as many different points as are contained in this interval, and consequently would have as many different measures. The real cause of this defect in the instrument, is the aberration of the rays of light that fall on different parts of the surface of the divided lens, and are so differently refracted, before they are incident on the great speculum, that they are not all united again in one focal point, but meet at different points in a straight line, so as to form a succession of images at different distances from the eye-piece. Hence, if the observer does not confine his

observation to the same image constantly, the result will be the same as if he measured with object-glasses of different focal lengths. In order therefore that observations made with an instrument of this description may be comparable with each other, it is necessary that they should be made by using images that correspond always to the same focal distance

7. The focal distance from which the preceding table was constructed, was ascertained from viewing a terrestrial object, which consequently was not at an infinite distance, and when we turn the telescope to a celestial object it becomes necessary to re-adjust the small mirror by its proper screw, we must not however stop at any place where we fancy that we have got the best vision, but we must displace the small mirror such a quantity only, as will enable us to see the heavenly object with the *same focal distance* that is necessary for giving the true measure. The problem for finding this exact quantity is resolved in a memoir on the *Theorie des Lunettes*, given by Lagrange, in the Memoirs of the Academy of Berlin for the year 1778; and the solution depends upon an equation of the second degree, to which we beg to refer the inquisitive reader. In the observation of the two lines above described, the distance of the small mirror, as computed by that theory, comes out 28 .96207, and in this instance the distance ought to be diminished by 0 .02053 inches, when a celestial object is to be observed with the same focal distance.

8. The author of the memoir which we have consulted on the subject of the heliometer, recommends that the axis of the screw, which moves the small mirror, should carry a vernier to read with a scale, engraved on the exterior surface of the metallic tube, which was the construction of the instrument he used, and with his vernier $\frac{1}{1000}$th of an inch could be distinctly read. When the measure of the two painted lines above described was taken by the telescope, the vernier indicated 0 .004 of an inch, and Reaumur's thermometer stood at 14°. Then to make an observation of a heavenly body under such circumstances as would correspond with the tabular quantities previously computed, by giving the image in its proper focal place, it was necessary to draw the vernier a small space measured by 0 .02053 towards the eye of the observer, or it should indicate 0 .02453, which quantity is = 0 .004 + 0 02053

9. When the length of the tube is altered by a change of temperature, the small mirror will be displaced a little, and when the quantity is computed, the vernier must be used to adjust the distance from the large mirror which the table requires. In the instrument of Mossotti for instance, when the temperature was increased 10° of Reaumur's scale, the small mirror was removed from its true place 0 .0075 of an inch. This may appear but a small quantity to be noticed, but the author shows that the effect produced on the measure thereby would be the same, as if the focal distance of the semi-lenses, composing the object-glass, should be increased from 511 .3357 to 514 .84 inches, and that when an angle of 30′ is measured, with the small mirror so displaced, the measure will exceed the true one tabulated by 13″.

10. Savery, of Exeter, who was the first person that contrived a heliometer, consisting of two object-glasses, in the year 1743, proposed to measure the difference of the sun's diameters when in the apogeal and perigeal points *, when the difference of temperature was found to affect the measures in a very sensible degree.

11. Another remarkable fact, discovered in using Short's telescope with the object-glass

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* Short's Account, in Vol XLVIII of the Philosophical Transactions of London

micrometer, was, that the distance at which the small mirror required to be placed, to produce distinct vision, varied with the quantity of the angle to be measured. When this angle was about 30′, it was found necessary to make the small mirror approach the eye by 18 or 20 thousandths of an inch more than when the angle was only 3′. To account for this necessary change of distance, we must consider that, when a small angle is measured, the rays of light pass through the central portions of the semi-lenses, but that, when a large angle is required to be ascertained, the images are formed by rays passing through their ends, and the difference in the aberrations is the cause of a small difference in the visible focal lengths.

12. This phenomenon is the same as that which was observed by Cesaris, and inserted in the Effemeridi di Milano of the year 1819, as producing a bad effect on the collimation of a telescope, by changing the position of the lines in the focus of the object glass. In the experiment made by this author, the central part of the object-glass was covered, and the extreme rays only permitted to enter the telescope, the aberration of which not only changed the length of the focus, but altered the line of collimation. This experiment however proved moreover, that the figure of the object-glass was not good.

13. When it is found that the focal distance of a divided object-glass varies with the separation of the semi-lenses, and that each object-glass will have its own changes, depending on the quantity of relative aberrations occasioned by the figure, the only remedy for the errors thus produced will be, to ascertain their amount at two or three given points of the scale of separation, and to interpolate proportional parts in a second table of corrections, to be applied to the tabular quantities computed from regular multiples.

14. The last source of error, against which it may be necessary to guard the observations made by the heliometer, is that which gives different measures by being taken in different directions, when the angle is the same. A defect in the adjustment of the mirrors, or in their figures, relatively to each other, may make the measure of the vertical diameter of a true circle, placed perpendicular to the axis of vision, somewhat different from the apparent measure of the horizontal diameter, which is said to have been noticed by Amici. An error of this description is detected by first getting a nice measure of a circle, erected at a considerable distance perpendicularly to the axis of the telescope, and then by turning the divided object-glass round, which operation will show, by a revolution of the two images of the circle round one another, whether all the diameters are of the same apparent dimensions.

15. From the consideration we have now given to the sources of error in the use of an heliometer, which probably contributed to diminish the confidence that might otherwise have been placed in this convenient but expensive instrument, we may recapitulate them under the four following heads, viz.

First, a variation in the place of the small mirror when placed by estimation of the eye only;

Second, a variation of the length of the tube by change of temperature, occasioning a corresponding change in the place of the small mirror, and a consequent inaccuracy of measure;

Third, a change of focal distance, when central and extreme rays are indifferently used, particularly when the aberration from the spherical figure of the semi-lenses is considerable;

Fourth, want of adjustment, or of perfect figures in the mirrors, as they regard each other, which defect may occasion the measures of the same angle to vary when taken in different directions.

§ XXX DOLLOND'S IMPROVED OBJECT-GLASS MICROMETER [Plate V]

1. The objections which have been shown to exist against the use of Dollond's object-glass micrometer, when applied in the form of a divided single lens, before the aperture of a reflecting telescope, have been obviated by an improved construction, which we shall now describe as briefly as the alteration in the instrument will admit of, by giving the same letters of reference that we have already used, in our twenty eighth section, to represent the same parts. The improved micrometer is represented by Fig. 3 of Plate V, but without the long handles it consists of two oblong or elliptic halves of a double glass, so compounded of crown and flint pieces as to be nearly achromatic, but not quite so, yet when used with an appropriate object-glass of a refracting telescope, the two used together constitute an achromatic object-glass, that is capable of producing double images free from the imperfections described by Mossotti, and capable of giving accurate measures, as a double-image micrometer. The divided compound glass is of the concave description, and therefore lengthens the solar focal length of the refracting telescope, of which it forms a portion. We have not ascertained the date of this improvement, but we have been assured by Mr. George Dollond, that it completely answers its intended purpose, when carefully made the expense however is necessarily considerable, from the complexity of the construction.

2. When a circular lens is divided into two halves, which slide by the sides of one another, as in the original construction, the metallic parts in which they are bedded, cover more or less of the telescope's aperture, accordingly as the centres of the semi-lenses are separated by a larger or smaller quantity, and consequently whenever a large angle is measured, the greater portion of the aperture is thus covered, and the light that is admitted, which ought always to be copious in double-image instruments, is proportionably diminished, this evil is cured in the improved object-glass micrometer, by the substitution of two long slices of glass, taken from the diametrical portion of a large lens, which, being nearly six inches in length, lie over the aperture of the tube in every state of longitudinal separation, and therefore impede not the admission of any incident rays of light To prevent unrefracted light gaining admission, it is necessary that the straight edges of the two portions of the divided compound glass, that come in contact, should be well ground, in doing which some portion of the central parts of the curved surfaces will be annihilated, and double images will be laterally formed very near each other, which can never be made to coincide; this vacant space, which has been made by grinding the edges straight, answers however an important purpose, by admitting a long brass scale and vernier to occupy it, of a thickness equal to the quantity of displacement of the true centres, and the only inconvenience, attending the interposition of the scale, is the loss of so much light, as might be admitted in the diametrical strip crossing the telescope's circular object glass.

3. The letter *A* in the figure shows a portion of the extreme end of the telescope's tube, and the rim of brass, attached to the micrometer, which surrounds the tube *A*, and fixes it to the micrometer, *C C'* is a moveable frame having teeth at the outer edge, and carrying one of the halves *G*, of the divided glass, while a similar moveable frame, not so well seen, carries

the other half F, and has also teeth on its remote edge, to the frame CC' the scale is made fast as an edge-bar, and divided into six inches, each being sub-divided into twenty parts, and reading with a vernier of 25 divisions, which is carried, as an edge-bar, by the other frame of the second portion of glass F It appears from the figure, as though each half of the divided glass would require a separate handle for moving it, but this is not the case · a fixed plate $H H'$, screwed to the rim B, and having a circular hole at the middle, forms a bed for the two sliding frames, and supplies them with grooves to slide in, into which the toothed parts enter, and are thereby in a great measure concealed, then a ring formed into a wheel, and cut so as to conceal no part of the aperture, which it surrounds, connects the two racks or toothed edge-bars, in such way, as to produce contrary motions in them, for the pinion held by the milled head, or long handle substituted for it, gives motion to the frame $C C'$ in the first place, and then to the second frame F, through the medium of the concealed wheel, with which it also acts, and a concealed second pinion acts with both likewise at the opposite edge, which mode of action may be more readily conceived than described, without a detached representation in another figure. There is also here the mechanism for giving the rotary motion for the requisite position of the micrometer in any given observation, which is also hid from the view, and which requires a second long handle to come within reach of the observer's hand. At I, is a cock and fine screw for adjusting the vernier to zero, when the two images of an object coincide.

4. The mode of using this object-glass micrometer is in every respect the same as that of the original instrument applied to a reflecting telescope, which we described in our twenty-eighth section, and a value can be given to its scale for the formation of a Table in the manner already explained in the said section.

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§ XXXI DIOPTRIC MICROMETERS BY RAMSDEN AND G DOLLOND. [PLATE V]

1. In consequence of the difficulty of measuring the diameter of a planet with a wire micrometer, in any other direction than in a line at right angles to its apparent path in the telescope, and also on account of the uncertainty attending the use of an object-glass micrometer, as constructed in Ramsden's time, this eminent artist contrived two double image micrometers, one *catoptric*, and the other *dioptric*, of which his description was published in the 69th volume of the Philosophical Transactions of London, in the year 1779. As the latter of these, in its construction and use, greatly resembles the instrument which we last described, we shall give an account of it, and of a similar one lately made by G. Dollond, in our present section.

2 "By the position of the (object-glass) micrometer," says Ramsden, "every error of its glass is magnified by the telescope, and if each surface of the micrometer glass has not, in every part, precisely the same radius (which opticians must allow to be exceedingly difficult to effect), there will be a considerable error in the angle to be measured, and the eye applied

to the different parts of the pencil will, without moving the micrometer, see the images of the object in the telescope fluctuating, sometimes appearing to overlap, and sometimes to separate from each other. But supposing the glass itself to be perfect in its substance and in its curvature, there will yet remain imperfections which arise from its principle A micrometer glass applied to a telescope causes a very considerable aberration If the focus of the glass is positive, the extreme aberration will be within the geometrical focus, if negative, it will be beyond it: and the aberration not only affects the distinctness of the image, but also the angle measured by the micrometer." "I therefore," says the author in a subsequent part of his paper, "have employed for the micrometer-glass one of the eye-glasses requisite in the common construction of the telescope, but if it should be found necessary to apply an additional eye glass for the conveniency of enlarging the scale, I am able thereby to correct both the colours and spherical aberration of the first eye-glass. This micrometer is applied to the erect eye-tube of a refracting telescope, and is placed in the conjugate focus of the first (innermost) eye-glass: hence arises its great superiority to the object-glass micrometer. It has before been observed, that, if a micrometer is applied at the object-glass, the imperfections of its glass are magnified by the whole power of the telescope; but in *this position*, the image being considerably magnified before it comes to the micrometer, any imperfection in its glass will be magnified only by the remaining eye-glasses, which (increase) in any telescope seldom exceeds five or six times. By this position the size of the micrometer glass will not be the $\frac{1}{100}$th part of the area, which would be required, if it were placed at the object-glass; and, notwithstanding this great disproportion of size, which is of great moment to the practical optician, the same extent of scale is preserved, and the images are uniformly bright in every part of the field of the telescope."

3. In this quotation Ramsden has fully explained his reasons for preferring the adoption of a divided lens in the eye-tube, and he then proceeds to explain the construction of his contrivance by a figure, that very much resembles our Figure 4 of Plate V, which is a perspective representation of a similar micrometer, recently constructed for us by the present Dollond. We will first describe the modern construction, and then point out how the original one differed from it.

4. In our figure *a b* is a brass plate six inches long and two and a half wide, which is concealed by the parts that lie over it, upon the sides of this plate are screwed longitudinally a pair of parallel pieces *c d*, and *e f*, which are so rabbeted lengthwise, as to allow the indented sides of another pair of brass pieces to pass under them, these indented rulers hold each a long slip of glass, *g h*, and *i k*, which are ground into long flat wedges apparently, but which we understand are radial slices cut from a large lens; the under sides of the indented rulers, or glass-holders, have each a string of brass that runs in a dove-tailed groove, made in the main plate, which keep the sides of the two glasses in contact throughout their whole length, whether in motion or at rest. As in Dollond's improved object-glass micrometer, an indented ring surrounds the central aperture made in the middle of the plate, and is at liberty to turn round an inner fixed ring, when actuated by either of two concealed pinions, the milled head of one of which is seen near *l*, and the other lies opposite it, and under it in our drawing. the ring which is indented at the outer circle is two inches and a half in diameter, is placed on the back face of the plate, and is kept in its place by a small square plate, like plate *p q*, seen

holding the eye-tube *n* in front, it reaches across both the indented glass-holders, the teeth of which are also separated two inches and a half on the front side of the plate; then each of two pinions, with teeth that will reach both the indented ring and glass-holders, act at opposite sides of the ring with both at the same time, so that if either of the two pinions is turned by its milled head, all the indented pieces, including the second pinion, being connected together, are obliged to move contemporaneously, but the teeth of the glass-holders, being situated at opposite sides of the indented ring, move in contrary directions, and thus separate longitudinally with a double velocity. The square piece of brass, *p q*, is screwed fast to the rabbeted side-pieces *c d* and *e f*, but has its upper edge cut away, to show the vernier and a portion of the divided scale marked on the piece *c d*, the divisions of which are too small to be inserted so as to be visible. The separation shewn by the scale is nearly five inches and a half, each of which inches is divided into ten parts, and as the vernier has ten divisions co extensive with nine-tenths of the inch, it reads $\frac{1}{10}$th of $\frac{1}{10}$th $= \frac{1}{100}$th of an inch by the coincident lines. The eye-tube *n* screws into the small square plate *p q*, and the second portion *l* screws into a similar plate, and when these two portions are detached, they will screw into one another, and form an ordinary terrestrial eye-piece, without the micrometrical apparatus. The long tube *n* contains the pair of eye-lenses at *o*, and the shorter portion *l* contains the field and amplifying lenses, the latter of which comes almost into contact with the surfaces of the long sliding glasses, that separate and produce the two images of any object, at a greater or smaller distance from each other, according to the quantum of separation. The part *m*, surrounding the second piece of tube *l*, is an adapting piece of tube, that screws into the telescope, and allows a circular motion to be given to the micrometer, as the position of the object to be measured may require. The scale extends about $\frac{4}{10}$ths of an inch behind zero, to allow of the index error being determined, and also of a small object being measured on both sides of zero. In this micrometer the quantity measured is very small in comparison with the total length of the scale, and the value of the scale, as in other constructions, varies inversely with the focal length of the telescope made use of.

5. Ramsden's dioptric micrometer differed from the one we have here described principally in these respects; the indented edges of the glass-holders were turned inwards, so that one pinion, placed between them, moved them both in opposite directions, the scale was figured from both ends to zero at the middle, and a separate vernier was placed at each side of zero, for reading observations made to the right and left of it; the divided slips of glass were placed at the other side of the amplifying lens, in the conjugate focus of the field lens, where a small diaphragm is usually inserted, and it had a graduated circle by which positions might be indicated, otherwise the external appearance of the original drawing resembles the representation we have given of Dollond's so much, as to render separate figures unnecessary for the purpose of description. We have not been able to ascertain whether or not Ramsden actually made the instrument of which he gave a description, as we do not find any record of measures taken with it, indeed the potential expressions used in his account seem to indicate, that the contrivance was one of those *projects*, which had its existence on paper only; and this probability is confirmed by the circumstance of G. Dollond having put into our hands his instrument, as one recently contrived, before we pointed out to him the account which Ramsden had written of a similar instrument about forty years before, at which observation he evidently was

surprised, and said he had not heard of it. The instruments however are so similar, that we are disposed to consider the two constructions the same. On learning that Mr. Davies Gilbert, for whom Dollond made his first instrument, had not made much use of it, we gave an order for a similar one for ourselves, that we might put it to the test of actual observations, and though the price was twelve guineas, we have no reason to be dissatisfied with our purchase

6. The weight of this micrometer was found so great, that it is unsuitable for being applied to an achromatic telescope of ordinary dimensions, mounted at the centre of gravity on a tripod in the usual way, because it bends the tube a little, and makes the eye-end greatly preponderate; but it may be used with a long telescope, that is supported at both ends, with great advantage, and still better with a Newtonian or Herschelian, to either of which it may become a good eye-piece, and a change of the eye-lenses, or a change of their distance from the micrometer, effected by a sliding pancratic tube, will produce a change of magnifying power.

7. In a former section (§ VII. 2) we have said that the modern terrestrial eye-piece forms a compound microscope, and accordingly we find that the dioptric micrometer, used as an instrument of this kind, will measure a small insect, or other diminutive object, in a beautiful manner, by means of the double images, when their opposite ends are brought into exact contact when an ivory scale, neatly divided into the hundredths of an inch, is made the object, we find that ten of those divisions or one tenth of an inch, in its natural state, will be measured by 4 8 inches on the scale, in its magnified state.

8 When we had applied Dollond's instrument to our Newtonian telescope numbered 4 (§ XI. 13.), of 71 75 inches solar focal length, the double images were beautifully formed, and the proper value was assigned to the scale by one of the methods explained in our nineteenth and twentieth sections; the sun's diameter being too large to be measured by this micrometer. The following Table was constructed from the value belonging to a single inch of the scale, which we subjoin as a specimen, that may be copied for any telescope of the same focal length, or may be altered by varying the scale of values to suit a telescope of a different focal length.

9. A TABLE

OF THE VALUES OF DOLLOND'S DIOPTRIC MICROMETER, APPLIED TO A NEWTONIAN TELESCOPE OF 71 75 INCHES FOCAL LENGTH

	For Inches	For Tenths.	For Hundredths
1	0′ 29″ 25	2″.925	0″.293
2	0 58 .50	5 .850	0 .585
3	1 27 .75	8 775	0 .878
4	1 57 .00	11 .700	1 .170
5	2 26 .25	14 .625	1 .463
6	2 55 .50	17 .550	1 .755
7	3 24 .75	20 .475	2 .048
8	3 54 .00	23 .400	2 .340
9	4 23 .25	26 .325	2 .633
10	4 52 .50	29 .250	2 .925

10. As an example, to illustrate the use of this micrometer and of the Table, we directed this Newtonian telescope to a window in a neighbouring parish, which had its wooden frame painted white, and, applying the micrometer, and obtaining good vision, we divided the two images just sufficiently to bring first the ends of the frame into exact contact vertically, and then the sides horizontally, and the measures were as follow

	FOR THE HEIGHT		FOR THE BREADTH	
Inches	3	1′ 27″ 750	4	1 57″.000
Tenths	.6	17 .550	.3	8 .775
Hundredths	03	0 .876	.03	0 .878
Whole measure		=1 46 176		2 6 .653

To prove the accuracy of these measures, we applied other instruments to the same object from the same station, and obtained the subjoined satisfactory confirmation.

		HEIGHT		BREADTH
With Tel. 4, and a glass scale we had	7 . 55 × 14″.11	=1′ 46″.53 and	9 × 14.″11	=2′ 6″ 99
With do and Troughton's micrometer	3 . 83 rev.	=1 46 .20 and	4 . 55	=2 6 .76
With Tel. 5, and do. [§ XIX. 6.]	4 . 08	=1 46 .48 and	4 . 85	=2 6 .58
With Tel. 2, and do.	2 .305	=1 46 .30 and	2 .745	=2 6 .58
With Tel 1, and do.	1 . 63	=1 46 .35 and	1 . 94	=2 6 .57
Average of the five measures		=1 46 .37		=2 6 .68

In the first of these measures the value of the divided glass scale, 14″.11 in each division, was obtained from the constant quantity 1012″.3 divided by 71 .75, the focal length of the Newtonian telescope, agreeably to our directions above given. [§ XXI. 5.] We have preferred taking these comparative measures from a stationary distant object, rather than from a heavenly body in motion, but the same mode will apply to taking the diameter of a planet, or the distance between any two contiguous bodies.

§ XXXII DIOPTRIC MICROMETER CONSTRUCTED BY T JONES [PLATE III]

1 MR. Troughton informs us, that when a Captain Countess, R.N. by accident broke the third lens of a four-glassed terrestrial eye-piece of his telescope, he observed that each piece of the lens produced a separate image in the field of view, more or less distinct, according to the size of the portion that produced each image This occurrence, it is said, led to the contrivance of the coming-up glass, that was first made by Nairne with a double screw, similar to Gascoigne's, to separate the halves of the amplifying lens; and from him Ramsden, it is thought, borrowed the idea of dividing a lens into two equal halves for the dynameter, and also for the double image dioptric micrometer which we described in our last section.

2. The first telescope of this kind that fell under our notice, was in the year 1819, in the Isle of Jersey, which we purchased at a toy-shop, and which had formerly belonged to a naval

officer in the English service It does not appear that this telescope, which was only of two and a half feet focal length, made by Watkins of Charing Cross, had been applied to any other than naval purposes, and indeed the workmanship was not sufficiently correct, to be applied to astronomical observations, even though a larger telescope had been used with the micrometrical eye-piece. It was however obvious, that the contrivance, well executed, might be serviceable as an astronomical micrometer, and Thomas Jones afterwards constructed us one for this purpose, which will measure as large an angle as any telescope will take into its field, with great convenience and accuracy, and which in our opinion is preferable to the divided object glass micrometer, as an heliometer

3. Two views of the micrometrical portion of the eye-piece made by T Jones are given in Plate III, in which fig. 7 exhibits the external appearance of the piece containing the third and fourth lenses, and fig. 8 shows the internal construction to an eye looking down the piece of main tube, when detached from the remainder of the eye-tube represented by fig 9. By a mistake of the draftsman the part 7 is placed in the plate at the wrong end of the piece 9, the end *a* of fig. 7, when applied to the piece 9, screws into its end *a*, and the end *b* of piece 7, where the fourth lens is placed, screws into the racked tube of the telescope, or rather into an adapter which allows of a revolving motion of the micrometer. In both the figures 7 and 8, which are similar to figures 14 and 15, in Plate XI. *e* is the milled head of a long screw, one part of which is held by a cock *m* at the outside of the tube, and the other passes into the tube, and is concealed in figure 7, but may be seen in figure 8; *f* is the head of the screw, divided into 100 parts, and revolves with it in such way that the graduated edge of the projecting index *g*, borne by the piece *n* attached to the cock, shows the number of the revolutions that the screw makes, when it is turned round, while the divided head *f* indicates the hundredth parts of a revolution. The index *g* is adjustable by friction to the cylindrical part of the holding piece *n*, and must be set to zero of the micrometer head, when the two semi-lenses *k* and *l*, seen in figure 8, are brought centre to centre to form one lens, which will always be known to be the case, when the image appears single and well defined. On the axis of the screw *h*, which is a fine screw of about 70 threads per inch, and at its interior end a second screw, of just one half the number of threads per inch, are formed for the purpose of giving motion to both the semi-lenses by a similar quantity, but in opposite directions, that the two images formed by them, may be equidistant from the centre of the field of view, which condition is important to the perfection of the vision.

4. The manner in which this purpose is accomplished may be explained thus, within the main tube a second tube of brass, of about two inches in length, and of a diameter to fit the interior diameter of the main tube, is fixed so that it may be occasionally taken out, and with it the apparatus contained within it, which regulates the contrary motions of the semi-lenses, a third tube of brass of less than one half of the diameter of the second tube, is divided longitudinally into two half tubes, each having a semi-lens burnished into its upper end, these half tubes are about two inches long, and descend down the middle of the second tube, till they enter a species of gimbol at its inferior end, that gives them each a separate pin as a pivot to turn upon, whenever they are separated from the state of a single tube into two halves, the pins *o* and *p*, seen in figure 8, forming the said pivots of the gimbol, appear to be at the upper end of the tubes, but are actually at the lower end; and the two half tubes may be considered as a pair of

levers, moving on those pivots when acted on The half tube, that holds the semi-lens *l*, has a stud fixed to its side, about half an inch from the semi-lens, against which the end of the screw rests, and a piece of strong watch-spring *q* made fast to the lower end of the second tube, ascends to this stud and presses against it, so as to produce a counteracting force in opposition to the screw, the other half tube, which holds the semi-lens *k*, has a piece *r* made fast to its edge, next the micrometer head, which is tapped into a female screw, of about 35 threads in the inch, to match the thick screw, that enters it. Now as the thick end of the screw, which contains the coarse threads, is connected with the half tube of the semi-lens *k*, and also presses against the stud on the half tube of the semi-lens *l*, it is obvious to any ordinary mechanic, that if there were no other threads on the screw but the coarse ones, and if the axis of the said screw had a shoulder before the piece *n*, attached to the cock *m*, the effect of turning the milled nut *e* would be, a quick motion in the semi-lens *k* towards the screw head, and by reason of the pressure of the spring at *q*, against the other half tube, the second semi lens *l* would also follow with the same velocity, and, as the two semi lenses would move together, no separation of them would take place; but instead of a shoulder on the axis of the screw, to confine it to the same position, as is usual in the tangent screws of ordinary clamping pieces, a second screw of double the number of threads acts on the same axis with the tapped nut *n*, carried by the external cock *m*, and thus the screw goes inwards with half the velocity that the nut *r* and half tube attached to it are drawn outwards, and the difference of the two contrary motions is the velocity given to the motion of the semi-lens *k*, in a direction towards the micrometer head; while at the same time the inward motion of the fine screw, that rests against the stud of the other half tube, carrying the semi lens *l*, pushes this semi-lens back towards the spring *q*, which now recedes, but keeps the screw in constant action, as well as prevents any motion in either semi-lens, but what is occasioned by the screws. The difference of the two screws moves the semi lens *k*, but the absolute motion of the fine screw alone moves the semi lens *l*, and yet they move to the right and left of the centre of the field by equal quantities. As the small screw proceeds inwards, it carries the micrometer head along with it, but not the index piece *g*, which therefore is graduated into divisions of 70 per inch, like the notches of the fine screw, and the edge of the graduated ring, forming the micrometer head, being nearly in contact with the scale of the index, shows how many revolutions the screw has made at each distance of the centres of the semi-lenses from each other.

5. The zero of the scale might have been made at one end of it, but, in the instance before us, there are twelve divisions at one side of zero, and only seven at the other, so that small angles may be measured on both sides alternately, to correct the index error or to adjust it; and yet larger angles may be measured on one side than could have been taken separately on both, which, when the sun or moon is the object, is a desirable property, if it were only to afford the means of obtaining a correct scale of measures, to be tabulated for any individual telescope.

6. At *c* and *d*, in fig. 9, are the lenses forming the eye end of the piece, and when they are screwed fast into the outer end of the terrestrial tube, the magnifying power of the telescope, to which it may be applied, will be invariable while it forms a part of the same telescope in this situation the piece forming the eye-end is said to be at home, and the value of the revolu-

tion of the micrometer's head will depend on the focal length of the telescope, conjointly with the fineness of the screw.

7. In the tube represented by fig. 9, the eye-end holding the lenses *c* and *d*, is attached to an inner tube, so fitting the outer one, that it will keep its position when drawn out any number of inches within its range, and, as the four lenses compose a compound microscope, the separation of the two pairs increases the magnifying power of the eye-piece, but alters not the value of the micrometer's scale of measures in any of the positions, though it affords the means of repeating any measure a great number of times with as many different magnifying powers.

8. In the micrometrical eye-piece before us, the third or divided lens is placed in the focus of the fourth lens, which is very nearly at the distance of three inches from it, and a small diaphragm of ¼ of an inch is placed at the end *a* of fig 9. When the inner tube is made to slide out, and is properly divided by points or strokes of a graver, it may be called *The Pancratic Dioptric Micrometer*

9. In determining a scale for this micrometer by the diameter of the sun or moon, it will be important to get distinct vision of the luminary before the lenses are separated, and not to alter the adjustment for vision after they are separated to any considerable distance, for when a measure of the sun is taken, though the contact be made in the middle of the field, which should always be the case, yet the least alteration of the focal distance will make the limbs overlap or separate very sensibly, according as the new adjustment is made inwards or outwards. This micrometer has all the advantages of the divided object-glass micrometer, with respect to double images, and to the largeness of the angle that it will measure, which may be greater than can be seen in the field at once, but when the telescope, to which it is applied, is too powerful to take in the whole diameter of the sun, it will be most safe to take a scale from a smaller telescope of known focal length, and to determine the constant product of the micrometer's value when multiplied by the known focal length of the short telescope (which will measure the sun with a smaller visible diameter), and then to divide the said constant by the focal length of the large telescope, to obtain the value of the micrometer when used with it, free from the error which might arise from the obliquity of a large measure. Agreeably to this plan the moon's longer diameter was measured on the night of April 19, 1826, and that of the sun at noon on the 20th of the same month, first by a telescope of 43.2 inches focal length, and again by a smaller one of 30 5 inches only, to which the micrometer before us was successively applied, and the following were the results.

10 The moon's diameter by the Nautical Almanack was 32′ 55″, or 1975″, which was measured by 8.16 revolutions of the screw, when the two images were in contact, and $\frac{1975''}{8.16} = 242''$ was the value of one revolution, when the telescope 43.2 was used with the moon; with the sun the value was given $\frac{1913'}{7\ 85} = 243''.7$, making the average of the two 242″.85. With the smaller telescope the moon gave after an interval of half an hour $\frac{1976''}{5.77} = 342''\,4$ as the value of a revolution of the same micrometer, and the sun gave $\frac{1913''}{5\ 615} = 340''.7$

making for an average of the two 341″.55. Now 242″.85 × 43.2 = 10491.12 is the constant product of the first telescope, and 341″.55 × 30.5 = 10417.275 is the constant product of the second telescope, and the average of the two will be 10454.2, as the corrected constant product derived from an average of both the solar and lunar observations, taken by both telescopes.

11. Then for the correct values of the micrometer's revolution, when used with our five telescopes [§ XI. 13.], we have

$$\text{Tel. 1.} \ldots\ldots \quad \frac{10454.2}{30.5} = 342''.76 = 5'\,42''.76$$

$$2. \ldots\ldots \frac{10454.2}{43.2} = 241.99 = 4\ \ 1\ 99$$

$$3. \ldots \quad \frac{10454.2}{67.5} = 154.88 = 2\ 34.88$$

$$4. \ldots\ldots \frac{10454.2}{71.75} = 145.70 = 2\ 25\ 70$$

$$5. \ldots\ldots \frac{10454.2}{76.25} = 137.10 = 2\ 17.10$$

12. To put the powers of the coming-up glass, used as a micrometer, to the test of actual measurement, and to show that the tables are properly adapted to give the true values, we have applied the same micrometrical tube, containing a divided amplifying lens, to four different telescopes successively, and have repeated the measures of the painted window-frame, taken in our last section, the similarity of the resulting quantities will appear from the statement which we subjoin.

		HEIGHT OF THE WINDOW		BREADTH OF THE WINDOW	
Tel. 1.	Measure	0.31	= 1′ 46″.25	0.37	= 2′ 6″ 81
2.	Ditto	0.44	= 1 46.48	0.5225	= 2 6.70
4.	Ditto	0.73	= 1 46.36	0.87	= 2 6.76
5.	Ditto	0.775	= 1 46.24	0.925	= 2 6.81
Average			1 46.33		2 6.77
Ditto by other micrometers in the last section			1 46.37		2 6.68

13 The following tables for the use of the said five telescopes were constructed from the preceding values of a revolution of the coming-up glass, used as a dioptric micrometer, when applied successively to each.

TABLE I.

ENTIRE REVOLUTIONS

	Tel. 1	Tel 2	Tel 3	Tel 4	Tel 5
1	0° 5′ 42″.76	0° 4′ 1″99	2′ 34″.88	2′ 25″70	2′ 17″.10
2	0 11 25 52	0 8 3.98	5 9.76	4 51 40	4 34.20
3	0 17 8.28	0 12 5.97	7 44.64	7 17.10	6 51 30
4	0 22 51.04	0 16 7.96	10 19.52	9 42 80	9 8.40
5	0 28 33.80	0 20 9.95	12 54.40	12 8.50	11 25 50
6	0 34 16.56	0 24 11.94	15 29.28	14 34.20	13 42.60
7	0 39 59.32	0 28 13.93	18 4.16	16 59 90	15 59 70
8	0 45 42.08	0 32 15.92	20 39.04	19 25.60	18 16.80
9	0 51 24.84	0 36 17.91	23 13.92	21 51.30	20 33.90
10	0 57 7.60	0 40 19.90	25 48.80	24 17.00	22 51.00
11	1 2 50.36	0 44 21 89	28 23.68	26 42.70	25 8 10
12	1 8 33.12	0 48 23.88	30 58.56	29 8.40	27 25.20
13	1 14 15.88	0 52 25.87	33 33.44	31 34.10	29 42 30
14	1 19 58 64	0 56 27.86	36 8 32	33 59.80	31 59.40
15	1 25 41.40	1 0 29 85	38 43.20	36 25.50	34 16 50
16	1 31 24.16	1 4 31 84	41 18.08	38 51.20	36 33 60
17	1 37 6.92	1 8 33.83	43 52.96	41 16.90	38 50 70
18	1 42 49.68	1 12 35 82	46 27 84	43 42.60	41 7.80
19	1 48 32.44	1 16 37 81	49 2.72	46 8.30	43 24 90
20	1 54 15.20	1 20 39.80	51 37.60	48 34.00	45 42.00

TABLE II.

HUNDREDTH PARTS OF A REVOLUTION

Parts	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5	Parts	Tel 1	Tel 2	Tel 3	Tel 4	Tel 5
1	0′ 3″ 43	0′ 2″ 42	0′ 1″ 55	0 1″,46	0 1″ 37	51	2′ 54″ 81	2′ 3″ 42	1′ 18″,99	1′ 14″ 31	1′ 9″ 92
2	0 6 86	0 4 84	0 3 10	0 2 92	0 2 74	52	2 58 24	2 5 84	1 20 54	1 15 77	1 11 29
3	0 10 29	0 7 26	0 4 65	0 4 38	0 4 11	53	3 1 66	2 8 26	1 22 09	1 17 22	1 12 66
4	0 13 72	0 9 68	0 6 20	0 5 83	0 5 48	54	3 5 09	2 10 68	1 23 64	1 18 68	1 14 03
5	0 17 14	0 12 10	0 7 75	0 7 29	0 6 85	55	3 8 52	2 13 10	1 25 19	1 20 13	1 15 40
6	0 20 57	0 14 52	0 9 30	0 8 75	0 8 22	56	3 11 95	2 15 52	1 26 74	1 21 59	1 16 77
7	0 24 00	0 16 94	0 10 85	0 10 21	0 9 59	57	3 15 38	2 17 94	1 28 29	1 23 05	1 18 14
8	0 27 43	0 19 36	0 12 40	0 11 66	0 10 96	58	3 18 80	2 20 36	1 29 84	1 24 50	1 19 51
9	0 30 86	0 21 78	0 13 95	0 13 12	0 12 33	59	3 22 23	2 22 78	1 31 39	1 25 96	1 20 88
10	0 34 28	0 24 20	0 15 49	0 14 57	0 13 71	60	3 25 66	2 25 20	1 32 93	1 27 42	1 22 26
11	0 37 71	0 26 62	0 17 04	0 16 03	0 15 08	61	3 29 09	2 27 62	1 34 48	1 28 88	1 23 63
12	0 41 14	0 29 04	0 18 59	0 17 49	0 16 45	62	3 32 52	2 30 04	1 36 03	1 30 33	1 25 00
13	0 44 56	0 31 46	0 20 14	0 18 94	0 17 82	63	3 35 95	2 32 46	1 37 58	1 31 79	1 26 37
14	0 47 99	0 33 88	0 21 69	0 20 40	0 19 19	64	3 39 37	2 34 88	1 39 13	1 33 25	1 27 74
15	0 51 42	0 36 30	0 23 24	0 21 86	0 20 56	65	3 42 80	2 37,30	1 40 68	1 34 70	1 29 11
16	0 54 84	0 38 72	0 24 79	0 23 31	0 21 93	66	3 46 23	2 39 72	1 42 23	1 36 16	1 30 48
17	0 58 27	0 41 14	0 26 34	0 24 77	0 23 30	67	3 49 65	2 42 14	1 43 78	1 37 62	1 31,85
18	1 1 70	0 43 56	0 27 89	0 26 23	0 24 67	68	3 53 08	2 44 56	1 45 33	1 39 07	1 33 22
19	1 5 13	0 45 98	0 29 44	0 27 68	0 26 04	69	3 56 51	2 46 98	1 46 88	1 40 53	1 34 59
20	1 8 55	0 48 40	0 30 98	0 29 14	0 27 42	70	3 59 93	2 49 40	1 48 42	1 41 99	1 35 97
21	1 11 98	0 50 82	0 32 53	0 30 60	0 28 79	71	4 3 36	2 51 82	1 49 97	1 43 45	1 37 34
22	1 15 41	0 53 24	0 34 08	0 32 06	0 30 16	72	4 6 79	2 54 24	1 51 52	1 44 90	1 38 71
23	1 18 83	0 55 66	0 35 63	0 33 51	0 31 53	73	4 10 22	2 56 66	1 53 07	1 46 36	1 40 08
24	1 22 26	0 58 08	0 37 18	0 34 97	0 32 90	74	4 13 64	2 59 08	1 54 62	1 47 82	1 41 45
25	1 25 69	1 0 50	0 38 72	0 36 43	0 34 27	75	4 17 07	3 1 50	1 56 17	1 49 27	1 42 82
26	1 29 11	1 2 92	0 40 27	0 37 88	0 35 64	76	4 20 50	3 3 92	1 57 71	1 50 73	1 44 19
27	1 32 54	1 5 34	0 41 82	0 39 34	0 37 01	77	4 23 92	3 6 34	1 59 26	1 52 19	1 45 56
28	1 35 97	1 7 76	0 43 37	0 40 80	0 38 38	78	4 27 35	3 8 76	2 0 81	1 53 64	1 46 93
29	1 39 40	1 10 18	0 44 92	0 42 25	0 39,75	79	4 30 78	3 11 18	2 2 36	1 55 10	1 48 30
30	1 42 83	1 12 60	0 46 46	0 43 71	0 41 13	80	4 34 20	3 13 60	2 3 90	1 56 56	1 49 68
31	1 46 25	1 15 02	0 48 01	0 45 17	0 42 50	81	4 37 63	3 16 02	2 5 46	1 58 02	1 51 05
32	1 49 68	1 17 44	0 49 56	0 46 63	0 43 87	82	4 41 06	3 18 44	2 7 00	1 59 47	1 52 42
33	1 53 11	1 19 86	0 51 11	0 48 08	0 45,24	83	4 44 49	3 20 86	2 8 55	2 0 93	1 53 79
34	1 56 53	1 22 28	0 52 66	0 49 54	0 46,61	84	4 47 91	3 23 28	2 10 10	2 2 39	1 55 16
35	1 59 96	1 24 70	0 54 21	0 51 00	0 47,98	85	4 51 34	3 25 70	2 11 65	2 3 84	1 56,53
36	2 3 39	1 27 12	0 55 76	0 52 45	0 49 35	86	4 54 77	3 28 12	2 13 20	2 5 30	1 57 90
37	2 6 81	1 29 54	0 57 31	0 53 91	0 50 72	87	4 58 20	3 30 54	2 14 75	2 6 76	1 59 27
38	2 10 24	1 31 96	0 58 86	0 55 37	0 52 09	88	5 1 62	3 32 96	2 16 30	2 8 21	2 0 64
39	2 13 67	1 34 38	1 0 41	0 56 82	0 53 46	89	5 5 05	3 35 38	2 17 85	2 9 67	2 2 01
40	2 17,10	1 36 80	1 1 95	0 58 28	0 54,84	90	5 8 48	3 37 80	2 19 39	2 11 13	2 3 39
41	2 20 52	1 39 22	1 3 50	0 59 73	0 56 21	91	5 11 91	3 40 22	2 20 94	2 12 59	2 4 76
42	2 23 95	1 41 64	1 5 05	1 0 19	0 57 58	92	5 15 33	3 42 64	2 22 49	2 14 04	2 6 13
43	2 27 38	1 44 06	1 6 60	1 1 65	0 58 95	93	5 18 76	3 45,06	2 24,04	2 15,50	2 7 50
44	2 30 80	1 46 48	1 8 15	1 3 10	1 0 32	94	5 22 19	3 47 48	2 25 59	2 16 96	2 8 87
45	2 34 23	1 48 90	1 9 70	1 4,56	1 1 69	95	5 25 61	3 49 90	2 27,14	2 18 41	2 10 24
46	2 37 66	1 51 32	1 11 25	1 6 02	1 3,06	96	5 29 04	3 52 32	2 28 68	2 19 87	2 11,61
47	2 41 09	1 53 74	1 12 80	1 7 47	1 4 43	97	5 32 47	3 54 74	2 30 23	2 21 33	2 12 98
48	2 44 52	1 56 16	1 14 35	1 8 93	1 5,80	98	5 35 90	3 57 16	2 31 78	2 22 78	2 14 35
49	2 47 95	1 58 58	1 15 90	1 10 39	1 7 17	99	5 39 33	3 59 58	2 33,33	2 24 24	2 15 72
50	2 51 38	2 1 00	1 17 44	1 12,85	1 8 55	100	5 42 76	4 1,00	2 34 88	2 25,70	2 17,10

§ XXXIII THE DIVIDED EYE-LENS MICROMETER [PLATES III AND XI]

1. IN describing the various methods of producing double-images, and of applying them to measurements, we must not pass in silence over the divided eye-lens, which may be used as the basis of a micrometer. When the dynameter, which we have described as now made by G. Dollond, [§ XI. 6] is fitted into an adapting piece of tube which will receive it, and screw into a telescope as an eye piece, it will not only produce double images, by the separation of its segments, but the screw which separates them, by the aid of its scale and divided head, will measure the distance between the centres of those images with considerable accuracy. Whoever therefore is possessed of a dynameter of this construction, may readily and with little expense convert it into a divided eye-lens micrometer, and adapt it to any of his telescopes, except such reflecting telescopes as are of the Gregorian and Cassegrainian constructions. There is however a practical inconvenience attending the use of such micrometer, which requires the addition of a concave lens to remedy it the line of separation of the segments must necessarily divide the pupil of the observer, otherwise one image only will be visible; and, as the dynameter is constructed, that central part of the field of view, which shows double images, is very small, and the observer must manage his telescope very dexterously, to keep both the images in sight, while he uses the screw for the measurement; indeed he must consider himself *fortunate*, when he succeeds in obtaining a good observation. This limit of the field is greatly enlarged by the introduction of a concave lens, immediately behind the divided eye-lens, which addition Mr. Dollond has made in several dynameters, for the express purpose of converting them into micrometers When this third lens is introduced, the distance between the pair of convex lenses must be re-adjusted, to restore the original value of the scale, when used as a dynameter, for the concave lens diminishes the power of the eye-piece, when not compensated either by a diminution of the distance between the lenses, or by the substitution of a second convex lens of shorter focal distance

2 The interior construction of the dynameter is shown in figure 18 of Plate XI., and its external appearance in figure 17 of the same plate, but figure 1 of Plate III, presents both an end and side view of the same instrument, on a diminished scale, and figure 2 shows it withdrawn from the adapting piece of tube, which we have spoken of, as being necessary for uniting it, as an eye-piece, to the tube of a telescope. The letters *a* and *b* refer to the same parts in both figures, *c* is the adapter, and *d* the concave lens, added to convert the simple dynameter into a micrometer, without interfering with the use of it as a dynameter. As the lens which is here divided is of small diameter, the number of divisions on the scale, placed on the side of the containing box, are few, but the micrometer head, being divided into 100 parts, will subdivide the division beautifully into the centesimal portions the value of the scale, as in other micrometers, will depend on the focal length of the telescope to which it is applied. An astronomical friend was highly pleased with an instrument of this description, and succeeded in taking some good measures of the distances between certain double stars, but with our dynameter we find it difficult to obtain images which are equally luminous, and in the case of a close double star, which must necessarily be measured at the line of separation of the semi-lenses, success appears to be impracticable, particularly when one of the two stars is very small.

When the stars are both of a considerable size, and upwards of 10″ apart, they may be measured, provided the telescope has a good slow motion in altitude, but not satisfactorily otherwise.

3. As we know that the number of revolutions made by the screw of the dynameter under our notice is about 50 in the inch, our rule for the determination of the value of one revolution (§ XX. 4.) will enable us to make the following computations without experimental measurements, viz.

First $3.1416 \times 2 \times 50 = 314.16$ is a constant number;

VALUES OF A REVOL.

Then for Tel. 1. we have $30.5 \times 314.16 = 9581.88$ and $\frac{1296000}{9581.88} = 135''.25$

Tel. 2. . . . $43.2 \times 314.16 = 13571.712$ and $\frac{1296000}{13571.712} = 95.5$

Tel. 4. . . . $71.75 \times 314.16 = 22540.98$ and $\frac{1296000}{22540.98} = 57.5$

Tel. 5. . . . $76.25 \times 314.16 = 23954.7$ and $\frac{1296000}{23954.7} = 54.1$

If we multiply the value of a single revolution of each telescope by its solar focal length in inches and parts, we shall have 4125″ for a constant product, and this product, divided by the focal length of any other telescope, will give the value of a single revolution of that telescope, when the value of the screw is $\frac{1}{50}$th of an inch; but when it makes 100 turns in measuring an inch, the tabular values above obtained must be divided by 2.

On applying the dynameter by Dollond as a micrometer to the four telescopes in succession, which were used in the last two sections, the same window frame had its breadth measured by each telescope, agreeably to the subjoined register, viz.

Tel. 1. $0.93 \times 135.25 = 123''.00$

2. $1.285 \times 95.5 = 122.72$

4. $2.15 \times 57.5 = 123.62$

5. $2.275 \times 54.1 = 123.08$

The telescope numbered 3 was at the optician's, at the time these and the preceding measures were taken.

4. It will be remarked here, that though these four measures agree pretty well with each other, they differ full three seconds from the measures of the same object, as given in the two foregoing sections. This want of accordance, we were persuaded, did not arise from any material inaccuracy in the operation of taking the measures by the dynameter, but yet was too great to be acquiesced in; we therefore measured the tenth of an inch on a finely divided scale with the concave lens added, and again when it was displaced, and the difference was sufficient to account for the apparent disagreement that has been noticed. When the dynameter was used without the concave lens, five revolutions exactly measured the tenth of an inch, but, when this lens was screwed into its place, the same space was measured by 4.85 revolutions, taken on an average on both sides of zero. But if we substitute 48.5 for 50 divisions per inch, the measure arising from the computation will be as much in the opposite extreme, or upwards of 129″. If however we take 49.6 instead of 50 divisions in the inch, the constant will be 311.65, and the resulting measures will accord with those previously determined by the average of other micrometers.

5. The ingenious Amici of Modena, we have been informed, has invented and used a micrometer for measuring, by means of a divided lens, with success, but we have not been able to learn whether it is applied before or behind the solar focus of the object-glass of his telescope; the manner in which it may be usefully employed will however be readily apprehended from the account we have now given of other micrometers, depending on the bisection of a single lens differently situated We understand that the description is given by Amici in one of the recent volumes *della Societa Italiana,* to which we have not at present access, and to which therefore we must necessarily refer our readers, who may be interested in his mode of application.

§ XXXIV RAMSDEN'S CATOPTRIC MICROMETER [PLATE VI]

1. We have already said that Ramsden contrived a catoptric or reflecting micrometer for measuring by means of double images [§ XXXI 1], as the inventor has described it very fully, and has given his reasons for executing such a construction in the 69th volume of the Philosophical Transactions of London, we will transcribe his own words, instead of offering any opinion of our own on an instrument which we have not seen used.—

"At the time I took up this subject," says Ramsden, "the divided object-glass micrometer was the only one, which measured angles by the separation of two images. Since that time a very ingenious application of the prism to this purpose has been invented by the Rev. Dr. Maskelyne, Astronomer Royal, and although experience has not yet ascertained the extent of its merit, it will always deserve great consideration from its ingenuity, but the more I considered the subject, I became more fully convinced, that the principle of reflection applied to micrometers would have great advantages over those hitherto constructed on the principle of refraction, and the catoptric micrometer I have the honour to describe, besides the advantage it derives from the principle of reflection, of not being disturbed by the heterogeneity of light, avoids every defect of other micrometers, and can have no aberration, nor any defect which arises from the imperfection of materials, or of execution, as the extreme simplicity of its construction requires no additional mirrors or glasses to those required for the telescope, and the separation of the image being effected by the inclination of the two specula, and not depending on the focus of any lens or mirror, any alteration in the eye of an observer cannot affect the angle measured.

2. "It has, peculiar to itself, the advantages of an adjustment to make the images coincide in a direction perpendicular to that of their motion, and also of measuring the diameter of a planet on both sides the zero, which will appear no inconsiderable advantage to observers, who know how much easier it is to ascertain the contact of the external edges of two images than their perfect coincidence. A short explanation of the annexed figure [fig. 8. of Plate VI.] will make the construction and the properties of this micrometer obvious.

3 "I divided the small speculum of a reflecting telescope, of Cassegrain's construction, into two equal parts, by a plane across its centre; and by inclining the halves of the speculum to

each other on an axis at right angles to the plane that separated them, I obtained two distinct images. The satisfaction I received on the first trial was checked by the apparent impossibility of reducing this principle to practice. The angular separation of the two images in this case being half the angular inclination of the two specula, it required an index of an unmanageable length, to allow the quantity of one second of a degree to become visible. Some time afterwards on revising the principle, I considered that if both the halves of the mirror turned on their centre of curvature, there could be no alteration in their relative inclination to each other from their motion on this centre, and that any extent of scale might be obtained, by fixing the centre of motion at a proportional distance from the common centre of curvature. This will be better understood from the drawing

4. "*A* represents the small speculum divided into two equal parts; one of which is fixed on the end of the arm *B*, the other end of the arm is fixed on a steel axis *X*, which crosses the end of the telescope *C*. The other half of the mirror *A* is fixed on the arm *D*, which arm at the other end terminates in a socket *y*, that turns on the axis *X*, both arms are prevented bending by the braces *a a*. *G* represents a double screw, having one part *e* cut into double the number of threads in an inch to that of the part *g*; the part *e* having 100 threads in one inch, and the part *g* 50 only. The screw *e* works in a nut *F*, in the side of the telescope, while the part *g* turns in a nut *H*, which is attached to the arm *B*; the ends of the arms *B* and *D*, to which the mirrors are fixed, are separated from each other by the point of the double screw pressing against the stud *h*, fixed to the arm *D*, and turning in the nut *H* on the arm *B*. The two arms *B* and *D* are pressed against the direction of the double screw *e g* by a spiral spring within the part *n*, by which means all shake or play in the nut *H*, on which the measure depends, is entirely prevented From the difference of the threads on the screw at *e* and *g* it is evident, that the progressive motion of the screw through the nut will be half the distance of the separation of the two halves of the mirror; and consequently the half mirrors will be moved equally in contrary directions from the axis of the telescope *C*

5. "The wheel [circle] *V* fixed on the end of the double screw has its circumference divided into 100 equal parts, and numbered at every fifth division with 5, 10, &c. to 100, and the index *I* shows the motion of the screw with the wheel round its axis, while the number of revolutions of the screw is shown by the divisions on the same index. The steel screw *R* may be turned by a key, and serves to incline the small mirror at right angles to the direction of its motion. By turning the finger head the eye-tube is brought nearer or further from the small mirror, to adjust the telescope to distinct vision, and the telescope itself has a motion round its axis for the convenience of measuring the diameter of a planet in any direction. The inclination of the diameter measured with the horizon is shown in degrees and minutes by a level and vernier on a graduated circle at the breech of the telescope

6. "The method of adjusting and using the catoptric micrometer is too obvious to require any explanation, it is only necessary to observe that, besides a table for reducing the revolutions and parts of the screw to minutes and seconds, it may require a table for correcting a very small error which arises from the excentric motion of the half mirrors. By this motion their centres of curvature will (when the angle to be measured is large) approach a little towards the large mirror the equation for this purpose in small angles is insensible, but when angles to be measured exceed ten minutes, it should not be neglected. Or the angle measured may be cor-

rected by diminishing it in the proportion that the versed sine of the angle measured, supposing the excentricity radius, bears to the focal length of the small mirror.

7. "The telescope to which the catoptric micrometer is applied is of Cassegrain's construction. The great speculum is about twenty-two inches focus, and bears an aperture 5 5 inches, which is considerably larger than those of the same focal length are generally made; indeed the apparent utility of this micrometer makes me wish to see the reflecting telescope meet with further improvements. I believe it would more tend to the advancement of the art of working mirrors, if writers on this subject, instead of giving us their methods of working imaginary parabolas, would demonstrate the properties of curves for mirrors which, placed in a telescope, will show images of objects perfectly free from aberration; or, what will yet be more useful in practice, of what forms specula might be made, that the aberration caused by one mirror may be corrected by that of the other. If mathematicians assume *data* which really exist, they must see, that when the two specula of a reflecting telescope are parabolas, they cause a very considerable aberration, which is negative, i. e. the focus of the extreme rays is longer than those of the middle ones If the large speculum is a parabola, the small one ought to be an ellipse; but when the small speculum is spherical, which is generally the case in practice, if concave, the figure of the large speculum ought to be an hyperbola, if convex, the large speculum ought to be an ellipse, to free the telescope from aberration.

8. "This will be more easily understood by attending to the positions of the first and second images, when a curve is of such form, that lines drawn from each image, and meeting in any part of the curve make equal angles with the tangent to the curve at that point, it is evident that such curve will be free from aberration. This is the property of a circle when the radiant and image are in the same place; but when they recede from each other, it is the property of an ellipse, of such form that the radiant and image are in the two *foci*, till, one distance becoming infinite, the ellipse changes into a parabola, and into an hyperbola when the focus is negative, i. e. when reflected rays diverge, and the focus is on the opposite side of the mirror.

9. "These principles made me prefer Cassegrain's construction of the reflecting telescope to either the Gregorian or Newtonian. In the former, errors caused by one speculum are diminished by those in the other.

10 "From a property of the reflecting telescope (which has not been attended to) that the apertures of the two specula are to each other very nearly in the proportion of their focal lengths, it follows that their aberrations will be to each other in the same proportion, and these aberrations are in the same direction, if the two specula are both concave, or in contrary directions, if one speculum is concave, and the other convex. In the Gregorian construction, both specula being concave, the aberration at the second image will be the *sum* of the two aberrations of the two mirrors · but in Cassegrain's telescope, one mirror being concave, and the other convex, the aberration at the second image will be the *difference* between their aberrations. By assuming such proportions for the foci of the specula as are generally used in the reflecting telescope, which is about as 1 · 4, the aberration in Cassegrain's telescope will be to that in the Gregorian as 3 · 5.

11. "I have mentioned these circumstances in hopes of recommending the demonstration of curves, suited to the purposes of optics, to the attention of mathematicians, which would be of great use to artists."

§ XXXV DR MASKELYNE'S PRISMATIC MICROMETER

1. When Dr. Maskelyne had pointed out the defect of Dollond's object-glass micrometer, which his cross wires were not calculated to remedy, namely, the supposed property of giving deviations from the true measure depending on the construction and state of the observer's eye, he contrived a micrometer to answer the same purpose, with a much longer scale, and nearly free from this defect the principle was that of measuring by means of double images, but instead of using a divided lens, he applied two broad and thin prisms of glass, placed side by side in reversed positions, as to their prismatic form; which he found was the most favourable mode of producing double images, and indeed the only one by which each image could be made equally bright at every part of the scale, which scale was co extensive with the tube of the telescope.

2 When a single prism was applied with its face to the external surface of an achromatic object-glass of any telescope, the Doctor found that, if it covered the whole of it, one image only was produced, which was thrown by the refraction, occasioned by the broad prism, into one side of the field of view, but that, if one portion of the object glass only was covered, two images of a distant object became visible, one achromatic, and the other tinged with colouration, the distance between these two images was found equal to the refraction produced by the prism, and when it was made to cover a little more than one half of the lens, the two images could be made equally bright

3 When the prism was divided lengthwise with a diamond, and the two halves reversed, the refraction of each, being in contrary directions, doubled the distance of the two images but though compound prisms of flint and crown glass could be made to destroy the colours of the images, the contrivance was incapable of giving more than one fixed measure of the subtense of an angle, while the pieces remained in the same position, and it required peculiar ingenuity to devise some practicable method of varying the measure, so as to admit of a scale that would descend down to zero.

4 By a remarkable coincidence as to time, it so happened, that both the abbots, Boscovich and Rochon, were devising the means of measuring small angles by means of double images, occasioned by the doubly refractive property of rock crystal, in the same year that gave birth to the invention under our present consideration; and, what is still more remarkable, the same mode of applying a scale to the doubly-refracting substances, occurred to all the three inventors within a short space of time from each other Boscovich candidly yielded the palm of priority of invention to his competitor Rochon, but there can be no doubt that Dr. Maskelyne's invention preceded Rochon's by some months In the 67th Vol. of the Philosophical Transactions of London (1777), the Doctor's "Account of a New Instrument for Measuring Small Angles, called the Prismatic Micrometer," appeared indeed at page 799, and Boscovich's "Account of a New Micrometer and Megameter" had been previously inserted at page 789 of the same volume, but from the testimony of both J. Dollond and Mr. Aubert, published at the same time, it appears that Dr. Maskelyne's micrometer was not only contrived, but

actually made and used in April 1776, which time was prior to the inventions of either of the foreigners. Indeed Boscovich's contrivance was spoken of in his paper only as a *project*, which had not been carried into execution, and it has not been proved, that Rochon's invention was so early as Dr Maskelyne's. It is more than probable that the report of Dr. Maskelyne's improvement of Dollond's heliometer in 1771, had excited on the Continent a desire to produce some contrivance to answer the same purpose, and a competition once excited to obtain the same object, might, as on other occasions, give rise to cotemporary inventions of similar means to produce the same effect, but the priority of time does not diminish the merits of the respective inventors.

5 Dr. Maskelyne found, that if the pair of inverted prisms of glass were made to pass along the inside of the tube of the telescope, the distance from the focus of the object-glass would become a scale of measurement for the observed angle, that is, that the focal length of the object-glass would be to the total refraction of the two prisms, taken in one sum, as their distance from the focus of the object glass, is to the measure given in that position, and that zero would always be at the focal point; which is also the case with Rochon's scale, as will be seen in its proper place. There are three positions in which a pair of prisms may be placed to produce double images, when the line of junction is at the thin edges, the images formed at the focus will have only one half of the light which falls on the object-glass, while the prisms remain in contact with it, and will be less and less enlightened as they approach the focus; on the contrary, when the line of junction is at the thick ends, though the light will be only one half of what falls on the object-glass while they remain in contact with the object-glass, yet the light will gradually increase as the prisms approach the focus, but when they have their line of junction along the tapering edges, and are reversed in position, the light, as we have said, will continue the same in every part of the scale, which is the reason why a preference was given to this mode of juxta-position. The prismatic colours however were objectionable, when the refracting pieces were formed of single pieces of glass, and the want of well defined edges of the images prevented the measures from being satisfactory, which defect could only be cured by making the prisms double, of flint and crown glass, so as to become achromatic. One of the Dollonds succeeded in making an achromatic prismatic solid for Dr Maskelyne, which performed very well, but as the instrument was his private property, his widow very properly took possession of it at his death, and it does not appear, that a second successful attempt has since been made. On enquiry of the opticians we learn, that there is some practical difficulty in the construction, which has prevented the general adoption of this micrometer, notwithstanding the sanguine expectations which were entertained of its utility by the inventor, and which were expressed in the following words. "Upon the principles", says Dr Maskelyne, "that have been here explained, a prism placed within the telescope of an astronomical instrument, adjusted by a plumb-line or level, to receive all the rays that pass through the object glass, may conveniently serve the purpose of a micrometer, and supersede the use both of the vernier scale and the external micrometer; and the instrument may then be always set to some even division before the observation. Thus the use of a telescope level may be extended to measure with great accuracy the horizontal refractions, the depression of the horizon of the sea, and small altitudes and depressions of land objects. Time

and experience will doubtless suggest many other useful applications of this instrument." We can only express our hope, that the expectation thus entertained by the learned astronomer may yet be realized.

6. Troughton informs us that the telescope, to which Dr. Maskelyne applied his prismatic micrometer, was one of only 30 inches focal length, and he remembers having taken the diameter of the sun with it, which it defined well, but as the scale was not good, he could take it as accurately with a sextant. We have not been able to learn, that any document is in existence, which has preserved the observations made by the prismatic micrometer at Greenwich, if any, nor can it be now ascertained what was the exact value of its scale.

§ XXXVI. THE CUNEIFORM MICROMETER. [Plate IV.]

1. When we first read Dr. Maskelyne's account of his prismatic micrometer, described in our last section, it occurred to us, that if the prisms of glass were made very thin wedges, and confined to move at the eye-end of the telescope within a tube, like the terrestrial tube of a refracting telescope, the range it would have, within reach of the hand, would afford a scale long enough for measuring the diameter of a planet, or the distance between two contiguous stars, without any apparent colouration of the double images, and on trial we found our expectation verified.

2. Fig. 1, of Plate IV. is a piece of brass tube $5\frac{1}{2}$ inches long, containing a cell *a*, like fig. 9 which holds either a single wedge as shown in fig. 7, or a pair of reversed wedges as seen in fig. 8, of the full dimensions, this piece of tube is divided into inches and tenths, and figured from left to right, and back again from right to left, for a purpose which will be explained. Fig. 2. is a longer tube of a calibre just sufficient to receive the shorter one within it, and to allow it to slide without much friction; the thumb-piece *b* screws into the end of the short tube, and, passing along a longitudinal opening made in the long tube, serves to move it into any position that the observation may require: about the middle of the long tube there are two larger openings at opposite sides, which allow the finger and thumb to take hold of the inner tube in turning it round, to present the tapped hole to the thumb-piece *b*, when it is screwed into its place; one edge of the visible opening serves also for a scale of graduations for the vernier, which reads the hundredths of an inch. At the end *c* a positive eye-piece is screwed into the large tube, which slides into another tube fixed to the telescope, as far as the end of the vernier, where distinct vision of a distant object is obtained, and the two opposite notches at the end *f*, allow the finger and thumb to handle the inner tube, in withdrawing it, when the thumb-piece *b* is unscrewed, that the cell *a* may be changed for another of different value, when the measure may require it.

3. Fig. 3. is a section of the tube shown in fig. 2, in which the lenses of the eye-piece, and the cell holding the wedge, are visible in their places: this figure shows the second position of the inner tube at fig. 1, after it has been reversed, end for end; in the first position the cell is brought so near the eye-piece, that the dividing line of the two contiguous wedges, or the

straight edge of the single wedge, may be exactly in its focus, so as to be distinctly visible; in which situation the zero of the vernier will coincide with the zero of the scale on the inner tube, and the object viewed will appear single, as it does without the micrometer: if this should not be the case, the eye piece may be pushed in, or drawn out, till it has good vision of the object, while the two zeros remain coincident, and when this adjustment is made, the instrument will be in a state for use. The effect of reversing the position of the inner tube is to make the scale equal to that of a tube of 11 inches; for it measures from 0 to 5.5 in the first position, and from 5.5 to 11 in the second, which is a convenience when the telescope is not a long one. When the instrument has a long focal distance, to admit of a lengthened exterior tube, the mode of fitting the inner tube will be more convenient as it is represented in fig. 4, where the same letters show the same parts, the inner tube here is not graduated, but is used to hold the cell into which the thumb-piece *b* and vernier are screwed, the opening of the long tube being 11 inches in length, and the inches and parts being engraved along its edge, while the vernier is the moving portion. The long opening in the tube terminates with a cranked part near the eye piece, which allows the thumb piece to bring the cell occasionally near enough the end of the tube, when the eye piece is unscrewed, to permit the cell to be changed without undoing any of the other parts, and the measure is always read on one fixed scale by means of the sliding vernier, otherwise the use of this construction is the same as that of the other, and either may be chosen which best suits the length of the telescope. The latter construction was contrived for our 12 feet refracting telescope of 7 inches aperture, for which it is well adapted; for the length of the tube affords the means of subdividing the second as low as to one tenth part.

4. We have used the micrometer represented by fig 2, with our 76.25 inches telescope (No. 5.), and when the measures are not very small, the results are quite satisfactory. Like the divided eye lens micrometer, this instrument requires the measures to be taken in the direction of the line of separation, and as near this line in the field of view as possible while the object is observed; otherwise it will frequently happen, that one of the images will disappear during the observation, when it is in motion. It will greatly facilitate the performance, if the tube be turned about till the line of separation is exactly in the direction of the object's apparent path, before the wedge is moved from zero, for then, if the telescope has a parallactic motion, the observation may be completed without trouble.

5. This instrument may also be used as a position micrometer, for a divided ring, that is graduated into four quadrants, is screwed to the exterior end of the fixed piece of tube, that moves by the rack-work for distinct vision, within which the graduated tube enters, and a vernier clamped on this inner tube will indicate the angle of position which the line of separation makes with the horizon, when the parallactic stand is properly adjusted in the meridian, and the position 90° made to be in a vertical line.

6. The micrometer which we have used, has three cells successively applicable to the inner tube, one containing two wedges, side by side, in a reversed position (fig 8), and two, holding each a semicircular wedge (fig 7), their values per inch are 54″.14, 33″.4, and 20″.74 respectively, as determined by methods that will be explained hereafter, and compared with the values of other micrometers, when used with the same telescope (No 5). We shall annex a table suitable for converting the measures taken by these wedges into minutes and seconds, which may be taken by inspection with the argument read on the scale.

A TABLE

OF THE VALUES OF THREE CUNEIFORM MICROMETERS USED WITH A TELESCOPE OF 76 25 INCHES FOCAL LENGTH

FIRST WEDGE = a				SECOND WEDGE = b				TWO WEDGES = $a + b$			
Scale	Inches	Tenths	Hundredths	Scale	Inches	Tenths	Hundredths.	Scale	Inches	Tenths	Hundredths
1	0 20" 74	2" 07	0 21	1	0 33".4	3' 34	0".33	1	0' 54" 14	5' 41	0" 54
2	0 41 48	4 15	0 41	2	1 6 8	6 68	0 67	2	1 48 28	10 83	1 48
3	1 2 22	6 22	0 62	3	1 40 2	10 02	1 00	3	2 42 42	16 24	1 62
4	1 22 96	8 30	0 83	4	2 13 6	13 36	1 34	4	3 36 56	21 66	2 17
5	1 43 70	10 37	1 04	5	2 47 0	16 70	1 67	5	4 30 70	27 07	2 71
6	2 4 44	12 44	1 24	6	3 20 4	20 04	2 00	6	5 24 84	32 48	3 21
7	2 25 18	14 52	1 45	7	3 53 8	23 38	2 34	7	6 18 98	37 90	3 79
8	2 45 92	16 59	1 66	8	4 27 2	26 72	2 67	8	7 13 12	43 31	4 33
9	3 6 66	18 67	1 87	9	5 0 6	30 06	3 00	9	8 7 26	48 73	4 87
10	3 27 40	20 74	2 07	10	5 34 0	33 40	3 34	10	9 1 40	54 14	5 41

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§ XXXVII ROCHON'S CRYSTAL MICROMETER [PLATES II, III, XIII]

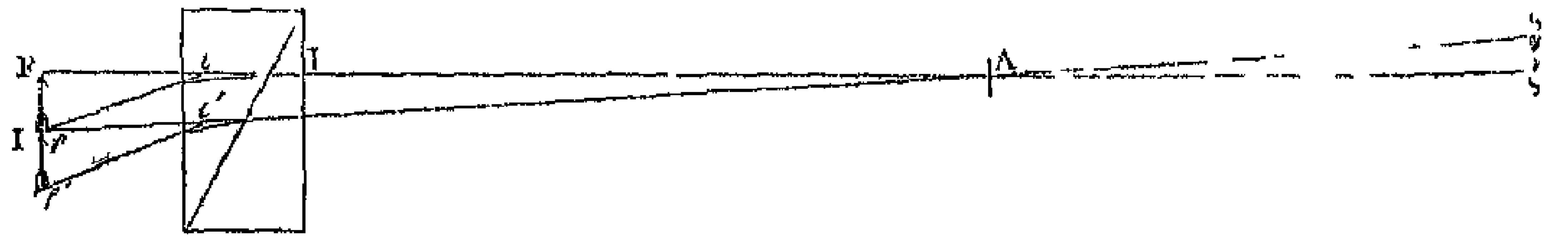
1. Having had occasion to mention the Abbot Rochon's application of the doubly-refracting property of rock-crystal to a telescope, for measuring the subtenses of small angles, by means of a scale which extends the whole length of the tube, in a manner similar to Dr. Maskelyne's contrivance, we may with propriety introduce our description of this ingenious Frenchman's invention in this section It is well known to mineralogists, that several crystallized bodies possess the singular property of dividing into two pencils the rays of light, which are transmitted through them in certain directions, and that each pencil, after converging to a focus, will form a separate image of the object from which the rays originally emanate. It is not our business to examine into the natural cause of this curious property, nor yet to investigate any law by which the double refraction is guided under different circumstances, but merely to state what is the groundwork of Rochon's application of the principle of double refraction to the measurement of small arcs, by the aid of a short telescope.

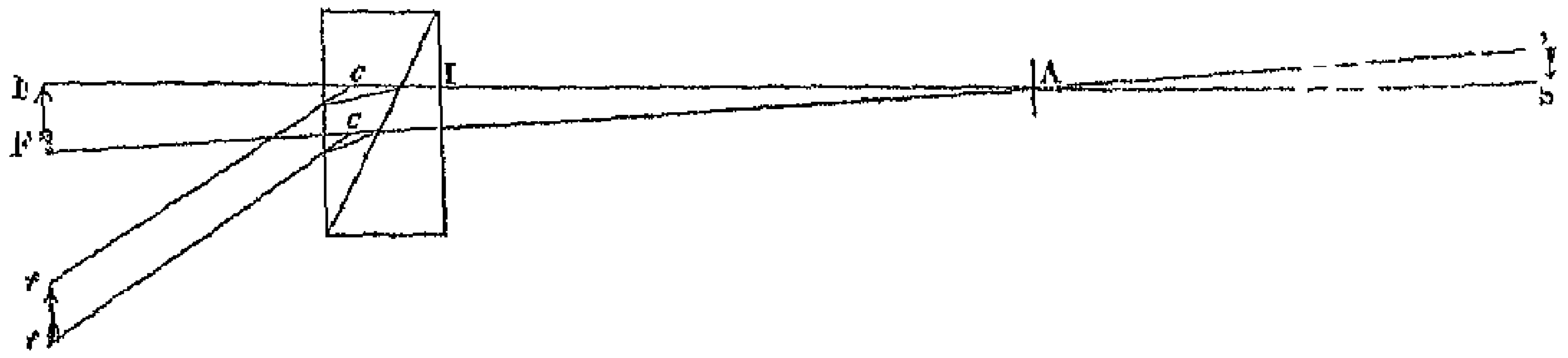
2. It had been previously observed that when the rays of light pass through a transparent piece of rock crystal lengthwise, or in the direction of its axis of formation, the pencil suffers only the ordinary refraction, as though it were transmitted through a single piece of glass; but when the rays are incident on the side of the crystal, or come in an oblique direction, the phenomenon of double refraction takes place during the transmission, one pencil of rays being in their natural state and the other polarized. Availing himself of this property, Rochon formed a prism of rock-crystal from an oblique section, and polished it, so as to produce two images in the common focus of a telescope, when placed on the external face of the object-

glass, but while it remained there, like Dr. Maskelyne's wedges of glass, it produced only one constant angular measure; and the question was in both cases, how a change in the measure could be effected by a mechanical contrivance. It was found that varying the angle of inclination from a state of contact, with the object glass, gave a corresponding variation in the quantity of refraction, or, which was the same thing, in the distance between the images of an object, viewed through the telescope; but this distance did not increase in the same ratio with the angular change of inclination, nor could a scale sufficiently extensive, and also correct, be obtained from this mode of varying the incidence and refraction of the rays.

3. On trying what the effect would be within the telescope, it was discovered, not only that double images would be formed in any position of the prism, between the object-glass and its focal point, but that the angular measure became altered in every new situation, and in such proportion, that if the whole focal length represented the whole constant angle, the different distances from the focal point would give proportional parts of that constant angle, and thus the length of the main tube became the scale for the equal divisions and subdivisions of the constant angle. This measure being ascertained to be correct, it became necessary to make a longitudinal opening or slit in the tube, to admit a button and sliding index attached to the piece which held the moveable prism. The single prism, in this case, as in Dr Maskelyne's, gave the prismatic colours to the images, and a second prism of similar dimensions, but cut so as to produce only the ordinary refraction, was applied, in a reversed position, to the prism of double refraction, and made with it a cubic solid; the two prisms were made to adhere with *mastic en larmes*, which has nearly the same refractive power as the crystal, and is also transparent. This cubic solid retained the property of giving double images, and became achromatic, so far as the image produced by ordinary refraction was concerned, but the image produced by the extraordinary refraction still retained a tinge of yellow at the edge of the image belonging to it, and no means have yet been devised for freeing it from this imperfection, which in a certain degree injures the delineation of the polarized images.

4. The annexed figures will explain the manner in which the pencils of rays differently refracted not only form two separate images of a distant object, seen through a telescope; but also appear at different distances from each other as the crystal of double refraction recedes from the focal point towards the object-glass;





In both these figures, we will consider the arrow *S' S*, an object placed at a distance beyond *A*, the place of the object-glass of the telescope, and the enlarged and inverted arrow *F F'* the image formed by the pencil of ordinary refraction at the focus of the object-glass when the two pencils coming from the ends *S'* and *S*, fall on the centre of the object glass, they cross one another and proceed diverging till they meet with the solid formed of the two prisms, cut and put together as above described, they proceed through the first prism, from the face *I*, without suffering any other than the ordinary refraction, but when they arrive at the inclined face of the second prism, formed by the diagonal line, one portion of each pencil is bent in an extraordinary manner towards its acute angle, which is called the refracting angle, and on emerging from this prism into air, the refraction is increased so much, that the pencils of extraordinary refraction proceed in a direction as though they came from the points *e* and *e'* within the prism, and not from the incident points on the inclined hypothenusal surface, there are now two pairs of pencils, separated from each other by this double refraction, and each forming a separate representation of the extreme ends of the magnified arrow; and pencils coming in like manner from the intermediate parts of the arrow *S'S*, will also suffer double refraction, and form the middle parts of the separated and enlarged arrows as seen in the figures.

5. When the prismatic solid is placed in the focus of the object-glass so that the points *FF'*, of the arrow coincide with the points whence the divergence of the rays of extraordinary refraction apparently emanate, there will be only one image of the arrow, but as the doubly refracting solid is moved gradually towards the object-glass at *A*, two images begin to appear, and at first overlap one another and produce a dark shade, but a further removal into the position exhibited in the first figure will separate the arrows by a quantity which will just measure the length of either of them, by putting their opposite ends into exact contact: a further removal from the original position at the focus will separate the arrows still more, until a space is seen between the images, the quantum of which, on the principle of similar triangles, will always be proportionate to the distance from the focal point, where the zero of the scale must be placed; and when the solid is removed as far as to the object glass *A*, the separation of the image becomes a maximum, and the scale terminates.

6. As the rays of light pass through both the prisms and interposed mastic, or other adhesive transparent substance, before they come to the focal point of the object-glass, the impurities of the substances passed through render the images less perfect than a single image formed by the object-glass alone would be, and the bipartition of the light, coming from the object, between the two images, diminish the brilliancy of its colours, independently of the polarization of the image formed by the extraordinary refraction, and the more the images are magnified, the more apparent will these defects become; particularly when the doubly-refracting solid is at a considerable distance from the focus of the telescope, and the divergence of the rays proportionably great. On these accounts the ingenious Abbot confined the length of his telescope to about 20 inches, which afforded a scale for minutes only, as he divided it, and its use was confined, like that of the English coming-up glass, to naval and military purposes.

7 Conceiving that a Rochon's micrometrical telescope, of a portable construction for the pocket, might be serviceable on many occasions, but particularly in observing solar and lunar eclipses, we took the opportunity, during a visit to Paris, in the summer of 1819, of getting an achromatised prismatic solid of rock crystal applied to a pocket telescope, similar to

the one represented by fig 10 of Plate II, by getting the cell which held it adapted to screw into the remote end of each sliding tube, as the measure might require, and three scales made along the exterior face of the outermost sliding tube, to suit the three positions of the crystal, afforded the means of measuring any angle from 0′ to 36′ by a vernier which indicated single seconds placed at the eye-end of the main tube.

8 The changing of the cell from one tube into another was found inconvenient, and the same solid of double refraction was applied more conveniently to the large finder of the telescope represented, in two different positions, by figures 2 and 3 of Plate XIII The main tube of this finder is 19 inches long, and holds an achromatic object-glass of an inch in diameter, an inner tube, of half the length, is moved by a pinion and rack, and contains a scale which will shew a measure as far as 32′, on which a fixed vernier indicates single seconds, by means of an opening made near the pinion at the middle of the exterior tube This was the identical micrometer used by Mr Baily in his observations of the solar eclipse of the 7th of September, 1820, which attracted universal notice throughout Europe.

9. Fig 8 of Plate II is a drawing of a 33 inches achromatic telescope, with an aperture of $2\frac{1}{6}$ inches, which we have had fitted up agreeably to Rochon's construction, with two separate solids of crystal, to be applied in succession. The tube is graduated from the solar focus into two scales, one at each side of the opening made down the middle, and the detached sliding piece, at figure 9, holds either of the two prismatic pieces within it, and also a pair of verniers attached to it by the two screws with milled heads, which pass through the opening, and serve to move the piece either backwards or forwards, when they are not screwed too tight. The thicker crystal, contained in the main tube, has a constant angle of 32′, and the detached thin one will measure only 5′; the vernier of the former indicates seconds, and of the latter the tenths of the second, which indeed is a more nice indication than the magnifying power of the telescope requires, which is only 35 .5. The instrument is mounted on a tripod, but is not free from the imperfections which have been noticed; it may however be used as an heliometer, or for observations of solar and lunar eclipses. The telescope is so mounted that it will turn round its bed, on its axis of vision, for adjustment to the line of position in which the measure is required to be made, which is a condition to be attended to in the mounting of all the double-image micrometers, where the length of the tube constitutes the scale of measurement.

10 When nice observations of very small arcs are required to be made, such as of double stars of the first and second classes, or of the diameters of planets, Rochon's principle may be confined to that end of the scale which is nearest to the focal point of the object-glass, where a section of the converging cone of light is diminished so as to be received by small prisms, which may be chosen out of pure and perfectly transparent pieces of crystal, and fitted into small cells, like fig 10 of Plate IV, and then they will apply to the eye-tubes represented by figures 1, 2, 3, and 4 of the same Plate, in the way that the wedges of glass are applied, which we have already explained in the thirty-sixth section.

11. We first adapted a crystal of double refraction to the small tube in fig 1, as prepared by Cauchoix, of Paris, the constant angle of which was 38′ 10″, and used it with our telescope numbered 5, which afforded a scale of 30″.033 in the inch, or $\frac{38' \, 10''}{76 .25}$ The vision within the

limit of the small tube was found tolerably good, and, what is a great advantage, the double images appeared distinctly in every part of the field of view, which is not the case with the wedges of glass and divided eye-lens Yet it was found, that the nearer the measure is taken to the middle of the field of view, the more correct in general will the measure be. With this prism, the scale of 11 inches, when the inner tube was reversed in position, would measure 5′ 30″, or, in one or other of the positions, any smaller angle, down to a single second The subjoined Table was constructed for the telescope and prism under our present consideration.

12. TABLE.

	Inches	Tenths	Hundredths
1	0 30 .0	0′ 3″	0″.3
2	1 0 .0	0 6	0 .6
3	1 30 .1	0 9	0 .9
4	2 0 .1	0 12	1 .2
5	2 30 .1	0 15	1 .5
6	3 0 .2	0 18	1 .8
7	3 30 .2	0 21	2 .1
8	4 0 .3	0 24	2 .4
9	4 30 .3	0 27	2 .7
10	5 0 .3	0 30	3 .0
11	5 30 .4		

13. When a celestial object is viewed by this micrometrical telescope, the positive eye-piece must be adjusted for distinct vision, and also to make single images at zero of the scale, if this cannot be effected, the index error must be ascertained, and applied with its proper sign to every observation. When terrestrial objects are measured, this will be indispensably necessary, because the elongation of the telescope alters the place of zero, and, indeed, in a certain degree, even the divisions of the scale, which are fractional parts of the solar focal length; and the longer the telescope is, the greater will be the error introduced into the scale, when near objects are observed.

14. An additional advantage which Rochon's construction has, in common with the preceding double-image micrometers, is, that it does not require the magnifying power of the telescope to be altered during the time of taking an observation, but in all of them, it must be allowed, the performance of the telescope to which they are applied is sensibly deteriorated.

15. The quantity of the constant angle, measured by any prism, depends on the magnitude of its refracting angle formed by the section, the number of minutes measured being somewhat fewer than the number of degrees of the prism's refracting angle. Rochon was of opinion that the prismatic shape of an Iceland crystal would not admit of a constant angle of separation of the images greater than 20′, and, to increase it to a quantity which would measure a greater subtense, he employed a Parisian artist, Narci, to cut two prisms in such way as would, when applied in conjunction, produce a constant angular measure of 40′. At present the

opticians who are able to prepare the achromatic crystals of double refraction, in Paris, are Cauchoix, Soleil, and Lenoir, and, as far as we know, the only instrument-makers in England who have been successful in their attempts to cut and polish them in their proper sections, are Dollond, and Robinson of Devonshire Street, London.

16. Dr. Wollaston has written a paper in the Philosophical Transactions of London of the year 1820, in which he has explained how two wedges must be cut to form an achromatic solid which shall have the constant angle doubled, in the way Rochon had mentioned without giving a full explanation, which paper will be found an useful guide to those opticians, who may be disposed to prepare such a solid. Monsieur Arago, according to Biot, has preferred increasing the constant angle of a prism by placing it in an oblique direction, as it regards the line of vision, which may have its advantage, by admitting of a reduction in the thickness of the prism.

17. As an experiment we had two prismatic solids of double refraction put into brass cells, and mounted one over the other, end to end, in the manner represented by figure 11 of Plate III., and seen in section in figure 12, so that one of the cells will revolve round the other, and a stroke *a*, made on the outer cell, will point to the plane face of the cap *b c*, which screws over an eye-piece of a telescope, and will indicate thereon the relative positions of the two prismatic solids as the upper one revolves. The two prisms have their constant angles very nearly alike, and when they are so placed that their refracting planes are parallel to each other, as in figure 12, the constant angle is doubled, or rather equal to the sum of the two taken singly; but when the upper prism is reversed by turning its cell through a semicircle, the constant angle is very small, being only the difference of the two single constant angles, and would be nothing if the two prisms were precisely alike. When the upper prism is turned only 90° either forward or backward, the compound constant angle seems to be one half the sum of the two: in all other relative situations of the two prisms there will be four images of the object, which will change their positions in a curious manner as the cell is turned round, till the stroke *a* points out the four quadrantal positions, in which the double images are exhibited, unless the two prisms are alike, in which case one of the four quadrantal positions will exhibit only a single image. At the middle point of each quadrant the four images will be equally luminous, but beyond and short of this point one pair will be more and the other less luminous by alternate changes, till in altering the position one of the pairs disappears, and two lines connecting the centres of each pair respectively will always be at right angles to each other, though they will alternately elongate and shorten while the cell is turning into the four positions which give double images only. If the refracting planes, however, are not made in their true sections for producing primary images, whenever a luminous object is viewed, there will be faint secondary images visible, which will render the prism unfit for micrometrical purposes. When the sun is the object viewed, a darkening glass may be placed either before or behind the doubly refracting solid, as may be most convenient, and if, with a single prism achromatised, there appear to be other refracted images of the sun, besides the two primary brilliant ones, and the faint reflected images, the direction in which the prism has been cut is not the true section, and the crystal so formed will not be serviceable as a micrometer.

18. Besides the prism from Cauchoix, which was originally applied to the telescope of

76.25 inches focal length, we have adapted four English prisms, prepared by Dollond, to the same telescope, and have tabulated the values arising from the division of their constant angles, by 76.25, which four Tables are here subjoined, as further specimens of this mode of gaining correct measures of small angular subtenses.

19.

THE TABULAR VALUES

OF

DOLLOND'S FOUR PRISMS,

USED WITH A TELESCOPE OF 76 25 INCHES FOCAL LENGTH

PRISM I

Scale	Measure for Inches	Tenths	Hundredths
1	0′ 10″.92	1″.09	0″.11
2	0 21.84	2.18	0 21
3	0 32.76	3.28	0.33
4	0 43.68	4.37	0.44
5	0 54.60	5.46	0.55
6	1 5.52	6.55	0.66
7	1 16.44	7.64	0.76
8	1 27.36	8.74	0.87
9	1 38.28	9.83	0.98
10	1 49.20	10.92	1.09

PRISM II.

Scale.	Measure for Inches.	Tenths.	Hundredths
1	0 17″.45	1″.75	0″.17
2	0 34.90	3.49	0.35
3	0 52.35	5.24	0.52
4	1 9.80	6.98	0.70
5	1 27.25	8.73	0.87
6	1 44.70	10.47	1.05
7	2 2.15	12.22	1 22
8	2 19.60	13.96	1.40
9	2 37.05	15.71	1.57
10	2 54.50	17.45	1.75

PRISM III.

Scale	Measure for Inches	Tenths	Hundredths
1	0′ 22″ 96	2″.30	0″.23
2	0 45.92	4.59	0.46
3	1 8.88	6.89	0.69
4	1 31.84	9.18	0.92
5	1 54.80	11.48	1.15
6	2 17.76	13.78	1.38
7	2 40.72	16.07	1.61
8	3 3.68	18.37	1.84
9	3 26.64	20.66	2.07
10	3 49.60	22.96	2.30

PRISM IV

Scale.	Measure for Inches	Tenths	Hundredths
1	0′ 31″.10	3″ 11	0″.31
2	1 2.20	6.22	0.62
3	1 33.30	9 33	0.93
4	2 4.40	12.44	1.24
5	2 35.50	15.55	1.56
6	3 6.60	18.66	1.87
7	3 37.70	21.77	2 18
8	4 8.80	24.88	2.49
9	4 40.90	28.99	2.80
10	5 11.10	31.10	3.11

20. The methods most convenient for determining the constant angles of a set of prisms will be explained in the subsequent section; we say a *set* of prisms, because in certain angular distances a prism of a small constant angle may give a better scale than a large one, but where the angular measure is comparatively great, or when a high power is used, a prism of a large constant angle will answer better, particularly where the length of the scale is limited to a few inches by the sliding prism-holder. Hence it will be desirable to have more prisms than one adapted to the sliding holder of the same telescope, to be used as occasion may require. When extreme accuracy is the principal object, which is generally the case in micrometrical measurements, each prism may be employed, according to Rochon's method, for getting a separate measure, and then an average of the whole, if they differ but little from each other, may be considered as a measure extremely correct. In this way the following measures were taken, with the instrument and prisms we have above specified, of the diameter of Jupiter in the direction of his belts, which is his larger diameter.

21. March 20, 1826 Prism 3 gave 1.94 inches and parts as the measure
Prism 4 gave 1.43 do
Then the Table for prism 3 affords ....1. =22″.96
.9 =20.66
.04= 0.92
measure =44.54

And the Table for prism 4 gives ....1. =31″.10
.4 =12.44
.03= 0.93
measure =44.47

Hence the average of the two for the said evening will be 44″.505, which measure exceeds the greatest diameter, at a mean distance, contained in the Table A. at page 265 of our first volume, the basis of which Table was probably the shorter or vertical diameter of this planet, which is the only one that can be correctly measured by the wire-micrometer.

INCHES
Again on March 29, Prism 3 appeared to give 1.93=44″.31 by the Table.
Prism 4................... 1.42=44.16 by do
March 30 ....,Prism 2 .... 2.54=44.33 by do
Prism 1..................... 4.07=44.34 by do.

Average of four measures=44.28.5

22. From these results it may be inferred that Jupiter's diameter was gradually diminishing at the times of observation, and accordingly if we compare the sun's longitude with the heliocentric longitude of the planet, as given in the Nautical Almanac for the said days, we shall find that the difference was becoming smaller, or that the planet was departing from opposition where its diameter, (if at or near the perihelion of its orbit also,) is a maximum.

In the present instance Jupiter was nearly two signs from the aphelion point of his orbit, and therefore his diameter would not be a maximum even at opposition this year.

23 It cannot be expected that micrometrical measures taken in this way by four different prisms will always accord so nearly, but if the solar focal point be well determined by finding the exact position of the prisms where well defined single images are formed (which will be on the section of the doubly refracting solid), the measures will have no other difference than what may be fairly imputed to a defect in the observation, provided the telescope be good, the constant angles truly determined, and the eye-piece of that construction which is usually denominated *positive*. We will conclude our section by subjoining some additional measures taken by the same means in some of the following months.

Jupiter, April 16, 1826.

PRISM 1	PRISM 2	PRISM 3	PRISM 4
3. … ..32″.76	2 . . ..34″.90	1 ……22″.96	1. .. …31″.10
.7 ….. 7 .64	.3 …… 5 .24	.8 .. . 18 .37	.3 …… 9 .33
.07.. … .76	.02…… .35	.06…… 1 .38	.07…… 2 .18
41 .16	40 .49	42 .71	42 .61

From an average of these four measures the long diameter of the planet was at this time 41″.74, still decreasing.

Jupiter, April 24.

$$\left.\begin{array}{r} \text{Prism 2. gave } 1.76 = 41''\,36 \\ 3. \quad .\ 2.37 = 40.41 \end{array}\right\} = 40''.88$$

Jupiter, May 2.

$$\left.\begin{array}{r} \text{Prism 1. gave } 3.51 = 38''.33 \\ 2\ . \quad 2.24 = 39\ 96 \\ 3.\ .\ .\ .\ 1.71 = 39.26 \\ 4. \quad .\ .\ 1.275 = 39.65 \end{array}\right\} = 39''.30$$

Mars, May 31.

PRISM 1	PRISM 3	PRISM 4	MEAN
1. .. .. 10″ 92	0 .8 ……18″.37	0 6 …..18″.66	
.7 …… 7 .64		.004….. 0 .12	
.01… .. 0 .11			
18 .67	18 .37	18 .78	18″.61

Mars, July 16

Prism 3. gave 0 .47 = 10″.79 only, which measure shows that Mars was apparently decreasing very fast.

Saturn's Ring, April 7, 1826

	LENGTH	BREADTH.
Prism 1 . .	3 82 = 41″.71	1.89 = 20″.64
2.	2.39 = 41.71	1.24 = 21.64
3.	1.84 = 42.25	0.91 = 20.89
4.	1.36 = 42.30	0.68 = 21.15
Average . . .	41.99	21 08

Nov. 26. Sun's diameter by Rochon's nineteen-inch telescope (8.) 32′ 30″
Ditto, by the Nautical Almanac 32 29

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§ XXXVIII ON THE DIFFERENT METHODS OF DETERMINING THE CONSTANT ANGLE OF A DOUBLY REFRACTING PRISM OF ROCK CRYSTAL, OR OF A GLASS WEDGE.

1. When a solid of double refraction is formed into a figure, which will make an object viewed through it appear double, two lines drawn from the centres of those two images to the eye of the observer, will include a small angle at the eye, the magnitude of which will always be the same whatever object be viewed, or whatever may be the distance of that object, hence this angle is called the *constant angle* of the doubly refracting body. We have already said that the shape usually adopted for the separation of the two images is that of a prism, and that the quantity of separation, constituting the constant angle of any prism, depends on its refracting angle included between the two principal sections of formation, which limit the thickness of the end opposed to this angle. When two prisms, one having single and the other double refraction, are placed in a reversed position so as to form a cubic, or oblong solid, it becomes achromatic, but retains the property of giving double images including a constant angle, as we have before noticed.

2. The accuracy of all measures taken by means of the constant angle we have above explained, however applied, depends on the correctness with which this angle is determined in the identical achromatised prism made use of, it therefore becomes a matter of importance to the practical astronomer, who has occasion to avail himself of the constant angle arising from double refraction, to be able to determine its quantity in the most satisfactory manner. During our investigation of this interesting subject, four different methods have occurred to us of ascertaining the constant angle of a prism of rock crystal achromatised in the way that has been described, which agree with one another in a remarkable manner, as will be seen from a comparison of the respective results.

3. *First method*.—When a card is fixed vertically against a wall and viewed through a prism of double refraction, the apparent distance between the two images of such card will depend, partly on the magnitude of the constant angle, and partly on the distance at which the eye of the observer is placed from the card, for the absolute angle that the object subtends varies inversely as its distance from the eye, while the constant angle of the prism remains un-

altered at all distances, if therefore the eye of the observer approaches or recedes from the card, accordingly as the two images of the card are separated or overlap one another, there will be a certain distance found by trial, where the opposite sides, or ends, of the images, according to the position of the prism, will just come in contact, in which situation the distance between the exact centres of the cards will be equal to the breadth or length of the card, as the case may be, and when the dimensions of the card are known, and the exact distance of its surface from the eye, the angle subtended by it at the eye is easily computed by a case in plane trigonometry. One condition is, that the eye be situated in the same horizontal line with the central point of the vertical card, so as to be at right angles to its plane, and another requisite is, that the face of the solid, presented to the card, be exactly parallel to its surface The method of taking the measure correctly when the place of the eye is marked on a suspended plumb-line, or on a fixed vertical staff, will be obvious to every observer. If the card is an exact inch, the distance may be measured in inches and decimal parts, but if a sheet of paper is cut to an exact foot, the measure may be taken in feet and decimal parts, for in either case, the computation will be equally easy Indeed we have already stated [§ XX. 8] that when a yard is the denomination of measure, the angle subtended by it, at any distance, may be immediately obtained from the logarithmic arithmetical complement of that distance, which is always equal to the logarithmic sine of the subtended angle, which in our case is the constant angle required.

4. Indeed our table, given in the Section just referred to, though computed for yards, will give the angle corresponding to feet, or inches, by inspection equally well, provided the measure of the object be taken as *unity*, and its distance be measured in numbers of the same denomination When inches are chosen, the constant angle of a prism may generally be taken in a good sized room, but if feet or yards be preferred, the measure of distance must necessarily be on level ground.

5 We will illustrate this method by four examples afforded us by the four prisms prepared for us by Dollond, for the express purpose of being applied to astronomical observations When an oblong card, just an inch wide, was made fast to a vertical black ground painted on a board, the distances at which the images of the card, made by the respective doubly refracting bodies, were laterally in exact contact, and the corresponding angles were as follow, viz

Prisms used	Distances in Inches	Logarithms of distances	Log Ar Co = the Sines	Const Angles determined
1	248 .5	2 .3953264	7 .6046736	13′ 50″
2	155 .	2 .1903317	7 .8096683	22 11
3	118 .	2 .0718820	7 .9281180	29 8
4	87 .1	1 .9400182	8 .0599818	39 28

If we examine our Table in Section XX, we shall find the distances in the columns, with the corresponding minutes at the side, and seconds at the head, exactly as above computed. The last distance 87 .1 is smaller than any that appears in the table, but we shall find 174 .2, the double of this number, pointing out 19′ 44″ as being half the required angle.

6 Monsieur Arago was probably the first astronomer who successfully applied prisms of rock crystal to the eye piece of a telescope for the measurement of very small angular subtenses his plan, we understood him to say, was to use the prisms placed obliquely, to increase the constant angles due to them, and to determine their respective values, thus increased, by means of concentric circles, placed vertically at a measured distance from his eye, when looking through the prism; for, as he knew the diameters of each circle, he could generally find one out of the number, which would come into exact contact with its image, and thus gain the value of the constant angle

7. In all measures taken by the unassisted eye, the coincidence of the opposite edges of contiguous images of a distant object, of whatever shape, may be expected to be liable to erroneous estimation in such degree, as may render computations founded on the approximate measures more or less doubtful; we will therefore proceed to show how the magnifying power of the telescope may be resorted to, in aid of the natural power of the human eye.

8. *Second method.*—When a prism of double refraction is placed on the exterior face of the object-glass of an ordinary sized telescope, in a way that will exclude all the light except what enters the prism, which may be done by means of a small hole in the cap, just large enough to receive the cell of the prism, a white staff or other light coloured body will be seen double, if not placed at too great a distance from the telescope, and the distance between the two erect images of such a staff, placed vertically, will be the subtense of the constant angle of the prism, in the same manner as if the staff were viewed through the prism by the naked eye. This circumstance affords the ready and accurate means of measuring the distance between the two images of the staff by a spider's line micrometer, used as the eye-piece of the telescope. The method has been explained at considerable length, in one of the earlier papers contained in the first volume of the Memoirs of the Astronomical Society of London, which indeed was the first paper read before the Society as a body, but in that explanation no notice was taken of the change of the micrometer's *scale,* which sometimes takes place in consequence of the shortening of the focal distance of the telescope, which change, being produced by surfaces not perfectly plane, will not be the same with different prisms. We propose to determine the constant angles of the same four prisms by the aid of a telescope, and at the same time to show how the effects of an alteration in the micrometer's scale may be avoided.

9. There is a coping stone painted white, eight feet long, on the pier which holds the two meridian marks, about 320 yards to the north of South Kilworth Observatory, which stone it was judged would be suitable for the subtense of an angle to be measured from either of the pillars, which carry the transit instrument and the circle respectively, as being equally distant from both The angle subtended by this stone, as measured by the azimuth circle, was 28′ 48″, and when measured by a spider's-line micrometer attached to a 43 2 inch telescope placed on the transit pillar, 37 .82 turns, of 46″ 12 each, gave 29′ 4′, from which, if we deduct 16″, the proper correction for the small distance, we shall have for this measure also 28′ 48″, so that the measure of the angle subtended by the length of the white stone may be considered as correct. In the next place the four prisms, which had been put into circular cells of equal diameters, were fitted into a hole in the centre of the cap, which covers the aperture of the said telescope; and when the eye-piece was pushed in, to obtain distinct vision with each prism in succession, the number of revolutions of the micrometer's screw which the stone now measured

were put down, and also the number corresponding to each measure of the constant angle, or separation of the images of the same end of the stone, agreeably to the subjoined register viz.

Prisms	Entire stone	Constant angle
1	35 .25	17 .00 Revolutions.
2	35 20	27 .15 do
3	34 .94	35 .45 do.
4	34 .77	47 .77 do.

10 Now as we knew that the true measure of the stone was 1728″, and as we were aware, that the scale of the micrometer might be changed from the solar value, we no longer used the tabular quantities belonging to the telescope here employed, but by direct proportion made the constant angles of the prisms to be in the same ratio to the revolutions of the micrometer measuring those angles, that the true measure of the stone had to the number of revolutions which respectively measured it, at the different positions for the different prisms; for the first prism shortened the solar focus about 2¼ inches, the second about 2½, and the third and fourth nearly 3 inches, so that the same scale, however modified, would not apply to the revolutions belonging to the different prisms. The work when abridged will stand thus; viz.

	REVOL	STONE	REVOL	CONST ANGLES
Prism 1.	As 35 .25	1728″	17 .00	13′ 53″.4
2.	35 .20	1728	27 .15	22 12 8
3.	34 .95	1728	35 .45	29 12 .7
4.	34 .77	1728	47 .77	39 34 .0

This determination of the constant angles of the four prisms accords in a surprising manner with the results of the preceding method, and as the magnifying power of the telescope with Troughton's micrometer is upwards of 44 times, we must feel disposed to give a preference to the latter determination

11. *Third method.*—If the telescope could have been shortened sufficiently to admit of distinct vision of the sun or other heavenly body, when the prisms were applied before the object glass, the two images of such body would probably have been preferable to the images of the white stone which formed the standard of the altered scale of measures, in the preceding method of gaining the constant angles of the four prisms, but this could not be done with the telescope which had its cap adapted to receive these prisms. Another arrangement however occurred, which admitted of the solar focus of the telescope being used at its usual length of 43 .2 inches, and also of the corresponding tabular measures being taken at once without alteration, from the Table previously constructed, we shall describe this as a third method, which is equally accurate with the second, and in most situations much more convenient in practice

12. A dial-plate of 2¾ inches in diameter, with a white enamelled ground, but without circles or figures, was procured as a disc to be attached to the top of a staff, and to be placed at any distance which circumstances might require; and when the said telescope was adjusted to

its solar focus, by viewing the sun, the thickest prism was inserted into the cap, and placed before the object-glass, and a near station was found by trial such, that the elongation of the telescope, occasioned by the smallness of the distance, was an exact compensation for the shortening occasioned by the refraction of the prism, this was known to be the case, when the images of the disc were well defined at the solar adjustment for vision of the telescope This compensation took place at the distance of 46 feet only from the object-glass of the telescope, with all the four prisms, or very nearly so, and the head of the small screw, which fixed the dial or disc to the staff, being blackened, formed such a contrast with the white ground of the enamel, as to become an object sufficiently well defined to admit of being bisected by the spider's lines. The extreme edges of the image formed by the rays of ordinary refraction, were also well defined, but the edges of the image, formed by the rays of extraordinary refraction, were always tinged with yellow on the side next to the other image, and with violet at the remote edge, so that a measure taken from either of its edges could not well be depended upon, though it was always tried as affording a verification, while the lines remained at the separation, which measured the distance from centre to centre of the two discs The measures turned out to be most satisfactory, as will appear from the annexed statement of results, which were examined and acquiesced in by two separate observers, before the corresponding tabular quantities were extracted from the Table appropriated to the micrometer, when used with the said telescope.

Prism 1. . .	18.0	revolutions =	13′	50″.16	§ XIX. 6
2 . . .	28.86	do.	22	11.66	
3. . . .	37 90	do.	29	7.94	
4. . . .	51.44	do.	39	32.32	

13. When a prism is applied before the object-glass of a telescope, as was done in the two last methods, the observer must be careful not to admit any extraneous light at the hole usually made at the side of the main tube for illumination, for otherwise the images, occasioned by the prism, will be very faint and imperfectly defined, which circumstance was discovered by accident, while a second observer was walking round the telescope, and in one particular situation intercepted the light, which had previously occasioned great difficulty in getting good measures

14 *Fourth method.*—If we place a prism of double refraction upon the surface of the first lens of the positive eye-piece of a spider's-line micrometer, and look through it, each line in the micrometer will appear double, and when the prism is turned round upon its face, a position will be found where the separation is a maximum: if, while in this position, the screw of the micrometer be turned, to separate its two coincident lines, supposed to be both at zero, four lines will now appear, and a further opening by the screw will lay the image of the line formed by the extraordinary refraction over the next contiguous line seen by means of the ordinary refraction, and the two lines so superposed will form a single line, much *blacker* than either of the remaining lines, to the right and left of it. In this situation the micrometer's screw measures the distance between either of the lines and its image, in parts of a revolution, the value of which we will call c, when the micrometer is used with a given telescope, then, as the constant angle of a prism is undiminished when in contact with the object-glass of the telescope, if we call it C, and put F and f for the focal distances of the object-glass and eye-piece, we shall have the following analogy

As $f : F :: c : C$

therefore $Fc = fC$, and, by transposition, $\frac{Fc}{f} = C$,

but [§ XI 1.] $\frac{F}{f} = P$, the magnifying power of the telescope;

and consequently $Pc = C$, the constant angle required.

For instance, when our prism 1 was placed on the eye-piece of Troughton's micrometer, and held over an inclined mirror which illuminated the spider's lines, the formation of the strong black line was at 0 41 of the micrometer's head, taken on both sides of zero, and since the tabular value of a revolution with telescope No. 2 is 46″.12 [§ XIX. 6.], we have $c = 0.41 \times 46''.12 = 18''\,9092$, then the magnifying power of this telescope, with the same micrometer, is 44.23, as determined by a good dynameter; hence $18''.9092 \times 44.23 = 13'\ 56''.35 = C$ is the constant angle required. The following Table contains the data and constant angles deduced therefrom, of all the four prisms.

15.

Prisms.	Micrometer	Tabular Value of Tel 2	c	P	C
1	0.41	46″.12	18″.9092	44.23	13′ 56″.35
2	0.65	46.12	29.9780	44.23	22 5.93
3	0 86	46.12	39.6632	44.23	29 14.30
4	1.16	46.12	53.4992	44.23	39 26.27

16 If we multiply the proper tabular value of Troughton's micrometer by the magnifying power of any telescope to which it may be applied, the constant product will be 2040″, or very nearly so, and the operation of determining the constant angle of any prism may be abridged, by multiplying the numbers contained in the second column respectively by this constant product only, according to the following Table viz.

1	2040″ × .41	13′ 56″.4
2	2040 × .65	22 6.0
3	2040 × .86	29 14 4
4	2040 × 1 16	39 26.4

17 AVERAGE OF THE FOUR METHODS.

	Prism 1	Prism 2	Prism 3	Prism 4.
First method ..	13′ 50″.0	22′ 11″.0	29′ 8″ 0	39′ 28″.0
Second do ...	13 53 4	22 12.8	29 12.7	39 34.0
Third do ...	13 50 2	22 11.7	29 7.9	39 32.3
Fourth do. .	13 56 3	22 5.9	29 14.3	39 26.3
Average...... .	13 52.5	22 10 4	29 10.7	39 30.2

These last are the constant angles which we have used in the construction of the four Tables inserted in our last Section, and also of the Tables which appear in a subsequent Section, for the use of our ocular crystal micrometer, and as the measure taken by a prism is always small, there can be no perceptible error arising from a fractional part of the whole angle thus determined

18. When the constant angle of a prism is once well determined, and also the value of c with any given telescope, we have the means of knowing the magnifying power without a dynameter for if C be equal to $P\ c$, we have $\frac{C}{c} = P$, and this will be the case with any prism when C and c are both known For instance, with respect to the four prisms under our examination, if we divide the constant angles contained in the sixth column, C, of our Table (15.) by the numbers under the fourth column, c, respectively, each quotient will separately give the power $P = 44\ 23$, which was determined by the dynameter

If we were to determine the magnifying power of our telescope of 76.25 inches focal length (No 5) in the same way, the tabular value of a revolution of Troughton's micrometer, when applied to it, is 26″.1 [§ XIX. 6], and we should have the following results

Prisms	Micrometer	c	C	$\frac{C}{c} = P$
1	0.41	10″.701	13′ 52″.5	78. nearly
2	0.65	16.965	22 10 4	78.4
3	0.86	22.446	29 10.7	78. nearly
4	1.16	30.276	39 30.2	78.2

19. When the prisms of an achromatic crystal of double refraction are thick, and have consequently a large refracting angle, it is necessary that the two prisms which compose the solid, should be of equal thickness, and have their faces perfectly flat, otherwise the solid will not be achromatic, nor the constant angle the same, when the solid is turned half round; neither will it remain unaltered when the faces of the solid are reversed, unless the constant angle is a compound where both the prisms have double refraction, and in this case the sections must be made in the proper planes precisely, or there will be several secondary images. When the measures of a certain thick crystal, formed of two prisms, of which one only has the doubly refracting property, were taken according to our third method, it was found that all the four, taken in the reversed and inverted positions, were a little different from one another, leaving a doubt undetermined, which of the angular measures ought to be considered the proper *constant angle* to be used in astronomical observations. The only way in which the prism in question can be used, with any certainty of giving accurate results, is, to have the four positions marked, and as many Tables computed, to correspond to these positions; the measures were as follow, when the cell had been marked with a dot of ink on one side viz.

Direct position .. with dot up........ 71.20 revolutions of micrometer =3284″
with dot down ..... 73 70 ditto ... . =3366
Reversed position with dot up. .... 69.30 ditto .... =3196
with dot down .... 73 20 ditto .. . . =3374
Mean constant angle in the direct position .. .. =3325
Do. .... in the reversed position .... =3285
Mean of the four positions .... =3305

20. The measures of the same thick prism, taken by our fourth method, differ also, not only among themselves, in the respective positions, but as they regard the measures taken by our third method, thereby increasing the uncertainty of gaining any good results from any of the constant angles of this crystal, that may be adopted with reference to any given position The subjoined are the measures of the constant angle of the same thick prism, resulting from our fourth method.

Direct position... with dot up......... 1.62 × 2040″ = 3305″
with dot down ..... 1.70 × 2040 = 3468
Reversed position with dot up. .... 1.63 × 2040 = 3325
with dot down .... 1.72 × 2040 = 3509
Mean constant angle in the direct position .... = 3315
Do. .... in the reversed position .... = 3488
Mean of the four positions............... .... = 3402

21. The measures, by both these methods, were taken with as much care, as the measures from which the constant angles of the preceding four prisms were determined, and it appears that there is a difference of 97″ in the mean angles, arising from an average of all the four positions, when the third and fourth methods, which we consider the most convenient, have their results compared. The discrepance points out the propriety of examining the properties of a prism of double refraction carefully, before it is fixed upon to constitute the part of an instrument for determining micrometrical measurements.

22. When the constant angle of a wedge of glass, or of a pair of wedges, is to be determined by applying the cell containing them to the diminished aperture of a telescope, the focal length of the telescope will not be affected by such application: the angles may therefore be determined by any object placed at a known distance, according to our *second method*, by means of a spider's-line micrometer applied to the measurement of the distance between the centres of the two images, and the values thus determined may afterwards be converted into the solar values by $\frac{f}{f''}$, as we have before explained [§ XX. 5]. To exemplify this method, our telescope of 43.2 inches focal length (No. 2) was adjusted to distinct vision of the painted coping-stone, containing the meridian marks, at the distance of 960 feet, and when the wedges *a*, *b*, and *a* + *b*, specified in our thirty-sixth section (6.), were successively inserted into the central hole made in the cap covering the object-glass, the following measures were obtained of the distances between the two images of one of the extremities of the said stone.

WEDGE *a*	WEDGE *b*	WEDGES *a* + *b*
Revol 34. = 26′ 8″.08 .28 = 12.90	Revol 55. = 42′ 16″.60 .23 = 10.60	Revol 50. = 38′ 26″.00 39 = 29 58.68 .51 23.51
Sum. . = 26 20.98	Sum .. = 42 27.20	Sum... = 68 48.19

The constant angle of the double prism *a* + *b* was found too large to be contained in the field of view of the telescope, and therefore marks were made on the stone, which afforded the means of measuring the distance between the double images at two operations, as stated in the third column. The accuracy of the measures was proved by the circumstance, that the sum of the measures of the constant angles of the wedges *a* and *b*, gives the same amount, as the measure taken of the wedge *a* + *b* in a separate determination.

23. When the constant angles thus determined were respectively divided by 76.25, the focal length of our telescope (No. 5), the quotients afforded the measures per inch of the scale applied on Dr. Maskelyne's principle, viz. for wedge *a* 20″.74, for wedge *b* 33″.4 and for *a* + *b* 54″.14, which numbers form the basis of our Table, given at the end of our Section XXXVI, but this Table is not adapted for giving accurate celestial measures, till it has been rectified by our formula $\frac{f}{f'}$, which we will now proceed to explain, by converting the said Table into a Table suitable for giving celestial measures. By our formula $\frac{f^2}{d-f} = e$, we find that the elongation of the telescope, when adjusted to the mark, was 1.63, and when we have substituted f' for $f + e$, we shall have $\frac{f}{f'} = \frac{43.2}{44.83}$ for finding the true constant angles 1523″.5, 2454″, and 3977″.5, and also for converting the Table for terrestrial measures into a Table of celestial arcs, as here annexed.

24.

A TABLE

OF THE CELESTIAL VALUES OF THREE CUNEIFORM MICROMETERS USED WITH A TELESCOPE OF 76.25 INCHES SOLAR FOCAL LENGTH

First Wedge *a*				Second Wedge *b*				Two Wedges *a* + *b*			
Scale	Inches	Tenths	Hundredths	Scale	Inches	Tenths	Hundredths	Scale	Inches	Tenths	Hundredths
1	0 10″ 98	2″ 00	0 20	1	0 32′ 18	3″ 22	0″ 32	1	0 52″ 16	5″ 22	0″ 52
2	0 39 96	4 00	0 40	2	1 4 36	6 44	0 64	2	1 44 32	10 43	1 01
3	0 59 94	5 99	0 60	3	1 36 54	9 65	0 96	3	2 36 48	15 65	1 56
4	1 19 92	7 99	0 80	4	2 8 72	12 87	1 29	4	3 28 64	20 86	2.09
5	1 39 90	9 99	1 00	5	2 40 90	16 09	1 61	5	4 20 80	26 08	2 61
6	1 59 88	11 99	1 20	6	3 13 08	19 31	1 93	6	5 12 96	31 30	3 13
7	2 19 86	13 99	1 40	7	3 45 26	22 53	2 26	7	6 5 12	36 51	3 65
8	2 39 84	15 98	1 60	8	4 17 44	25 74	2 57	8	6 57 28	41 73	4 17
9	2 59 82	17 98	1 80	9	4 49 62	28 96	2 90	9	7 49 44	46 91	4 69
10	3 19 80	19 98	2 00	10	5 21 80	32 18	3 22	10	8 41 60	52 16	5 22

In all *celestial* observations this Table must be substituted for the Table given at page 201 of this volume.

25. As an exemplification of the use of this Table, we will take the measures of the longer diameter of Jupiter, taken by the three cuneiform micrometers, on the 2d of May, 1826, at the same time that the prisms of crystal were used for the same purpose; which measures, with their respective tabular values, will stand thus

Wedge $a+b$ measured the disc at 0 .78 . . . . = 40″.68
b . . . . 1 .32 . . . . . = 39 .26
a . . . . 2 015 . . . . = 40 .26

Mean of the whole . . . . 40 .03

On turning back to page 209, we shall find that the same diameter, taken by an average of four prisms, on Rochon's principle, on the same evening, was 39″.30, in which deductions the difference is less than three quarters of a second.

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§ XXXIX THE OCULAR CRYSTAL MICROMETER [PLATE III]

1. After having shown how Rochon applied the *constant angle* of an achromatic prism of rock crystal, situated between the object-glass and eye-piece of a telescope, as a micrometer with a long scale, and having explained the different methods of determining this angle with sufficient precision, we are now prepared to convey to our readers an idea, how any similar but smaller prism may be employed between the eye and the eye-piece of a refracting, Newtonian, or Herschelian telescope, for measuring small angles with as much correctness as the powers of the human eye will admit of We have had occasion [§ XXXVIII. 14.] to show, that, when a prism of double refraction is placed contiguous to the exterior face of the first lens of the eye-piece of a spider's-line micrometer, the apparent distance between one of its lines and the image of that line, may be measured in the field of view, when the micrometer is applied to a telescope having a Table of values appropriated to it. This value with any given prism we called c, and the constant angle C, and when the known magnifying power of the telescope is designated by P, it has been seen that $P\,c = C$, and consequently $\frac{C}{P} = c$. The principle of this micrometer is, to determine the value of c, or measure of an object at any time, by its equivalent $\frac{C}{P}$, found by means of the prism alone, when the spider's-line micrometer is not used. As the value of C is constant, we have only to apply an eye-piece with variable powers, such as we described in our sixth section, or as that in the ninth paragraph of our seventh, to find P at all times; the former of which eye pieces gives the object inverted, and the latter erect, but as the celestial construction employs fewer lenses, and consequently has more light, we will confine our description principally to it, as being preferable, as well as more convenient to use

2 If our reader has not perused our sixth section with attention, we must refer him back to it for a description of the celestial eye piece with variable powers, to the use of which we shall now have occasion to introduce him We have there stated that figures 13 and 14 of Plate III explain the most simple construction of an eye-piece possessed of various powers depending simply on the separation of its two lenses, but the eye-piece represented by figure 15, and in section by figure 16, with a portion detached in fig. 17, is an improved one, which includes a graduated circle and vernier, shown separately in figures 18 and 19, for giving

angles of position, which the line of direction of the measure makes with any given line. In the improved construction, *a* is the second or constant lens, which slides by the rack-work *c*, and has a single very fine diametrical line, cut by a diamond, which is capable of being adjusted into a horizontal or vertical position, by turning round the cell that holds it; *b* is the eye lens, which may be changed for another of longer or shorter focal length, to give a new scale of tabulated magnifying powers; of these there may be three or four successively screwing into the same place; *d* is the graduated circle, which may be screwed to a circular shoulder near the eye-lens, when required, or may be displaced if not wanted; *e e* are two pins entering the vernier-piece, seen detached in fig. 17, which serve to take hold of in turning the vernier, which has a free motion round a circular shoulder, formed on the circle; *g* is the brass cell which holds the prism in all the figures, and at first was made to screw into the end of the eye-piece, but it has since been found more convenient to make it slide into a plain piece of tube made fast to the vernier-piece, when the circle is used, or to the eye-piece when the circle is omitted. This method affords facility, and saves time in changing the prisms, if, during the observation, the angle is found too large or too small for a given prism, which happens to be applied. We have seen an eccentric circular holder of brass, made by Mr. Dollond, which holds four different prisms, and by turning it round the centre of motion, it will present any one of the four prisms in succession, without their being displaced, in the way a compound microscope has sometimes several magnifying lenses successively applied. When this mode of fixing the prisms is used, there ought to be an adjustment for allowing the prisms to turn round, when the index of the circle is at zero, that the horizontal line on the second lens and its image may coincide in that position. We prefer slipping in the cell, holding the prism, in the way represented in figure 19, where *g*, having no screw, is at liberty to turn round for this adjustment, and can also be readily removed and changed for another.

3. The method of using this variable eye-piece, which, in conjunction with a prism of double refraction, constitutes the *ocular crystal micrometer*, will require some explanation. When the micrometer has been screwed, without its prism, into the small tube of a telescope, in the place of an ordinary eye-piece, till it is at home, the nut *c* must be turned till the line on the face of the second lens, *a*, is clearly seen, and if it be vertical, as compared with a plumb-line, seen by the second eye looking out of the telescope, the position of this lens is right; otherwise it must be turned round a little, by degrees, till it is found to present its diametrical line in a vertical position. This adjustment however is of no importance, if an angle of *position* is not intended to be measured. In the next place, turn the telescope to the object to be measured, and obtain distinct vision by its proper screw, and put zero of the vernier to 90° on the circle, in which case 0° will lie in the horizontal line, provided the figures run 10, 20, 30, &c. on each side of zero; insert now one of the prisms into the holder of the vernier-piece, and turn it round separately, till the diametrical line on the lens *a*, and its image, form one single line, the zero of the vernier still remaining at 90° of the circle; the identical points in this vertical line will not however coincide with their images, though they appear to do so, for in this position of the eye-piece, of the vernier, and prism, a single star will appear double, and a line joining the two will be vertical. When the vernier now is gradually moved, by the pins *e e*, along the circle's limb towards either 90°, the line joining the star, and its image, will form corresponding angles with the vertical, the complements of which will be the

angles made with the horizontal line, and as the figures go backwards from 90 to 80, 70, 60, &c. it is the latter angle which is always indicated by the vernier when a quadrant is thus passed over, the star and its image will appear in a horizontal line, to which the diametrical line of lens *a* might have been adjusted, but that a vertical stroke is more easy to verify, by comparison with a plumb-line.

4. When we spoke of the adjustment of the second lens, and the corresponding position of the prism, to render the diametrical stroke and its image a single line, we supposed the telescope placed in the meridian of the place, and elevated in its plane, when the single star was supposed to be seen double, and this is the situation which a telescope must occupy, when it is not mounted on an equatorial stand, and duly adjusted for the latitude, in order to measure angles of position. Let us now suppose a *double* star passing the telescope in its adjusted position, here we shall have a pair of double stars before us, and if the vernier stands at 90°, as in the first instance, the lines joining each separate pair will both be vertical, and the four luminous points will form the angular points of an oblong four-sided figure, extending horizontally if the prism has too great a constant angle, or vertically if its angle is too small, but if the figure is nearly a square, it may be made exactly so, as near as the eye can guess, by moving the lens *a* by the micrometer's rack, and adjusting again for vision by the telescope's rack, then this position of the vernier will give the argument for an *approximate* measure, depending on the tabulated corresponding power of the telescope used But, to render the observation perfect, the vernier must now be moved gradually round the limb of the circle, with its included prism, till the two pairs of stars are seen in an exact straight line, in which situation it will indicate the complement of the angle made with the vertical line, which is the distance moved over from the first position; and if the sides of the square were truly estimated to be equal to each other, three stars only will now appear, because the right hand star will coincide with the image of the left hand one; but if this is not exactly the case, a further alteration of the place of lens *a* will make the coincidence perfect, when good vision is restored When the two stars which compose the double one are nearly of an equal magnitude like the stars of Castor, the perfect coincidence can be judged of in a state of superposition; but when one of them is considerably smaller than the other, as in the case of Polaris, a small motion given to the vernier alternately to the right and left, will show whether the small one transits the centre of the large one or not, when it does, the *distance* and *position* are both truly obtained at the same operation, the tabular argument of the former will be indicated on the scale of the tube, and the vernier will give the true position of the latter without tabulation. In observations where there is a great difference of the apparent magnitudes of the two stars, the position may be read at the ingress and egress of the smaller star into and from the disc of the large one, and the middle point will be the angular measure of the central position. This method may be observed in all cases where the distance between two given points is required to be measured, and the tables which we have computed will serve to illustrate it by suitable examples.

5 When the length or breadth of any small surface, such as the diameter of a planet, or of a lunar spot, is required to be measured, it must be taken for granted, that both the images viewed are precisely of the same magnitude, which they must necessarily be, from both becoming distinctly visible at the same focal distance: in this case the distance between the central points of the two images is always equal to the length or breadth of the object mea-

sured, accordingly as the contrary ends or sides are brought into exact contact. This is obvio in a circle which has two images touching one another at their circumferences; the distan between their centres is evidently the sum of their radii, and as they are equal to each oth this distance must be the measure of a diameter of either of them. When therefore there is space between the two images of a planet or other body, when the two lenses are brought in contact by the rack *c*, where the magnifying power is a maximum, the prism has too large constant angle, and a prism with a smaller angle must necessarily be substituted, but, on t contrary, when the limbs of the body overlap with the smallest power, or when the lenses a removed to their greatest distance from each other, a prism with a larger constant angle mu be applied. Hence arises the necessity of having a set of prisms with different constant angl When the lenses are at their mean distance from each other, and the images are overlappi or separating only by a small quantity, in general this quantity can be reduced to nothing, b a corresponding alteration in the magnifying power; for an increase of this power, by enlargi the discs, lessens the apparent distance between their limbs, and the contrary, till an exa contact is obtained.

6. This micrometer has two important advantages, common indeed to all double-imag micrometers, first, that it requires no illumination, and secondly, that an object in motio may have its dimensions ascertained in any direction, as well as if it were at rest. It mu however be acknowledged, that it has, like most others, some disadvantages, the change magnifying power during the observation requires alternate adjustments for power and fo distinct vision; the scale of powers must be tabulated for the same eye that used the dynam ter for their determination, and as the eye alters by age, the table may require some little co rection on this account. When compared with Rochon's micrometer, this mode of applyin the prism of double refraction makes more perfect images, which is material in making corre measurements; because in his mode of application the converging rays of light, which are tendin to form a good single image, pass through an imperfect material, when they pass through th prism, before the images are formed, and the edges of the images are frequently less perfectl defined, than when a perfect image, made by a good telescope, is afterwards rendered doubl by a subsequent passage through the prism; as is the case in the use of this micromete Otherwise Rochon's principle requires no change of magnifying power for obtaining th measure, and the scale is co-extensive with the focal length of the telescope. In both con structions the double images are seen in every part of the field of view, which renders the use pleasant, though the measure is most correct when taken as near as may be to the centr of the field of view, to avoid the effect of oblique incidence, which will separate contiguo images a little, when they are viewed at either extremity of the field. Another advantage whic this micrometer has over Rochon's is, that the accuracy of a small angle measured does no depend on any adjustment for zero of the scale, and therefore is free from the influence of a index error.

7. As this micrometer has not yet been brought into general use, and is probably bu little understood, though several, we learn, have lately been constructed, we shall give th tabular values of two sets of prisms which we have obtained, one French and the oth English, to be used with two separate telescopes, of which we have also computed the variab powers due to the respective variable eye-pieces which we have described, as contained in ou third plate. The shorter object glass is that which is now applied to Troughton's three-fe

altitude and azimuth circle, and the longer one is the one we have denoted by the numeral 5, in some of the preceding tables.

8 The method of using this micrometer practically will be best explained by a reference to the register, which we have found it convenient to adopt, according to the subjoined plan, in which are recorded actual observations, reduced by the following Tables respectively.

A REGISTER

OF OBSERVATIONS MADE BY THE OCULAR CRYSTAL MICROMETER WITH THE FRENCH PRISMS

Date	Object	Telescope	Eye lens	Distance of Lenses	Prism	Power	Measure	Remarks
1822.								PLAIN SLIDING MICROMETER
Feb. 21	Mars	44 .5	2	48	5	96 .5	13″.57	Good measure of the vertical diameter.
26	Jupiter	44 .5	2	100	7	74 .7	31 .45	Diameter in the direction of the belts.
	Ditto	44 .5	2	80	7	83 .1	28 .27	Shorter or vertical diameter.
	Saturn's Ring	44 .5	2	100	7	74 .7	31 .45	Longer diameter of the ring.
		44 .5	2	86	5	80 .5	16 .28	Shorter diameter of ditto.
	Saturn	44 .5	2	60	5	91 .5	14 .32	Body of Saturn.
28	Mars	44 .5	2	40	5	99 .9	13 .12	The object-glass damp.
	Ditto	44 .5	2	40	5	99 .9	13 .12	The object-glass wiped: vision good.

OBSERVATIONS MADE WITH THE ENGLISH PRISMS.

Date	Object	Telescope	Eye lens	Distance of Lenses	Prism	Power.	Measure.	Remarks
1826.								MICROMETER WITH A CIRCLE AND RACK.
March 20	Jupiter	44 .5	3	19	4	53 .5	44 .30	Long diameter.
30	Ditto	44 .5	3	17	4	53 .8	44 06	Ditto
April 16	Ditto	44 .5	3	89 .5	3	41 .5	42 .20	Ditto.
	Ditto	44 .5	2	144	4	56 .2	42 .27	Ditto.
24	Ditto	44 .5	3	82	3	42 .8	41 .02	Ditto.
	Ditto	44 .5	2	140	4	57 .8	41 01	Ditto.
	Ditto	44 .5	3	58 .5	3	46 .8	37 .42	Short diameter.
	Ditto	44 .5	4	127	4	63 .3	37 .45	Ditto.
	Mars	76 .25	4	81 .5	2	79 .45	16 .70	Horizontal diameter.
27	Ditto	76 25	4	110	2	72	18 .50	Ditto.
	Ditto	76 .25	4	23 5	3	94 .55	18 .50	Ditto.
May 31	Ditto	76 .25	4	32 5	3	92 .2	18 .98	Ditto.
	Ditto	76 .25	3	66 .6	4	125	18 .96	Ditto.

9. Though we have explained the method of computing the tabular powers of a variable eye-piece in sections VI and XXIII, when the data have been determined by a dynameter, agreeably to the directions given in section XI, yet it may be satisfactory to our readers to be informed, what were the steps of our process, by which we arrived at the tabular numbers constituting the respective series of magnifying powers, which we have computed as specimens of tabulation, that may be adopted for the use of the ocular crystal micrometer, applied to a telescope of any solar focus whatever. For it is as necessary to have the variable magnifying powers of the telescope used correctly assigned, as it is to know the exact constant angles of the prisms made use of in the micrometer. The accuracy of the resulting measure is alike dependent on both. After the explanations we have already given in detail, the following statement will be deemed sufficient.

Telescope 44.5 with *Plain Micrometer*.

Lens 1. $\frac{2.75}{.059 \times 2} = 233 = P$ at 10 on the scale.

$\frac{2.75}{.0865 \times 2} = 159 = P$ at 80 Ditto

Differences 74 70

Then as 70 74 . . 100 105.7, or 1 1.057

Lens 2. $\frac{2.75}{138 \times 2} = 99.9 = P$ at 40

$\frac{2.75}{.238 \times 2} = 57.9 = P$ at 140

Difference . . . . . . 42 in 100

Lens 3 $\frac{2.75}{.25 \times 2} = 55 = P$ at 10

$\frac{2.75}{.3655 \times 2} = 38 = P$ at 110

Difference . . . . . 17 in 100

Telescope 76.25 with *Circular Micrometer*.

Lens 1. $\frac{3.24}{.064 \times 2} = 253 = P$ at 10 on the scale

$\frac{3.24}{.098 \times 2} = 165 = P$ at 110 Ditto

Difference . . . . . 88 in 100

Lens 2. $\frac{3.24}{.093 \times 2} = 174 = P$ at 10

$\frac{3.24}{.1365 \times 2} = 119 = P$ at 110

Difference . . . . . . 55 in 100

Lens 3. $\frac{3.24}{.1075 \times 2} = 151 = P$ at 10

$\frac{3.24}{.154 \times 2} = 105 = P$ at 110

Difference . . . . . . 46 in 100

Lens 4. $\frac{3.24}{.165 \times 2} = 98 = P$ at 10

$\frac{3.24}{.225 \times 2} = 72 = P$ at 110

Difference . . . . . . 26 in 100

From the powers, and their differences in 100 divisions of each scale above determined, the interpolations were made for every successive unit, as contained in the tables. The dividends 2.75, and 3.24 were the apertures made use of, and the divisors are the respective measures taken by a dynameter, having a scale of exact fiftieths of an inch, which must therefore be doubled, to convert them into decimal quantities of an inch. The magnifying powers are taken to the nearest unit, to accommodate the computation of the tables, but are sufficiently near the truth to give exact results, when the power is not very small.

10 The cells containing the glass wedges, exhibited in figures 7, 8, and 9 of our Plate IV, will fit the same eye-holes which hold the prisms, represented by fig. 10 of the same plate, and when thus applied to the vernier-piece of the eye-piece with variable powers, may be used in precisely the same way, and for the same purposes as the crystals of double refraction, when an eye cap with a small hole in the centre has so limited the point of sight, that the pupil of the observer's eye is obliged to be bisected by the side of the wedge which divides the field of view, which position ensures the appearance of double images. We had computed a table for the wedges *a*, *b*, and *a* + *b* when used as *ocular cuneiform* micrometers, but when the magnifying power which effects the measure is known from a table, such as we have given in this section, numbered I and II, it will be sufficient for the purpose to divide by it the proper *constant angle* 1523″ 5, 2454″, or 3977″.5, which are the angles due to the respective wedges, when used in celestial observations, instead of constructing a table.

11. OBSERVATIONS

MADE WITH

THE GLASS WEDGES AND A VARIABLE EYE-PIECE

Date	Object	Telescope	Eye lens	Scale	Table	Wedge	Power	Measure.	Remarks.
1826 May 2	Jupiter	44 5	2	130	II	*b*	62 1	2454″ ÷ 62 1 = 39″ 51	Long diameter
	Jupiter	44 5	2	30	II	*a* + *b*	100 3	3977″ 5 — 100 3 = 39 71	Ditto
	Jupiter	76 25	4	1	I	*a* + *b*	100 4	3977″ 5 — 100 4 = 39 61	Ditto
	Jupiter	76 25	3	119	I	*a* + *b*	100 9	3977″ 5 — 100 9 = 39 42	Mean of the four 39″ 59
Nov 20	Mizar	44 5	1	131	II	*a*	105 1	1525″ 5 — 105 1 = 14 51	Distance between the stars
	Mizar	76 25	2	18	I	*b*	169 0	2454″ — 169 0 = 14 46	Mean of the two 14″ 48

This register will require no explanation further than a comparison of the numbers given in the fifth and eighth columns, with the Tables I. and II. respectively, from which those numbers are extracted. The seven first columns, and the last, are filled up at the time of making the observations, but the eighth and ninth may be filled at any subsequent period. The method of taking the angle of position is the same by the wedges as by the prisms. In one respect the wedges of glass have the advantage over the prisms of crystal; the images are more distinctly defined, and are free from that discolouration which the polarized rays of the extraordinary refraction occasion, but this recommendation is counterbalanced by the circumstance, that the images must be near the straight diametrical edge of the wedge *a* or *b*, or near the line of junction of *a* + *b*, in order to be both visible; if any observer should, on trial, prefer the glass wedges, he may easily compute a table after the model of the following tables III and IV, when the constant angles are previously determined.

TABLE I.

THE MAGNIFYING POWERS OF A VARIABLE CELESTIAL EYE-PIECE,

WITH FOUR SEPARATE EYE LENSES, AND A CIRCLE FOR POSITIONS, USED WITH A TELESCOPE OF 78 25 INCHES FOCAL LENGTH

Scale	Eye-lenses				Scale	Eye-lenses				Scale	Eye-lenses			
	1	2	3	4		1	2	3	4		1	2	3	4
0	261 9	179 5	155 6	100 6	41	225 8	157 0	136 8	90 0	82	189 7	134 4	117 9	79 3
1	261 0	179 0	155 2	100 4	42	224 9	156 4	136 3	89 7	83	188 8	133 9	117 5	79 1
2	260 1	178 4	154 7	100 1	43	224 0	155 9	135 9	89 5	84	187 9	133 3	117 0	78 8
3	259 2	177 9	154 3	99 9	44	223 1	155 3	135 4	89 2	85	187 0	132 8	116 5	78 5
4	258 3	177 3	153 8	99 6	45	222 2	154 8	134 9	88 9	86	186 2	132 2	116 1	78 3
5	257 4	176 8	153 3	99 3	46	221 4	154 2	134 5	88 7	87	185 3	131 7	115 6	78 0
6	256 6	176 2	152 9	99 1	47	220 5	153 7	134 0	88 4	88	184 4	131 1	115 2	77 8
7	255 7	175 7	152 4	98 8	48	219 6	153 1	133 6	88 2	89	183 5	130 6	114 7	77 5
8	254 8	175.1	152 0	98 6	49	218 7	152 6	133 1	87 9	90	182 6	130 0	114 2	77 2
9	253 9	174.6	151 5	98 3	50	217.8	152 0	132 6	87 6	91	181 8	129 5	113 8	77 0
10	253 0	174 0	151 0	98 0	51	217 0	151 5	132 2	87 3	92	180 9	128 9	113 3	76 7
11	252 2	173 5	150 6	97 8	52	216 1	150 9	131 7	87 1	93	180 0	128 4	112 9	76 5
12	251 3	172 9	150 1	97 5	53	215 2	150 4	131 3	86 9	94	179 1	127 8	112 4	76 2
13	250 4	172 4	149 7	97 3	54	214 3	149 8	130 8	86 6	95	178 2	127 3	111 9	75 9
14	249 5	171 8	149 2	97 0	55	213 4	149 3	130 3	86 3	96	177 4	126 7	111 5	75 7
15	248 6	171 3	148 7	96 7	56	212 6	148 7	129 9	86 1	97	176 5	126 2	111 0	75 4
16	247 8	170 7	148 3	96 5	57	211 7	148 2	129 4	85 8	98	175 6	125 6	110 6	75 2
17	246 9	170 2	147 8	96 2	58	210 8	147 6	129 0	85 6	99	174 7	125 1	110 1	74 9
18	246 0	169 6	147 4	96 0	59	209 9	147 1	128 5	85 3	100	173 8	124 5	109 6	74 6
19	245 1	169 1	146 9	95 7	60	209 0	146 5	128 0	85 0	101	172 9	124 0	109 2	74 4
20	244 2	168 5	146 4	95 4	61	208 2	146 0	127 6	84 7	102	172 1	123 4	108 7	74 1
21	243 4	168 0	146 0	95 2	62	207 3	145 4	127 1	84 5	103	171 2	122 9	108 3	73 9
22	242 5	167 4	145 5	94 9	63	206 4	144 9	126 7	84 3	104	170 3	122 3	107 8	73 6
23	241 6	166 9	145 1	94 7	64	205 5	144 3	126 2	84 0	105	169 4	121 8	107 3	73 3
24	240 7	166 3	144 6	94 4	65	204 6	143 8	125 7	83 7	106	168 6	121 2	106 9	73 1
25	239 8	165 8	144 1	94 1	66	203 8	143 2	125 3	83 5	107	167 7	120 7	106 4	72 8
26	239 0	165 2	143 7	93 9	67	202 9	142 7	124 8	83 2	108	166 8	120 1	106 0	72.6
27	238 1	164 7	143 2	93 6	68	202 0	142 1	124 3	83 0	109	165 9	119 6	105 5	72 3
28	237 2	164 1	142 8	93 4	69	201 1	141 6	123 9	82 7	110	165 0	119 0	105 0	72 0
29	236 3	163 6	142 3	93 1	70	200 2	141 0	123 4	82 4	111	164 2	118 5	104 6	71.7
30	235 4	163 0	141 8	92 8	71	199 4	140 5	122 9	82 2	112	163 3	117 9	104 1	71 5
31	234 6	162 5	141 4	92 6	72	198 5	139 9	122 5	81 9	113	162 4	117 4	103 7	71 3
32	233 7	161 9	140 9	92 3	73	197 6	139 4	122 1	81 7	114	161 5	116 8	103 2	71 0
33	232 8	161 4	140 5	92 1	74	196 7	138 8	121 6	81 4	115	160 6	116 3	102 7	70 7
34	231 9	160 8	140 0	91.8	75	195 8	138 3	121 1	81 1	116	159 8	115 7	102 3	70 5
35	231 0	160 3	139 5	91 5	76	195 0	137 7	120 7	80 9	117	158 9	115 2	101 8	70 2
36	230 2	159 7	139 1	91.3	77	194 1	137 2	120.2	80 6	118	158 0	114 6	101 4	70 0
37	229 3	159 2	138 6	91 0	78	193 2	136 6	119 8	80 4	119	157 1	114 1	100 9	69 7
38	228 4	158 6	138 2	90 8	79	192 3	136 1	119 3	80 1	120	156 2	113 5	100 4	69 4
39	227 5	158 1	137 7	90 5	80	191 4	135 5	118 8	79 8	121	155 4	113 0	100 0	69 2
40	226 6	157 5	137 2	90 2	81	190 6	135 0	118 4	79 6	122	154 5	112 4	99 5	68 9

TABLE II.

THE MAGNIFYING POWERS OF A VARIABLE CELESTIAL EYE-PIECE,

WITH THREE SEPARATE EYE LENSES, WITHOUT A RACK, USED WITH A TELESCOPE OF 41.5 INCHES FOCAL LENGTH

Scale	Eye-lenses			Scale	Eye-lenses			Scale	Eye-lenses			Scale	Eye-lenses		
	1	2	3		1	2	3		1	2	3		1	2	3
0	243.6	116.7	56.7	44	197.0	98.2	49.2	88	150.5	79.7	41.8	132	104.0	61.2	34.3
1	242.6	116.2	56.5	45	196.0	97.8	49.1	89	149.5	79.3	41.6	133	102.9	60.8	34.1
2	241.5	115.8	56.4	46	194.9	97.3	48.9	90	148.4	78.9	41.4	134	101.9	60.4	33.9
3	240.5	115.4	56.2	47	193.9	96.9	48.7	91	147.4	78.4	41.2	135	100.8	60.0	33.8
4	239.4	115.0	56.0	48	192.8	96.5	48.6	92	146.3	78.0	41.1	136	99.8	59.5	33.6
5	238.3	114.6	55.9	49	191.7	96.1	48.4	93	145.3	77.6	40.9	137	98.7	59.1	33.4
6	237.3	114.1	55.7	50	190.7	95.7	48.2	94	144.2	77.2	40.7	138	97.7	58.7	33.3
7	236.2	113.7	55.5	51	189.6	95.2	48.0	95	143.2	76.8	40.6	139	96.6	58.3	33.1
8	235.1	113.3	55.4	52	188.6	94.8	47.9	96	142.1	76.3	40.4	140	95.6	57.9	32.9
9	234.1	112.9	55.2	53	187.5	94.4	47.7	97	141.1	75.9	40.2	141	94.5	57.4	32.8
10	233.0	112.5	55.0	54	186.5	94.0	47.5	98	140.0	75.5	40.1	142	93.5	57.0	32.7
11	231.9	112.0	54.9	55	185.4	93.6	47.4	99	138.9	75.1	39.9	143	92.4	56.6	32.5
12	230.9	111.6	54.7	56	184.4	93.1	47.2	100	137.9	74.7	39.7	144	91.3	56.2	32.3
13	229.8	111.2	54.5	57	183.3	92.7	47.0	101	136.8	74.2	39.5	145	90.2	55.8	32.1
14	228.8	110.8	54.4	58	182.2	92.3	46.9	102	135.7	73.8	39.4	146	89.2	55.3	31.9
15	227.7	110.4	54.2	59	181.2	91.9	46.7	103	134.7	73.4	39.2	147	88.1	54.9	31.7
16	226.6	109.9	54.0	60	180.1	91.5	46.5	104	133.6	73.0	39.0	148	87.1	54.5	31.6
17	225.6	109.5	53.8	61	179.1	91.0	46.3	105	132.6	72.6	38.9	149	86.0	54.1	31.4
18	224.5	109.1	53.7	62	178.0	90.6	46.2	106	131.5	72.1	38.7	150	85.0	53.7	31.2
19	223.5	108.7	53.5	63	177.0	90.2	46.0	107	130.4	71.7	38.5	151	83.9	53.2	31.0
20	222.4	108.3	53.3	64	175.9	89.8	45.8	108	129.4	71.3	38.4	152	82.8	52.8	30.9
21	221.3	107.8	53.1	65	174.9	89.4	45.7	109	128.3	70.9	38.2	153	81.8	52.4	30.7
22	220.3	107.4	53.0	66	173.8	88.9	45.5	110	127.3	70.5	38.0	154	80.7	52.0	30.5
23	219.2	107.0	52.8	67	172.8	88.5	45.3	111	126.2	70.0	37.8	155	79.7	51.6	30.4
24	218.2	106.6	52.6	68	171.7	88.1	45.2	112	125.1	69.6	37.7	156	78.6	51.1	30.2
25	217.1	106.2	52.5	69	170.6	87.7	45.0	113	124.1	69.2	37.5	157	77.6	50.7	30.0
26	216.0	105.7	52.3	70	169.6	87.3	44.8	114	123.0	68.8	37.3	158	76.5	50.3	29.9
27	215.0	105.3	52.1	71	168.5	86.8	44.6	115	122.0	68.4	37.2	159	75.5	49.9	29.7
28	213.9	104.9	52.0	72	167.5	86.4	44.5	116	120.9	67.9	37.0	160	74.4	49.5	29.5
29	212.9	104.5	51.8	73	166.4	86.0	44.3	117	119.9	67.5	36.8	161	73.4	49.0	29.3
30	211.8	104.1	51.6	74	165.3	85.6	44.1	118	118.8	67.1	36.7	162	72.3	48.6	29.2
31	210.7	103.6	51.4	75	164.2	85.2	44.0	119	117.7	66.7	36.5	163	71.3	48.2	29.0
32	209.7	103.2	51.3	76	163.2	84.7	43.8	120	116.7	66.3	36.3	164	70.2	47.8	28.8
33	208.6	102.8	51.1	77	162.1	84.3	43.6	121	115.6	65.8	36.1	165	69.2	47.4	28.7
34	207.6	102.4	50.9	78	161.1	83.9	43.5	122	114.6	65.4	36.0	166	68.1	46.9	28.6
35	206.5	102.0	50.8	79	160.0	83.5	43.3	123	113.5	65.0	35.8	167	67.1	46.5	28.3
36	205.5	101.5	50.6	80	159.0	83.1	43.1	124	112.5	64.6	35.6	168	66.0	46.1	28.2
37	204.4	101.1	50.4	81	157.9	82.6	42.9	125	111.4	64.2	35.5	169	64.9	45.7	28.0
38	203.4	100.7	50.3	82	156.9	82.2	42.8	126	110.3	63.7	35.3	170	63.9	45.3	27.8
39	202.3	100.3	50.1	83	155.8	81.8	42.6	127	109.3	63.3	35.1	171	62.8	44.9	27.6
40	201.2	99.9	49.9	84	154.8	81.4	42.4	128	108.2	62.9	35.0	172	61.8	44.5	27.4
41	200.2	99.4	49.7	85	153.7	81.0	42.3	129	107.2	62.5	34.8	173	60.7	44.0	27.3
42	199.1	99.0	49.6	86	152.7	80.5	42.1	130	106.1	62.1	34.6	174	59.7	43.6	27.1
43	198.1	98.6	49.4	87	151.6	80.1	41.9	131	105.1	61.6	34.4	175	58.6	43.2	26.9

TABLE III

A GENERAL TABLE OF MEASURES ADAPTED TO EIGHT FRENCH PRISMS, AND TO VARIOUS POWERS OF ANY TELESCOPE

Powers	PRISMS								Powers	PRISMS							
	1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8
	Const angle 178"	Const angle 330'	Const angle 586"	Const angle 906"	Const angle 1310"	Const angle 1440"	Const angle 2349'	Const angle 2596'		Const angle 178"	Const angle 330"	Const angle 586"	Const angle 906'	Const angle 1310"	Const angle 1440'	Const angle 2349"	Const angle 2596'
48	3 71	6 87	12 21	18 88	27 29	30 00	48 91	54 08	103	1 73	3 20	5 69	8 79	12 71	13 98	22 80	25 20
49	3 63	6 73	11 96	18 50	26 74	29 38	47 91	52 98	104	1 71	3 17	5 64	8 71	12 59	13 84	22 58	24 96
50	3 56	6 60	11 72	18 12	26 20	28 80	46 93	51 92	105	1 70	3 14	5 58	8 63	12 47	13 71	22 37	24 72
51	3 ,49	6 47	11 49	17 76	25 69	28 24	46 06	50 91	106	1 68	3 12	5 53	8 55	12 36	13 58	22 16	24 48
52	3 42	6 35	11 27	17 42	25 19	27 69	45 17	49 93	107	1 67	3 08	5 48	8 47	12 24	13 45	21 95	24 26
53	3 36	6 23	11 06	17 10	24 72	27 17	44 32	48 98	108	1 65	3 05	5 43	8 39	12 13	13 33	21 75	24 04
54	3 30	6 11	10 86	16 78	24 26	26 66	43 50	48 08	109	1 63	3 02	5 38	8 31	12 01	13 20	21 55	23 82
55	3 24	6 00	10 67	16 47	23 82	26 19	42 71	47 21	110	1 62	3 00	5 31	8 24	11 91	13 09	21 36	23 60
56	3 18	5 89	10 48	16 18	23 39	25 72	41 95	46 36	111	1 60	2 97	5 29	8 16	11 80	12 97	21 17	23 39
57	3 12	5 79	10 29	15 90	22 98	25 26	41 21	45 54	112	1 59	2 94	5 24	8 09	11,69	12 86	20 98	23 18
58	3 07	5 69	10 10	15 62	22 58	24 82	40 50	44 76	113	1 57	2 91	5 19	8 02	11 59	12 71	20 79	22 97
59	3.01	5 59	9 92	15 35	22 19	24 41	39 82	44 01	114	1 56	2 89	5 15	7 95	11 49	12 63	20 61	22 77
60	2 96	5 50	9 76	15 ,10	21 83	24 00	39 15	43 27	115	1 54	2 87	5 10	7 88	11 39	12 52	20 43	22,57
61	2 91	5 41	9 61	14 86	21 48	23 60	38 53	42 61	116	1 53	2 84	5 05	7 81	11 29	12 41	20 25	22 38
62	2 87	5 32	9 45	14 62	21 13	23 23	37 88	41 87	117	1 51	2 81	5 00	7 74	11 19	12,30	20 08	22,19
63	2 82	5 23	9 30	14 38	20 80	22 86	37 29	41 22	118	1 50	2 79	4 96	7 68	11 09	12 20	19 91	22 01
64	2 78	5 15	9 16	14 15	20 47	22 50	36 73	40 57	119	1 49	2 77	4 92	7 61	11 00	12 10	19 74	21 82
65	2 73	5 07	9 ,02	13 93	20 16	22 15	36 14	39 94	120	1 48	2 75	4 88	7 55	10 92	12 00	19 58	21 64
66	2 69	5 00	8 88	13 72	19 85	21 82	35 59	39 34	121	1 46	2 72	4 84	7 49	10 ,83	11 90	19 42	21 47
67	2 65	4 92	8 75	13 52	19 55	21 50	35 07	38 75	122	1 45	2 70	4 80	7 43	10 74	11 80	19 26	21 30
68	2 62	4 85	8 62	13 32	19 26	21 18	34 55	38 18	123	1 44	2 68	4 76	7 37	10 65	11 71	19 10	21 12
69	2 58	4 78	8 49	13 13	18 98	20 87	34 05	37 63	124	1 43	2 66	4 72	7 31	10 56	11 62	18 94	20 94
70	2 54	4 71	8 37	12 94	18 71	20 57	33 56	37 09	125	1 42	2 64	4 68	7 25	10 48	11 52	18 79	20 77
71	2 50	4 64	8 25	12 76	18 44	20 28	33 09	36 54	126	1 41	2 62	4 65	7 19	10 40	11 43	18 64	20 61
72	2 47	4 58	8 14	12 58	18 19	20 00	32 63	36 02	127	1 ,40	2 59	4 61	7 13	10 31	11 34	18 49	20 45
73	2 43	4 52	8 03	12 41	17 94	19 73	32 18	35 53	128	1 39	2 57	4 58	7 08	10 23	11 25	18 35	20 29
74	2 40	4 46	7 92	12 24	17 70	19 46	31 74	35 07	129	1 38	2 55	4 54	7 02	10 15	11 ,16	18 21	20 13
75	2 37	4 40	7 81	12 08	17 47	19 20	31 32	34 61	130	1 37	2 53	4 51	6 97	10 08	11 ,08	18 07	19 97
76	2 34	4 34	7 71	11 92	17 21	18 95	30 91	34 16	131	1 36	2 51	4 47	6 91	9 99	10 99	17 93	19 82
77	2 31	4 28	7 61	11 76	17 01	18 70	30 51	33 71	132	1 35	2 50	4 44	6 86	9 92	10 91	17 79	19 67
78	2 28	4 23	7 51	11 61	16 79	18 46	30 12	33 28	133	1 34	2 48	4 41	6 81	9 84	10 83	17 66	19 52
79	2 25	4 18	7 42	11 47	16 59	18 22	29 71	32 86	134	1 33	2 46	4 38	6 76	9 77	10 ,75	17 53	19 38
80	2 23	4 12	7 33	11 33	16 38	18 00	29 37	32 45	135	1 32	2 44	4 34	6 71	9 70	10 67	17 40	19 23
81	2 20	4 07	7 23	11 19	16 18	17 77	29 00	32 05	136	1 31	2 42	4 31	6 66	9 63	10 59	17 27	19 09
82	2 17	4 02	7 14	11 05	15 98	17 56	28 64	31 66	137	1 30	2 40	4 28	6 61	9 56	10 51	17 14	18 95
83	2 14	3 97	7 05	10 91	15 77	17 35	28 30	31 28	138	1 29	2 39	4 25	6 56	9 49	10 43	17 02	18 82
84	2 12	3 93	6 98	10 78	15 57	17 14	27 97	30 92	139	1 28	2 37	4 22	6 51	9 42	10 36	16 90	18 68
85	2 09	3 ,88	6 90	10 66	15 40	16 94	27 64	30 55	140	1 27	2 35	4 19	6 47	9 35	10 29	16 78	18 55
86	2 06	3 84	6 82	10 54	15 23	16 74	27 32	30 18	141	1 26	2 33	4 16	6 42	9 28	10 21	16 66	18 41
87	2 04	3 79	6 74	10 41	15 06	16 55	27 00	29 84	142	1 25	2 32	4 13	6 38	9 22	10 14	16 54	18 27
88	2 02	3 75	6 66	10 29	14 89	16 36	26 69	29 50	143	1 24	2 ,30	4 10	6 33	9 15	10 07	16 42	18 14
89	2 ,00	3 70	6 58	10 17	14 72	16 18	26 39	29 17	144	1 23	2 29	4 07	6 29	9 09	10 00	16 31	18 01
90	1 98	3 66	6 51	10 06	14 56	16 00	26 10	28 84	145	1 22	2 27	4 04	6 24	9 03	9 93	16 20	17 88
91	1 95	3 62	6 44	9 91	14 40	15 82	25 82	28 53	146	1 21	2 26	4 01	6 20	8 97	9 87	16 09	17 76
92	1 93	3 59	6 37	9 84	14 24	15 65	25 54	28 22	147	1 21	2 24	3 98	6 16	8 91	9 80	15 98	17 64
93	1 91	3 55	6 30	9 74	14 08	15 48	25 26	27 91	148	1 20	2 23	3 96	6 12	8 85	9 73	15 87	17 53
94	1 89	3 51	6 24	9 64	13 93	15 32	24 99	27 ,61	149	1 ,19	2 21	3 93	6 08	8 79	9 66	15 76	17 42
95	1 87	3 47	6 18	9 54	13 78	15 16	24 73	27 32	150	1 18	2 20	3 90	6 04	8 74	9 60	15 66	17 31
96	1 85	3 43	6 12	9 44	13 64	15 00	24 47	27 04	151	1 17	2 19	3 88	6 00	8 68	9 54	15 55	17 19
97	1 83	3 39	6 05	9 34	13 50	14 84	24 22	26 76	152	1 17	2 17	3 86	5 96	8 62	9 48	15 45	17 07
98	1 81	3 36	5 98	9 25	13 37	14 69	23 97	26 49	153	1 16	2 16	3 83	5 92	8 56	9 41	15 35	16 ,96
99	1 80	3 33	5 92	9 15	13 23	14 54	23 72	26 25	154	1 15	2 14	3 80	5 88	8 50	9 35	15 25	16 85
100	1 78	3 30	5 86	9 06	13 10	14 40	23 47	25 96	155	1 15	2 13	3 78	5 84	8 44	9 29	15 15	16 74
101	1 76	3 26	5 80	8 97	12 97	14 27	23 25	25 ,70	156	1 14	2 11	3 76	5 80	8 38	9 23	15 06	16 64
102	1 74	3 23	5 75	8 88	12 84	14 12	23 03	25 45	157	1 ,14	2 10	3 74	5 76	8 32	9 17	14 96	16 ,53

TABLE IV

A GENERAL TABLE OF MEASURES ADAPTED TO FOUR DIFFERENT ENGLISH PRISMS, AND TO VARIOUS POWERS OF ANY TELESCOPE

Powers	PRISMS				Powers	PRISMS				Powers	PRISMS			
	1	2	3	4		1	2	3	4		1	2	3	4
	Const angle 833″	Const angle 1330′	Const angle 1751″	Const angle 2370″		Const angle 833″	Const angle 1330″	Const angle 1751″	Const angle 2370″		Const angle 833″	Const angle 1330″	Const angle 1751″	Const angle 2370″
40	20 83	33 25	43 78	59 25	97	8 59	13 71	18 05	24 43	180	4 63	7 38	9 73	13 18
41	20 32	32 43	42 71	57 80	98	8 51	13 57	17 87	24 18	185	4 50	7 19	9 47	12 81
42	19 81	31 66	41 70	56 42	99	8 42	13 43	17 69	23 94	190	4 38	7 00	9 22	12 48
43	19 38	30 93	40 73	55 11	100	8 33	13 30	17 51	23 70	195	4 26	6 82	8 98	12 17
44	18 91	30 22	39 80	53 86	101	8 25	13 17	17 34	23 46	200	4 16	6 65	8 75	11 85
45	18 52	29 56	38 92	52 66	102	8 17	13 04	17 17	23 23	205	4 06	6 48	8 54	11 56
46	18 11	28 91	38 07	51 52	103	8 09	12 91	17 00	23 00	210	3 96	6 33	8 34	11 28
47	17 73	28 29	37 26	50 42	104	8 01	12 78	16 84	22 78	215	3 87	6 18	8 15	11 02
48	17 35	27 71	36 48	49 37	105	7 93	12 66	16 68	22 56	220	3 78	6 04	7 96	10 77
49	17 01	27 14	35 74	48 36	106	7 86	12 54	16 52	22 35	225	3 70	5 91	7 78	10 53
50	16 66	26 60	35 02	47 40	107	7 79	12 43	16 37	22 15	230	3 62	5 78	7 61	10 30
51	16 34	26 08	34 33	46 47	108	7 71	12 32	16 22	21 94	235	3 54	5 66	7 45	10 08
52	16 02	25 57	33 62	45 58	109	7 64	12 20	16.07	21 74	240	3 47	5 54	7 29	9 87
53	15 72	25 09	33 04	44 72	110	7 57	12 09	15.92	21 55	245	3 40	5 43	7 15	9 67
54	15.43	24 63	32 43	43 89	111	7 51	11.98	15 78	21 36	250	3 33	5 32	7 01	9 48
55	15 15	24 18	31 84	43 09	112	7 44	11 87	15.64	21 16	255	3 27	5 22	6 87	9 29
56	14 88	23 75	31 27	42 32	113	7.37	11 77	15 50	20 97	260	3 20	5 11	6 73	9 11
57	14 62	23 33	30 72	41 58	114	7 31	11 66	15 36	20 79	265	3 14	5 02	6 61	8 94
58	14 37	22 93	30 19	40 86	115	7 25	11 56	15 23	20 61	270	3 08	4 92	6 49	8 78
59	14 12	22 54	29 68	40 17	116	7 18	11 46	15 10	20 43	275	3 03	4 83	6 37	8 62
60	13 88	22 16	29 17	39 50	117	7 12	11 36	14 97	20 26	280	2 97	4 75	6 25	8 46
61	13 66	21 80	28 70	38 85	118	7 06	11 27	14 84	20 08	285	2 92	4 66	6 14	8 31
62	13 41	21 45	28 24	38 22	119	7 00	11 17	14 72	19 92	290	2 87	4 58	6 04	8 17
63	13 22	21 11	27 79	37 62	120	6 94	11 08	14 59	19 75	295	2 82	4 51	5 94	8 03
64	13 02	20 78	27 36	37 03	121	6 88	10 98	14 47	19 59	300	2 77	4 43	5 84	7 90
65	12 82	20 46	26 94	36 46	122	6 83	10 90	14 35	19 42	305	2 73	4 36	5 74	7 77
66	12 62	20 15	26 53	35 91	123	6 77	10 81	14 24	19 27	310	2 69	4 29	5.65	7 64
67	12 43	19 85	26 13	35 37	124	6 72	10 72	14 12	19 11	315	2 64	4 22	5 56	7 52
68	12 25	19 56	25 75	34 85	125	6 66	10 64	14 01	18 96	320	2 60	4 15	5 47	7 40
69	12 08	19 27	25 38	34 35	126	6 61	10 55	13 90	18 81	325	2 56	4 09	5 39	7 29
70	11 90	18 99	25 01	33 86	127	6 56	10 47	13 79	18 66	330	2 52	4 03	5 31	7 18
71	11 73	18 73	24 66	33 38	128	6.51	10 39	13 68	18 51	335	2 49	3 97	5 23	7 07
72	11 57	18 47	24 32	32 91	129	6 46	10 31	13 58	18 37	340	2 45	3 91	5 15	6 97
73	11 41	18 22	23 98	32 46	130	6 41	10 23	13 47	18 23	345	2 41	3 85	5 08	6 87
74	11 26	17 97	23 66	32 02	131	6 36	10 15	13 37	18 09	350	2 38	3 80	5 00	6.77
75	11 11	17 73	23 35	31 59	132	6 31	10 08	13 27	17 95	355	2 34	3 75	4 93	6 68
76	10 96	17 50	23 04	31 18	133	6 26	10 00	13 17	17 82	360	2 31	3 69	4 86	6 58
77	10 82	17 27	22 74	30 77	134	6 22	9 92	13 07	17 68	365	2 28	3 64	4 79	6 50
78	10 68	17 05	22 46	30 38	135	6 17	9.85	12 97	17 55	370	2 25	3 59	4 73	6 41
79	10 55	16 83	22 17	29 99	136	6 12	9 78	12 88	17 42	375	2 22	3 54	4 67	6 33
80	10 41	16 62	21 89	29 62	137	6 08	9 71	12 78	17 30	380	2 18	3 50	4 61	6 24
81	10 29	16 42	21 62	29 25	138	6 04	9 64	12 69	17 17	385	2 16	3 45	4 55	6 16
82	10 16	16 22	21 36	28 90	139	5 99	9 57	12 60	17 05	390	2 14	3 41	4 49	6 08
83	10 04	16 02	21 10	28 55	140	5 95	9 50	12 51	16 92	395	2 11	3 36	4 43	6 00
84	9 92	15 83	20 85	28 21	142	5 86	9 36	12 33	16 71	400	2 08	3 32	4 38	5 93
85	9 80	15 64	20 61	27 88	144	5 78	9 24	12 16	16 47	410	2 03	3 24	4 27	5 78
86	9 69	15 46	20 37	27 56	146	5 70	9 11	11 99	16 25	420	1 98	3 16	4 17	5 65
87	9 58	15 29	20 13	27 24	148	5 63	8 99	11 83	16 03	430	1 94	3 09	4 07	5 52
88	9 47	15 11	19 90	26 93	150	5 55	8 87	11 68	15 80	440	1 89	3 02	3 98	5 39
89	9 36	14 94	19 68	26.63	152	5 48	8 75	11 52	15 59	450	1 85	2 95	3 89	5 27
90	9 26	14 77	19 46	26 33	154	5 41	8 63	11 38	15 38	460	1 81	2 89	3 81	5 16
91	9 16	14 61	19 25	26 04	156	5 34	8 52	11 23	15 19	470	1 77	2 83	3 73	5.05
92	9 06	14 45	19 04	25 76	158	5 27	8 41	11 08	14 99	480	1 74	2 77	3 65	4 95
93	8 96	14 30	18 83	25 48	160	5 21	8 31	10 94	14 81	490	1 70	2 69	3 57	4 84
94	8 86	14 14	18 63	25 21	165	5 05	8 06	10 61	14 36	500	1 66	2 66	3 50	4 74
95	8 77	14 00	18 44	24 95	170	4 90	7.82	10 30	13 94	510	1 63	2 61	3 43	4 65
96	8 68	13 85	18 24	24 70	175	4 76	7 59	10 00	13 55	520	1 60	2 56	3 37	4 56

§ XL. THE SPHERICAL CRYSTAL MICROMETER BY G. DOLLOND. [Plate V.]

1. Mr. George Dollond contrived a micrometer of rock crystal, which differs in its construction from those we have described, principally in this respect, that its form is a sphere instead of a prism. His account of this contrivance was read before the Royal Society of London on the 25th of January 1821, and published in their volume of the same year. The sphere of crystal supplies the place of the eye-lens in the celestial eye-piece, which constitutes this micrometer; and consequently the difficulty of effecting the proper section required in a prism is avoided: the measure of any object is proposed to be taken by double images, produced by turning the sphere about 45° round its axis of motion, which axis lies at right angles to the axis of vision of the telescope, to which it is applied. An index with a vernier indicates the angle of revolution passed over in turning either to the right or left from zero, which is at the middle of the divided arc. When the crystal is ground and polished into a perfect sphere, and fixed in a bed formed by two half holes, at the middle of the axis of motion, it is capable of being adjusted to any position, before the fixing screws make it fast, and when it is turned till its natural axis of formation lies parallel to the axis of the telescope's vision, there will be but one image of any distant object, and when the sphere is once adjusted to this position, the index must be put to zero of the divided arc; and then the images will gradually separate, as the index carries the sphere round, along with the axis of motion, to which they are both attached, whether the direction be to the right or left, and this property affords the means of annihilating the index error.

2. Mr. Dollond has constructed for our use a micrometer of this description, which we have applied to an achromatic telescope of 43.6 inches solar focal length, and 3.2 inches aperture; which micrometer has also a divided circle for measuring the position of any given line, as it regards an horizontal or other line. This micrometer may be more intelligibly described by a reference to figures 5 and 6 of our plate V, the former of which presents a perspective view of the micrometrical eye piece, and the latter exhibits a view of the interior part of the tube, when the covering cap at the eye end is removed. The piece of brass tube *a*, seen in fig. 5, is loosely surrounded by an indented wheel *b*, to which a ring *c* cut into a coarse screw is made fast, which ring also surrounds the tube *a* in the same manner, and screws into the drawer of the telescope, by which the wheel is held fast; to the outer end of the said tube *a* the surrounding piece *d* is screwed fast, and carries two potences, one for holding the graduated plate *e*, and the other for carrying an opposite pinion and thumb-screw, not seen, to act with the wheel *b*. The axis which carries the index bar pointing to the opposite ends of the graduated plate *e*, passes diametrically through the eye piece, as seen in fig. 6. and holds the crystal sphere *f* in its hollow bed, so that when the index is moved along the plane of the graduated plate, this sphere revolves, and produces the double images at different distances from each other, depending partly on the position of the index, and partly on the magnifying power of the telescope used. The eye-hole at *g* is very small, and close to the sphere, and the diaphragm limiting the field is also contracted and close to its opposite side, to exclude the light not passing

through the sphere, which has a diameter of only $\frac{28}{100}$ths of an inch, and consequently a very short focal distance. From this description it will be perceived, that when the concealed pinion, acting with the fixed wheel, is turned by its milled head, the whole piece *d*, which bears it and the graduated plate, must necessarily revolve, carrying with it the vernier *h* and the tube *a*, as well as the said plate and axis holding the sphere, which axis can therefore be turned into any required position that an observation may require, and this position will be indicated by the vernier, when its zero is adjusted for the horizontal or other given line.

3. To enlarge the field of view, and to vary the magnifying power of the telescope, a field lens is screwed into the interior end of another piece of tube *i*, which fits the tube *a*, and may be graduated into parts of an inch, to show the distances of the lens from the sphere, which together constitute the eye-piece In our eye piece the scale on the inner tube *i* is an inch and a half, divided into 15 parts, for so many different magnifying powers. The measures taken by the crystal sphere vary inversely as the magnifying powers, so that the graduated opposite arcs have different values belonging to each separate magnifying power, and the instrument is capable of taking 45 × 15, or 675 different measures on each side of zero, without subdividing the spaces of either the scale of powers or of the arc of measures.

4. As the crystal of double refraction is formed into a sphere, we found it no easy matter to ascertain its constant angle, which indeed could not be done by any of the methods applicable to a prism with flat faces, which admits of being applied to the eye, in viewing a distant object; but, by reversing one of the operations, we at length succeeded. When a small black circular disc was placed on a white ground at 322 yards distance, and the micrometrical eye-piece applied to the telescope already specified, with the index at 45°, we found that the opposite limbs of the black disc came into contact when the inner tube *i* was removed from its home position a certain distance, at which the magnifying power was found by a good dynameter to be 137.5, the eye-piece was then applied to a smaller telescope of 30.5 inches focal length (No. 1.) and the same black disc was exactly measured by it, when its power was found to be 137. the disc itself was next measured from the same station by a Troughton's micrometer with a large telescope (No. 5.), and its diameter was determined to be 9″.39 = *c*, which measure multiplied by the mean power, gave 1289″ nearly for the constant angle $C = P c$ (§ XXXVIII. 14) The following table contains the magnifying powers of the telescope 43.6, obtained at the 15 positions of the field-lens, and the corresponding greatest angles which the crystal sphere will measure, which must be at the position 45°, are obtained by dividing the constant 1289″ successively by the respective powers, determined by a dynameter in the ordinary way, and arranged in column 2, opposite the respective positions of the inner tube contained in column 1, which positions indicate the distance of the field-lens from the sphere of crystal.

5. THE TABLE OF POWERS

AND CORRESPONDING GREATEST VALUES OF THE ANGULAR MEASURE OF THE SPHERE, WITH A TELESCOPE OF 43 6 INCHES FOCAL LENGTH

1 Scale	2 Powers	3 Values	4 Diff
0.0	191.0	6.75	
0.1	184 6	6.98	23
0 2	178.2	7 23	.25
0 3	171.8	7.50	27
0.4	165.4	7 79	.29
0.5	159.0	8.11	.32
0.6	152.6	8.45	.34
0.7	146.2	8.82	.37
0.8	139.8	9 22	40
0.9	133.4	9.66	.44
1.0	127 0	10.15	49
1.1	120.6	10 69	.54
1.2	114.2	11.29	.60
1.3	107.8	11.96	67
1.4	101.4	12.71	75
1 5	95.0	13.57	.86

6. We have said that the seconds contained in the third column are those which belong to the corresponding magnifying powers, when the index on the graduated plate points to 45°, but as this index is carried towards zero, the quantity gradually diminishes, till the double images coincide in one at the point zero. each maximum value, here given, is therefore the basis of a new circular scale of graduations, and the 15 positions of the field-lens admit of 15 such scales, all differing in value from one another.

7. Mr. Dollond seems to have been aware of the great variety of measures which his micrometer is capable of taking, but has not given any directions either how the constant angle ' of the crystal, forming the sphere, may be ascertained, or how to find by what law the measures taken by the circular graduations of the plate vary. In a conversation which we had with him on the subject, he seemed persuaded, that the arc is a scale of equal parts, which opinion is not founded on any theory or experiment, but taken for granted, though it is opposed by the experiments of Dr. Maskelyne, Rochon, and Boscovich, which induced them to adopt a straight scale having the desirable property of affording measures proportionate to the equal spaces passed over.

8. The focal distance of the field lens is about three inches, and to determine the exact focal distance of the sphere of double refraction, we removed the field-lens, and applied it singly to the eye-piece, and found the magnifying power of the telescope, adjusted for a distant object, just 202, and as we knew its solar focal length to be 43.6 inches, we had $\frac{43.6}{202} =$ 0 215 for the focal length of the sphere; which determination agrees with the theory, by giving the focal point, with parallel rays, at one half the radius of curvature from the face of the sphere for if we put the radius $= \frac{.28}{2} =$ 14, and add .07, we shall have 21, very nearly as we determined by our experiment and in this way the focal distance of any small lens may be determined by an object-glass of known focal length

9. Our next object was to contrive some method of ascertaining, experimentally, whether or not the graduated arc will give measures proportioned to the number of divisions passed over by the double index, when carried forwards or backwards from zero, where the image ought to be single, but we found, on trial, an index-error in terrestrial measures of $3\frac{1}{2}$ divi-

sions. For this purpose, a card was placed vertically at eighty-six feet from the object-end of the telescope, on which were inserted seven small round dots of ink, at exactly one tenth of an inch from each other, and, when the micrometer was applied, the field lens was so pushed in by degrees, while the index stood at 45° on the arc, that the six spaces of one tenth of an inch each, were made to appear thirteen, included within double the number of dots, and the experiment was, to make all the thirteen intervals apparently equal to one another, of which the eye could easily judge, when the spaces were so small as one-twentieth of an inch. This effect took place at the position of the lens 0 .95, where, by our Table, the measure taken by proportion is 9″.901, which may be considered as the true measure subtended by one-twentieth of an inch, when the magnifying power was 130 .2. Now as a larger power than this would not measure so small an angle, we had only five other positions of the inner tube, from 1 .0 to 1 4 inclusive, in which the inner tube could be put, to retain its place, and as they all afford measures exceeding 9″ 901, at the position 45° on the arc, there must necessarily be a division on the arc, at some point of it, which will give a measure of 9″ 901 at each of the five positions of the lens. our business was to put the dots into the same relative situations, to make equal intervals of one-twentieth of an inch, at each of those five positions of the lens, by simply moving the index from 45° towards zero, at each side of it, till this appearance took place, and then to register the degrees respectively pointed to by the index. The subjoined register of very careful observations will show the result.

Place of the Lens	Index at home.	Measure due to it	Observed place of the Index	Proportional Measure
0 .95	45°	9″.901	45 .0	9″ 901
1 .0	45	10 .15	42 .5	9 .58
1 .1	45	10 .69	34 .5	8 .19
1 .2	45	11 .29	32 .0	8 .02
1 3	45	11 .96	29 .5	7 .84
1 .4	45	12 .71	26 .0	7 .34

In this register, the first column shows the place of the field lens at each successive observation, the second shows the position of the index corresponding to the greatest measure given in the third column, the fourth column contains the degrees indicated on the divided arc, when the dots were equally distributed to form intervals of one-twentieth of an inch, and the last column gives the proportional measure for such indication, thus, at the place 1 .0 we have 45° . 10″ 15 : 42°.5 9″.58, and in like manner the other numbers in the sixth column were obtained, which should have been all *alike*, viz. 9″.9 very nearly, if the scale of the arc indicated equal differences in the measures, as the construction supposes. But it appears that the error in the measure increases with the decrease of the arc indicated, and the contrary, for which imperfection we see no remedy

10. At a time when the diameter of the planet Mars was about 9″, we attempted to measure it by this micrometer, but its limbs were so imperfectly defined, that no satisfactory observation could be made When the telescope is very good, and the crystal free from specks, we

have no doubt, but that the two images of a large star may be better defined; yet while the nature of the scale of measurement remains unknown, observations of their apparent diameters, taken by this micrometer, can afford no satisfactory results

11. From the examination, however, which we have given of this crystal sphere, we perceive that it may be substituted, in a celestial eye-piece with variable powers, such as we have used and described, for both the eye-lens and achromatic prism, since it will perform the office of both at the same time, and in its different fixed positions may also answer the purpose of several successive prisms, provided that the vision of the telescope can be made good, because the plane, where the proper section ought to be made to produce double refraction, by revolving on an axis, takes a succession of oblique positions, corresponding to as many prisms of different refracting angles.

12 Since the preceding part of this section was written, we have availed ourselves of an opportunity of observing the pole-star, with the spherical micrometer applied to the telescope for which the two Tables were computed, and though this star was not very well defined, yet we found the measures of its diameter, taken with a diminished aperture, to the right and left of zero, corresponding with one another more exactly than we expected, when reduced on a supposition of the arc of 45° giving measures proportional to the angle passed over from zero The index-error was found much greater than in our terrestrial measures, and the observations, we find, require to be made at the centre of the field of view, to agree with one another; for when a good contact was produced at the centre of the field, the two images ran into one at the right hand side of the field, and separated considerably when viewed at the left side. The subjoined observations, taken with much care, and repeated, will show the uncertainty of the index-error, at as proper an adjustment for vision as the eye could judge of, as well as the unexpected correspondence of four different observations, taken at different positions of the field lens, and consequently with as many different magnifying powers: viz.

OBSERVATIONS OF THE DIAMETER OF POLARIS.

Position of the Lens	Arc to the right	Arc to the left	Mean Arc	Value at 45°	Resulting Measures arising from the Observations
0.0	15° 0	9°.0	12°.0	6″75	$\frac{12}{45}$ of 6″75 = 1″.80
0.5	13.0	7.5	10.25	8.11	$\frac{10.25}{45}$ of 8.11 = 1.85
1.0	9.0	6.5	7.75	10 15	$\frac{7.75}{45}$ of 10.15 = 1 75
1.4	10.0	3.0	6.5	12.71	$\frac{6.5}{45}$ of 12.71 = 1.83
					Mean 1.81

If we could venture to affirm, that the mean of the measures above given is the correct measure of the diameter of Polaris, we should recommend the spherical crystal micrometer as a very convenient instrument for measuring the diameters of the larger stars, but as small stars are not visible through it, its application would be limited to the four or five magnitudes of the principal stars, in some of which it might detect a variation of the disc, when measured at distant intervals by the same eye, with the same telescope, and at the centre of the field of view But we are disposed to infer, that the similarity of the four measures, above exhibited, may be attributed in some degree to the smallness of the arcs used, as they have reference to the zero point, where one image only is seen. With our French prism 1, and a magnifying power of 99 .6, with lens 1 at the position 41, the same telescope gave a measure = 1".79; but the contact appeared somewhat too close, as if the images were flattened by pressure against one another. We shall be happy to see the nature of the scale proposed by Mr. Dollond more fully developed, which we may be able to do by future observations made on different parts of the arc of measures, and compared with each other.

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§ XLI DR BREWSTER'S PATENT MICROMETRICAL TELESCOPE [PLATE II]

1 In Dr. Brewster's *Treatise on new Philosophical Instruments,* the ingenious author has explained a variety of contrivances by means of which a good telescope may be rendered micrometrical, some of which may be made useful in practical astronomy, though the use for which he has principally adapted them is to measure distances. The instrument for which he took out a patent is made by the Tulleys, and sold by Harris, mathematical instrument-maker, of Holborn, London, and may be used either as a single-image or double-image micrometer, in taking the measure of small angles subtended by lines of known dimensions, either in naval, military, or geodetical operations. The telescope to answer these purposes is made of a portable size for the pocket, with sliding tubes and without a stand, as represented by fig. 10. of our Plate II which is a longitudinal section of all the parts in a state for use. We will first describe the instrument as constructed on its ordinary dimensions, and then show how an enlargement of its tube, and some alteration in the proportions of the optical parts, will convert it into an astronomical micrometer, that may be useful for some purposes.

2 The principle of the instrument used as a single-image micrometer, and which had been previously applied by Romer and De la Hire, is that of introducing two separate object-glasses, capable of being separated by mechanical means, and of thus occasioning a change of magnifying powers, with a corresponding scale of measures to be obtained by simple inspection The author has proved, both from theory and by computations, that the scale of measures, depending on the distance between the separated object-glasses, is a scale of equal parts; and therefore the length of the telescope, as in the constructions of Dr. Maskelyne and Rochon, may be graduated into equal divisions, as far as the separation extends. In Dr. Brewster's scale however there is no zero, and the extremities of the scale must have the values of the measures, taken at those points, determined experimentally. It will be sufficient for our purpose to recur to our formula $\frac{Ff}{F+f-d}$ for finding φ, the focal length of a single

lens, which is equivalent to the compound focal length of two lenses, in different states of separation (§ VI. 2) when used as an eye piece, the same formula will give us the focal length Φ of a lens which is equivalent to two object-glasses, of which the separate focal lengths are respectively F and f, and d the distance between them, but $\frac{\Phi}{\varphi} = P$ is the magnifying power of the telescope, and when Φ is invariable, and φ variable, d, the distance between the lenses, affords a scale of equal parts for the *powers*, but when φ is invariable and Φ variable, because the measures are *inversely* as the powers, the distance d affords an equal scale of *measures.* Hence a variation in the magnifying powers, occasioned by a separation of the object glasses, requires no values to be tabulated, but gives the scale of measures at once, and on this account would be a more convenient principle than that which depends on a separation of the lenses of an eye piece, if in practice a sufficient separation of the two object glasses could be as well effected, without interfering with the adjustment for good vision, and their radii of curvature proportioned to measure very small angles

3. In our instrument the external tube A is 10.6 inches long, the next, marked B, is 7.93, the third or C, also 7.93, and the fourth, D, containing the four lenses of the eye-piece, is 5.85 The fixed object-glass has a focal length of 18.25, and a diameter of 1.9 inches, but the moveable one, which screws into the end of tube C, next the object-end, has a focal length of 13.8, and a diameter of 1.4 inches, they are both achromatic. When the tubes B and C are both pushed home, and the eye-tube D drawn out as far as it will go, the two object-glasses are then in contact, or very nearly so, and a distant object is visible under the smallest magnifying power, the compound focal distance Φ, being then a minimum, viz according to one formula $\frac{18.25 \times 13.8}{18.25 + 13.8 - 0} = \frac{251.85}{32.05} = 7.86$ inches only, but when both the tubes B and C are drawn out to their full extent, and the eye-tube pushed home, a distant object becomes visible under the greatest power, the compound focal distance Φ being then a maximum, viz. $\frac{251.85}{32.05 - 15.86} = \frac{251.85}{16.19} = 15.55$. In the former position, which constitutes one end of the scale, 217′.6 are indicated, and the latter position indicates only 110′, but the product of the measure by the focal length is the same in both cases, viz. $217.6 \times 7.86 = 1710.336$, and $110 \times 15.55 = 1710.5$, and as the distance between the two opposite ends of the two tubes, B and C, is 15.86 inches, this length of scale must be divided into 107.6 equal spaces $(117.6 - 110)$ and each inch ought to contain very nearly 7 intervals $\left(\frac{107.6}{15.86}\right)$ in the inch, which we find to be actually the case. These divisions of one-seventh of an inch afford a scale for variations of single minutes, or of 30 when subdivided into two.

4. When the tubes B and C are both at home, the tube C must be drawn out first, for the scale engraved on it is indicated by the end of the surrounding tube B, from 217′.6 to 163′.5; after which tube B must be gradually drawn out, and its divisions from 163′.5 to 110′ will be indicated by the end of its surrounding tube A. The image of the body thus measured is referred to two pointed conical pins projecting from the opposite sides of the eye piece at its focus, and when the distance between the said points just includes the body measured, the magnifying power of the telescope is exactly suited for the observation; but if the body appears

too small to fill the whole distance between the fixed points, the telescope must be elongated, by drawing out tube *C* as far as it will move, if necessary, and then tube *B* also to complete the measure, during which operation the eye tube will require to be adjusted for vision, at every new adjustment of the distance between the object-glasses; which is the greatest inconvenience in using the instrument. Besides the two metallic points, there are a pair of parallel wires stretched across the field of view, which, being placed at half the distance separating the points, may be used with the same scale, by taking one half only of the quantity indicated. Small as the powers of this instrument are, used in this manner, it may be advantageously applied to measure the distances of a comet from two known stars, that may appear in the same large field of view, which this telescope supplies. Dr. Brewster has denominated this instrument a *new wire micrometer*, and from our description it is obvious, that when the focal length of the principal object-glass is considerable, a second object-glass may be made to slide for a foot or more, near the eye-end, with its focal length so proportioned, that a scale of divisions may measure to great accuracy, provided a well divided disc of glass, like that in our POLYMETRIC MICROMETER, were substituted for the points or wires, and properly illuminated. When the moveable object-glass has comparatively a short focal distance, the value of the scale will vary more rapidly than when it is nearly of the same length as that of the principal object glass, and when its length exceeds that of the fixed glass, the divisions of the scale will be enlarged, and may be so much increased, as to indicate smaller differences than the errors of observation.

5. The author has computed that, when the fixed object-glass has a focal length of 36 inches, and a distance between the parallel wires, such as will measure 26′ with a power of 40, a diminution to 30 will measure 34′ 40″, and also if a moveable object-glass of longer focal distance be used, such as will reduce a power of 40, measuring 29′, to 35 only, measuring 33′ 9″, in the former case a scale of 10 inches will measure 8′ 40″ (34′ 40″ − 26′), and in the latter only 4′ 9″ (33′ 9″ − 29′), and one minute will be measured by 1.15 and 2.41 inches respectively. Either of these constructions would be proper for measuring the diameter of the sun or moon, but particularly the latter. As an example for making a scale of 10 inches measure single seconds, by spaces of an inch to the second, let a telescope be taken which magnifies 300 times, with two parallel lines which will include 40″ with such power; and let the reduced power be 240 occasioned by a separation of the two object-glasses equal to 10 inches; then the corresponding measure will be 50″, for 300 × 40 = 12000, and likewise 240 × 50 = 12000.

6. This method of increasing or diminishing the magnifying power of a telescope, we are persuaded, would afford a very convenient scale of *measures* for an ocular crystal micrometer, though we have not yet had an opportunity of trying it. The method of determining the angle subtended by any body, at each extreme point of the scale, will require no further explanation than what we have already given. (§ XX. 8, 9.)

7. We may now proceed to describe the variation made in this micrometrical telescope, when it measures by means of double images. The method of doing this we believe to be original; it consists in substituting a *divided* achromatic object-glass for the entire one we have described, with the centres of the two halves removed from each other and permanently fixed, as seen in figure 12 of the plate above referred to; and again edgewise in figure 12, to show the screw which attaches it to the inner end of tube *c*, when the other moveable object glass is removed. In all other respects the telescope remains as we have described it, and the two

graduated scales, lying parallel to the tubes, are those which belong to the respective moveable object-glasses, the one next the tubes being the one used with the entire object-glass, and the remote one, which has the degree subdivided into three parts, belongs to the one which is divided they both extend along the tubes B and C without interruption, except what is occasioned by the interposition of the milled ring at the outer end of tube B, which is not counted. The measure on the second scale, belonging to the divided object-glass, begins with $11'.5$ and ends with $75'$, so that the space for each minute is $\frac{63\ 5}{15\ .86}$, or very nearly four in the inch, and each subdivision shows $20''$, or by the estimation of half a subdivision $10''$. The focal length of this divided object-glass is about .35 of an inch shorter than that of the entire glass, giving a somewhat greater variation of magnifying power than the former one, but the distance at which the centres of the half lenses are fixed, regulates the total quantity of the measure, while the change of magnifying power increases or diminishes it, till a contact of the two images of the object of observation shows that the measure is exact. In this scale also there is no zero, when a terrestrial eye-piece is used, and in both constructions the accuracy of the scale will depend on the correctness with which the values of its extreme points were in the first instance determined. Like all other double-image micrometers, this divided object-glass requires no illumination, nor subjects the observer to any inconvenience arising from an apparent motion of the body observed If the telescope had a large aperture, and a long focal distance, this method of measuring by double-images might become an useful substitute for those which are taken by the application of crystals of double refraction, when the angle is not very small. We have succeeded in measuring the diameters of the sun and moon with one divided glass within a few seconds, notwithstanding the smallness of the telescope's power, which varies from about seven to fourteen. In this latter construction the metallic points are of no use, and the wires serve only to show the middle of the field of view, near which the double images should always be situated in an observation.

8 With respect to the theory of the sliding semi lenses, if we conceive them to be in contact with the principal object-glass, and two parallel pencils of light passing through this glass, and then falling, in their refracted direction, on the centres of the two semi lenses respectively, they will proceed in the same straight lines without further refraction, till they cross one another in a point p, in the axis of the telescope's vision, and the distance between the centres of the semi lenses, which distance we will call δ, is the subtense of the angle formed at the said point p, then as each pencil after crossing will come to a focus and form an inverted image, the length of this image will also be a subtense to the same angle at p, but longer than δ, inasmuch as the distance of the principal focus, F, of the fixed object-glass, from the point p, is greater than the distance of the line δ from the same point, but this latter subtense is the measure of the angle at p, which we will call v, then if we denominate the shorter distance a, and the longer distance b, we shall have the following analogy, as $v \quad \delta \quad b \,.\, a$, again, if we call the focal length of the semi-lenses f, we shall have $a = \frac{fb}{f+b}$, and therefore as $v \quad \delta \quad b : \frac{fb}{f+b}$; consequently $v = \delta + \frac{\delta f}{f}$, from which formula, we may compute the different amounts

of v at equal variations of b, and show the nature of the scale, thus, suppose $f=10$, $\delta=2$, and $b=1$, 2, 3, &c. successively, we shall then have

$$v=2+\frac{2\times 1}{10}=2.2$$

$$v=2+\frac{2\times 2}{10}=2.4$$

$$v=2+\frac{2\times 3}{10}=2.6$$

from which it is evident, that the values of the measures taken between the centres of two images seen in the field of view, are proportionate to the spaces, b, b', b'' passed over. For as the semi-lenses recede from the principal object-glass, the point p of crossing also recedes, and the angle subtended there diminishes, till the said point coincides with F, where the images will unite in one. When the point p of crossing of the pencils, coming from the different semi-lenses, coincides with the compound focal point Φ of the two object-glasses at any time, the two images will touch, and this is the situation that gives the true measure.'

9 The length of the scale between the extreme angular measures depends entirely on the focal length of the semi-lenses, but the value of the smallest angle is regulated by the distance between their centres. each of these may be determined by computation. If we take f as the focal length of the semi-lenses, and b their distance from F, the principal focus of the fixed object-glass, when the object-glass and semi-lenses are in contact, b is then equal to the principal focal length of the former, and we have $\frac{Ff}{F+f}=\Phi$ the focal length of the combined glasses. Let us suppose the focal length, F, of the principal object-glass of a telescope to be just thirty inches, and let it be required to find such a focal length for the semi-lenses, that the extreme angular measures of the scale may be to one another in the ratio 24 : 6, since the angular measures are to each other inversely as the compound focal lengths Φ and Φ' respectively, we have 24 : 6 :: 30 : 7.5, so that 7.5 inches must be the length of Φ', when Φ is 24; now the formula $\Phi=\frac{Ff}{F+f}$ becomes by reduction $f=\frac{\Phi F}{F+\Phi}$, which in our example is $f=\frac{7.5\times 30}{30-7.5}=10$ In the next place a value must be assigned to the smallest angular measure, which we will call α, and as δ, the distance of the centres of the semi-lenses, subtends the angular measure belonging to the point p when it coincides with Φ', at 7.5 inches from the semi-lenses, we have $\frac{1}{2}\delta=\frac{\Phi.(\tan.\frac{1}{2}\alpha)}{\text{rad.}}$; then assuming 50′ for α, the smallest angular measure, we have this operation; viz.

Log. 7.5.................... 0.8750613
Log. tan. 25′................ 7.8616738

$\frac{1}{2}\delta=0.054542$.............. 8.7367351

and $\delta=0.109084$ of an inch, taken for the distance of the semi-lenses, will fulfil the condition. The greatest angular measure may now be found thus;

$$6 : 24 :: 50' : 200' = 3^\circ 20'.$$

If 50″ had been chosen instead of 50′, the corresponding distance of the centres, computed in the same manner, would have been only .001818, which quantity is too small for practical adjustment, and therefore, when very small angles require to be measured, this construction will not be competent to the operation, though it may afford a scale of large measures, differing from each other by very small quantities, even by less than single seconds.

10. Dr. Brewster has shown that a Gregorian or Cassegrainian telescope may have its power increased by means of an enlarged eye-tube drawing out to different distances from the small speculum, and a divided scale placed at the focus of the outer lens might thus be made polymetric.

BINOCULAR MICROMETERS.

§ XLII THE LAMP MICROMETER BY SIR WILLIAM HERSCHEL [Plate XIII]

1. The third class of micrometers which we proposed to describe is that which requires the use of both eyes in taking the measure, and is therefore called *binocular*. Micrometers of this kind owed their invention to the difficulty of measuring the distance between two stars, when one of them is so small, that it becomes invisible with the smallest illumination capable of rendering the wires of a common micrometer visible enough for useful observations, Sir William Herschel was probably the first astronomer who made a successful use of a contrivance of this kind, when he was prosecuting his immense undertaking of measuring the angular distances between the different pairs of double stars, as well as of triple and quadruple ones, which, with his powerful telescopes, he observed in various parts of the celestial expanse. He had reason to be dissatisfied with his wire, or silken thread micrometer on various accounts, which he has enumerated under the five following heads

2. In the first place, when two stars were included within his lines, their diameters would vary with different telescopes as much as 2″, and their spurious diameters would often change, even with the same telescope, according to the state of the air, and length of time occupied in viewing them with so high a power as 227, secondly, the deflection of light incident on the wires, prevented the stars, or at least one of them, from passing exactly along the wire, thirdly, there was much uncertainty about the *zero* of the micrometer, depending on the quantity and direction of light, and also on the change of position of the wires, fourthly, the screw or rack-work cannot be made perfect, and an error of a single thousandth of an inch in the measure of a disc or angular distance, may cause an error of several seconds in using most instruments, but fifthly, the greatest imperfection of all was, that the light necessary for illuminating the wires, made small stars entirely disappear.

3. To remedy these inconveniences, the inventive powers of this skilful searcher of the starry regions produced the *lamp micrometer*, which forms the subject of our present section, the construction and use of which we cannot better describe, than by giving an abridgment of his own paper, published in the 72nd volume of The Philosophical Transactions of London (1782), and by using his own words as far as an abridgment will allow. The representation of the instrument is that given in our first figure of Plate XIII, in which we have retained the author's letters of reference, in order to have the benefit of his own description. When he had applied the amazing power of 3168 to his seven-feet Newtonian telescope, he obtained a scale of measurement of ten inches for the distance between the two contiguous stars of *α* Geminorum, and this experiment pointed out to him the method of constructing a new micrometer, which should be free from the imperfections of the wire micrometer; the construction of which he thus described.

4. "*ABGCFE* is a stand nine feet high, upon which a semicircular board *q h o g p* is moveable upwards and downwards in the manner of some fire-screens, as occasion may require, and is held in its situation by a peg *p* put into any one of the holes of the upright piece *A B* This board is a segment of a circle of 14 inches radius, and is about three inches broader than a semicircle, to give room for the handles *r D*, *e P* to work. The use of this board is to carry an arm *L*, thirty inches long, which is made to move upon a pivot at the centre of the circle by means of a string, which passes in a groove upon the edge of the semicircle *p g o h q*; the string is fastened to a hook at *o* at the back of the arm *L* (not seen), and passing along the groove from *o h* to *q*, is turned over a pulley at *q*, and goes down to a small barrel *e*, within the plane of the circular board, where a double-jointed handle, *e P*, commands its motion. By this contrivance we see the arm *L* may be lifted up to any altitude from the horizontal position to the perpendicular, or be suffered to descend by its own weight below the horizontal to the reverse perpendicular situation. The weight of the handle *P* is sufficient to keep the arm in any given position; but, if the motion should be too easy, a friction spring applied to the barrel will moderate it at pleasure. In front of the arm *L* a small slider, about three inches long, is moveable in a rabbet from the end *L* towards the centre, backwards and forwards A string is fastened to the left side of the little slider, and goes towards *L*, where it passes round a pulley at *m*, and returns under the arm from *m n*, towards the centre, where it is led in a groove on the edge of the arm, which is of a circular form, upwards to a barrel, raised above the plane of the circular board at *r*, to which the handle *r D* is fastened. A second string is fastened to the slider, at the right side, and goes towards the centre, where it passes over a pulley *n*, and the weight *w*, which is suspended by the end of this string, returns the slider towards the centre, when a contrary turn of the handle permits it to act, *a* and *b* are two small lamps, two inches high, one and a half in breadth, by one and a quarter in depth. The sides, back, and top are made so as to permit no light to be seen, and the front consists of a thin brass sliding door. The flame in the lamp *a* is placed $\frac{3}{10}$ths of an inch from the left side, $\frac{3}{10}$ths from the front, and $\frac{1}{2}$ an inch from the bottom. In the lamp *b* it is placed at the same height and distance, measuring from the right side. The wick of the flame consists of a single very thin lamp cotton thread, for the smallest flame being sufficient, it is easier to keep it burning in so confined a place. In the top of each lamp must be a little slit, lengthways, and also a small opening in one side near the upper part, to permit air enough

to circulate to feed the flame. To prevent every reflection of light, the side opening of the lamp *a* should be to the right, and that of the lamp *b* to the left. In the sliding door of each lamp is made a small hole with the point of a very fine needle, just opposite the place where the wicks are burning, so that when the sliders are shut down, and every thing dark, nothing shall be seen but two fine lucid points of the size of two stars of the third or fourth magnitude. The lamp *a* is placed so that its lucid point may be in the centre of the circular board, where it remains fixed. The lamp *b* is hung to the little slider which moves in the rabbet of the arm, so that its lucid point, in a horizontal position of the arm, may be on a level with the lucid point in the centre. The moveable lamp is suspended upon a piece of brass fastened to the slider by a pin, exactly behind the flame, upon which it moves as on a pivot. The lamp is balanced at the bottom by a weight of lead, so as always to remain upright, when the arm is either lifted above, or depressed below the horizontal position. The double jointed handles *c D*, *e P*, consist of light deal rods, ten feet long, and the lowest of them may have divisions marked upon it, near the end *P*, expressing exactly the distance from the central lucid point, in feet, inches, and tenths.

5. "From this construction a person at a distance of ten feet may govern the two lucid points, so as to bring them into any required position, south or north, preceding or following from 0° to 90°, by using the handle *P*, and also to any distance from $\frac{6}{10}$ths of an inch to five or six and twenty inches, by means of the handle *D*." The two lamps *a* and *b* have the sliding doors open, to show the wicks. *W* is the leaden weight with a hole *d* in it, through which a wire is to be passed when the lamp is to be fastened to the slider.

6. "It is well known to opticians and others, that we can with one eye look into a microscope or telescope, and see an object much magnified, while the naked eye may see a scale upon which a magnified picture is thrown. In this manner", says the author, "I have generally determined the power of my telescopes; and any one who has acquired a facility of taking such observations, will very seldom mistake so much as one in fifty in determining the power of an instrument, and that degree of exactness is fully sufficient for the purpose.

7. "The Newtonian form is admirably adapted to the use of this micrometer, for the observer stands always erect, and looks in a horizontal direction, notwithstanding the telescope should be elevated to the zenith. Besides, his face being turned away from the object to which his telescope is directed, this micrometer may be placed very conveniently without causing the least obstruction to the view, therefore, when I use this instrument, I put it at ten feet distance from the left eye, in a line perpendicular to the tube of the telescope, and raise the moveable board to such a height that the lucid point of the central lamp may be upon a level with the eye. The handles lifted up, are passed through two loops fastened to the tube, just by the observer, so as to be ready for his use. I should observe, that the end of the tube is cut away, so as to leave the left eye entirely free to see the whole micrometer. Having now directed the telescope to a double star, I view it with the right eye, and at the same time with the left see it projected upon the micrometer; then by the handle *P*, which commands the position of the arm, I raise or depress it so as to bring the two lucid points to a similar situation with the two stars, and by the handle *D*, I bring nearer or farther off the moveable lucid point to the same distance of the two stars, so that the two lucid points may be exactly covered by, or coincide with the stars. A little practice in this business soon makes it easy, especially to one who has

already been used to look with both eyes open. What remains to be done is very simple. With a proper rule, divided into inches and fortieth parts, I take the distance of the lucid points, which may be done to the greatest nicety, because the perforations are very small. the measure thus obtained is the *tangent* of the magnified angle under which the stars are seen to a *radius* of *ten feet;* therefore, the angle being found, and divided by the power of the telescope, gives the real angular distance of the centres of a double star.

8. “For instance, September 25, 1781, I measured α Herculis with this instrument. Having caused the two lucid points to coincide exactly with the stars, centre upon centre, I found the radius, or distance of the central lamp from the eye, 10 feet 4.15 inches, the tangent, or distance of the two lucid points 50.6 fortieth parts of an inch, this gives the magnified angle 35′, and dividing by the power 460, we obtain 4″ 34‴ for the distance of the two stars With this power the scale of the micrometer at this distance is upwards of a quarter of an inch to a second. As another example: On Nov. 28, 1781, the diameter of the Georgium Sidus was measured with a power of 227. The radius on that occasion was 35 feet 11 inches, and the measure of the magnified diameter 2.4 inches, making the angle 19′, and consequently the real diameter $\frac{19'}{227} = 5''.022$; in which measure the scale was 0.474 of an inch to the second.

9. “The measures of this micrometer, however, are not confined to double stars, but may be applied to any other objects that require the utmost accuracy, such as the diameters of the planets, or of their satellites, the mountains of the moon, the diameters of the stars, &c. For instance,” continues the author, “October 22, 1781, I measured the apparent diameter of α Lyræ, and judging it of the greatest importance to increase my scale as much as convenient, I placed the micrometer at the greatest convenient distance, and took the diameter of this star by removing the two lucid points to such a distance as just to inclose the apparent diameter When I measured my radius, I found it to be twenty-two feet six inches. The distance of the two lucid points was *about* three inches, for I will not pretend to *extreme* nicety in this observation, on account of the very great power I used, which was 6450. From these measures we have the magnified angle 38′ 10″; which divided by the power gives 0″.355 for the apparent diameter of α Lyræ. The scale of the micrometer on this occasion was no less than 8.443 inches to a second, as will be found by multiplying the natural tangent of a second by the power and radius in inches.”

10. It may be proper to mention by way of appendage to this last example, that the magnifying power, above stated to be 6450, was by a subsequent experiment determined to be smaller, viz. 5786, and that therefore the apparent diameter of α Lyræ must be increased in the ratio of the former to the latter. The first method of determining the powers of the seven-feet telescope with the respective single eye-lenses, was by means of a disc of half an inch, the image of which was viewed at the same distance with the second eye, as we described in our section XI, under the paragraph 16, entitled *Double Vision* When the absolute magnifying power had been thus determined with a lens of pretty long focal distance, it was applied to a diagonal eye-piece called by the author a *Camera eye-piece*, and the image of a small object was projected downwards upon a sheet of paper, and the projected image measured, then each of the lenses with shorter focal lengths, was successively applied in the same way, and the images projected

by them being also measured, showed the comparative magnifying powers of all the rest, which were those annexed to the observations

11 The second method of gaining the powers was, by first measuring the solar focal length of a standard lens, of the longest focal distance, and applying it as the lens of a microscope, and obtaining the image of a wire projected on paper at eight inches and a half from the lens and eyes, and then by comparing the images of the other lenses with this by the measures of their projected images, and when their absolute focal lengths were thus obtained, the sidereal focal length of the great speculum being known to be 85.2 inches, it was easy to obtain the respective powers.

12. In another communication, which has reference to the example of α Lyræ, given above, Sir William says that, "whatever may be the cause of the apparent diameters of the stars, they are certainly not of equal magnitude with different powers in the same telescope. In my instruments I have ever found less diameter in proportion the higher I was able to go in power, and never have I found so small a proportional diameter as when I magnified 6450 times."*

13. Another paragraph in the same communication deserves to be quoted also, with reference to high powers. "Notwithstanding opticians have proved that two eye-glasses will give a more correct image than one, I have always (from experience) persisted in refusing the assistance of a second glass, which is sure to introduce errors greater than those we would correct. Let us resign the double eye glass to those who view objects merely for entertainment, and must have an exorbitant field of view. To a philosopher this is an unpardonable indulgence. I have tried both the single and double eye-glass of equal powers, and always found that the single eye-glass had much the superiority in point of light and distinctness. With the double eye-glass I could not see the *belts on Saturn*, which I very plainly saw with the single one. I would, however, except all those cases where a large field is absolutely necessary, and where power joined to distinctness is not the sole object of our view" †

14. When the two stars composing a double one are very near one another, as in the first and second classes, an estimation of their distance may often be made with a good telescope, giving a round well defined image, and magnifying upwards of 200 times, by a comparison of that distance with the apparent diameter of one of them, without any micrometer being applied, and several of our author's measurements are registered in terms of this denomination, though the exact diameter of the compared star has not been attempted to be ascertained.

15. Another substitute for a micrometer, when a very high power was used, was a *lucid disc*, made of oiled paper or other semi-transparent material, placed before the light of a lamp in the front part of a lantern, removable to any distance, until its diameter appeared equal to that of the planet viewed in a magnified state, or until the two stars observed would just include its diameter. In this way the diameter of Georgium Sidus was determined to be about 4″ in the year 1781. The colour of the disc was found to have an influence on the measure, and when it was all strongly illuminated, the measure was too small; on which account a black disc was used, surrounded by a narrow illuminated border, which had the same effect; and therefore a dark disc placed on an illuminated ground was tried, but gave the measure in the opposite extreme, so that a mean of the two was considered as the true measure.

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* Vol LXXII p. 102.

† Ibid p 94, 95

§ XLIII BINOCULAR SPIDER'S-LINE AND GLASS-DISC MICROMETERS [Plate III]

1. When an observer is using a lamp-micrometer, he has two inconveniences to contend with . first the management of the lantern with respect to both distance and position of the lucid points, as well as of the telescope, and secondly, the management of his eyes, which must have the adjustment of their pupils adapted to objects at different distances, for the stars must be seen close to one eye, and the lantern ten or more feet from the face by the other. Some practice only can overcome the latter difficulty To avoid these practical impediments, we have contrived a small apparatus which enables the observer to have the star and the scale at the same distance from the respective eyes, which view them separately; and which holds the scale within reach of the hand, for effecting its adjustments for distance and position. But in the use of this contrivance, as well as of the lamp-micrometer, the perfect use of both eyes is an indispensable requisite.

2. The different parts of the binocular eye-piece that we contrived, and had made by Jones of Charing Cross about seven years ago, are represented by figures 3, 4, 5 and 6 of Plate III, and the instrument is so constructed, that it will allow either a spider's line micrometer, or a divided disc of glass to be applied as the scale of measurement; but whichever may be made choice of, the eye-piece, being of the variable kind already described (§ VI. 4.), and seen separately in figures 13 and 14 of the same plate, has the further property of being *polymetric.* The measure may be taken any number of times with as many different powers of the telescope, and with corresponding alterations of the scale, affording the means of using that power which is best suited to the observation, or of taking a mean of several independent measures. *A* and *B* in figure 6 are the two tubes constituting the plain variable eye-piece, as in the figures 13 and 14, and holding each a lens at their ends next the eye, the scale of distances between the lenses, on which the magnifying power with a given telescope depends, is marked on the exterior surface of the tube *A*, and indicated from 0 to 180 by the inner end of tube *B*, which distance is the argument in our following table for giving the powers of a telescope of 76.25 inches focal length, with either of two separate lenses, which screw successively into the eye hole of tube *B*, and a similar table may be constructed for any other telescope by the aid of a good dynameter. The scale-holder, seen in figure *5*, has a clamp and fixing-screw *a* at one end, and a hole *b*, containing a female screw, at the other, into which hole the piece of tube *c*, fig. 6, holding a positive eye-piece and divided disc of glass, or the spider's-line micrometer *d*, seen in fig. 4, will either of them screw, accordingly as one or the other may be at hand, or be preferred. The distance between the centres of those holes must be equal to the distance between the pupils of the observer's eyes, to enable him to see at the same time through the eye-piece of the telescope and the eye piece of the micrometer, when both eyes are brought nearly into contact with the respective eye-ends, the nose having room between them; but that the same scale-holder may suit more persons than one, a joint is made in the middle of it at *e*, and a screw *f*, with a milled head, opens or shuts the hinge by acting on the tail piece *g*, attached to the half bearing the micrometer, and by regulating the distance of the eye ends of the two eye-pieces to suit any observer. The piece *h*, fig. 6, is a reflector turning on a wire, screwed into the scale-holder, and illuminating the lines of either micrometer, by being placed in the requisite angle for modifying the light coming from a candle or lamp, when standing at a

greater or smaller distance, as the intensity of necessary illumination may require for different observations.

3. With respect to the mode of using the spider's-line micrometer as a part of the binocular eye-piece, and of obtaining a measure of any object thereby, when the tube *A* of the variable monocular eye-piece has been screwed into the telescope, and good vision has been obtained of the object under examination, the spider's line micrometer must be screwed into the remote end of the scale-holder at *b*, and the holder clamped to the outer tube *B*, at such a distance from the eye-end as will allow both eyes to come nearly in contact with their respective eye-lenses, and if the distance between the eyes is too great or too small, the adjustment, by the screw *f*, must accommodate the distance between the two eye pieces accordingly, so that while the right eye views the object magnified by the telescope, the left may see the spider's lines apparently in the same field of view as the object is seen in by the other eye there is that intimate connection in the joint use of the two eyes, that, when both are used together, the observer is not aware which eye sees the spider's lines, and which sees the object to be measured; they appear as if seen by only one eye, as in the ordinary mode of observing, and the opening of the lines by the micrometer's screw will easily be made to include the diameter of the object to be measured, or the angular distance between two stars

4. In this operation there seems at first to be something of a magical nature, and nothing but a little practice will convince the observer, that the appearance is not an optical deception; it is in fact the lamp-micrometer brought closer to the eye, but changed into a more convenient form. The horizontal spider's line may be turned into any oblique position for giving the longest distance between two points thus seen, which will always be the case when the lines opened stand at right angles to the line of distance to be measured The peculiar advantage of the binocular micrometer is, that no illumination is required within the telescope, and however small a visible star may be, it will appear in the illuminated field of the micrometer as a speck of superior light distinctly discernible, and the micrometer can be used with the same convenience with small stars in this way, as if large stars were to be viewed and measured by the ordinary monocular method. It must however be admitted, that there is the same difficulty in measuring the distance between two stars in a line parallel to the equator, or nearly so, which there is in using the same micrometer in the usual way

5. Besides the advantage of being able to see a small star and the spider's lines at the same time, with suitable illumination, the observer has the further advantage of obtaining a new scale for the micrometer, at every change of power given to the telescope by the variable eye-piece attached to it, and is thereby enabled to take a mean of several distinct measures taken on different scales, and with the same telescope. If the two eye pieces, used with the telescope and with the micrometer, were precisely alike, the value of a revolution of the screw would be the same, used either as a monocular or binocular instrument, and the tabular values would be those contained in our tables printed in Section XIX, according to the telescope employed; but the exact similarity of two eye pieces cannot be depended upon, whatever care may be taken in their construction We have shown that the value of a micrometer's screw, multiplied by the focal length of any telescope to which it may be applied, is always a *constant quantity*, and that this quantity with the eye-piece in our Troughton's micrometer is about $1991''$, but as the powers with ordinary eye-pieces vary with the focal lengths of the object glasses, we may substitute the magnifying powers for the focal lengths, which, with Troughton's eye-piece

in question, will give a product = 2040″, as we have also had occasion to notice; if therefore, in any position indicated by an eye-piece of variable power we have a knowledge of the telescope's corresponding power, *P*, the value, *V*, of a revolution of the screw, will be $\frac{2040''}{P}$, and this value, multiplied by the number and parts of the revolutions effecting the measure, will be the said measure reduced to seconds of a great circle. The *constant product* for any other spider's-line micrometer, used with any telescope whatever, may be easily determined, by first finding the value of a revolution of its screw with the telescope in question by the Sun's diameter, or any of the other methods (§ XIX, XX), and then by determining the magnifying power with a dynameter (§ XI.), for the product of the determined value in seconds, multiplied by the power now found, will be the *constant number* required to be divided by the power (*P*) belonging to any position of the lenses in a variable eye-piece having a table of powers computed. For instance, when Troughton's micrometer is used with our telescope 76 .25 (5), the power is 78 16, and the value of one revolution 26″.1, the product of which factors will be 2040″, as nearly as may be, and this *constant*, divided by any of the powers produced by the variable eye-piece, will give the value in seconds of a revolution of the same micrometer, used according to the binocular method.

6. In like manner the *constant product* may be determined for a divided disc of glass, or for any other scale, which may be applied in the binocular manner to a given telescope. When our positive eye-piece, containing a disc divided by Turrell, was adapted to the same telescope, the product of the magnifying power by the value of one division of the scale was found to be 1530″, which is the *constant* belonging to this eye-piece and disc, used in either the monocular or binocular method with any telescope whatever We have computed a table of magnifying powers for our sliding variable eye-piece used with either of two eye-lenses, with the telescope at present under our consideration, which we shall subjoin to this section, after having given a few examples of its application, as it has reference to the binocular use of the two micrometers we have now treated of The eye-lens numbered 1 has been omitted.

7. When we had measured the angle subtended by a distant object with a Troughton's micrometer in the usual way, and found it 4 .80 revolutions, or 125″.28, taken from the table adapted for this purpose (§ XIX. 6) we took this known angle as a test of the measures to be taken by the binocular method, and obtained the following experimental measures; viz.

THE DIVIDED DISC

With Tel 76 25

Lens	Scale	Power	Value	Divisions	Measure
3	10	90	17″ .0	7 .4	125″.80
3	70	74 4	20 43	6 .0	125 .80
3	110	64	23 .90	5 .25	125 47
Mean by the divided disc.........125 .69					

SPIDER'S-LINE MICROMETER

With Tel. 76 25

Lens.	Scale	Power	Value	Revol	Measure.
3	10	90	22″ 66	5 .53	125″.31
3	70	74 .4	29 .14	4 .29	125 01
3	110	64	18 .55	6 .75	125 .21
Mean by the spider's lines.........125 .18					

A TABLE

OF THE MAGNIFYING POWERS OF A TELESCOPE OF 76 25 INCHES FOCAL LENGTH,

USED WITH A BINOCULAR VARIABLE EYE PIECE HAVING TWO DIFFERENT EYE LENSES.

Scale	Lens 2	Lens 3	Scale	Lens 2	Lens 3	Scale	Lens 2	Lens 3	Scale	Lens 2	Lens 3
1	187 8	92 3	46	156 9	80 6	91	126 1	68 9	136	95 3	57 2
2	187 1	92 0	47	156.2	80 3	92	125 4	68 6	137	94 6	56 9
3	186 4	91 8	48	155 5	80 1	93	124 7	68 4	138	93.9	56 7
4	185 7	91 5	49	154 8	79 8	94	124 0	68 1	139	93 2	56 4
5	185 0	91 3	50	154 1	79.6	95	123 3	67 8	140	92 5	56 2
6	184 4	91 0	51	153 5	79 3	96	122 7	67 6	141	91 8	55 9
7	183 7	90 7	52	152 8	79 0	97	122 0	67 3	142	91.1	55 6
8	183 0	90 5	53	152 1	78 8	98	121 3	67 1	143	90 4	55.3
9	182 3	90 2	54	151 4	78 5	99	120 6	66 8	144	89 7	55 1
10	181 6	90 0	55	150 7	78 3	100	119 9	66 6	145	89 0	54 8
11	181 0	89 7	56	150 1	78 0	101	119 2	66 3	146	88 4	54 5
12	180 3	89 4	57	149 4	77 7	102	118 5	66 0	147	87 7	54 3
13	179 6	89 1	58	148 7	77 5	103	117 8	65 8	148	87 0	54 0
14	178 9	88 9	59	148 0	77 2	104	117 1	65 5	149	86 3	53 8
15	178 2	88 6	60	147 3	77 0	105	116 4	65 3	150	85.6	53 6
16	177 6	88 4	61	146 6	76 7	106	115 8	65 0	151	85 0	53 3
17	176 9	88 1	62	145 9	76 4	107	115 1	64 7	152	84 3	53 0
18	176 2	87 9	63	145 2	76 2	108	114 4	64 5	153	83 6	52 8
19	175 5	87 6	64	144 5	75 9	109	113 7	64 2	154	82 9	52 5
20	174 8	87 4	65	143 8	75 7	110	113 0	64 0	155	82.2	52 3
21	174 1	87 1	66	143 2	75 4	111	112 4	63 7	156	81 6	52 0
22	173 4	86 8	67	142 5	75 1	112	111 7	63 4	157	80 9	51.7
23	172 7	86 6	68	141 8	74 9	113	111 0	63 2	158	80 2	51 5
24	172 0	86 3	69	141 1	74 6	114	110 3	62 9	159	79 5	51 2
25	171 3	86 1	70	140 4	74 4	115	109 6	62 7	160	78 8	51 0
26	170 7	85 8	71	139 8	74 1	116	109 0	62 4	161	78 1	50 7
27	170 0	85 5	72	139 1	73 8	117	108 3	62 1	162	77 4	50 4
28	169 3	85 3	73	138 4	73 6	118	107 6	61 9	163	76 7	50 2
29	168 6	85 0	74	137 7	73 3	119	106 9	61 6	164	76 0	49 9
30	167 9	84 8	75	137 0	73 1	120	106 2	61 4	165	75 3	49 7
31	167 3	84 5	76	136 4	72 8	121	105 5	61 1	166	74 7	49 4
32	166 6	84 2	77	135 7	72 5	122	104 8	60 8	167	74 0	49 1
33	165 9	84 0	78	135 0	72 3	123	104 1	60 6	168	73 3	48 9
34	165 2	83 7	79	134 3	72 0	124	103 4	60 3	169	72 6	48 6
35	164 5	83 5	80	133 6	71 8	125	102 7	60 1	170	71 9	48.4
36	163 9	83 2	81	132 9	71 5	126	102 1	59 8	171	71 3	48 1
37	163 2	82 9	82	132 2	71 2	127	101 4	59 5	172	70 6	47 8
38	162 5	82 7	83	131 5	71 0	128	100 7	59 3	173	69 9	17 6
39	161 8	82 4	84	130 8	70 7	129	100 0	59 0	174	69 2	47 3
40	161 1	82 2	85	130 1	70 5	130	99 3	58 8	175	68 5	47 1
41	160 4	81 9	86	129 5	70 2	131	98 7	58 5	176	67 9	46 8
42	159 7	81 6	87	128 8	69 9	132	98 0	58 3	177	67 2	46 5
43	159 0	81 4	88	128 1	69 7	133	97 3	58 0	178	66 5	46 3
44	158 3	81 1	89	127 4	69 4	134	96 6	57 8	179	65 8	46 0
45	157 6	80 9	90	126 7	69 2	135	95 9	57 5	180	65 1	45 8

§ XLIV. ON THE USE OF POSITION-MICROMETERS

1. In several of our preceding descriptions of micrometers, we had occasion to remark that when a graduated circle forms a part of the instrument, such addition will enable the observer to measure the angle of position which a line joining the centres of two adjoining stars will make with the horizon, when they pass the meridian, or with their apparent path, when out of the meridian, but as the methods of applying a graduated circle to such purpose have not been sufficiently explained, we will resume the subject in this section, and supply such information as may be useful to the unpractised observer.

2. In the earlier observations of Sir William Herschel, made in the year 1779 and three subsequent years, the wire micrometer used by this first observer of double and triple stars, as made by Nairne (Philosophical Transactions, Vol. 71. p. 500) was by no means perfect, but when he re measured them after the lapse of twenty years, he used improved micrometers; and in the recent measures taken by Messieurs I. F. W. Herschel and J. South, spider's line micrometers, in their most improved form, were applied to superior achromatic telescopes, supported and directed by equatorial axes, and furnished with all the necessary adjustments, to facilitate the labour, and insure the accuracy of their operations. We cannot better direct the exertions of the unskilled observer, who wishes to use a position-micrometer with spider's lines, than by referring to the account given by the last-mentioned astronomers, of "Observations of the Apparent Distances and Positions of 380 Double and Triple Stars", &c. which constitutes an entire part of Volume 114 (1824), of the Philosophical Transactions of London As we shall have occasion to describe the EQUATORIAL INSTRUMENT in its place hereafter, we will satisfy ourselves here with noticing the principal difficulties which the observer may expect to encounter in using the spider's-line position-micrometer, and the methods of obviating or of overcoming those difficulties.

3. The first requisite to be attended to is to obtain a telescope of such length and aperture, whether reflecting or refracting, as will afford at the same time considerable magnifying power and a sufficiency of light to divide close stars, and to render small ones visible; in the next place, the foundation, on which such telescope is to be placed, must be firm enough to obviate tremors, thirdly, the motions in right ascension and in declination, must be independent of each other, fourthly, the clamps must be strong enough to preserve the position of the telescope unaltered, and the tangent screws of slow motion competent to produce easy regular motions, as well as be within reach of the hand, by means of handles or otherwise; fifthly, some method of illuminating the lines in the eye-piece in any given quantity, and within reach of the observer will be indispensable, sixthly, a change of eye-pieces for the regulation of the magnifying power, or otherwise a variable eye-piece, will be required to suit the different classes of double stars, and lastly, the convenience of a meridian, or of an eastern mark, by which the astronomical adjustments may be regulated, will be essential, and, when the latter is used, the axis of motion in declination should be formed into a telescope, having central cross wires at both ends. An equatorial stand of the best construction has most of these pro-

perties, and may be substituted for an equatorial axis having all the appendages on a suitable scale, but will be liable to occasional derangements from its locomotive construction.

4. When the micrometer has been applied to a telescope having all the advantages above enumerated, it may easily be turned to any star within its reach, at a given time, when the adjustments are complete, by simply elevating the telescope to the known declination of the star, and turning the polar axis round, till its distance from the meridian is indicated on the equatorial circle; as a celestial eye-piece is usually applied to this purpose, the star thus found will be apparently *below* the centre of the field of view, by reason of the effect of refraction, though it is actually elevated; in fact the position of the star will be reversed with respect to east and west, and inverted with respect to north and south; which appearances may not at first be familiar, but will gradually become so when such a power has been applied as affords a good view of the two stars to be observed, the larger of them must be brought to run along the equatorial spider's-line, and a short time will show whether its position is at right angles to the axis of the earth, which will be the case, if the star, particularly near the meridian, continue bisected through the field; then if the same appearance continue while the polar axis is turned slowly by its proper screw, this axis may be considered as duly adjusted to be parallel to the earth's axis; and if the moveable parallel lines are by construction at right angles to the equatorial line, they become *horary* lines while the micrometer remains in the adjusted position. When the telescope is directed to a star of known declination, or, which is better, of no declination, the value of the micrometer's screw may now be obtained by our second method already described [§ XX. 2.]. While the micrometer is in this adjusted position, the vernier or other index must be put to zero; or have the arc indicated made an index error, accordingly as the construction may require; then a circular motion must be given to the equatorial spider's-line till it lies over the centres of both stars, as nearly as the eye can estimate, which will require to be done carefully several times, as the stars are in apparent motion; during which operations one hand must be at liberty to move the tangent-screw, regulating the equatorial motion of the polar axis, in such a delicate manner as may keep exact pace with the earth's motion round its axis, and preserve the relative position of the spider's-line and stars covered by it, which faculty can only be learnt by practice; then a mean of all the repeated readings corresponding to the different measurements read on the arc, and corrected for the index error, will be either the angle of position or its complement, accordingly as the figuring of the spaces are engraved on the circle: if the figures increase by 1, 2, 3, &c. from 0 to 90° to the right and left from each horizontal zero, the angles of position will be read truly; but if the four quadrants follow one another successively from 0 to 90° in the same order round the circle, two of the quadrants only will indicate the positions, and the other alternate two will give complements of the angles measured.

5. The line to which the positions of stars have usually been referred is a circle of declination, along which the larger of two contiguous stars is supposed to run when the measurement is made, and the *position* of the small one, as it regards the large one, is that which is registered; there are four quadrantal positions in which the small star may be situated, viz. *s p*, *s f*, *n p*, and *n f*, which abbreviations imply south preceding, south following, north preceding, and north following, accordingly as the small star precedes or follows the large one, and is on the south or north side of the large star's line of declination; on the opposite ends of which the two

zeroes of the circle are situated. Thus if we read *55° 30′ s f* in any register, we may understand that the small star was on the south side of the large star's circle of declination and following it, and that the line uniting their centres made an angle of *55° 30′*, or its complement, with that circle. In registering the observations it will be convenient to adopt certain initials, as denoting certain relative magnitudes and colours, such as *L* for large, *S* for small, *w* for white, *r* for red, *n* for north, *s* for south, &c which plan was adopted by Sir W. Herschel, who also put down *equal, unequal,* and other indications of the appearances observed, together with the magnifying power used, and the date of the observation, as well as right ascension and declination of the object observed, as nearly as could be ascertained, in order that the object might be again identified. This plan of registering all the observable particulars, when carefully taken, is of great importance, as furnishing data for comparisons with distant preceding or following observations; which comparisons will point out in a convincing manner the *changes* which are taking place, and their quantities, as well as direction of motion, from epoch to epoch, either in the measured distances or angles of position. If the question regarding the annual parallax of the stars should ever be satisfactorily settled, it will probably be from observations of this kind, carefully taken at distant intervals of time, and compared with each other. Much indeed has recently been done in this way by Messrs. Herschel, South, Struve, and Amici, but much remains yet to be done, as well in comparative observations of the double and triple stars, as in fixing their mean absolute places, as to right ascension and declination.

6. The difficulties which occur in making practical measurements arise from either the closeness or inequality in the magnitudes of the two stars to be observed, or from their invisibility in that state of illumination which is necessary for rendering the spider's-lines perceptible by the eye. It is indeed a curious fact, that a small star of a blue colour, will bear more illumination than stars of the same magnitude of any other colour; at least when the illumination is caused by the ruddy light of a lamp. Nay, some small stars have their telescopic appearance even improved by illumination. Of such very small stars as bear illumination well, Messrs. Herschel and South have given us the following list, viz.: σ Scorpii, η Lyræ, ι Trianguli, μ Persei, 59 Serpentis, 22 Monocerotis, θ Virginis, and 51 Piscium. The same persevering observers have availed themselves occasionally of a singular method of obtaining a view or transient glance of a very faint star, by turning the eye into another direction, and then catching an oblique impression made laterally on the less fatigued portion of the pupil, and a rough measure might sometimes be thus effected, when the minuteness of the star eluded direct vision. But when the two stars are both very close, and very unequal in apparent magnitude, the difficulty in getting a good angle of position is extreme, and can only be obtained by a repetition of measurements giving a tolerable mean, especially when the stars differ in colour. In some cases a line drawn from the small star to become a tangent, first to one limb and then to the other of the large star, may be used with advantage, to get extreme measures, when high magnifying powers are used: for with most of the refracting telescopes the spurious discs of large stars, greatly magnified, have limbs sufficiently well defined. In general the discrepancies of repeated measures may be considered as indications of the difficulty of the observation, as well as of its uncertainty. When the equatorial line lies across both stars, the measure of the distance must also then be taken by opening the parallel spider's-lines, and this operation requires likewise considerable practical tact in the management of the equatorial motion, the micrometer's screw, and the illumination at the same instant.

If the centres of the stars are not bisected, the diameter of one or both of the two may require to be allowed for, which, being conjectural, may vitiate the observation. As the observations of Messrs. Herschel and South are before the public we will subjoin only one example, and take that which first occurs in their valuable labours

"No. I. R.A 0ʰ 6ᵐ, Decl 7° 49′ N.

35 Piscium, Struve 4; III 62,

Large white · small blue, bearing illumination very well.

POSITION		Nov 27, 1821	DISTANCE IN PARTS	
90° − 30° 9′	S	Five feet equatorial *s f*	36 .0	S
29 30			35 .8	
29 0			37 .0	
28 52		Position = 60° 46′ *s f*	36 .0	
29 43	II	Distance = 11″.168	34 .5	II
28 46			35 .0	
29 47			34 .1	
28 38			38 .0	
			34 .9	
Mean = −29 14			Mean = 35 .70	
			Z.... − 0 .28	
			35 .42	

Sir William Herschel measured this star on the 30th of June 1783, and his measures, as recorded in his second catalogue, Philosophical Transactions, 1785, are

Position 58° 54′ *s f.* Distance 12″.50.

So that this star has undergone no material alteration. M. Struve (Dorpat. Obs. III.) has four sets of measures, the mean result of which is

1821.45, Position 62° 12′ *s f*, Δ declin. = 9″.875, whence distance = 10″.591."

7. In the preceding register the complements of the angle of position are measured, and the discrepancies in the single observations show, that it is more difficult to obtain this angle, than to determine the distance correctly. According to the Table given for this purpose, we have for the distance .... 35 .0 parts = 11″.054

.4 .. ... = 0 .126

.02 ...... = 0.006

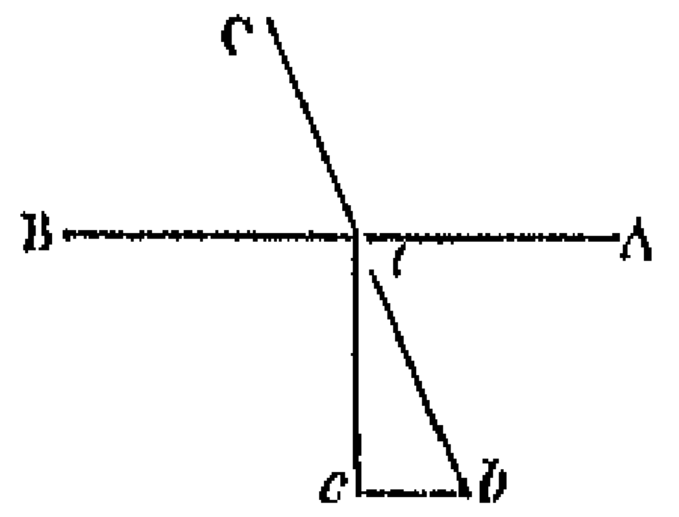
Sum = 11 186

Hence there has been some mistake either in making the reduction, or in transcribing .168 for .186.

8. When the micrometer has a revolving spider's-line distinct from the equatorial line, it must first be made to coincide with this line or lie parallel to it, when the vernier is at zero; whether the circle or the scale shall move with this line is of no importance, and will depend on the construction, but in either case the measure must be taken by the moveable

line, as before directed. In our altitude and azimuth circle, the toothed circular scale carrying the spider's line moves by a pinion, and when the instrument is used on the meridian, the index-error applied to the reading gives the angle of position at all times when the instrument is in due adjustment: and the same will be the case with any single line cut on glass or other transparent substance, which has a circular motion and a graduated arc of any description.

9. When the parallel lines which separate by the screw are placed to coincide with a circle of declination, and the large star made to run along them, the difference of declination of the two stars may be more easily measured, in most cases, than the direct distance between them, because the small one will run along the second line, when the measure is well taken, at the same time that the large one is seen on the first; and this seems to have been the way in which Struve has taken his measure of Δ declin. $= 9''.875$, which he afterwards converted into the distance $10''.591$, by the help of his angle of position $62^\circ\ 12'$; and the same may be done by observing the difference of right ascension and the angle of position, or by observing the differences of right ascension and of declination only, but not so accurately, since one of the latter data must be taken in *time*. We will give the formulæ for the different cases, which will be found useful as a check on the measures of both the distance and angle of position taken by the micrometer, and may often supersede the use of this instrument when the stars are not very close together.



10. If, in the preceding diagram we make *B A* a parallel of declination, and *C b* the line having a circular motion lying over the stars *a* and *b*, as the larger star *a* is proceeding from *A* towards *B*, the position of the small following star *b* will be denoted by *n f*, because it is on the north or lower side of the line *A B*, which is the apparent path of *a*; the angle *A a b* will be the angle of position, and *b a c* its complement, or angle made with the horary circle; *a b* will be the distance to be measured, *a c* the difference of declination, and *c b* the difference in right ascension when reduced to the equator. As the triangle is always small, the lines connecting the points *a*, *b*, and *c*, may be taken as straight lines without apparent error, and therefore when any two of the quantities *a b*, *b c*, *c a*, and *b a c* are given, the rest may be found by plane trigonometry, or by formulæ depending on it. Let *a c*, the difference of declination, and *b a c*, the complement of the angle of position be given to determine *a b*, the distance, and the difference of right ascension in time depending on *b c*, then we have

$$\text{(1.)} \qquad ab = \frac{ac}{\cos bac} = ac \,.\, \sec \angle\ bac$$

$$\text{and diff. of } R\,A \text{ in time} = \frac{bc}{15 \cos \text{dec} *} = \frac{ac \,.\, \text{tang.} \angle\ bac}{15 \cos \text{dec} *} = \frac{ac \,.\, \text{tang} \angle\ bac \sec \text{dec} *}{15}$$

Or let the distance *a b* and ∠ *b a c* be given, to find the differences of right ascension and declination, and *vice versâ*, and we have

(2.)
$$ac = ab . \cos \angle\, bac$$
$$bc = ab . \sin \angle\, bac$$

and $$\frac{bc}{15 . \cos \text{dec} *} = \frac{ab \ \sin \angle\, bac \ \sec \text{dec} *}{15}.$$

As an example we will take the data recorded by Struve in his Observation of 35 Piscium, which is 4 in his list; and has its declination − 7° 49′, or is so much south of the equator;

(1.)	Given $ac = 9''.875$	log. 0.994537
	sec ∠ 27° 48′, or bac	0.053262
	To find $ab = 11''.164$ nearly	1.047799

Hence it appears that there is some mistake also in Struve's determination of the distance when it is quoted = 10″.591, our present computation brings it more into accordance with the mean measure given by Messrs. Herschel and South.

	Given ac as before	0.994537
	Tang. 27° 48′	9.722008
	Sec. dec. 7° 49′ (reduction)	10.004054
	To find bc in arc reduced = 5″ 2553	0.720599
	Subtract log. 15	1.176091
	Diff. of R. A. in time = 0′.35035	9.544508
(2.)	Suppose to be given $ab = 11''.164$	1.047799
	Cos. ∠ 27° 48′	9.946737
	To find Diff of dec ac = 9″ 875	0.994536
	Suppose again to be given $ab = 11''.164$	1.047799
	Sin. ∠ 27° 48′	9 668746
	Sec dec. 7° 49′ (reduction)	10.004054
	To find bc in arc reduced = 5″ 2553	0.720599
	Sub log. 15	1.176091
	Diff of R. A. in time = 0.35035	9.544508

If we take the difference of right ascension in arc, bc, = 5″.2065 unreduced, and of declination, ac, = 9″875, to find the distance ab, and angle of position abc, because the triangle is right angled at c, we can obtain the side ab without logarithms more readily thus.

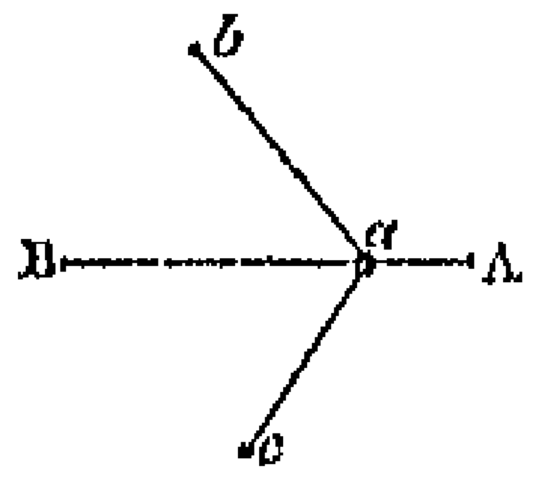
$$\sqrt{(ac^2 + bc^2)} = ab = \sqrt{125.1338} = 11''.164,$$

Lastly, as $\frac{ac}{ab}$ is equal sin. ∠ abc, or angle of position, we have

Log. ac 9″.875	0.9945371
− Log. ab 11.164	1.0477995
Sin. ∠ 62° 12′	9.9467376

11. With respect to the double-image position micrometers, we can best explain their application by reference to five diagrams such as we here annex:

FIG. 1



If we consider the line AB the circle of declination of a star a, having S. dec., and render the stroke cut across the field-lens of our variable eye-piece exactly coincident with it, by making the star pass along it, either on the meridian, or out of it when the telescope has an equatorial motion, and if bc be two small stars forming a triangle with the larger star a, the position of b will be sp, and that of c will be np, then let it be required to determine the angles of position baB and caB respectively, by means of the ocular crystal micrometer described in § XXXIX when the position of the telescope has been adjusted so that the declination line, made across the plain face of the field-lens, is rendered visible by the micrometer's rack, and turned so that the star will pass along it, the vernier piece holding a prism with a large constant angle may be put to its place and turned to one of the zeroes, then six stars will appear in the field of view and two parallel lines or images of AB, the tube or drawer of the telescope, into which the micrometer screws, must now be turned gradually round, without altering the position of the vernier, till the two parallel lines unite in one; which effect may be produced by turning the prism only, if the drawer will not turn round and admit of being clamped. Then the appearance of the stars will be as shown in the second figure, in which the triangle abc will be repeated by the formation of a similar triangle $a'b'c'$, and the two large stars a and a' will lie in the declination line, which is now a black single line; this is the zero position of the micrometer, and must remain unaltered so long as the star a is seen upon it. Turn now the vernier forward, and the secondary stars $a'b'c'$ will commence their respective circuits round their primaries abc, and when the arc due to the angle baB has been passed over, the appearance will be as represented by figure 3, annexed, where b, b', a, and a' will constitute a straight line, and the original double line AB will be separated into two parallel lines, having a line joining their ends pointing in the same direction as the line formed by the four stars; and also a line joining cc', though at a distance, will point in the same direction, the two triangles being still similar, though in different relative positions; in this situation the vernier will indicate the angle of position in degrees and mi-

FIG. 2.

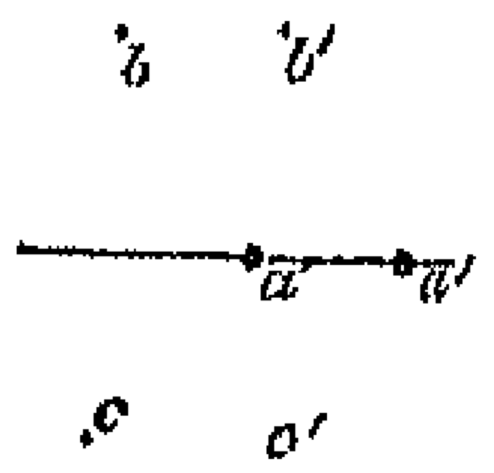


FIG. 3

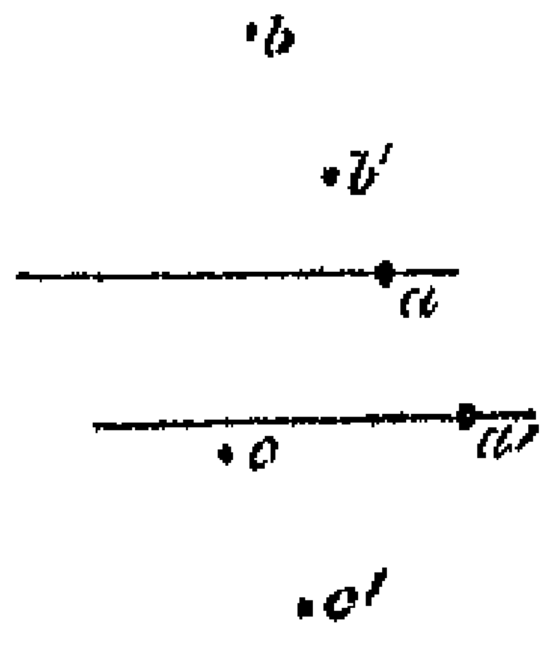


FIG 4

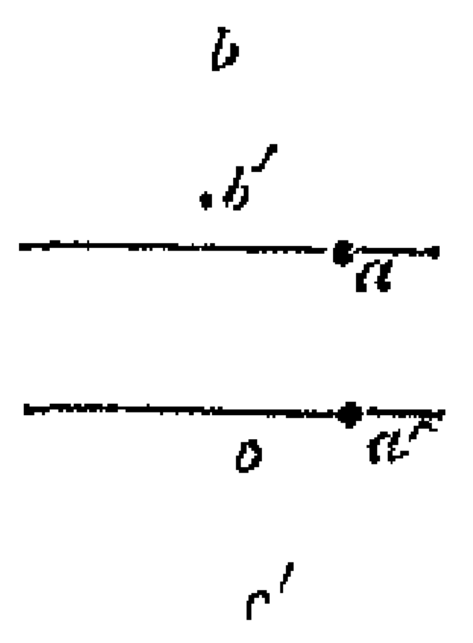
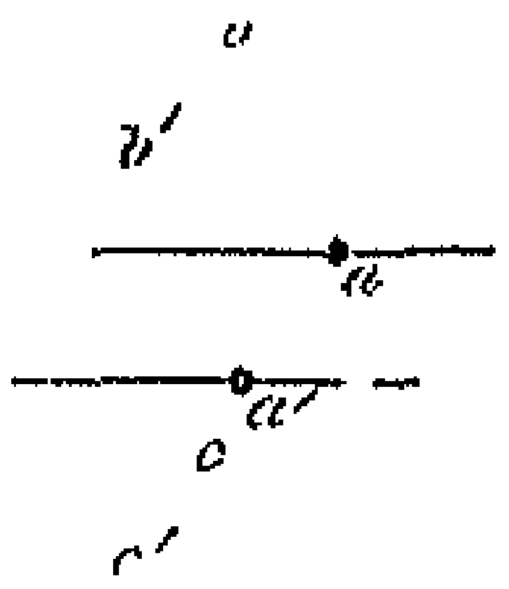


FIG 5



nutes when the vernier has been moved over a quadrant, the parallel lines will have opened to their greatest extent, and the four small stars b, b', c, and c' will be nearly in a straight line as in the fourth figure, and would be exactly so if they had the same right ascension, when they form an exact straight line the vernier will indicate the angle of position which a line connecting them forms with the declination line AB, which in this case would be 90° ± x, and x would be the angle formed with the vertical or horary line A further motion of the vernier will produce the appearance exhibited in our fifth figure, where a straight line is formed by the stars a, a', c, and c', b, b', being at a distance, and the parallel lines having changed the position of their ends, but the connecting lines will as before point in the same direction as the line formed by the four stars, and in this position the vernier will as before indicate the true angle of position caB When either of the two straight lines, formed by the four images of either pair of stars, has the two middle stars a' and b, or a' and c near each other, a change of power given to the eye-piece will make them coincide, and then the distance between the said pair will be obtained at the same time that the angle of position is indicated; the adjustments and position of the vernier must be the same for both determinations, which therefore may always be contemporary when both are observed It will always be desirable to make the parallel lines unite again, after the observations have been completed, to try if there has been any alteration in the position of zero, that its error, if any, may be applied. When the vernier has been carried over a semi-circle of the micrometer's limb, the same appearances will be repeated respectively, and when the measures are taken on opposite limbs of the circle, the index-error will merge in the average of the two measures

12. What has been above stated with respect to a prism of double refraction, used as an ocular micrometer, will be equally applicable to a prism applied on Rochon's principle, where a graduated circle is made an appendage to the tube bearing the sliding prism, the glass wedges, or cuneiform micrometers, have also the same property of measuring angles of position as well as distances at the same time, when substituted for prisms, in either mode of application. The most proper prism, or glass wedge, to be used in any observation, is that which with the given telescope and eye-lens will produce nearly equal intervals between the four images, constituting the straight line, which determines the angle of position, for as each secondary image a', b', or c' revolves round its corresponding primary a, b, or c, as the vernier proceeds along the arc, the two middle images have the advantage of apparent contrary motions, and a small displacement of the vernier is obeyed by a considerable change of relative situations among the four images of a pair of stars, which circumstance is important in contributing to accuracy. The double images have the further recommendation of affording great facility in gaining the measure of the angle of position, as well as of the distances, while the object of observation is in motion, since no fixed scale of measurement is required in the field of view by which the measurements are to be effected, and hence the equatorial motion may even be dispensed with, when the declination-line of the field lens is adjustable to the path of the principal star. The angle of position requires no alteration in the magnifying power, or adjustment for vision, and is therefore obtained in a very short time, and when that is determined, the distance to be measured by a change of power will not require an altered position

of the vernier during the time necessary for completing the measure, provided a prism, or wedge be applied that is competent to effect both purposes.

13. That we might form a comparative estimate of the accuracy of the single-image and double-image methods of obtaining the angles of position, by means of objects not in motion, we placed a card at the distance of 86 feet, having the points on it constituting our first diagram, which were distinctly seen by our refracting telescope of 43.6 inches focal length, and 3.2 inches aperture, when the variable eye-piece, holding the English prisms and cuneiform discs, was applied to it. After having adjusted the prism 4, and after that the wedges $a+b$ to the zero position successively, and proceeded with obtaining the appearances represented by the preceding figures respectively at the different positions, it occurred to us, that, as the two angles of position baB and caB may be taken together as the whole angle bac of the triangle, and as the three angles formed at a, b, and c are together = 180°, by placing the declination line on the plain surface of the field-lens so as to lie over the points b and c, the two remaining angles, abc and acb might be measured by the same appearances, before exhibited, of the rectilinear arrangement of the respective images; and on making the trial our experiment proved highly satisfactory. We then applied Troughton's spider's-line micrometer and the divided disc of glass or net-micrometer in succession, and measured the angles of position with each of these, and found that the double-image micrometers had the claim to our preference on the score of both accuracy and convenience. We confined our experiment to a single observation of each angle with each instrument, as the best test of the powers of each, and the following table contains the resulting respective measures

Angles	Prism 4	Wedges $a+b$	Troughton's Micrometer	Net-Micrometer	Mean of the four
baB	61° 30′	60° 55′	61° 30′	60° 17′	61° 3′
caB	49 0	48 55	49 5	49 20	49 5
abc	29 30	32 0	32 30	33 0	31 45
acb	39 30	38 25	39 15	39 30	39 10
Sums	179 30	180 15	182 20	182 7	181 3

From this single experiment it appears, that the wedges and prisms give the sum of all the angles more correctly than the single-image micrometers, and yet in measuring the angle abc the double-image micrometers differ from one another, more than the other micrometers do in any of the angles. With any of the micrometers an exact measurement cannot be procured but by a repetition of the observation, in order to gain a good mean. We may however find out which of the angular measures are most faulty by means of the angle made with the vertical by the line bc, which we determined to be 3° only; for when the single triangle is considered as divided into two triangles by the line AB, the two angles at the intersection of this line with the line bc (fig. 1.) will be 90° − 3°, and 90° + 3°, and with a knowledge of these angles we shall have the different angles of each separate triangle given, as expressed in the subjoined table

<table>
<tr><th colspan="6">UPPER TRIANGLE</th></tr>
<tr><th>Angles</th><th>Prism 4</th><th>Wedges</th><th>Troughton's Micrometer</th><th>Net-Micrometer</th><th>Mean</th></tr>
<tr><td>Computed</td><td>87° 0'</td><td>87° 0'</td><td>87° 0'</td><td>87° 0'</td><td>87° 0'</td></tr>
<tr><td><i>b a B</i></td><td>61 30</td><td>60 55</td><td>61 30</td><td>60 17</td><td>61 3</td></tr>
<tr><td><i>a b c</i></td><td>29 30</td><td>32 0</td><td>32 30</td><td>33 0</td><td>31 45</td></tr>
<tr><td>Sums</td><td>178 0</td><td>179 55</td><td>181 0</td><td>180 17</td><td>179 48</td></tr>
<tr><th colspan="6">LOWER TRIANGLE</th></tr>
<tr><td>Computed</td><td>93 0</td><td>93 0</td><td>93 0</td><td>93 0</td><td>93 0</td></tr>
<tr><td><i>c a B</i></td><td>49 0</td><td>48 55</td><td>49 5</td><td>49 20</td><td>49 5</td></tr>
<tr><td><i>a c B</i></td><td>39 30</td><td>38 25</td><td>39 15</td><td>39 30</td><td>39 10</td></tr>
<tr><td>Sums</td><td>181 30</td><td>180 20</td><td>181 20</td><td>181 50</td><td>181 15</td></tr>
</table>

From this comparison of the sums of all the angles of each separate triangle, the glass wedges seem to give more correct results than any of the other micrometers, or than an average of the whole. But the experienced astronomer will not fail to judge for himself of the preference to be given to an individual micrometer, when he has put them to the test of actual measurements of the double stars. We have not yet laboured much in this interesting field, and can therefore only point out the variety of tools which have been adapted for the labourer's use his own experience will enable him to decide which instrument is in all respects best fitted for his particular purpose We have tried Dollond's sphere of crystal in our experiment, but the smallness of its constant angle, even when a maximum, affords not a separation of the images sufficiently great to produce a straight line such as may be useful in determining the angle of position. The main advantage of this double-image micrometer is, that the arrangement of four points into a straight line of some length supersedes the use of a visible spider's line requiring illumination, when the adjustment of the declination line to the star's path has been made

14. There will, it must be acknowledged, be always some uncertainty in the determination of the vernier's position on its arc, which shall produce an exact straight line of the four luminous points constituting this line, and the error in the angle of position may be considered to be

proportionate to this uncertainty, which therefore can only be diminished by taking a mean of several measures but when the telescope is furnished with an apparatus for varying the quantum of illumination, a spider's line or a single fine stroke cut on a disc of glass, or, what is still better, a divided disc, constituting a net micrometer, may be fixed in the focus of the eye-piece, and the prism or wedge of double refraction may be so situated, that this line, or one of the lines of the net-micrometer, may be laid over the said four points, whenever they form a straight line, and thus render the observation of the angle of position as certain as the motion of the double star will admit of. Such application of a visible line to the four luminous points, removed from one another by the operation of the double refraction, will ensure the direction of that line, as a measure of angular position, and the co operation of the two micrometers will diminish much of that uncertainty which attends the measurement by either method singly; particularly when the two stars composing the double one are very close together.

15. The divided circle represented by figures 18 and 19 of Plate III. was contrived so as to admit of being used as a position-micrometer with single images, with double ones, or with both jointly, as we have here suggested, and is equally subservient to any of the three purposes When it is used with double images to effect the angular measure, by placing the four luminous points in a straight line, as has been described, the prism of crystal, or the glass wedge, is placed before the eye-piece, in the manner explained in figure 19; or may be situated between the two lenses which compose it, but when the micrometer is used in the ordinary way without the prism, or wedge, the disc of glass containing the single stroke, or net work cut on its face, is placed on the other side of the eye-piece at *a*, and the line to be used is first turned round, while the vernier is kept at zero, till the larger of the two stars in question runs along it, as seen in the telescope with the said eye piece applied to it, after which adjustment to the path of the star, the motion of the vernier along the divided arc carries the glass disc into the position that makes the same line pass over both the stars, and thereby affords a measure as good as if a spider's line micrometer, with a graduated circle for giving positions, had been the instrument used. When the reticulated disc is used, the line passing over the two stars being followed by a succession of parallel lines, over which the two stars pass together in succession, precludes the necessity of altering the position of the telescope, until the adjustment of the vernier for giving the angle is finally settled When both these means are used in conjunction, which in most cases will be found superior to either of them used separately, the prism or wedge of double refraction is fixed a few inches within the tube of the telescope, and produces double images on the principle of Rochon's construction, which images will be seen upon the disc of glass which contains the single line, or net, in the common focus of the object and eye glasses.

16. A contrivance for affording the joint application of the two micrometers, with the same graduated circle, is easily managed in this manner, instead of the eye-piece represented in figure 19, a tube of the same diameter, about four inches long, more or less according to the magnitude of the constant angle to be employed, is made to fit the piece of central tube, made fast to, and moving round with the vernier, and is substituted for the eye-piece, this tube contains at the end next the eye a positive eye-piece with the disc of glass fixed in its focus, but in such way that this eye-piece can be turned round while the tube remains fast to the vernier, for the purpose of adjusting the line of measure to the star's path, without moving the vernier

from zero, then the opposite or interior end of the tube holds the cell of the prism, or wedge, which is also adjusted, by having the means of being turned round without altering the vernier's position on the circle; it is easy therefore to comprehend without further explanation, that, when the vernier is carried from its adjusted position round the divided arc of the circle, both the disc of glass, having the stroke or net-work, and also the solid of double refraction will be carried along with it in the same tube, and whenever the four luminous points, or images of the double star, are brought into a straight line, the stroke on the glass will lie over them, and show whether or not the line so formed is a perfectly right line, and if not, will afford the ready means of making it so, by the necessary adjustment of the vernier's position on the arc of measurement. This device has very recently suggested itself, and when the apparatus for modifying the illumination admits of its application, it is well adapted for perfecting the measures of angular positions, particularly when the telescope has sufficient light.

17. A second method of effecting an union of the net and double-image micrometers, and of supplying a visible line to coincide with the four luminous points constituting a right line, in obtaining the measure of angles of position, has also recently occurred to us, and is not only more simple than the preceding one, but capable of giving the measure of the distance between two stars at the same time that it measures the angle of position. This method consists in fixing a graduated circle round the end of the drawer of the telescope, which is moved by the rack; and in clamping a vernier on the graduated tube sliding into it, and carrying the positive eye piece and sliding prism within it. For when the eye piece has a reticulated net cut on a glass disc in its focus, the double images of any object and the net are both seen together in the common focus. The adjustments for this construction (which is nothing more than a circle added to Rochon's application of a prism confined to the eye-end of a telescope) are these; first, the prism is put to zero of its scale, and the eye-piece is adjusted in its cell till the object viewed is seen *single* and well defined; secondly, the prism is carried forwards a short distance into the tube, and the eye piece unscrewed to receive its reticulated disc, as an appendage to it; then being replaced, it is adjusted to its place of distinct vision without altering its position in its cell; when the eye piece is replaced, both the double images and a stroke in the net-work will be visible together, but not in the same line, till it and the disc contained in it are turned round to make them appear so; when this is once done, the graduated tube carrying them is itself turned round, till both images of the principal star to be observed run along some individual line of the disc, and when that is the case, the vernier is set to zero and clamped to the circle, which operation completes the adjustments; then, if the tube now carrying the vernier, as well as the prism and eye-piece with its attached net, be turned round, till the four images of the double star form a right line covered by a stroke of the reticle, the vernier will indicate the angle of position. The reason, why the prism is *carried forwards* in the graduated tube, before the eye-piece is detached to receive its disc of net-work, is, that when the eye-piece is replaced, its adjustment, as it regards the divided scale of distances, indicated by the prism's vernier, is not disturbed, which adjustment could not be made after the disc has occupied the place of the focal point, which it is necessary the prism should occupy in being adjusted to its zero. When due care has been taken in completing the adjustments above specified, the telescope is in a situation to measure both angular distances and angles of position at the same operation, while yet the *visible* parallel lines of the disc will ensure the

true position of the circle's vernier, on which the accuracy of the observation will depend When one of the two stars will bear but little illumination, the apparatus for modifying the light must necessarily be managed accordingly

18 Lastly, the compound prism of double refraction may be united with the spider's-line micrometer, by simply fixing it between the two lenses which form its positive eye-piece, particularly when the constant angle is comparatively large; for the separation of the images takes place, as in Rochon's construction, and either the horizontal or vertical line of the micrometer, being illuminated properly, will be in the place of the stroke on a disc of glass, in the common focus, and the divided circle of the micrometer, having a wheel and pinion, or circular rack and endless screw, will be found very convenient for obtaining the angle of position, at the same time that the lines opening by the screw will measure the distance. This is a beautiful union which we have tried with perfect success, and can therefore recommend it, as well as the preceding contrivances, all which we have brought to the test of practical application The adjustment of the eye-piece, containing the prism in this construction, must be such, that, whether the horizontal or vertical line be chosen, as the line of indication of the vernier's position, that line must be made *single* by turning the loose eye piece round, till the line and its image coincide, in which situation the line at right angles with it, will be *double*, and at the maximum distance. When the distance of two contiguous stars is to be measured, as well as the angle of position, the *stationary line* of the micrometer must be made the single one, having its image laid over it; and must be first placed in the apparent path of the principal star, or parallel of its declination; when the two images of that star will both move along it as two beads on a string, then, the vernier being put to zero, if it has such adjustment, or its index error being noted, this double line must be carried round by the pinion, or screw, till the four images of the double star form a right line covered by the strong black line of indication, in which situation the angle of position, or its complement, as the case may be, will be indicated after allowance is made for the index error, which may happen to consist of several degrees. we have thus taken the angles *b a B* and *B a c* shown in figure 1. (11) and have successively found them 61° and 49° within a few minutes: while the black line of indication remains over the four images, one of the moveable lines may be separated by the screw, to measure the distance, and in doing this four slender lines will appear, viz.: two lines as usual, and two extraordinary images of those lines, each pair of which lines will separately measure their own distance, or the corresponding pair of images, and it will be of no importance which pair affords the measure, except that one of them may be more distinctly seen than the other, and will therefore be preferred. In fact, there will be a pair of measures taken at one operation. Troughton fitted us an eye-piece constructed in this manner to his micrometer, which completely answers our expectation.

§ XLV. EXPERIMENTAL COMPARISON OF SEVERAL MICROMETERS

1. DURING the time that our descriptions of the different micrometers have been in the press, we have had the large object-glass, which lately excited considerable interest among the members of the Astronomical Society and others, mounted in a brass tube, supplied with all the appendages requisite for making a complete telescope, and have also had most of the micrometers recently adapted to it, as so many eye-pieces. This addition to our former stock of instruments has induced us to make some experiments with various micrometers, when successively applied to three excellent achromatic telescopes; namely, to those which we have had occasion to denominate 3 and 5 [§ XI. 13], and to the twelve feet one, which we shall now call 6. Our objects in making comparative measurements were two-fold, in the first place to ascertain how far the different micrometers will accord with each other, when tables computed for the different telescopes are separately used, and secondly, to determine under what circumstances the largest telescope has the advantage over the smaller ones. For these purposes we fixed a blank enamelled dial of 2.75 inches in diameter, with a white ground, on a board previously painted green, and suspended it in a vertical position at the distance of about 332 yards, that it might be distinctly seen with all the three telescopes, and that the angle subtended might be between 40 and 50 seconds. We were aware that the long telescope would give the *apparent* angle greater than the shorter ones would do, at this short distance, owing to its greatest proportional elongation, but that when the reduction is applied, the resulting measures, if carefully taken, ought to be so nearly alike, that the differences may be considered as the errors of observation belonging to the different micrometers, taken conjointly with the errors of the tables. We have therefore computed a series of tables for our largest or sixth telescope, from the same data, with respect to the *constants*, which were the ground work of our former tables, having determined its solar focal length to be 145.8 inches, when the convex lens of English glass is used in conjunction with the German flint-glass, which is not the same that was in use when the report of a select committee was made upon it, but which is considered equally good by the maker, though it shortens the focal length about a couple of inches. These tables will be subjoined to this section, for the purpose of being referred to in reducing the measures. We will first give the micrometrical measures afforded by the dial placed at the short distance above specified, as we took them in succession by each of the three telescopes, each single observation being put down without coaxing, and the whole being registered before the reductions were computed, or extracted from the tables. As the weather was unfavourable for viewing celestial objects in December 1826, when the experiments were made, and as none of the planets were near the meridian in the evening, a stationary object was fixed upon, which could be viewed by day-light: but it was found necessary to confine the observations with single images chiefly to those hours, when the sun was obscured more or less by clouds, to keep the object steady in its position.

2. The measures taken by the telescopes 5 and 6 were reduced by means of their respective tables, but those made by the telescope 3 were reduced from the tabular quantities belong-

ing to telescope 6, by increasing them in the inverse ratio of the respective focal lengths of the two telescopes, or by a factor equal to that ratio. We might have computed the reductions at once for telescope 3 from the proper data due to its own focal length, or otherwise might have constructed a series of tables for it also, but we preferred using the tables already constructed, to show how the reductions, suitable for any individual telescope, may be equally useful for any other telescope, when the ratio of their focal lengths is known, for when the tabular quantities are extracted from a table, depending on the focal length of a given telescope, the proper factor will convert the amount into the quantity required for the second telescope by a simple multiplication. In our case the ratio $\frac{145.8}{76.25}$ gives the factor 1.912, and $\frac{145.8}{67.5}$ 2.16, by one of which the amounts of the tabular quantities are respectively multiplied, when derived from the Tables I. II. III. &c. in our annexed series. The elongations of the respective telescopes, when adjusted to distinct vision of the dial at 332 yards, are 1.8, 0.4884, and 0.38 in inches and parts, as determined by our formula $\frac{f^2}{d-f}$, and also very nearly so by actual measurement on the tubes; whence we have the factors for reducing the apparent mean angles to the true angles from $\frac{145.8}{147.6} = 0.9878$, from $\frac{76.25}{76.7384} = 0.9936$, and from $\frac{67.5}{67.88} = 0.9944$. When the power of any telescope is proposed to be derived from either of the tables given at pages 131 and 132, which are computed for variable eye-pieces, the same reticulated discs must be used, and the same position of the eye-lens with both telescopes; for then the power corresponding to that position may be taken from the table, and afterwards increased or diminished by a proper factor, equivalent to the ratio of the focal lengths of the telescopes in question, for instance, when the power of the telescope 67.5 is wanted, the factor $.885 = 67.5 - 76.25$ will be proper for giving the reduction, but when the telescope 145.8 is used, then $1.912 = 145.8 - 76.25$ will be the suitable factor; for, as we have before asserted, the magnifying powers of telescopes are directly to each other as their focal lengths when the *same* eye-piece is used; and in this case the eye-piece will be the same at the *same position* of the eye-lens.

3

ANGULAR MEASURES OF THE DIAL

TAKEN AT 332 YARDS DISTANCE, BY A TELESCOPE OF 145 8 INCHES FOCAL LENGTH, AND 6 8 INCHES APERTURE

Micrometers used	Solar Power	Measure	Reduction	Angle	Difference from the Mean	Remarks
Spider's-line micrometer	149 4	3 55	Tab I	48" 47	+ 27	Vision good
Dioptric micrometer by Jones	170	0 675	Tab II	48 40	+ 20	Images well defined
Dioptric micrometer by Dollond	104	3 32	Tab III	47 77	− 43	Vision imperfect
Dynameter used as a micrometer	300	1 70	Tab IV	48 25	+ 05	
Rochon's method, Prism 1 ..	200	8 32	Tab V	47 53	− 67	Images well defined
2	200	5 22	Tab VI	47 61	− 69	Ditto
3 .	200	4 04	Tab VII	48 52	+ 32	Ditto
4 .	125	2 96	Tab VIII	48 11	− 09	Ditto
Cuneiform micrometer, Wedge *a*	200	4 45	Tab IX.	48 26	+ 06	Vision more perfect
b	200	2 35	Tab X	47 55	− 65	Ditto
a + *b*	200	1 70	Tab XI	48 13	− 07	Ditto
Polymetric micrometer, No 2	120	4 at pos. 122	× 12 .19 (= *V*)	48 76	+ 56	Vision good
Ditto	149 78	5 at pos 81	× 9 77 (= *V*)	48 95	+ 75	Constant $\frac{1463''}{P} = V$.
Ditto	181 35	6 at pos 38	× 8 06 (= *V*)	48 36	+ 16	See Table
Ditto	200 06	6 8 at pos 6	× 7 1 (= *V*)	48 48	+ 28	at page 132
Mean apparent angle				48 20	± 17	Mean difference
True angle 48" 20 × 9878 . ... =				47 61		$.9878 = \frac{f}{f'} = \frac{145\ 8}{147\ 6}$

4

ANGULAR MEASURES OF THE DIAL

TAKEN AT 332 YARDS DISTANCE, BY A TELESCOPE OF 67 5 INCHES FOCAL LENGTH AND 3 75 INCHES APERTURE

Micrometers used	Solar Power	Measure.	Reduction	Angle	Difference from the Mean	Remarks
Spider's-line micrometer.	69	1 64	2 16 × Tab I	48".34	+ .66	Single observation
Dioptric micrometer by Jones	79	0 31	2 16 × Tab II	48 02	+ 34	Ditto
Dioptric micrometer by Dollond	48	1 52	2 16 × Tab III	47 24	− 34	Ditto
Dynameter used as a micrometer	138	0 775	2 16 × Tab IV	47 52	− 16	Ditto
Rochon's method, Prism 1.	92	3 80	2 16 × Tab V	46 87	− 81	Ditto
2 ..	92	2 42	2 16 × Tab VI	47 69	+ 01	Ditto
3 .	92	1 82	2 16 × Tab VII	47 22	− 46	Ditto
4. .	58	1 35	2 16 × Tab VIII	47 39	− 29	Ditto
Cuneiform micrometer, Wedge *a*	92	2 05	2 16 × Tab IX	48 02	+ 34	Ditto
b	92	1 20	2 16 × Tab X	48 66	+ 98	Ditto
a + *b*	92	0 79	2 16 × Tab XI	48 27	+ 59	Ditto
Polymetric micrometer, No 2	61 87	2	× 1463 − 61 .87	47 29	− 39	Position 99
Ditto	92 75	3	× 1463 − 92 75	47 32	− 36	Position 7
Mean apparent angle...				47 68'	± 22	Mean difference
True angle 47' 68 × 9914				47 41		$9914 = \frac{f}{f'} = \frac{67\ 5}{67\ 88}$

5

ANGULAR MEASURES OF THE DIAL,

TAKEN AT 332 YARDS DISTANCE, BY A TELESCOPE OF 76 25 INCHES FOCAL LENGTH, AND 1 4 INCHES APERTURE

Micrometers used	Solar Power	Measure	Reduction	Angle	Difference from the Mean	Remarks
Spider's-line micrometer . .	78	1 86	§ XIX 6	48."28	+ 55	Single observation
Dioptric micrometer by Jones	80	0 35	§ XXXII 13	47 98	+ 25	Ditto
Dioptric micrometer by Dollond	54	1 70	1 .912 × Tab III	46 77	— 96	Ditto
Dynameter used as a micrometer	156	0 875	1 912 × Tab IV	47 49	— 24	Ditto
Rochon's method, Prism 1	104	4 36	§ XXXVII. 19	47 62	— 11	Ditto
2	104	2 72	Ditto	47 .47	— 26	Ditto
3 . .	104	2 06	Ditto	47 30	— 43	Ditto
4	65	1 53	Ditto	47 .58	— 15	Ditto
Cuneiform micrometer, Wedge *a*	104	2 36	§ XXXVIII 24	47 .15	— .58	Ditto
b	104	1 .50	Ditto.	48 27	+ .54	Ditto
a + *b*	104	0 91	Ditto	47 46	— 27	Ditto
Polymetric micrometer, No 2	90 46	3	× 16″ 173 (= *V*)	48 52	+ 70	See page 132
Ditto	60 37	2	× 24″.231 (= *V*)	48 .47	+ 74	See Ditto
			Mean apparent angle......	47 73	± 23	Mean difference
			True angle 47″.73 × 9936	47 42		$9936 = \frac{f}{f'} = \frac{76\ 25}{76\ .7384}$

6

ANGULAR MEASURES OF THE DIAL,

PLACED AT NEARLY A MILE DISTANCE, TAKEN BY A TELESCOPE OF 115 8 INCHES FOCAL LENGTH, AND 6 8 APERTURE

Micrometers used	Solar Power.	Measure.	Reduction	Angle.	Difference from the Mean.	Remarks.
Spider's-line micrometer	149 4	0 70	Tab I	9″ 56	+ .35	Single observation.
Dioptric micrometer by Jones .	170	0 13	Tab II	9 32	+ .11	Ditto.
Dioptric micrometer by Dollond	104	0 61	Tab III.	8 77	— 44	Ditto
Dynameter as a micrometer ...	300	0 31	Tab IV	8 80	— .41	Ditto
Rochon's method, Prism 1	200	1 60	Tab V.	9 14	— 07	Ditto
2 . ..	200	1 06	Tab VI	9 67	+ 46	Ditto
3 ..	200	0 78	Tab VII	9 37	+ 16	Ditto.
4	125	0 56	Tab VIII.	9 10	— 11	Ditto
Cuneiform micrometer, Wedge *a*	200	0 83	Tab IX	9 01	— 20	Ditto
b	200	0 53	Tab X.	9 26	+ 05	Ditto
Ocular micrometer, Prism 2*	143 1	Lens 4	Tab IV p 220.	9 29	+ 08	*Position 103 Tab XII
3 .	188 5	Lens 4	Tab Ditto	9 28	+ 07	Position 14. Ditto.
4	255 2	Lens 3.	Tab Ditto	9 28	+ 07	Position 56. Ditto.
Reticulated disc	211	1 30	× 7″ 0 (= *V*)	9 10	— .11	*V* = 126″ — 18 divisions.
			Mean angle	9 21	± .09	Mean difference.

7.

ANGULAR MEASURES OF THE DIAL,

PLACED AT THE DISTANCE OF NEARLY A MILE, TAKEN BY A TELESCOPE OF 76 25 INCHES FOCAL LENGTH, AND 4 1 APERTURE

Micrometers used.	Solar Power	Measure	Reduction	Angle.	Difference from the Mean	Remarks
Spider's-line micrometer	78	0 36	§ XIX 6	9 39	+ 16	Single observation.
Dioptric micrometer by Jones..	89	0 70	§ XXXII 13	9 59	+ 36	Ditto
Dioptric micrometer by Dollond	54	0 30	1 012×Tab III	8 77	— 46	Ditto
Dynameter, used as a micrometer	156	0 065	1 012×Tab. IV	8 91	— 32	Ditto
Rochon's method, Prism 1	104	0 80	§ XXXVII 19	8 74	— 49	Ditto
2	104	0 53	Ditto	9 25	+ 02	Ditto
3 .. .	104	0 40	Ditto	9 18	— 05	Ditto
4 . .	65	0 305	Ditto	9 48	+ 25	Ditto
Cuneiform micrometer, Wedge *a*	104	0 47	§ XXXVIII 24	9 39	+ 16	Ditto
b	104	0 20	Ditto	9 34	+ 11	Ditto
Ocular micrometer, Prism 1*	91 3	Lens 4	Tab IV p 229	9 13	— .10	*Position 36, in Tab I p 226
2 .	143 7	Lens 3	Ditto	9 28	+ 05	Position 26, in Ditto
2 ..	143 .8	Lens 2	Ditto	9 29	+ 06	Position 65, in Ditto
3 .	186 2	Lens 1	Ditto	9 44	— 21	Position 86, in Ditto
4	258 3	Lens 1	Ditto	9 23	00	Position 4, in Ditto
			Mean angle	9 23	± 13	Mean difference

8.

ANGULAR MEASURES OF THE DIAL,

PLACED AT THE DISTANCE OF NEARLY A MILE, TAKEN BY A TELESCOPE OF 67 5 INCHES FOCAL LENGTH, AND 3 70 APERTURE

Micrometers used	Solar Power	Measure	Reduction.	Angle	Difference from the Mean	Remarks
Spider's-line micrometer... .	69	0 35	2 16×Tab I	10 32	+1 18	Before and behind zero
Dioptric micrometer by Jones..	70	0 055	2 16×Tab II.	8 51	— 63	Before and behind zero
Dioptric micrometer by Dollond	48	0 30	2 16×Tab III	9 33	+ 19	Before and behind zero
Dynameter, used as a micrometer	138	0 15 ±	2 16×Tab, IV	9 20	+ 06	Vision imperfect
Rochon's method, Prism 1. . ..	92	0 72	2 16×Tab V	8 88	— 26	Vision better
2 .. .	92	0 43	2 16×Tab VI	8 47	— 67	Ditto
3 .. .	92	0 31	2 16×Tab VII	8 04	—1 10	Ditto
4	58	0 28	2 16×Tab VIII	9 83	+ 69	Ditto
Cuneiform micrometer, Wedge *a*	92	0 39	2 16×Tab IX	9 14	00	Measure difficult
b	92	0 23	2 16×Tab X	8 66	— 48	Measure more difficult
Ocular micrometer, Prism 1 ..	87 8	Lens 4	832"5 — 87 8	9 48	+ 34	Position 4
1 ..	90 9	Lens 3	832 5 — 90 9	9 15	+ 01	Position 113
2 ..	139 8	Lens 2	1330 4 — 139 .8	9 51	+ 37	Position 24
3 .	184 4	Lens 1	1750 7 — 184 4	9 49	+ 35	Position 56
			Mean angle.	9 14	± 227	Mean difference

9. In making the six preceding series of observations we could not avail ourselves of Dollond's object-glass micrometer, because it is adapted to a smaller telescope, nor of his spherical prism, because the greatest angle it will measure was too small. With respect to the catoptric micrometer, we could not procure one, and the binocular was not attempted, as being found liable to give results depending on the state and management of both eyes, that require some practice to regulate, and as being intended only for observations of very small stars that will not bear illumination, which conditions apply also to the lamp-micrometer. Dr. Brewster's patent micrometer is not competent to measure very small angles, even if it had sufficient magnifying power; and the reticulated diaphragms and annular or circular micrometers are adapted to determine the measure of an arc from the time in which some motion is performed by the body observed, otherwise we should have had all these different appendages applied to our telescopes. If it should be remarked that the large telescope does not exceed in accuracy the two smaller ones so much as might have been expected, it must be recollected that the three instruments are equally perfect in their construction, and differ only in the powers they possess of producing a large and luminous image well defined; in measurements of this kind, as an increase of magnifying power is accompanied with a corresponding decrease of light, as well as of indistinctness at the edges of the object, the magnifying power may be carried too far to be useful; the main object is to have an image large and distinct enough to admit of measurement. In the first position at 332 yards the dial was well seen, and sufficiently large to be correctly measured by any of the telescopes, which therefore at that distance agree in a surprising manner; but at the greater distance of 1709 yards, or at somewhat more than five times the first distance, the intensity of light was diminished more than twenty-five times, and here the advantage of superior power and light was manifest, though the dial was still visible in full day-light to the smallest telescope. The curved face of the dial proved to be unfavourable to the accuracy of the observations, by dispersing the light so as to render the circumference less luminous than the central portion, which, with some of the micrometers, particularly those that have divided lenses, rendered the contacts somewhat dubious at the greater distance, especially when the smallest telescope was used. On this account we determined to measure the breadth of a single pane in a blank window, having a black ground inclosed by four white boundary lines like those of a modern sash, and the following resulting measures taken by our largest telescope will show what confidence may be placed in its powers, when employed under favourable circumstances.

10 ANGULAR MEASURES OF AN OBJECT,

SEEN AT THE DISTANCE OF 1709 YARDS, TAKEN BY A TELESCOPE OF 145.6 INCHES FOCAL LENGTH AND 6.3 INCHES APERTURE

Micrometers used.	Solar Powers	Measure.	Reduction	Angle.	Difference from the Mean	Remarks
Spider's-line micrometer .	140 4	2.00	Tab I	27" 31	− 01	A mean of several
Dioptric micrometer by Jones	170	0 38	Tab II	27 25	− 05	Ditto
Dioptric micrometer by Dollond	104	1 90	Tab III.	27 34	+ 04	Ditto
Dynameter used as a micrometer	300	0 96	Tab IV	27 24	− 06	Ditto
Rochon's method, Prism 1......	200	4 78	Tab. V	27 31	+ 01	Ditto
2. ...	200	3 00	Tab VI	27 36	+ 06	Ditto
3	200	2 27	Tab VII	27 26	− 04	Ditto
4 ..	125	1.08	Tab VIII	27.30	00	Ditto
Cuneiform micrometer, Wedge *a*	200	2 52	Tab IX	27.33	+ 03	Ditto
b	200	1 50	Tab X	27 26	− 04	Ditto
a+*b*	200	0 965	Tab XI	27 32	+ 02	Ditto
Ocular micrometer, Prism 4 .	173 7	lens 4	Tab IV p 220×2	27 29	− 01	Position 43 Tab XII
Ocular wedges *a*+*b*	145.6	lens 4.	3977" 5−145 6	27 32	+ 02	Position 98 Tab XII
Net micrometer..	211	3 9	×7.00 (=*V*)	27 30	.00	*V*=126".—18 squares
Polymetric micrometer No 2...	160.56	3	×9.11 (=*V*)	27 33	+ 03	Pos 62 Tab p 132×1 012
			Mean Angle	27 30	± 0133	Mean difference

11. When the ocular micrometer was used with its prism 4, having the largest constant angle, its greatest measure did not extend much beyond one-half of the pane, which formed the object, on which account it was bisected by estimation, and the resulting tabular measure was doubled thus, 13".645 × 2 = 27" 29. An estimation of one-half of a small object is capable of greater accuracy than an inexperienced observer may imagine, for the apparent difference of two halves is affected by the slightest quantity + or −, as we may perceive from the use of a reading micrometer, that has a fine line bisecting a small angle with more accuracy, than if it were placed over a dividing stroke.

12.

MICROMETRICAL TABLES

ADAPTED TO A TELESCOPE OF 145 8 INCHES FOCAL LENGTH

TABLE I TROUGHTON'S SPIDER'S-LINE MICROMETER

Revolutions				Parts of a Revolution									
1	0′ 13″ 65	21	4′ 46″ 75	1	0″ 14	21	2″ 87	41	5″ 60	61	8″ .33	81	11″ 05
2	0 27 31	22	5 0 .41	2	0 27	22	3 00	42	5 .73	62	8 46	82	11 19
3	0 40 96	23	5 14 06	3	0 41	23	3 14	43	5 87	63	8 .60	83	11 33
4	0 54 62	24	5 27 72	4	0 55	24	3 27	44	6 00	64	8 73	84	11 46
5	1 8 27	25	5 41 37	5	0 68	25	3 41	45	6 14	65	8 87	85	11 60
6	1 21 93	26	5 55 03	6	0 82	26	3 55	46	6 27	66	9 .00	86	11 73
7	1 35 58	27	6 8 68	7	0 96	27	3 68	47	6 41	67	9 14	87	11 87
8	1 49 24	28	6 22 34	8	1 09	28	3 82	48	6 55	68	9 28	88	12 01
9	2 2 89	29	6 35 99	9	1 23	29	3 96	49	6 69	69	9 .42	89	12 15
10	2 16 55	30	6 49 .65	10	1 37	30	4 10	50	6 83	70	9 56	90	12 .29
11	2 30 20	31	7 3 30	11	1 51	31	4 .23	51	6 96	71	9 69	91	12 43
12	2 43 86	32	7 16 96	12	1 64	32	4 37	52	7 10	72	9 83	92	12 57
13	2 57 51	33	7 30 61	13	1 78	33	4 51	53	7 23	73	9 96	93	12 70
14	3 11 17	34	7 44 .27	14	1 92	34	4 64	54	7 37	74	10 10	94	12 84
15	3 24 .82	35	7 57 92	15	2 .05	35	4 78	55	7 .51	75	10 23	95	12 97
16	3 38 48	36	8 11 58	16	2 18	36	4 .92	56	7 .64	76	10 .37	96	13 .11
17	3 52 13	37	8 25 23	17	2 33	37	5 05	57	7 78	77	10 51	97	13 .24
18	4 5 79	38	8 38 89	18	2 46	38	5 19	58	7 92	78	10 65	98	13 38
19	4 19 44	39	8 52 54	19	2 60	39	5 33	59	8 05	79	10 78	99	13 .52
20	4 33 10	40	9 6 20	20	2 73	40	5 46	60	8 19	80	10 92	100	13 .65

TABLE II. DIOPTRIC MICROMETER BY T JONES

Revolutions		Parts of a Revolution									
1	1′ 11″ 7	1	0″ 72	21	15″ 06	41	29 40	61	43″ 74	81	58″ 08
2	2 23 .4	2	1 43	22	15 77	42	30 11	62	44 .45	82	58 79
3	3 35 1	3	2 15	23	16 49	43	30 83	63	45 17	83	59 51
4	4 46 8	4	2 87	24	17 21	44	31 55	64	45 89	84	60 23
5	5 58 5	5	3 58	25	17 93	45	32 27	65	46 60	85	60 94
6	7 10 2	6	4 30	26	18 64	46	32 98	66	47 32	86	61 66
7	8 21 9	7	5 02	27	19 36	47	33 .70	67	48 04	87	62 38
8	9 33 6	8	5 74	28	20 08	48	34 42	68	48 76	88	63 10
9	10 45 .3	9	6 45	29	20 79	49	35 13	69	49 47	89	63 81
10	11 57 0	10	7 17	30	21 51	50	35 85	70	50 .19	90	64 53
11	13 8 7	11	7 89	31	22 23	51	36 57	71	50 .91	91	65 .25
12	14 20 4	12	8 60	32	22 94	52	37 .28	72	51 .62	92	65 96
13	15 32 1	13	9 32	33	23 66	53	38 00	73	52 .34	93	66 68
14	16 43 8	14	10 04	34	24 38	54	38 .72	74	53 .06	94	67 40
15	17 55 5	15	10 76	35	25 10	55	39 44	75	53 .77	95	68 12
16	19 7 .2	16	11 47	36	25 81	56	40 15	76	54 49	96	68 83
17	20 18 9	17	12 19	37	26 53	57	40 87	77	55 .21	97	69 55
18	21 30 .6	18	12 91	38	27 .25	58	41 59	78	55 93	98	70 27
19	22 42 3	19	13 62	39	27 96	59	42 30	79	56 .64	99	70 .98
20	23 54 0	20	14 34	40	28 .68	60	43 .02	80	57 .36	100	71 .70

TABLE III.

DIOPTRIC MICROMETER BY G DOLLOND

	Inches.	Tenths.	Hundredths
1	0′ 14″ 39	1″ 44	0″ 14
2	0 28 77	2 88	0 29
3	0 43 ,10	4 32	0 43
4	0 57 55	5 76	0 58
5	1 11 94	7 19	0 72
6	1 26 32	8 63	0 86
7	1 40 71	10 ,07	1 01
8	1 55 10	11 51	1 15
9	2 9 48	12 95	1 30
10	2 23 ,87	14 39	1 44

TABLE IV

DYNAMETER AS A MICROMETER

	Divisions	Tenths	Hundredths.
1	0′ 28″ 38	2″ 84	0″.28
2	0 56 76	5 68	0 57
3	1 25 15	8 52	0 ,85
4	1 53 53	11 35	1 ,14
5	2 21 91	14 19	1 42
6	2 50 ,29	17 03	1 70
7	3 18 67	19 87	1 99
8	3 47 06	22 71	2 27
9	4 15 44	25 54	2 55
10	4 43 82	28 38	2 84

TABLE V

ROCHON'S PRISM 1

	Inches	Tenths	Hundredths.
1	5″ 71	0″ 57	0″.06
2	11 42	1 14	0 ,11
3	17 13	1 71	0 17
4	22 85	2 28	0 ,23
5	28 56	2 86	0 29
6	34 27	3 43	0 34
7	39 99	4 00	0 40
8	45 70	4 57	0 46
9	51 41	5 14	0 51
10	57 13	5 71	0 57

TABLE VI

ROCHON'S PRISM 2

	Inches	Tenths	Hundredths
1	9′ 12	0″,91	0′ 09
2	18 ,24	1 82	0 18
3	27 ,36	2 74	0 27
4	36 48	3 65	0 ,36
5	45 61	4 56	0 46
6	54 73	5 47	0 55
7	63 85	6 38	0 64
8	72 97	7 30	0 73
9	82 09	8 21	0 82
10	91 22	9 12	0 91

TABLE VII

ROCHON'S PRISM 3

	Inches.	Tenths	Hundredths
1	12″ 01	1″ 20	0″ 12
2	24 02	2 40	0 24
3	36 03	3 60	0 36
4	48 04	4 80	0 48
5	60 05	6 00	0 60
6	72 06	7 21	0 72
7	84 07	8 41	0 84
8	96 08	9 61	0 96
9	108 09	10 81	1 08
10	120 10	12 01	1 20

TABLE VIII

ROCHON'S PRISM 4

	Inches	Tenths.	Hundredths
1	16″ 25	1″ 63	0′ 16
2	32 51	3 25	0 32
3	48 76	4 88	0 49
4	65 02	6 50	0 65
5	81 27	8 13	0 81
6	97 53	9 75	0 97
7	113 78	11 38	1 ,14
8	130 04	13 00	1 30
9	146 29	14 63	1 46
10	162 55	16 25	1 63

TABLE IX

WEDGE *a*

	Inches	Tenths	Hundredths
1	10″ 84	1″ 08	0″ 11
2	21 69	2 17	0 22
3	32 53	3 25	0 33
4	43 ,38	4 34	0 43
5	54 22	5 42	0 ,54
6	65 07	6 51	0 65
7	75 91	7 59	0 76
8	86 76	8 68	0 87
9	97 ,60	9 76	0 98
10	108 44	10 84	1 08

TABLE X.

WEDGE *b*

	Inches	Tenths	Hundredths.
1	17′.47	1″ 75	0″.18
2	34 91	3 49	0 35
3	52 41	5 24	0 52
4	69 88	6 99	0 70
5	87 35	8 74	0 87
6	104 82	10 48	1 05
7	122 29	12 23	1 22
8	139 76	13 98	1 40
9	157 23	15 72	1 57
10	174 70	17 47	1 75

TABLE XI

WEDGES *a*+*b*

	Inches	Tenths	Hundredths
1	28″ 31	2′ 83	0″ 28
2	56 63	5 66	0 57
3	84 94	8 49	0 85
4	113 26	11 33	1 13
5	141 57	14 16	1 42
6	169 89	16 99	1 70
7	198 20	19 82	1 98
8	226 51	22 65	2 27
9	254 83	25 48	2 55
10	283 14	28 31	2 83

TABLE XII.

THE MAGNIFYING POWERS OF A VARIABLE EYE-PIECE,

WITH FOUR SEPARATE EYE LENSES, AND A CIRCLE FOR POSITIONS, USED WITH A TELESCOPE OF 115 8 INCHES FOCAL LENGTH

Scale	Eye-Lenses				Scale	Eye-Lenses				Scale	Eye-Lenses			
	1	2	3	4		1	2	3	4		1	2	3	4
0	505 5	341 6	307 3	195.6	42	436 2	299 2	268 2	174 2	84	366 9	256 8	229 1	152.8
1	503 9	340 6	306 4	195 1	43	434 5	298 2	267 3	173 7	85	365 2	255 8	228 2	152.3
2	502 2	339 6	305 5	194 6	44	432 9	297 2	266 4	173 2	86	363 6	254 7	227 3	151 7
3	500 5	338 6	304 5	194 1	45	431 2	296 2	265 4	172 7	87	361 9	253 7	226 3	151 2
4	498 9	337 6	303 6	193 6	46	429 6	295 1	264 5	172 1	88	360 3	252 7	225 4	150 7
5	497 2	336 6	302 7	193.1	47	427 9	294 1	263 6	171 6	89	358 6	251 7	224 5	150 2
6	495 6	335 5	301 7	192 5	48	426 3	293 1	262 6	171 1	90	357 0	250 7	223 6	149 7
7	493 9	334 5	300.8	192 0	49	424 6	292 1	261 7	170.6	91	355 3	249 7	222 6	149 2
8	492 3	333 5	299 9	191 5	50	423 0	291 1	260 8	170 1	92	353 7	248 7	221 7	148 7
9	490 6	332 5	299 0	191 0	51	421 3	290 1	259 9	169 6	93	352 0	247 7	220 8	148 2
10	489 0	331.5	298 0	190 5	52	419.7	289 1	258 9	169 1	94	350 4	246 7	219 8	147 7
11	487 3	330 5	297 1	190 0	53	418 0	288 1	258.0	168 6	95	348 7	245 7	218 9	147 2
12	485 7	329.5	296.2	189 5	54	416 4	287 1	257 1	168 1	96	347 1	244 6	218 0	146 6
13	484 0	328 5	295 2	189.0	55	414.7	286 1	256 1	167 6	97	345 4	243 6	217 1	146 1
14	482 4	327 5	294 3	188 5	56	413.1	285 0	255 2	167 0	98	343 8	242 6	216 1	145 6
15	480 7	326 5	293 4	188 0	57	411 4	284 0	254 3	166 5	99	342 1	241 6	215.2	145 1
16	479 1	325 4	292 4	187 4	58	409 8	283 0	253 3	166 0	100	340 5	240 6	214 3	144 6
17	477 4	324 1	291 5	186 9	59	408 1	282 0	252 4	165 5	101	338 8	239 6	213 4	144 1
18	475 8	323 4	290 6	186 4	60	406 5	281 0	251 5	165 0	102	337.1	238 6	212 4	143 6
19	474 1	322 4	289 6	185 9	61	404 8	280 0	250 5	164 5	103	335.5	237 6	211 5	143 1
20	472 5	321 4	288 7	185 4	62	403 2	279.0	249.6	164 0	104	333 9	236 6	210 6	142.6
21	470 8	320 4	287 8	184.9	63	401 5	278 0	248.7	163 5	105	332 2	235 6	209 6	142.1
22	469 2	319 4	286 8	184 4	64	399 9	277 0	247 7	163 0	106	330 6	234 5	208 7	141 5
23	467 5	318 4	285 9	183 9	65	398 2	276 0	246 8	162 5	107	328 9	233.5	207 8	141 0
24	465 9	317.4	285.0	183 4	66	396 6	274 9	245.9	161 9	108	327 3	232 5	206 9	140 5
25	464 2	316 4	284 0	182 9	67	394 9	273 9	244 9	161 4	109	325 6	231 5	205 9	140 0
26	462 6	315 3	283 1	182 3	68	393 3	272 9	244 0	160 9	110	324.0	230 5	205 0	139 5
27	460 9	314 3	282 2	181 8	69	391 6	271 9	243.1	160 4	111	322 3	229 5	204.0	139 0
28	459 3	313 3	281 2	181 3	70	390 0	270 9	242 2	159 9	112	320 6	228.5	203 1	138 5
29	457 6	312 3	280 3	180 8	71	388 3	269 9	241 2	159.4	113	318 9	227 5	202 2	138 0
30	456 0	311 3	279 4	180 3	72	386 7	268 9	240 3	158.9	114	317 3	226 5	201 3	137.5
31	454 3	310 3	278 4	179 8	73	385 0	267 9	239 4	158 4	115	315 7	225 5	200 3	137 0
32	452 7	309 3	277 5	179 3	74	383 4	266 9	238 4	157 9	116	314 1	224 4	199 4	136 4
33	451.0	308 3	276 6	178 8	75	381 7	265 9	237 5	157 4	117	312 4	223 4	198.5	135 9
34	449 4	307 3	275 6	178 3	76	380 1	264 8	236.6	156 8	118	310 8	222 4	197 5	135 4
35	447 7	306 3	274 7	177 8	77	378 4	263 8	235 7	156 3	119	309.1	221 4	196 6	134 9
36	446 1	305 2	273 8	177 2	78	376 8	262 8	234 7	155 8	120	307.5	220 4	195 7	134 4
37	444 4	304 2	272 8	176 7	79	375 1	261 8	233 8	155 3	121	305 8	219 4	194 7	133 8
38	442 8	303 2	271 9	176 2	80	373 5	260 8	232 9	154 8	122	304 2	218 4	193 8	133 3
39	441 1	302 2	271 0	175 7	81	371 8	259 8	232 0	154 3	123	302.5	217 4	192 9	132 8
40	439 5	301 2	270 1	175 2	82	370 2	258 8	231 0	153 8	124	300 9	216 4	191 9	132 3
41	437.8	300 2	269 2	174 7	83	368 5	257.8	230 1	153 3	125	299 2	215 4	191 0	131 8

§ XLVI. ON CLAMPS AND TANGENT SCREWS. [Plate IV.]

1. In describing the various micrometers that have fallen under our notice we have purposely passed in silence over those appendages of the telescope, which retain it in a given position, and which afford the convenient means of giving a slow motion in any direction that may be required. This desirable purpose is accomplished frequently by a racked wheel and endless screw, by a wheel and pinion, or by a pinion and racked bar, all which are contrivances of common occurrence, that need no particular explanation: but when the observation requires a very slow and delicate motion, the tangent screw, in conjunction with a clamp of some kind, is found the most safe and commodious mechanism for holding one part of an instrument, while motion is imparted to another in a smooth and gradual manner. We have therefore reserved our account of clamps and tangent screws to be given in a separate section, which mode of describing will save the trouble of repeated details respecting this important part of the more complex instruments, when they present themselves in their turns to our attention. Clamps considered singly, and applied merely to the secure fixing of an instrument in a given position, may be of any shape or strength that the work it has to perform may require, and the mechanic employed will never be at a loss how to proceed with the formation of his materials, and application of his clamping screw; but when an adjustment to the position, or an equable steady motion is moreover to be effected by mechanical means, he will do well to avail himself of the ingenuity of some mechanist of superior skill, either a predecessor or contemporary, if he has not sufficient resources within himself. The astronomer also, who is to use an instrument requiring the aid of a tangent-clamp, will be glad to have his choice directed by a comparative statement of the different clamping apparatus that modern improvements have produced. For these reasons we have devoted our attention, among other practical matters, to the construction of the different contrivances that come under the denomination of clamps, the most useful of which we will now endeavour to describe in the order of their priority of invention.

2. *Old Clamp.*—The first clamp, having a tangent-screw worthy of notice, is that which was applied by Graham, the Sissons, and Bird to the astronomical quadrants, that preceded our circular instruments. As the quadrant remained stationary, and the telescope alone moved along the graduated arc, the clamp was necessarily attached to the eye-end of the telescope, and during the quick motion of elevation was made to slide along the divided limb of the instrument, in conjunction with the vernier indicating the rough measure. A sketch of such clamp is given in figures 11 and 12 of our Plate IV, which will suffice to explain its construction and mode of action, the same letters denoting the same parts. A portion of the quadrant's limb lies between the letters *a* and *b*, to which the clamping plates *c* above and *c'* below, forming a hold-fast, are screwed occasionally by the screw *d* with a milled head visible in both figures; *e* shows the eye-end of the telescope's tube, lying at right angles to the curve of the limb, and having its centre of motion at the centre of the curve; *g h* is a metallic axis, with a milled head, turn-

ing in a tapped ball *m*, made fast to the clamping-plate *c*, the remote end *h* of this tangental axis is formed into a screw, acting in a spherical nut *i* attached to the telescope's tube *k*, holding the eye-piece *l*; the axis has a shoulder at each side of the fixed ball *m*, and cannot recede or advance by the operation of the screw, therefore the eye-end of the telescope obeys the force applied by its action, and approaches the clamping screw *d* gradually, as the tangent-screw revolves, carrying with it the annexed vernier-plate *n o*, in contact with the graduated quadrantal arc, thereby indicating minutes of a degree. The tapped nut *i*, in which the screw on the axis acts, has a circular motion round a pivot, that accommodates the tangental position of the screw, as it regards the clamp, which is favourable to its quick motion, when the clamping-screw is released; but in some quadrants which we have examined, long usage has rendered the fittings very imperfect. In quadrants of a large radius the end *g* of the tangent-screw, has a small circular plate divided so as to subdivide the minute into small portions of 2″, 5″, or 10″, as the radius will allow, which quantities are indicated by a fixed pointer; but as this screw has to sustain and pull all the weight of the preponderating end of the telescope, the threads of the screw in long use make large indentations in the nuts, and become too loose to be serviceable in giving small additional measures; and therefore this method of reading is very properly discontinued. The two screws *p* and *q*, seen under figure 12, attach an elastic slip of brass to the vernier-plate, which, by being bent, applies its remote ends to the under face of the limb, and produces pressure enough to regulate the uniformity of the vernier's forward motion, produced by the action of the tangental screw, but has not the effect of preventing a loss of motion in the commencement of a retrograde revolution of the screw. In this application of a vernier plate, the middle part is so covered by the telescope, that it has frequently, as in our figure 11, two vernier-scales, one of which reads the minutes on the arc of excess, when the observation is made near either the zenith or horizon, as the case may be.

3. *The common Clamp*.—The figures 13, 14, and 15, present different views of an ordinary sextant clamp, which is also sometimes applied to circular instruments. Figure 13 is a representation of the parts seen in connexion with the reverse face of the limb, having no graduations; figure 14 shows the same parts as seen when the clamping screw and friction spring *f* are detached; and figure 15 exhibits a lateral view to an eye directed towards the centre of the instrument. To prevent confusion in our description of this clamp, we will adopt the same letters of reference, which we have used for explaining its predecessor, and will apply them, as before, to the same parts in each figure. The leading features of difference in the two clamps are, that the clamping plates embrace only the circumference of the limb, and that the clamping takes place at the middle of the vernier's scale, in the apparatus now before us, whereas, in the older clamp, one of the plates embraced the whole breadth of the limb, and was clamped at such a distance from the vernier-plate, as admitted of distortions in their relative positions, in the act of clamping or of unclamping. In this common clamp the parts are more compactly united, but still the screw has the same work to perform, and the friction opposed to its action will sometimes interfere with the uniform smoothness of the vernier's motion, which is its principal objection. In figures 13 and 14 the extreme end of the radial bar, usually called the vernier-bar, terminates with a fork, over which the two ends of the tangent-screw's axis lie, and have each a milled head for the convenience of the observer, who may thus use either hand, as may be most convenient. The clamping-plate *c* has its sides formed of parallel curves which confine it to

move between the exterior edge of the limb *a b*, and an edge-bar *r s*, which connects the extreme ends of the forks already mentioned, and the space it can move over by the tangent-screw is limited by the distance between the forks. at the middle of this clamping-plate *c*, a part of the inner edge of which lies over the outer edge of the graduated face of the limb, there is a square notch forming a bed for the second clamping-plate *c'*, which takes hold of the under face of the limb, as seen in figure 14, and the clamping-screw *d*, passing through the hole made in this small plate *c*, enters and acts with the tapped hole made in the plate *c*, so that turning this screw forwards or backwards will fix or release the vernier in regard to its connexion with the limb. The spherical nut *m* is screwed fast to one of the forks of the vernier bar, and the similar nut *i* is in like manner made fast to the clamping-plate *c*, therefore when the piece *c* is clamped to the limb, and the tangent-screw turned in either direction, by one of the milled heads, *g* or *h*, the vernier must necessarily move in the same direction, accordingly as the two spherical nuts approach or recede from one another by the slow motion. But when the clamping-screw *d* is released, the vernier is at liberty to be pushed by a quick motion to any required position, in which it may be fixed, and then finally adjusted by the slow motion of the tangent-screw, as the observation may require. A portion of the graduated face of the limb is represented by figure 19, together with the scale of the vernier attached to a similar clamp, in which figure is seen an oblong plate covering the ends of the vernier's forks, to which it is fixed by the four screws that appear, a long opening is made lengthwise at the middle of this plate, which allows the fifth or middle screw to enter, that fixes the small elastic oblong slip of brass, covering the opening, to the contiguous clamping plate *c*, and keeps it in its place as the vernier moves. The double pressure occasioned by the slip of brass just described, and by the friction-spring *f*, seen detached at the side of figures 13 and 14, and opposed to the other, keeps all the parts of the clamp compactly in their places, while they conceal the interior and acting portions of the mechanism.

4. *Clamp with Balls.*—The clamp represented by figures 16, 17, and 18, is of an improved construction, and is lately brought into use in England by T. Jones and Simms, with some variations from the clamp that has been used on the Continent. In this construction the tapped nuts are not only globular, but are slit half way through to give elasticity to the female screw, and to ensure close action, and are bedded in a corresponding concave cup, having a central hole into which a pivot under the ball enters, and affords the means of self-adjustment to the direction of the tangent-screw. In this clamp the cup for the ball *i* is attached to the solid part of the vernier bar, as seen in figure 18, where the ball is displaced to show its pivot, but the cup *m'* of the ball *m* is placed on the inner end of the clamping portion of the mechanism, seen in its place in figure 17. The cock *l*, represented by figure 17, is screwed to the vernier-bar by its capstan screw, as shown in figure 16, and the upper cup is formed on its inferior face, the pressure of which may at any time be regulated by the capstan-screw, placed near a pair of steady pins fixed at the extreme end of the cock, of which only one is visible in the figure, for the lower face of the crank is not in contact with the vernier-bar, except at its remote end. This adjustment of the pressure of the two cups containing the ball constitutes the principal improvement, in conjunction with the slit of the ball, for by their joint means, there is not the least loss of motion in the vernier in changing the direction of the tangent-screw's motion, neither is there any tendency in such fittings to produce jerks in the motion, to which the older clamps

are more or less liable. The clamping-plate *c* (fig. 16) receives the end of the clamping screw *d*, which is below the limb, after it has passed through the lower clamping-piece not seen, and the outer edge of the limb is embraced between them, when the screw is made fast; the two steady pins, near the end of the clamping-screw, ensure the proper position for biting. At right angles to this clamping-piece, which carries another cock for the second cup of the ball *m*, an arm, *r s*, formed by a pair of circular edges, is rabbeted into a groove formed on the limb, which keeps the vernier to its place, while the tangent-screw draws the ball *i*, and with it the vernier, towards the ball *m*, by a smooth and uniform motion. If an extremely slow motion were required, it is obvious that this apparatus would admit of the application of a Hunter's screw.

5. *Dollond's Clamp.*—When the astronomer royal found some inconvenience in the use of the ordinary clamp, as applied by Troughton to the Greenwich circle, the clamp which we described in our last paragraph (4.) was substituted by T. Jones; but still there appeared to be room for further improvement, so far as the free motion of the circle was concerned; and G. Dollond was employed to adapt a clamp having a strong spiral spring, which relieves the mechanical action of the screw from the load which it had to pull in the former constructions, and ensures the immediate obedience of the circle to the least possible turn of the screw. This clamp, which we have called Dollond's, because it is known by that name at Greenwich, borrows its principle from an old clamp constructed by Ramsden, inasmuch as the spring is concerned, but the mode of applying it with success may be considered as a distinct contrivance. This clamp is represented in miniature by the figures 22 and 23, the former of which exhibits the external appearance, and the latter a section of it across the middle. The strong brass plate *a b* is made fast to the masonry penetrated by the axis of the circle, and always keeps its station: to this plate two strong bearing pieces *m* and *f* are made fast by screws entering the concealed face, and carry each a tube of brass, *i* and *l*, the latter of which is represented as being transparent, to shew the strong spiral spring coiled within it, and the former has a female screw formed within it, to admit and act with the screw cut on the axis *g h*, which holds a milled nut at each end of it: the two tubes *i* and *l* are made fast to their respective bearing-pieces *m* and *f*, and an inner tube *e*, enters the latter till it comes in contact with the spring. The pieces *c* and *c′*, as before, are the clamping-pieces which contain the edge of the circle between them, and the screw *d*, taking hold of the piece *c′* draws it towards the stronger piece *c*, and fixes the circle by making them bite its edge. The four pins seen apparently uniting the two parts *c* and *c′*, not only guide the biting parts in opening and shutting, by being attached to one piece, and entering holes made to receive them in the other, but two of them have small concealed spiral springs, lying at the bottoms of the holes, opposed to their extremities, which push one of the biting-pieces, *c*, from a state of contact with the limb of the circle, whenever the clamping-screw *d* is released; and to allow of a due separation in opening, the solid part of *c*, denoted by *n*, through which the tangent-screw *g h* passes, is perforated with an elongated hole. The plate *a b* has an oblong opening cut along it, through which the piece *k* passes, which carries an elastic slip of metal on the posterior face of the plate, and which comes nearly in contact with the extreme edge of the circle's limb, as seen in fig. 23. The biting piece *c′* is directed in its motion by its connexion with the edge-bar *r s*, which slides along the inner edge of the circle's limb, while the tangent-screw revolves. This

screw has a shoulder and collar in contact with that side of the solid stem *n*, of the clamping-piece *c*, which is nearest to its threads, against which collar the spiral spring at the opposite side constantly presses, through the medium of the inner tube *e*, for accordingly as this screw is turned forwards or backwards, the spiral spring contained between the two ends of the interior and exterior tubes, *e* and *l*, always exerting its power to relax itself, forces the opposite end of the inner tube against the solid part *n*, and keeps it close to the collar of the screw, as the screw itself enters the tube *i*, hence the biting-pieces, *c* and *c'*, holding the limb of the circle, and being connected with the part *n* thus acted on by the spiral spring, partake of its motion, and carry the circle with them in a forward direction, the pressure of the spring being constantly exerted. On the contrary, when the tangent-screw is turned back, it recedes from its tube *i*, and the collar now acts in its turn against the end of the inner tube *e*, and consequently against the spiral spring urging it, which spring it again compresses, and carries the solid piece *n* back again, together with the biting-pieces *c* and *c'*, which embrace the limb of the circle, and therefore a backward motion, in opposition to the force of the spring, is produced in the circle, without any loss of motion from a change of direction, and this property, derived from the spring's constant exertion of its force, constitutes the improvement in the performance of the clamp for, by the use of this clamp, a star can be brought into a position to be bisected by a horizontal line in the eye-piece with the greatest ease, and, what is equally important, the star will retain its place when the action of the screw terminates. The distance through which the circle can be moved by the tangent-screw, depends on the length of the concealed oblong perforation made in the plate *a b*, along which the piece *h* slides, and the length of this opening again depends on the distance between the inner ends of the two fixed tubes *i* and *l*, which determine the limits of the screw's action. From this description it is easy to perceive, that, when the clamp is released from the circle, a force, superior to that exerted by the screw, applied by the hand against the solid part *n* of the clamp, in a direction from *g* towards *h*, will compress the screw, and carry the clamping portion of the apparatus towards *h*, till *n* meets with the end of the fixed tube *j*, which stops its further motion. hence, when the clamp holds the limb, the circle itself will admit of a small backward motion at any time, when acted upon in that direction by a power greater than the force of the spiral spring, but this is no objection to the good performance of the clamp, and may become a safeguard from any blow that the circle may receive by accident in a direction opposite to the spring's action. Mr Dollond has lately reduced the size of this clamp to nearly the dimensions of the figure we have described, and, by making a slight alteration, has applied it to his sextants and reflecting circles, as an improvement on the common clamp. The alteration made for this purpose is, that the tangent-screw is shortened into one half of its length, so as to have only one milled head, and does not pass through the clamping-piece *n*, but only presents its point against it, where the shoulder and collar are placed in the larger clamp, but the tubes and spiral spring are retained, and the action remains the same in both constructions

6. *Clamp by T Jones*—When Mr. Jones constructed the second mural circle, now in use at Greenwich, in compliance with the astronomer royal's wish, he applied a set of clamps having the same property of communicating uniform motion, by a spring in constant action, which Dollond's clamp possesses This clamp is represented by figures 20 and 21, which correspond to figures 22 and 23, which have been just described, and as we have put the same

letters of reference to the same corresponding parts, we shall not have occasion to enter so fully into detail, in our description of this modification The plate a b here is also fixed to the wall, but a second and smaller plate, a' b', stands on short strong pillars attached to it, one of which is seen at p in fig. 21 ; this smaller plate, thus brought a short distance forwards from the large one, has the oblong perforation in it, as shown in fig 20 , the biting-pieces c and c' have each a pair of pins urged by small spiral springs, which make them both recede in opposite directions, when the clamping-screw d is released, so that neither face of the limb is in contact with the clamp, when the circle takes its quick motion. The fixed tube, which is held by one of the pillars in this construction, is the inner one, and is also the one containing the female screw, the exterior tube, as before, containing the spiral spring, which is compressed between them, and which constantly urges the stem n of the clamp, through which the clamping-screw d, which is here of the capstan kind, passes before it reaches the biting-piece c'. Instead of a shoulder and collar on the tangent screw's axis, at the stem n is formed a ball occupying a small cup on the side of the said stem, and the oblong perforation is visible in our figure in the plate, a' b', behind which is an elastic slip of metal carrying the squared piece of brass that fits the sides of the opening, and guides the direction of motion. What was before said about the advantage of a spiral spring, applied to Dollond's clamp, is equally true of this, and the circle has the same property of yielding backwards a small distance by the application of an opposite force superior to that of the spiral spring. This clamp may be dismounted without displacing the large plate from its fixed position in the wall.

7. *Circular Clamp*.—The last clamp that we have to notice, is one which may be applied to various purposes, and used either with or without a screw of slow motion. This clamp is seen in fig. 24, and will require but little explanation beyond what the figure suggests Its common construction is that of a clamping-ring, with its ear divided, and closed by a thumb-screw passing across the slit; for the elasticity of the ring usually suffices to open it when the screw is turned back when the slit is made quite through the solid ear, it has been found that the clamping does not make a close fitting all round the cylinder, which it is intended to hold fast, but if the exterior end be left undivided, as in our figure, the fitting becomes good all round, particularly when the cylinder's diameter is nearly equal to the interior circle of the ring. A clamp of this kind, which we have lately seen forming part of a repeating table at Greenwich, answers the full expectations of the astronomer royal, and may be recommended to general adoption, in cases where a clamp of similar construction is required When it is destined to retain any axis which it embraces in a fixed position, it must itself be first made fast to a fixture by strong screws, but if it be intended to regulate a slow motion, as that of an equatorial's axis, it must clamp the axis, and a screw of slow motion, acting with some projecting part of it, may be made fast to the fixture. In the construction of Captain Huddart's equatorial, now the property of Mr. South, the circular clamp of the polar axis is cut into three portions, and two of them united again by joints to allow of closer fitting, and the screw of slow motion is at the remote end of a long lever, made fast to what we have called the ear of the clamp. In our portable transit-instrument, a clamp, having two joints moving in different directions, holds the graduated circle fast to the frame, while the verniers and level are reversed in position, for the sake of increasing the number of readings of a single observation in altitude, but as we shall have occasion to describe this instrument in its place, we will defer our further notice of it.

§ XLVII ON THE VERNIER.

1. In describing the different clamps and tangent-screws, we had occasion to mention the *vernier*, as being in connexion with the said screw, and as having its motion regulated by its means; we come now, therefore, to describe the principle and application of this scale, as affording an ingenious and accurate method of subdividing a division on the limb of a quadrant, sextant, or circle, into quantities of a lower denomination than can be obtained by the operation of the dividing engine The number of spaces, into which a degree is usually divided, depends on the length of the radius of the divided limb, and the scale now generally known by the term vernier, which is the name of its inventor, has its length and number of divided spaces determined from the smallest assumed quantity that it is intended to measure. Mathematicians have been accustomed to explain the principle of the vernier scale thus; if we denominate any portion of a graduated arc A, that shall contain a given number of degrees, or of other divided spaces, being fractional parts of a degree, and take an equivalent arc described on the edge of the vernier with nearly the same radius, then if one of these arcs be divided into any even number of equal parts, and the other be divided into the same number + unity, or − unity, the difference between the value of a space on one arc and the value of a space on the other, will be the smallest quantity that is measurable by a pair of such co-extensive arcs, and its value may be ascertained by one of the two annexed formulæ,

$$\frac{A}{n}-\frac{A}{n+1}=\frac{A}{n\ (n+1)},\ \text{or}\ \frac{A}{n-1}-\frac{A}{n}=\frac{A}{(n-1).\,n}$$

in which notation, A being the common arc, n is put for the number of divisions on the vernier, and $n+1$ or $n-1$ denotes the same number increased or decreased by unity, as counted on the graduated limb of the instrument.

2 Let us first suppose the arc $A=7°$, and each degree divided into three divisions, or spaces of the value of $20'$ each, and let it be required to divide the vernier so that single minutes may be indicated, or that the smallest value be one-twentieth of a space, in this case, an arc of twenty divisions on the vernier $\left(\frac{A}{n}\right)$ must be taken equal to 21, or 7×3, on the limb $\left(\frac{A}{n+1}\right)$ and $\frac{A}{n\ (n+1)}$ will give $\frac{7°}{20\times 21}=\frac{420'}{420}=1'$. This arrangement of the scale's value, which was formerly in use, was found inconvenient in practice, as the coincidences of the lines of the scale with the lines on the limb were to be counted in a retrograde direction, as they regarded the figures on the limb; and therefore $\frac{A}{n-1}$ was adopted instead of $\frac{A}{n+1}$, for the divisions on the limb, which adoption affords a succession of coincidences to take place in a direct order, without altering the value of the scale. For if, as before, we suppose the degree divided into three parts, and the vernier to have twenty divisions, for the purpose of reading single minutes, as values of the lowest denomination, $n-1$ will represent nineteen divisions,

of each 20′, for the whole arc *A*, and we shall have $\frac{6^\circ\ 20'}{19 \times 20} = \frac{380'}{380} = 1'$ derived from a shortened vernier, but reading in the proper direction.

3. When we speak of the *reading* of a vernier, it must be understood that when an account has been taken of the degrees and fractional divisions of a degree, as constituting together the approximate measure of an observed arc, the final additional quantity must be obtained from the place of the coincident stroke of the vernier with some line on the limb, as that place regards the zero of the vernier; when the coincidence takes place at the first or nearest stroke, counting forwards from zero, the value is 1′, when at the second stroke the value is 2′, when at the third 3′, &c. till all the strokes have come successively under the eye, during a motion given to the tangent-screw in passing over one division, at the end of which both the stroke at zero and also at 20′, in this case, become coincident at the same time, when the measurement falling in the next following division commences, and is stepped over by a succession of coincidences, as the screw is turned gradually, in the manner we have described. When a contact has been taken by means of the slow motion of the screw, the place of the coincidence will readily be found by first casting the eye to zero, and noticing what part of the division under examination lies opposite to the stroke at that point, for this notice will guide the eye immediately to that part of the vernier's scale, where the existing coincidence may be expected to be found.

4. If, in the instance we have given, the vernier should be required to indicate 30″, as its lowest value, it must have its length increased so as to take in 13° or 39 divisions of the limb, co-extensive with forty on its arc, for then $\frac{13^\circ}{39 \times 40} = \frac{780'}{1560} = 0'.5$ or 30″ will be the smallest indication, and the figures 1′, 2′, 3′, &c. must be placed at each alternate stroke. Otherwise the degree may be divided into four spaces, and then the vernier may have thirty divisions, figured alternately 1, 2, 3, &c. up to 15, on an arc of 7° 15′; for $\frac{7^\circ\ 15'}{29 \times 30} = \frac{435}{870}$ will give the same value, $0.5 = 30''$, but will require a higher magnifying power for detecting the true coincident pair of lines. In the same manner a vernier may be applied to measure a still smaller fraction of a minute, partly by increasing the number of subdivisions of the degree on the limb, and partly by giving an increased length to the vernier's scale to admit of more subdivisions. In Troughton's reflecting circle of five inches radius, though a space of 30′ reads as a degree, the vernier indicates 20″ in an arc = 19° 40′, divided into fifty-nine parts co-extensive with sixty on the vernier; and in a brass sextant which he divided, and which is now in our possession, an arc of 7° 6′, with ten divisions in the degree, has seventy-one divisions co-extensive with seventy-two on the vernier, which therefore indicates $25560'' - 5112 = 5''$. And in our eighteen-inch circle attached to a portable transit telescope, a vernier with 100 divisions measuring ninety-nine on the limb, having twelve divisions in the degree, indicates a quantity so small as 3″.

5. The application of the principle, which we have here attempted to explain, will be more clearly apprehended by the young astronomer on reference to fig. 25. of Plate IV, where the degree is seen divided into three parts, and where twenty divisions on the edge of the vernier, or inner arc, are commensurate with nineteen on the limb, or outer arc; the coincidence

takes place at 15′ on the vernier with 25° on the limb, and as the zero, or stroke 0 on the vernier, points to three-fourths of the first division of the twenty-first degree, the reading is 20° 15′; but if this first stroke had fallen in the second or third space of the same degree, the corresponding reading must have been increased by 20′, or 40′ in the latter case, so that the angle measured would have been either 20° 35′ or 20° 55′, accordingly as the second or third subdivision stood opposed to the vernier's zero. Sometimes the zero of the vernier is at the middle of its scale, and the figures running off at one end commence at the other; and when the vernier is used alternately at both sides of zero on the arc of a complete circle, there are two series of numerals reading in opposite directions, as the case may require. The vernier of the reflecting circle is usually figured in this manner. We have hitherto confined our observations to the principle and application of a single vernier, such as forms a part of a quadrant or sextant, but we shall have occasion hereafter to point out the advantages to be derived from the joint co-operation of several verniers, placed at equal distances from one another, for the purposes of correcting the effects of changeable temperature, and the errors of division or excentricity of the graduated circle, to which they may be attached. The greatest objection to the use of verniers is, that they are apt to scratch the divisions of the limb by being moved in contact with intervening particles of adhering dust. The Continental instrument-makers avoid this inconvenience by making the graduations of the verniers on a circular revolving plate depressed to the same plane with the graduated circle to be indicated, in which construction the chamfered edge is not necessary, the parallax in reading is in a great measure prevented, and the effects of varying temperature are supposed to be less prejudicial.

6. The vernier represented in fig. 19. has a diminished scale of the same value as the enlarged scale given in fig. 25., and must be read in the manner we have above explained, but requires a magnifying lens to render the coincidences legible. The arc in figure 11. which is a portion of an astronomical quadrant, has a similar vernier at each side of the telescope on the same plate, to read with the arc of excess when observations require to be taken near the horizon or zenith; in other situations either one or both verniers may be read, though no advantage will be gained from reading both, when they are so near together, provided the quadrantal arc be well divided. These verniers are represented as having their coincidences at both ends, to show that the arcs of the limb and of the vernier are co-extensive. In fig. 16. the short subdividing strokes of the vernier read to 30″. In many of the best instruments single microscopes, with reflectors of plaster of Paris, are applied to the vernier-bar to illuminate the divisions of the limb and vernier, which would otherwise be scarcely legible. When more verniers than one are used, the distance between any two of them may be advantageously used for examining the large arcs of a divided circle, and the length of the vernier's scale may be made to step the smaller arcs, so as to detect any inaccuracy there may exist among the neighbouring divisions.

7. Before we dismiss the consideration of verniers it may be proper to notice, that the principle is frequently applied to the subdivision of a straight line, forming a scale of a micrometer or barometer. In fig. 26. is a portion of a rectilinear scale that will serve to explain such application, in which the left hand portion is the scale to be subdivided, and the right hand portion the vernier, consisting of ten divisions, for the convenience of affording decimal notation. In this figure nine quarters of an inch are opposed to 2.25 inches on the vernier

divided into 10, and the coincidences are represented as taking place at both ends, to show the gradual increase of the subdivided spaces from 0 to 10 on the vernier. If we consider the point or stroke 0 as the index, and begin to reckon forwards from it, we shall see the smallest space that the vernier will measure lying between the figures 1 on the scale and 1 on the vernier, which is $\frac{1}{10}$ of $\frac{1}{4}$ of an inch $= \frac{1}{40} = .025$, or, which is the same thing, $\frac{1}{10}$ of $\frac{1}{9}$ of $2.25 = \frac{1}{90}$ of 2.25, or $.025$ as before, then ascending the scale we perceive the space contained between 2 and $2 = \frac{2}{40}$, between 3 and $3 = \frac{3}{40}$ and so on, till at the coincidence of 10 with 9 on the scale the subdivision of another divided space commences. We have given an enlarged scale for the express purpose of exhibiting to the eye the successive differences between $\frac{1}{40}$, $\frac{2}{40}$, $\frac{3}{40}$, &c. but in general a scale is divided into smaller portions, before the subdivision by the vernier takes place. Most usually the inch is divided into tenths by the dividing strokes, and the vernier then subdivides one of these into ten more, or into hundredths of an inch, but if the inch is first divided into hundredths, then the vernier will subdivide it into thousandths of an inch, which may be rendered legible by a magnifier.

§ XLVIII. ON THE READING MICROSCOPE

1. When a circle is graduated into spaces of 5′ each, the rough reading of the degrees and fractional parts of the degree, as far as to the last 5′ space, is performed by an index having a single stroke upon a piece of metal that is adjustable, and capable of being fixed, when properly adjusted, on the cock that holds the reading microscope, or otherwise a second microscope, of smaller power and a larger field of view, is placed contiguous for the sole purpose of reading the degree and divisions, which modern methods preclude the necessity of marking points of discrimination at the intermediate strokes of the divisions, as was the practice of Ramsden. The reading microscope, as it is now constructed by Troughton and Jones, is a species of compound microscope consisting of three lenses, one of which is the object lens, and the other two are formed into a positive eye piece, the amplifying lens being omitted, as the field is not required to be extensive, and the measure is usually made near its centre. The microscope is usually fixed in a stationary position on a cock, or potence, attached firmly to some strong part of the frame-work, or to a wall, and when its powers are well adapted to the space it is destined to measure, and all its adjustments are of a permanent kind, the accuracy and facility with which a small arc may be measured by it, give it a preference in our estimation over every other mode of subdividing the minute into seconds, it possesses all the advantages which arise from an union of great magnifying power with micrometrical nicety of measurement; while at the same time the eye of the observer is removed far enough from the divisions to be examined, to admit of the light of a wax taper being interposed without danger to the face, which is not the case with the single lens, or simple microscope composed of a positive eye-piece, placed close to a vernier.

2. This microscope consists of a number of component parts, requiring adjustments for their relative positions, to render the measurement of a given arc perfectly correct, the arc

usually measured is a divided space of the value of 5′, to be subdivided into 300″ by five revolutions of the micrometer's screw, which therefore has its circular head divided and numbered from zero, by 10″, 20″, 30″, &c up to 60″, every fifth dividing stroke being longer than those of the other units, and terminating with a lozenge instead of the figure 5. Now as the magnitude of a 5′ space depends on the length of the radius of the divided circle, it is obvious that the microscope must be constructed to suit the division it is destined to subdivide, and that, where the screw is the same, the magnifying power must necessarily increase as the circle diminishes, that five of its revolutions may exactly measure a 5′ without an excess or defect of a single second The change of magnifying power that is necessary to render the screw competent to its office, may be effected in two different ways, whatever may be the focal length of the object-lens, the image that it forms of an object, which in this case is the divided limb of the circle, will be more remote, or ascend higher into the body of the microscope, the nearer it is brought towards its solar focal distance, measured from the limb upwards, and when it is placed exactly at its solar focal distance, the rays passing through it become parallel, and the image, in optical language, is said to be formed at an infinite distance but remove the lens beyond the solar focus, so that an image may be formed within the body of the main tube, the situation of which, to be visible, must be in the focus of the positive eye piece, and then the two points where the divided limb and its image are situated are called the two *conjugate foci* of the lens, the latter of which recedes from its place upwards as the other descends gradually towards the limb, then if we call the distance of the object or graduated limb from the object-lens f, and the distance of its image from the same lens F, the length of the image will exceed that of the object in the ratio $F : f$, or $\frac{F}{f}$ will represent the magnified state of the image. Hence it is manifest that the expression $\frac{F}{f}$ will have a new and increased value, if we either increase F or diminish f, or, which may be necessary in the original formation, if both be altered, till the condition required of the screw be within the compass of the adjustments. In our three feet altitude and azimuth circle, which has three reading microscopes by Troughton at equal intervals, with each circle, the parts of the microscope and their dimensions are nearly as given in the following description, and the letters of reference will point out those parts in figures 9, 10, 11, and 12 of our Plate XI, the first of which exhibits the external appearance, and the others show a longitudinal section of one of the said microscopes, and the internal parts detached.

3. *Description of the Reading Microscope*

a is a spider's-line micrometer with one screw, which we will describe hereafter

b is a tube two inches long, and three quarters bore, having a coarse male screw on the exterior face, and a fine female screw on the interior, and is screwed fast to the lower plate of the micrometer at right angles to it.

c is the cock to which a short tube of half an inch in length is fixed, that admits the tube *b* just to pass into it·

d and *d′* are two circular nuts with milled heads, having each a female screw acting with the coarse male screw formed round the tube *b*, one above the cock and the other below;

the use of which nuts is, to fix the tube *b* in any required situation within the small tube of the cock

e is the cell that receives the eye-piece, and screws into the outer plate of the micrometer

f is the eye-piece, an inch and a half long, that slides by friction into the cell, for producing distinct vision of the spider's lines within the micrometer :

g is a cone of brass, holding the object lens at its inferior end, and terminating at its superior end with a cylinder of half an inch in length, round which a fine screw is cut that acts with the fine screw in the interior part of the body tube *b*, the diameter of the lens is about three tenths of an inch, and its solar focal length upwards of an inch

h is the small cell that holds the object-lens, and screws into the truncated end of the cone *g*, having at its interior end a small diaphragm, at the distance of a quarter of an inch, of about one eighth of an inch in diameter.

i is a third circular nut with a milled head, that acts with the fine screw on the cylindrical end of the conical piece *g*, and fixes the object lens at any given distance from the image of the graduated circle.

The two pieces in fig 10 lie over one another in the oblong box shown in fig. 12, one having the scale of notches on the edge of its large opening, and the other carrying the crossed spider's lines immediately above the said scale · they are both formed to fit the sides of the box, which keeps them parallel to one another when moved separately over each other. The fork, having its prongs connected by an end-piece *k*, is moved by means of the milled nut and divided head seen in fig. 9, and the crossed lines and included piece of wire forming an index to the notches, travel over the scale *m*, the place of which is adjusted by a small screw *n*, passing through an *X* spring, as shown in fig. 12, and entering the end-piece of the fork, which it pulls in one direction, when turned forwards, while the spring pushes it back and keeps it in its adjusted place, as the zero of the scale has reference to the crossed lines and the divided head, which are thus adjustable to one another A strong pin rises from the under side of the oblong box at *o*, fig. 12, and passing within the fork and slit made at *p*, through the plate carrying the notched scale, presents its upper end to the small spiral spring, lying between it and the inner face of the end-piece of the fork, which spring therefore acts in opposition to the micrometer's screw *l*, and prevents any loss of motion in turning it backwards or forwards. The fixed piece *q*, seen in figures 9 and 11, is the index to the divided head *r*, turning with the screw, and showing exact seconds. This head is kept fast to the screw by an interior nut, but so as to admit of being turned by the finger and thumb, in its adjustment to zero, while the milled head is held fast in its situation. The revolving nut connected with the divided head, retaining its place, makes the screw and fork advance the space of one notch of the scale at each revolution, while the small spiral spring *o* opposes the motion, the scale *m*, when once adjusted, keeping always its station

4. *Adjustments.*—After having described the different parts of the reading microscope, we may now proceed to explain the adjustments for putting it into a proper state for performing its office These will be known to be right when the image of the divided limb and the spider's lines are all so distinctly visible together, that no parallax takes place by varying the position of the eye, and also when, in this state of good vision, five revolutions of the screw will exactly measure one of the 5′ spaces on the limb, which measurement is effected by making each stroke

of the included division successively bisect the two external and internal angles, formed by the intersection of the two spider's lines, which move as one in obedience to the screw; as the circular divided head of the screw is adjustable by friction, when a given stroke bisects the spider's lines, the zero point *o*, may easily be put to the fixed index *q*, that points to the divisions as the screw revolves, and the notches seen within the micrometer, on a delicate serrated scale, will watch the number of revolutions which have at any time been made by the screw, when counted from a small hole in the place of a notch, which is its zero. In the first place then, supposing the microscope to be altogether out of adjustment, let the object-lens be screwed home, and the body, or coarse cylindrical screw *b*, be inserted into the short tube attached to the cock, its collar being on, and let nut *d′* be screwed up till it rests against the second collar just above it, supposed to be also in its place at the lower part of the tube, in this situation make the spider's lines visible by putting the eye-piece a proper depth into its cell *e*, and examine whether or not, in that position of the micrometer, the graduated limb is also visible, if not, make it so by means of the nuts *d* and *d′*, unscrewing one and screwing the other as the distance from the limb may happen to require, which will soon be ascertained on trial. Secondly, when the spider's lines and graduations are seen distinctly together, bring the point of bisection of the spider's lines upon a stroke of the limb, no matter which, and turn the divided head to zero; then on turning the screw just five revolutions, it will be seen whether the magnifying power is too great, or the contrary, *i. e.* whether the divided space is too wide or too narrow to be measured by the given number of revolutions. If the five revolutions have not included the whole of one space, the object-lens must be screwed up towards the image of the limb, to diminish the power, by turning the cone *g* a little, and then by altering the position of the microscope by means of the nuts *d* and *d′*, till distinct vision is again obtained of both the spider's lines and the divisions of the limb; the measure of the space must then be repeated in the way just described, when it will be discovered what approximation has been made towards the due adjustment of the magnifying power. It may require repeated trials of the alternate process, before the eye is satisfied that all the requisite conditions are completely fulfilled, and when that is the case the three nuts *d*, *d′* and *i*, must be screwed tight home to render the adjustments permanent. The value of the measure depends entirely upon the distance of the object lens from the image of the divisions, and if the plane of the circle should happen to deviate from a right angle with its axis of motion, this distance may vary a little as the circle turns round, but it will never amount to a quantity that can interfere with distinct vision, or with the measure. When the micrometer is properly fixed, the zero of the circle's divisions must be brought to the bisecting point of the spider's lines, and the divided head of the micrometer turned till its zero is pointed to by its index, and then if the zero on the scale of the notches within the micrometer be covered by the fine pin included between the spider's lines, the micrometer is in a situation for measuring angles of altitude, zenith distance, or azimuth, accordingly as the circle is adapted for one or other of these purposes; but if the zero of the notched scale do not correspond to the zero on the divided circle exactly, in a given position, it must be adjusted by the screw *n* entering the end of the micrometer's box or frame, which is the proper screw of regulation. When the measure is to commence from any point previously given in the horizon, the circle must be brought to its position, by turning the tangent-screw of slow motion till the telescope's vertical or horizontal line, as the case may be, is coincident with the said point of departure, while all the adjustments remain unaltered, and then the operation of measuring may proceed, provided one micrometer only be used, but if two, three, or four micrometers be applied to the same circle, each micrometer must be previously treated in the way that has been described, and their positions must be at equal distances from one another

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§ XLIX ON THE PLUMB LINE

1. The plumb-line takes its name from the leaden weight which was formerly appended to a fine thread of silk, and suspended by a fixed pin to indicate the direction of a vertical line. When applied as an index to a graduated arc in an astronomical instrument, or to ensure the vertical position of an axis of motion, it has been found a desirable improvement to substitute a fine silver wire for the thread, and a small bucket of brass with several holes round its side to be filled, or nearly filled with shot, and to be immersed in water, to prevent a long continuance of vibrations after concluding the operations connected with the plumb line. The point of suspension is adjustable usually, in the best instruments, by two pairs of screws acting at right angles to each other, and the wire rests on a deep stroke, made as a bed for it, on the sloping face of a piece of brass attached to the cock of suspension, from the lower end of which sloping surface the vertical direction takes place, without making so great an angle as to endanger the safety of the wire, which is liable to break at the angular point when sufficiently loaded When the bucket is immersed in water its load of shot should be such, as is just sufficient to break the wire when the water vessel is empty, for without such a load a long wire would not be sufficiently stretched to become perfectly straight, but when immersed in water the loss of weight will be a security against breaking the wire, and its tension and disposition to assume a vertical line will both be ensured otherwise the scum of particles of dust, that will frequently cover the surface of the water, will prevent the line from taking a direction perfectly vertical, particularly when a portion of the bucket is not immersed in the water, by reason of its diminution occasioned by gradual evaporation

2. The use of the plumb-line was probably coeval with the construction of the first graduated instruments, but its application became most serviceable, when it was used as a guide to the position of a transit instrument's horizontal axis, of an astronomical quadrant's vertical axis, and particularly as an index to the limb of a zenith sector, in conjunction with the micrometer screw. By means of the plumb-line applied to Dr. Bradley's zenith sector, the discovery of both the aberration of light, and of the nutation of the earth's axis, was fortunately made, which data now supply corrections for reducing the apparent to the mean places of the heavenly bodies. In those indications the position of the plumb line was referred to a very fine point made on the surface of a divided arc, at the point *zero*, when the point of suspension was on or above the centre of the graduated limb, or otherwise a line parallel to the said line was substituted, for the sake of convenience. The permanency of the adjustments made by means of the plumb-line, however, could not be depended upon without repeated applications of the line at short intervals, and the changes that took place were not immediately detected; besides, as the line was necessarily suspended at some distance from the point, or stroke, with

which it was required to be in coincidence, the accuracy of the estimation depended on the situation of the observer's eye, and it was difficult to avoid a certain degree of parallax. These inconveniences were at length removed by the ingenuity of Ramsden, who substituted the *image* of a point for the point itself, and made it fall exactly at the place of the plumb-line, by means of an optical contrivance, grounded on the principle of a compound microscope; which contrivance has been denominated, aptly enough, Ramsden's *ghost*, as presenting the appearance without the substance of an object. This ghost has been adopted and cherished by Troughton, and in his hands has become a very vigilant and useful monitor to an observer possessed of one of his azimuth and altitude circles, by constantly watching and pointing out any the slightest deviation of the vertical axis of motion from a state of perfect verticality. The plumb-line of this eminent artist descends down a tube of brass that protects it from accidents, and that turns with the azimuthal motion of the instrument to which it is attached; and in whatever situation the instrument may be used, or left, an inspection of the ghost will show the nature, and by estimation the amount of the error of position of the vertical axis, to which the line must necessarily be parallel, when in all the reversed positions the central part of the ghost is covered or bisected by it. The substance, from which the ghost derives its appearance, is a circular disc of mother-of-pearl, having a point marked at a small distance out of its true centre, in order that a motion given, in turning it round, may afford the means of adjusting the point to be observed to a state of bisection, after the plumb line and axis of the azimuth circle have been adjusted to a state of perfect parallelism.

3. This application of the plumb-line is calculated to afford it every advantage that it is capable of receiving; and its remaining constantly in its place, and pointing out at all times, when the instrument is at rest, the dependence to be placed on an altitude taken under its sanction, is a source of consolation to the anxious observer. but still there are two defects in the services of the plumb-line, which, we apprehend, are without a perfect remedy, in the first place, it does not give the exact *quantity* of deviation from the true position, though the value of the disc's diameter may be ascertained, and proportional parts thereof estimated, and secondly, when put in motion, by reversing the position of the instrument, the *vibrations* thereby unavoidably occasioned, notwithstanding the submersion in water, will frequently require more time to come to a state of rest, than the exigencies of the observation will admit of. With respect to the former of these defects, we have sometimes thought, that the disc of pearl might have very close and fine divisions marked on it as a scale of equal parts, and that its diameter might be enlarged to admit of a scale of sufficient extent, but this has not yet been attempted to be put in practice, and with respect to the second objection, it has occurred to us, that a contrivance might be introduced into the side of the water vessel, that might assist the return of the bucket to a state of quiescence, in a manner similar to what is done, in bringing the vibrations of a magnetic needle to a state of rest The best substitute for a perfect plumb line, to be applied in various situations, and for all the different purposes, without the first of these inconveniences, and in a great degree free from the latter, is the spirit-level, which forms the subject of our next section. On comparing the run of the level of our circle with the disc and thickness of the silver wire forming the line, we find that the diameter of the disc measures 33″, and that the thickness of the magnified wire subtends 18″, so that when the line bisects the disc, each segment that remains uncovered is equal to 7″.5.

§ I. ON THE SPIRIT-LEVEL

1. There is no appendage to an astronomical instrument, particularly when portable, so useful, and at the same time so convenient for ensuring the adjustments, with reference to a horizontal line, as a spirit level, when its accuracy can be depended on; but implicit confidence can seldom be placed on any level coming from a maker, who has not tried its liability to give different indications under different degrees of temperature. Its utility must therefore depend on the best mode of using it. At one period cylindrical tubes of glass were selected, from their apparent uniformity of caliber, and smoothness of interior surface, for the purpose of being converted into levels, without further preparation than being hermetically sealed, after the alcohol had been inclosed in the proper quantity to form a suitable bubble, and frequently some one face of the glass was found on trial to be so uniform, when the tube was turned round to make the bubble run along it at different faces, that considerable dependence might be placed on the scale of such bubble, while the surrounding tube retained that identical shape and position. But while the accuracy of the readings depended so much on casual changes, it was not to be expected that the level could be held in high esteem, and the plumb-line, though less convenient in use, was more to be depended on for the uniformity of its indication. But since levels have been ground to an even surface, and made to possess a certain degree of curvature, proportioned to the intended run of the scale, they have risen in estimation, and are now, when properly registered, to be put in competition with the plumb line, even on the score of accuracy. Indeed the repeating circle of Borda, under all its various progressive improvements, is indebted to the assistance of a good level for much of its celebrity, notwithstanding the boasted omnipotence of the repeating principle.

2. But the art of making levels has not yet arrived at its summit of perfection; generally speaking, too little attention has been paid to the uniformity of caliber in the tubes, from an idea, that the perfection of the curve is the chief object of attention. The usual way of giving the requisite curve in the interior surface of a tube, is by means of a cylinder of brass used as a tool, with fine emery and water, and the degree of curvature in this operation will depend on the relative lengths of the tool and hollow cylinder of glass. If these are of the same length, and the friction is continued through the whole surface of the tube, the run of the bubble will generally be too sensible, but working with a shorter cylinder will increase the concavity of the curve, and render the bubble less sensible. The cylinder of a level is usually from four to eighteen inches long, according to the uses for which it is intended, but a foot will afford a good scale for most purposes, and the delicacy may be made such, that one twentieth of an inch in the scale will indicate a second of altitude. It is perhaps a determination not easily made, to assign the limit of most useful length either of the scale or bubble of a level, for a given diameter of the bore; nor is it an easy matter to ascertain what is the best diameter for different liquids respectively.

3. The sensibility, as well as uniformity of the run of a bubble, is usually determined by an instrument called a *bubble-trier*, which is a bar of brass furnished with two feet at one end, and

a screw of known value, as a single leg at the other, and having a pair of angular beds for the tube to lie in, for the turns of the screw having a divided head will show, not only whether equal quantities of elevation by the screw will produce equal spaces in the run of the bubble, but also how many inches on the scale are equal to one minute of an arc. The same determinations may be even more satisfactorily made, by fixing the level to the tube of a telescope connected with a vertical well-divided circle, having a very fine vernier, or reading microscope that will determine seconds, for if the readings on the circle be examined at both ends of the run of the bubble, as guided by the tangent screw of slow motion, the difference of these readings in seconds, divided by the number of inches and parts in the whole run, will be the number of seconds per inch for a scale suitable for such level. When the telescope has no connexion with a divided circle, but has a spider's line micrometer applied to it, the value of each inch of the bubble's run may be ascertained by taking a distant point as a mark, when the telescope is nearly levelled, for the space above or below that mark, passed over by the horizontal line of the micrometer during the bubble's run over the scale, as the telescope's elevation is gradually altered, may afterwards be measured by the micrometer's screw, and then the measure in seconds divided by the whole run measured on the scale, will give the value of one division of such scale, as well as if a graduated circle had been used.

4. When the value of an inch on the scale of any level is determined by any of the methods above explained, it will be easy to determine the radius of curvature of the interior face of that level by the following simple formula,

$$r = \frac{21600\,n}{6.2832},$$

where r is the radius of curvature, 21600 the number of minutes contained in the circumference of a circle, n the number of inches and parts run over by the bubble in one minute of elevation, and 6.2832 the double of 3.1416, or the measure of the circumference to radius 1. For instance, La Lande had a level, on the scale of which one tenth of an inch indicated a single second, or six inches a minute, and the radius of curvature attributed to it by him was = 1719 feet: now, according to our formula, we have

$$r = \frac{21600 \times 6}{6.2832} = 20626.45 \text{ inches}, = 1718.87 \text{ feet, or } 572.95 \text{ yards};$$

which result accords very nearly with La Lande's determination. If one inch had measured exactly a minute, then one sixth part, or 286.48 feet, would have been the radius of curvature. In the level attached to our altitude circle, a minute is measured by 2⅞ or 2.875 inches, and therefore we have its radius of curvature thus;

$$r = \frac{21600 \times 2.875}{6.2832} = 9883.5 \text{ inches, } 823.625 \text{ feet, or } 274.54 \text{ yards.}$$

In like manner, when the radius of curvature is known, we may determine the value of the scale of any level by a transformation of our formula into

$$n = \frac{6.2832\,r}{21600}.$$

Thus in our last example we have $\frac{9983.5 \times 6.2832}{21600} = n = 2.875$ as before. Professor Littrow, in his paper "On the Correction of the Transit-Instrument"*, informs us that Reichenbach constructed levels so accurately, that, according to his own assertion, the radius of curvature of one of them is *two hundred English miles!* Let us try how far this extraordinary assertion is correct in 200 miles we have 352000 yards, or 12672000 inches, and therefore $n = \frac{12672000 \times 6.2832}{21600} = 3686$ inches, or 307.17 feet, will afford a scale for one minute, or 5.119 feet for measuring a *single second!*

5 But the confidence to be placed in a level is not simply that which an uniform run of the bubble gives at equal inclinations in a given temperature, nor even that which arises out of an apparent good run in all changes of temperature, for the run, depending on the curvature given in grinding, may continue to be uniform, while the *zero* of the scale changes by variations of temperature, and on this account the measures may be false. It is important that the astronomer should detect this source of error where it exists, and particularly as it is generally unknown to the maker, who is not extremely particular in examining the interior column of his tubes, in the same way that he is obliged to scrutinize his tubes destined for barometers, and more particularly for thermometers. Whenever there is a sensible difference in the capacities of the opposite ends of the tube, which may be judged of by inverting a column of water in it, when corked at both ends, it is quite unfit for a level; because, as the bubble is only the vacant space left by the displacement of the enclosed spirits, whenever the bulk of the liquid is contracted by a diminution of heat, the bubble is elongated, and the elongation will not take place alike at the opposite ends of a bubble so circumstanced, hence, without any alteration in the position of the level, a new zero of measurement will necessarily take place, and the bubble will become insidious in its indications. This effect may be detected by adjusting the level to a perfectly horizontal and permanent line in a high temperature, when the bubble is short, and leaving it to show the nature of its elongation by a subsequent diminution of heat; the difference between noon and night will frequently be sufficient to show the disposition to elongate in one direction more than in the other, and when this happens to be the case with any level, it must either be laid aside as a dangerous guide, or the zero point must be ascertained and marked with a pencil or sliding index, by reversing the level and adjusting or allowing for the true place of zero, as near to the time of making the observation depending on the bubble, as can be effected, or otherwise both ends of the bubble must be read on the graduated scale in the reversed positions; for the error may be made to merge in the correction, when the value of one division in seconds has been ascertained. When a tube is properly ground and found to have an uniform and sufficiently sensible run, it will not be necessary to polish the interior face, but care must be taken that the same surface be uppermost when the instrument is applied, which was uppermost when placed on the bubble-trier, or attached to a telescope with a graduated circle or micrometer; lest the curvature be not the same at different sides of the tube. This, it may be said, is the business of the maker, but as the effect of variable temperature of the brass tube, which surrounds and guards the glass, may alter the

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* Part II of Vol I. of the Memoirs of the Astronomical Society of London

curvature and place of the bubble, at a given inclination, if the glass be under the command of the more changeable substance, by being in too close contact, it seems desirable that the English artists should copy the precaution of the French in this respect, and leave the glass as much at liberty as possible, consistently with its safety, by fixing it at the middle only; and in doing this, it will be necessary to know always which part of the tube ought to be uppermost.

6. It appears to have been thought a necessary condition in the construction of a level, that the bubble be not made short, lest it should be sluggish in its motion; and also that the curve, of whatever radius, be circular, in order that the scale may be divided into equal parts to correspond to equal variations of inclination. In several good levels which have fallen under our examination, the proportion between the length of the tube and of the bubble is about 3 : 2; and great sensibility may be obtained in tubes from nine to fifteen inches in length, according to the modes in which they are intended to be applied. T. Jones has lately determined to shorten his bubbles with reference to the length of his glass tubes, from a conviction that they are equally sensible in indication, but less liable to change their zero points; he also has adopted the French plan of fixing his ground tubes into the brass hollow cylinders at the middle only, to prevent any alterations taking place in the ground curve of the level, by freeing its ends from contact of the cylinder.

7. The levels used in practical astronomy may be divided into two classes, the *riding*, and the *hanging* or revolving, as they are more commonly denominated. In the riding level, which is generally used for levelling the axis of a portable transit, of an equatorial telescope, or zenith micrometer, the end pieces, connected with the protecting tube of brass, terminate below with each an inverted Y, that rest upon the pivots of the horizontal axis, as shown in figures 4, 5, and 6, of Plate XI.; and when the bubble will rest at the same divisions of the scale attached to the brass tube, in the reversed positions, the angles are considered as being adjusted to their proper relative depths, for in this case not only will the pivots be shown to have equal diameters, but a line forming a tangent to the middle of the bubble, considered as a curved surface, will be parallel to the imaginary line joining the upper faces of the pivots, which line must consequently be horizontal. If this is not the situation of the bubble after reversion on a truly horizontal line, that angle must be enlarged a little by a fine file or scraper, towards which the bubble ascends in both positions; and if, when the angle is so altered, the bubble is disposed to run towards the same pivot in both the reversed positions of the telescope's horizontal axis, it may be concluded that the pivots themselves have not equal diameters; hence, that which requires to be diminished must be ground a little in a blank die, turned gently round it, with a little fine emery included, when properly moistened, provided the instrument-maker is too distant to be applied to. In detecting any inaccuracy in the pivots of an axis, by means of a reversed level, great care must be taken that the adjustable end-pieces in which the inverted angles or Ys are formed, stand truly vertical; for sometimes, particularly if the edges of the angles are not rounded, a little inclination or declination of these end pieces, or of either of them from a vertical line, will send the bubble to one end of the tube, by making one side only of the angle rest on the pivot. This is a source of error which must be guarded against, by giving such proper faces to the angles as will allow the bubble to remain stationary while the level is turned back and brought forwards a little out of the true vertical position of the end-pieces. In the riding levels of some makers, one of the angles is usually adjustable by a

screw, but when the angle is once properly made, it will seldom require to be altered, and therefore is better without such screw. It is not quite certain whether alcohol or ether included in the prepared tube, hermetically sealed, forms the better level; if the former be less sensible, it is perhaps less liable to decomposition, and on that account is more usually chosen. In many of the best levels having a fixed graduated scale of ivory placed over the bubble, with a zero at each end, a pair of moveable indices put to the ends of the bubble, at the points where the bubble will settle at both the reversed positions, become a pair of temporary zeroes for the respective ends of the bubble, whatever may be the state of the atmosphere; or otherwise the graduations are figured both ways from the central point as a single zero, without such indices.

8. The hanging or revolving levels have also their angles inverted, but are at the superior ends of the end pieces; they may be suspended either on the pivots themselves, if the axis is short, or, if long, on two equal cylinders placed parallel to the line joining the faces of the pivots, and borne by the axis, as seen in the Plates XIV and XV. The angles are sometimes cut in the end-caps of the brass tube which contains the level, and sometimes form crank-pieces by which they are suspended, agreeably to circumstances; but however the angles are formed, the levels must be capable of reversion in position, as well as of preserving their parallelism, by making a revolution as the axis of the telescope turns round. The same attention is due to the formation of the angles of a hanging as of a riding level, and the convenience of the latter over that of the former consists in its exhibiting at all times, by simple inspection, any the slightest deviation in the axis of an instrument from the true horizontal line. In some of the modern levels there are a pair of screws entering the brass tube near one end, by means of which an adjustment of the parallelism of the brass tube and bubble may be effected without filing or scraping the angles, which adjustment is very convenient, especially as it may be made while the level remains suspended. We have only further to remark on this subject of levels, that whether they be used for verifying the position of a telescope's axis, or for pointing out and watching the zero of graduations in any instrument, for both which purposes they are equally adapted, the proper adjustments of the level itself must precede all other adjustments that depend upon its accuracy for what the instrument has to indicate, otherwise the results will be uncertain, and probably erroneous.

9. The reader will now be prepared to form an estimate of the properties of his level, and when he has examined it he must bear in mind, that the qualities which constitute its excellence may be enumerated under the following heads; viz.

First, the bubble must be long enough, compared with the whole tube, to admit of quick displacement, and yet not too long to admit of its proper elongation by low temperature.

Secondly, the curve must be such, that the sensibility, and uniform run of the bubble, will indicate quantities sufficiently minute, while those quantities correspond exactly to the changes of inclination, as read on the graduated limb of the instrument of which it forms a part.

Thirdly, the bubble must keep its station when the angles are moved a little round the pivots of suspension.

Fourthly, The opposite ends of the bubble must vary alike in all changes of temperature, or, in other words, the ends of the bubble must elongate or contract alike in opposite directions, so that the middle point may always be stationary

Fifthly, the angles of the metallic end pieces must be so nicely adjusted, that reversion on horizontal pivots that are equal, will not alter the place of the bubble

Sixthly, the distance between the two zeroes of a fixed scale, when such a graduated scale is used, should be equal to the length of the bubble at the temperature of 60° of Fahrenheit's scale, and should be marked at equal distances from the visible ends of the glass tube. Then as the bubble lengthens by cold, or shortens by heat, its extreme ends may always be referred to these fixed marks 0, 0, on the scale, and will fall either within, upon, or beyond them, according to the existing temperature The number of subdivisions of the scale that each end of the bubble is standing at, counted from the fixed zero marks, at the instant of finishing an observation, must always be noted, that an allowance may be made for the value of the deviation in seconds, + or –, as the case may require

Seventhly, when the two ends of the bubble are not alike affected by a change of temperature, the scale should be detached and adjustable to the new zero points, by an inversion of the level

Eighthly, when the scale has only one zero at its centre, which is a mode of dividing the least liable to misapprehension, the positions must be reversed at each observation, and both ends of the bubble read in each position, for in this case, if any change has taken place in the true position of this zero, the resulting error will merge in the reduction of the observation. This mode of graduating is generally practised on the Continent.

10. It now remains that we explain the proper methods of registering and reducing the indications of a level in taking actual observations. We will first consider a scale that counts both ways from one zero at the centre of the level's tube, and that has the places of the extreme points of the bubble indicated by its divisions; a scale of this most useful description is figured both ways, 5, 10, 15, 20, &c. to the right and left of zero, and when the level is known to be perfect and in due adjustment, its indication may be taken in the first position only for common observations, where great accuracy is not a principal object, and then, when h, the value of one division is known, and the error in *arc* is required to be ascertained thereby, it may be had by the formula

$$\text{Correction} = \frac{o \backsim e}{2}.h,$$

in which expression o denotes that end of the level which points to the object-end, and e that which points to the eye end of the telescope, and the readings must be registered accordingly: for instance, when a level having one of its divisions $= 0''.75$ was used with a circle, the reading at the end o, no matter whether pointing to the north or south, was 42, and that at the end e 54, and $\frac{54 - 42}{2} \times .75 = 4''.50$ will be the reduced correction of the instrument's position, and must be applied with the sign + or – accordingly as the instrument is measuring altitudes or zenith-distances. When the bubble runs more towards o, the object end, it is known that the telescope has too great an elevation, admitting it to have a connexion with the level, and therefore the sign of the correction must be –, to diminish the altitude given by it;

but if zenith distances are read on the graduated arc of the instrument, this distance will be given too small, and the correction must therefore be used with the sign +. But if the bubble runs more towards the eye-end e, the signs must be reversed, in applying the corrections respectively. With common attention the due application of the signs of the correction can hardly be misunderstood, as they have no reference to the cardinal points, but only to the direction in which the numerals marked on the graduated arc of the circle succeed one another, viz. whether they count from the point horizon or the point zenith

11 But the accurate astronomer will not often depend on his level till he has reversed his position, end for end, and read its indications also in the second position; in this second position the level changes its ends, with reference to the ends of the telescope; let the readings now be denoted by o' and e', as the ends of the bubble have reference to the object and eye-ends of the telescope, and the formula for giving at once the sum of the two corrections, depending partly on the state of the level, and partly on its relative positions, will be compounded of all the four readings thus

$$\text{Correction} = \frac{(o + o') \backsim (e + e')}{4} . h$$

Suppose the readings to be $o = 50$, $e = 63$, $o' = 57$, and $e' = 56$, then $o + o' = 107$, and $e + e' = 119$, and $119 - 107 = 12$, hence we have $\frac{12}{4} \times 0.75 = 2''.25$ for the correction; and as the object end of the level has the smaller quantity, the telescope is not sufficiently elevated, and the correction will be + for altitudes, and − for zenith distances. We shall come to the same conclusion if we derive the two corrections separately from each of the positions, and then take a mean of the two, by using half their sum, in the following manner, $\frac{o \backsim e}{2} = \frac{13}{2} = 6.5$ towards e, and $\frac{o' \backsim e'}{2} = \frac{1}{2}$ towards o', then $\frac{6.5 - 0.5}{2} = 3$, and $3 \times 0.75 = 2''25$ as before, with the excess at the end e. In this scale $o + e$ and $o' + e'$ will each of them be equal to the total length of the bubble, and if the sums are not the same, either the bore of the tube varies, or the readings have been recorded incorrectly

12. When the level is applied to the horizontal axis of a transit-instrument, the inclination which it points out has reference to the east and west points of the horizon, in the direction of which this axis lies at all times, and therefore we substitute the letters w, w', and e, e' for the letters which have reference to the opposite ends of the telescope, when altitudes and zenith distances are measured, and as the effect of an inclination in the transit-instrument's axis is to give the time erroneously, by a quantity depending on this inclination, + or −, as the case may be, it has become usual to change the formula, for giving the error of position in arc, into one that will at once give it in time, viz. instead of

$$\frac{(w + w') - (e + e')}{4} . h,$$

the formula becomes

$$\frac{(w + w') - (e + e')}{4 \times 15} . h, \text{ or } \frac{h}{60} (w + w') - (e + e'),$$

which is of the same value, but more convenient, because with any given level $\frac{h}{60}$ is a constant factor. When the reduction of this error has been made, as we shall have occasion to do when we treat of the transit-instrument, the sign will be + or − according to the end of the axis that is elevated above the horizon.

13. When the level's scale has two zeroes including the length of the bubble at a mean temperature, the distances from those points marked on the fixed scale, may be put down with their signs + or −, instead of the whole absolute distances from the middle of the scale, and, by attending to the application of the signs, the same results will follow as are derived from one zero, whether the level be used in one or both of the reversed positions. If we take our first example for one position only, and assume the zero points 0 and 0 to be at 45 divisions from the middle, the readings would be $o = -3$, and $e = +9$, the difference between which is 12, and one half of 12 multiplied by 0.75 will give 4″.50 as before. And in the second example, where both positions of the level were used, if we assume the zero points at 55 divisions each from the middle of the scale, we shall have $o = -5$, $o' = +2$, $e = +8$, and $e' = +1$; therefore $(o + o') = -3$, and $(e + e') = +9$, the difference between which is again 12, then $\frac{12}{4} \times 0.75 = 2''.25$ will afford the correction before given. Carlini has affirmed, in a recent publication, that a perfect level will become imperfect by long use: such deterioration may probably be occasioned by frequent exposure to the sun.

§ LI. ON ARTIFICIAL HORIZONS. [PLATE XXIX.]

1. THAT large circle of the celestial sphere, which separates the upper from the lower hemisphere, and which is called the horizon, is of no use to the practical astronomer, as a line of departure, to which his observed altitudes may be referred; the indentations made by the inequalities of the earth's surface, render the distinction between the rational and sensible horizon quite unnecessary, and it is only in nautical observations made by reflecting instruments, where an allowance is made for the curvature of the aqueous surface, that the natural horizon can be available. But a reflecting plane, situated at right angles to a perpendicular line, may be substituted for the natural horizon, and is therefore called an *artificial horizon.* It is a well known optical fact, that when a ray of light comes from a luminous body, and falls obliquely on a reflecting plane surface, the angle of incidence on that plane, as it has reference either to the plane itself or to a line perpendicular to it, is always equal to the angle caused by reflection: and consequently when a heavenly body is viewed, as seen reflected from a perfectly horizontal plane, its apparent situation is just as much below the rational horizon as its real situation, when viewed by direct vision, is above the same. Hence all observations of a heavenly body, taken by reflection, give *double* the apparent altitude, and various natural as well as artificial reflecting surfaces have been accommodated by artificial arrangements, to afford the means of obtaining double angles, that may be either taken as absolute measures, as checks on single measures, or as tests of the index-error of a graduated circle. When the heavenly body moves very slowly, as is the case with the pole-star, the horizontal position of the axis of a

transit-instrument, or of an astronomical circle, may be verified by an immediate comparison of its direct and reflected places, as referred to one of the vertical lines of the eye-piece. We will therefore give a brief account of the different contrivances which have been used for these purposes, and which have obtained the appellation of artificial horizons

2. *Horizontal circular Spirit-level.* The most convenient portable horizon which we have seen used, is that which is represented by fig. 2 of our Plate XXIX, it consists of a circular box of brass 3 .75 inches in diameter, and half an inch in depth, containing alcohol, and having an air-tight cover of glass with its interior face ground into a concave portion of a sphere, but not polished, and its exterior face ground and polished into a perfectly plane surface, it stands on three equi-distant feet, *a*, *b*, *c*, formed of screws that serve at the same time for adjusting its position, and when the stopple at *d* is withdrawn, a small funnel inserted into the tube allows the spirit to be admitted into the box in such quantity, that a bubble of air only remains at the top, not sufficient to fill the whole cavity; which bubble will take a circular shape, and remain in the centre of the cavity, when the external surface of the circular glass plate is perfectly horizontal in all directions. It may be brought into this situation by the screws constituting the feet, when standing on a firm basis at a proper height and distance, as it regards the object-end of the telescope, that requires to be directed upon it in taking an observation. As the concave face of the glass is left rough, but not so as to prevent the bubble from being visible, there are not two images formed by two reflections and refractions, and when there is reason to suspect that the upper face is not parallel to a tangent at the vertex of the interior cavity, the box may be turned half round more or less, and re adjusted for a repeated observation of the pole-star at its meridian passage, when it will appear what confidence may be placed in a horizon thus constructed. It is hardly necessary to observe, that a change of temperature will affect the size of the bubble, and that when used with the sun in summer, it may be necessary to withdraw the stopple, and permit some of the spirit to escape.

3. *Circular Plate* with a *detached Level.* The alteration produced in the preceding artificial horizon by a change of temperature, has been avoided by Mr. Troughton, who has placed a circular plate of black glass upon three legs, with screws of adjustment formed upon them, and applied a short spirit-level, hermetically sealed at both ends, and having a plane face ground under the tube, where it lies in contact with the plate's reflecting surface. In this construction, which is made an appendage to the small box sextants, double images are avoided, and the level, being capable of reversion in position, will put the exterior polished surface, by means of the feet screws, into a state of perfect horizontality. These horizons have mostly been used with sextants or reflecting circles on shore by nautical men and others, but if the polished plates were of larger dimensions, they might be used in some cases with advantage in the observatory. In bringing this reflecting plate of the horizon into a plane which shall be horizontal in all directions, the observer will avoid some trouble and loss of time, if he will attend to the following directions. Try if the bubble will reverse in position on any horizontal plane that may be chosen, and if it will, it is fit for its purpose; otherwise the flat face must be rectified by grinding, till it will reverse and keep the bubble at the middle in both positions, when this face is known to be parallel to the bubble, apply the level across the plate to be levelled in a direction parallel to the line joining two of the three feet, as *a* and *b*, and move one of these screws till the bubble will remain in the middle of the tube in both the reversed

positions, and the plate will be horizontal in that direction. Then, because a line drawn from the third foot *c* to the middle of the line joining *a* and *b* is perpendicular to it at that point, the level must secondly be placed upon this perpendicular line, and the screw of the third foot turned back or forwards, till the bubble will rest at the middle of its tube as before, and the former levelling will not be altered thereby, provided the three feet form an equilateral triangle, which the construction professes; hence the plate being now horizontal in two directions, at right angles to each other, will necessarily be horizontal in all other directions, and the bubble will remain in the middle of the tube if it be placed on any part of the reflecting surface. If this method of managing the screws be not adopted, the observer may turn first one foot-screw and then another, as the place of the bubble may appear to require adjustment, till his patience is exhausted, and the opportunity of making his observation lost.

4. Various fluids have been proposed as substitutes for artificial surfaces, of which water, oil, and mercury well purified have been found the most useful on trial, when the containing vessel has been placed on a solid basis, and protected from the influence of the wind. Our fig. 4 shows the shape of a roof, consisting of two plates of glass with parallel faces, that Troughton places over an oblong vessel of mercury, to prevent its being agitated by the wind, which protection enables an observer to take the altitude of the sun or moon by a reflecting circle or sextant out of doors; and when the roof has its position reversed at a second observation, any error occasioned by undue refraction at either plate of glass will be corrected. At Greenwich, where one of the two circles now constantly measures by reflection, wooden troughs of different shapes and dimensions are used for containing the mercury, which have covers and openings at the ends to admit the entrance of the incident rays, and exit of the reflected ones, according to the different altitudes of the stars observed. The positions of the troughs are regulated by varying their heights and distances, as they regard the object-end of the telescope, and experience has proved, that observations may be made in this way by a careful observer with the greatest accuracy. Dollond has constructed a stand for a mahogany trough, shown in fig. 5, which has a system of strong tubes, drawing out to two feet and a half, and adjustable to any convenient height, without varying the distance, and capable of being fixed by the cylindrical nuts *a*, *b*, and *c*, embracing their respective holding tubes, that have slits about two inches downwards in three equi-distant places, and screws cut to fit the nuts which close the slits, and fix the trough at its required height from its loaded pedestal. When the altitude of the star or other body is previously known within a few minutes, the instrument may be first set to its depressed situation, and then the trough adjusted to suit it; but if the altitude is quite unknown, the eye looking down the side of the telescope, or through the finder, if there be one, to the reflected image, must direct the position approximately, and then the final adjustment may be made by the assistance of the telescope itself. The situation of the trough containing mercury must be far enough removed from a public road, to avoid tremors on the reflecting surface, which would be occasioned by the motion of a carriage. Professor Gauss of Gottingen succeeded in taking the altitude of the pole-star, as reflected from a vessel of water, even in the day-time; and his deduction of its zenith distance taken from both the superior and inferior culminations, accorded so exactly with observations taken by direct vision, from which the place of the polar point on his new circle was determined, that no doubt can be entertained of the accuracy which observations by reflection from water are capable of. Such observations com-

pared with others taken by direct vision, are not only useful in determining the apparent declination or polar distance of a star, but also in detecting errors in the construction and adjustments of the larger instruments.

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§ LII ON FLAMSTEED'S AND LA CAILLE'S METHODS OF OBSERVING.

1. When Flamsteed was appointed to the Royal Observatory of Greenwich, his first object was to supply himself with instruments proper for ascertaining the places of the heavenly bodies. He had previously contracted an acquaintance with the founder, Sir Jonas Moor, while he lived at Derby, and had been presented by him with Townley's micrometer, which we have had occasion to mention [§ XVIII. 13.], and which he used there in the years 1671, 1672, 1673, and 1674. At that time, the clock-maker was the only person to whom the first astronomer royal could apply for the construction of his instruments, and the only models before him were those of Hevelius and Tycho Brahe: the transit-telescope had not been invented The most eminent clock-maker of the day, Tompion, was employed, at the expense of Sir Jonas, to make a sextant of upwards of six feet radius, with a limb racked at the edge to work with a screw, and having telescopic sights. with this instrument, and two annual clocks, furnished by the benevolence of his friend, together with one weekly clock belonging to himself, Flamsteed entered the observatory in August 1676, and began his observations in the following month. Of this instrument the author has given a description in the *Prolegomena* of his *Historia Cœlestis*, with engraved figures at the end, showing the different parts of the mechanism, from which it appears that it had two telescopes, one fixed from the centre to the extreme end of the limb, and the other moveable along the limb from its parallel position. The sextant moved on a polar axis, so that it could be placed and fixed in the plane of the meridian, with the zero put to any altitude; hence both altitudes, or zenith distances, and right ascensions, could be taken at the same time, but as it was found difficult to verify the position, the instrument had moreover the means of turning the limb into any plane passing through two given stars out of the meridian, the distance between which it could measure in any direction, by means of the two telescopes, which included the measured angle between them, while each viewed its own star. In the year 1677, the threads of the screw, guiding the motion and sustaining the weight of the moveable telescope, were worn so unequally, and in some places so much, that the measures afforded by its revolutions could be no longer depended on, and therefore the limb was divided into degrees, and subdivided by a diagonal scale, as a check on the readings of the screw, which had been tabulated into the sexagesimal denomination, the Table was then rectified by suitable corrections, in the form of equations.

2 With this apparatus, in December 1677, the greatest and smallest zenith distances of several circumpolar stars were observed, by determining their mutual distances from one another as they passed the meridian, and from these observations, cleared of refraction, the latitude of Greenwich Observatory was determined to be 51° 28′ 10″; the pole-star being at the same time determined to be 2° 25′ 10″ from the polar point. But to put this determination

of the latitude to the test of another method, this father of English astronomers measured the zenith distance of η Ursæ Majoris, and also its distance from the pole-star, with which distances, and the difference of their observed right ascensions, he obtained the following results. viz.

Zenith distance of η Ursæ Majoris	32°	10′	0″
The distance of ditto from the pole-star	41	23	0
The difference of their right ascensions	164	9	0
The apparent latitude thus deduced	51	29	22
The latitude corrected for refraction	51	28	30

This latter deduction differs only 9″ or 9″.5 from the most recent determination made with modern instruments, with the advantage of all the best tabular corrections, and the improvements in their construction, which have gradually taken place during a century and a half: and yet this approximation to the truth is highly important to astronomical deductions depending on such an element of computation. With this sextant observations continued to be made till the 15th of September 1680, when, on account of the weight of some of the moveable parts, and weakness of others, affecting the steadiness of the position, and of the difficulty of fixing it firmly in the plane of the meridian, it was determined to dismount it, and to replace it with another made more firmly, of nearly the same radius, which this astronomer now completed at his own expense, and divided with his own hands. With this new instrument several meridional zenith distances of stars and planets were taken by Flamsteed between the years 1683 and 1687, both inclusive, assisted by his amanuensis, J. Stafford. From these and former observations he made a catalogue of the right ascensions and declinations, as well as of the longitudes and latitudes of 150, which was used as a list of reference, with which to compare the places of comets, the superior planets, and stars, until the year 1696, when he determined on forming an extended and more accurate catalogue. This first catalogue was confidentially shown to Sir Isaac Newton, and other leading members of the Royal Society, while the author was computing the enlarged one, when it was discovered that the structure of this second instrument was still too weak for preserving its different positions unaltered, and therefore it was taken from the meridian wall, to which it had been made fast, and strengthened by iron bracing-pieces, and received a new limb of brass.

3. After the death of Stafford, which happened in May 1688, Abraham Sharp, a skilful mechanic, was engaged as an assistant in the following August, and was employed in fitting up and graduating the dismounted instrument. Soon after the introduction of Sharp, a mural arch, of 79½ inches radius, was begun, which engaged this ingenious servant fourteen months before it was finished, at an expense to Flamsteed himself of £120. The arch was extended to about 135°, that it might reach beyond the north pole, and when it was ready for receiving the graduations, it was fitted to its place on two meridian walls, one facing the east and the other the west, in order that, by means of a plumb-line, and the reversed observations of γ Draconis, the zero point might be determined, from which the divisions of the limb were to commence, as well as the first notch for racking its edge by the screw, which was to conduct the telescope and index along with it. This arch was divided by Sharp into spaces of 5′, between two circular lines drawn at the distance of an inch and $\frac{17}{100}$ths from each other, and diagonals were drawn from point to point in such way, that the fiducial wedge of the index-bar

pointed out, by its intersection, the nearest minute, and as the spaces between each minute were subdivided into six parts, each 10″ or by estimation each 5″ could be distinguished. This mural arch being at length firmly fixed to the meridian wall, and furnished with a single telescope, was the admiration of every spectator. With this instrument the astronomer royal determined his latitude, taken from the two zenith distances of the pole-star, to be 51° 28′ 34″, approximating still nearer to the truth. From the summer solstice of 1690 he determined the obliquity of the ecliptic to be 23° 28′ 57″, and from the winter solstice of the same year he made it 23° 29′ 4″, taking his latitude in both cases at 51° 28′ 30″ only, as he had at first determined it. Now if we compute the obliquity from Bessel's annual diminution of 0″.457, and begin with 23° 28′ 17″.65 for the year 1750 (Vol. I. p. 436), we shall have 23° 28′ 45″ for the mean obliquity of 1690, as given by recent observations. By a comparison of various observations of the sun and of certain stars, obtained by this mural arch, with the observations antecedently made by Bernard Walter, Tycho Brahe, Riccioli, and Hevelius, Flamsteed determined that the equinoctial point of the ecliptic recedes 50″ in every year.

4. Though the mural arch was made fast to a wall which had been built fourteen years, yet it was supposed to sink a little every year, so as to require an annual correction; but as the corrections depending on aberration and nutation were not then known, such annual correction of the wall could not be accurate. The instrument was fixed in the plane of the meridian in the first instance by means of the passages of the pole-star, and its deviation was corrected by equal altitudes, taken to the east and west, by a sextant of four feet radius, made for the occasion, but after all it was found that the index connected with the telescope, sliding along the plane of the divided arc, did not guide the object-end in a circle of declination that was in all points of altitude perfectly vertical, and that consequently a perfect right ascension instrument was still a desideratum in practical astronomy. The observations, however, made with this mural arch were continued for thirty years, and nearly 3000 stars were computed and arranged from them, constituting the BRITISH CATALOGUE, and affording data for the projection of a large volume of celestial maps, which bear testimony to the skill and industry of their author, and which may, even now, be consulted with advantage on many occasions.

5. After the death of Flamsteed, which happened in 1719, Dr. Halley succeeded to the situation of astronomer royal, and in the year 1722, though then sixty-five years of age, commenced his *Saros*, or observations of nineteen successive years, principally of the moon's motions, which were published down to the year 1738, and the remainder are in manuscript, deposited at Greenwich Observatory. It was probably under Halley's direction that a transit-instrument was first constructed and erected at Greenwich, with which, and the iron mural quadrant by Graham, he made his observations This transit-instrument had the telescope near one end of its horizontal axis, and is still preserved at Greenwich, but has long been out of use. This aged astronomer died in his eighty-sixth year, in 1742. We will give a specimen of Flamsteed's mode of registering his observations, which will explain his method of observing better than any particular description, that we can give in words only; and, that we may give a faithful transcript, we will copy his language itself verbatim.

A SPECIMEN OF FLAMSTEED'S REGISTER

Ann Chris 1695 mense die styl. vet.	Tempora per horologium oscillatorium	Tempora vera apparentia	ANNO MDCXCV	Dist a vertice numeratæ		Distantiæ a vertice correctæ.
				Per lineas diagonales	Per strias cochleæ	Er 05′ 30″
	h ′ ″			° ′ ″	Revol Cent	° ′ ″
♃ Feb. 14	0 01 39 .5		Solis limbus primus transit, centro	60 47 30	1377 96	60 42 00
	02 39½		centrum transit, remoto......	61 03 15	1383 02	60 57 45
	03 51		limbus sequens transit, centro	60 47 10	1377 85	60 41 40
	7 57 44½		Geminorum .... ξ	38 22 20	869 95	38 16 50
	8 13 21		.... ω	26 57 10	610 65	26 51 40
	15 33		.... ζ	30 35 00	693 02	30 29 30
	20 22½		.... ο	35 11 00	797 35	35 05 30
	30 03		.. . λ	34 30 25	782 00	34 24 55
	31 23		.... δ	29 03 20	658 45	28 57 50
	44 31		.... α	19 02 40	431 28	18 57 10
	49 43	8 46 33	Lunæ limbus primus intrat.........			
	51 08	47 58	proximus integer ...	32 05 00	727 07	31 59 30
	51 43	48 35	remotus parum fract.	32 35 30	738 60	32 30 00
	52 38	49 28	primus transit, centro	32 21 05	733 10	32 15 35
	53 42	50 32	centrum transit, proximo ..	32 06 00	727 45	32 00 30
	54 28	51 18	sectio transit, remoto.........	32 36 20	738 87	32 30 50
	55 11	52 01	limbus proximus	32 06 20	727 58	32 00 50
	55 45	52 35	remotus	32 36 20	738 88	32 30 50

6. In the year 1743 the French astronomer, La Caille, commenced his observations on the southern stars, which he continued at Paris and the Cape of Good Hope till the year 1752. He had remarked that the two methods of observing right ascensions, which had been practised before his time, were both liable to considerable errors; the mural arch, or quadrant, and the transit telescope, both required the greatest care in getting them into the plane of the meridian correctly, and the instability of their adjustments produced such a want of confidence in their permanency, that it was always found necessary to correct their positions by equal altitudes taken at each side of the meridian, and to compare the middle time, thus obtained, with the time of the observed meridian passage of the same body taken on the same day. This circumstance induced La Caille to reject meridian passages altogether, and to rely solely on equal eastern and western altitudes, and their corresponding times, for determining the middle of the interval as the apparent time of the meridian passage. This mode of observing required only an equal altitude instrument and a good clock, in addition to a tolerable stock of patience and perseverance; for frequently, during the period of observing at the Mazarine College in Paris, from the year 1743 to 1750, the eastern observations were rendered ineffectual by the intervention of clouds impeding the corresponding western ones; but still the few determinations obtained under such disadvantage were deemed more desirable, than the mean right ascensions derived from several doubtful transits over an uncertain meridian. The

instrument applied to this purpose was an iron quadrant of three feet radius, the motions of which, we are told, were easily guided by a screw*. In the years 1751 and 1752 the observations were carried on at the Cape of Good Hope, and in 1753 at the Isle of France; the zenith distances being also observed with a six feet sector These observations, being made after Dr. Bradley had discovered the aberration and nutation, had the advantage of corrections for converting the apparent into the mean places, and furnished data for the catalogue of 398 stars contained in this author's *Astronomiæ Fundamenta*, published at Paris in the year 1762, as reduced to the year 1750. It was this astronomer's original intention to observe all the brighter stars first, and to compute their places as so many fixed points, to which the places of the smaller stars might afterwards be referred by comparison, and the limit of magnitudes was proposed to be those stars which were visible with using a telescope of only two feet focal length, but the former part of the plan only was accomplished. The telescope used at Paris had a focal length of nearly five feet, but it was shortened to three feet and a half, as being more manageable, for the observations made at the Cape. The quadrant was adjusted for taking altitudes by a plumb line of silver wire, protected from the wind by a long gutter made of brass (orichalcico canali), and the suspended weight was immersed in a water-vessel, while the divisions on the limb were read by a microscope placed immediately over the line, by the assistance of a lamp. Besides the fixed horizontal wire in the common focal point, the telescope had a micrometer with a moveable wire, which was separated a short distance from the fixed one, and the altitudes were taken at both these wires in each corresponding observation. The error of zero on the limb was determined by several experimental observations made successively in the years 1743, 1745, and 1748, from which it appeared that the constant error to be applied to the altitudes given by the limb was – 19″. At the Cape, when the telescope had been shortened, the optical axis was found to have undergone an alteration, and from a mean of three trials the error of zero required a correction of – 51″. We mention these particulars to show what precaution was deemed necessary in the use of instruments even where exact graduations were not of so much importance as in modern observations, for, in the method of observing by equal altitudes, it is not the absolute, but the relative altitudes, on which the time deduced depends, though the measures are none of them quite correct, yet if the corresponding ones are *alike*, this is sufficient for the purpose; and the method might be revived with great advantage, where opportunities for determining the time in an unknown latitude would admit of its being practised. We subjoin a few examples extracted from La Caille's own register, that will explain, better than any descriptive rules, the process by which the apparent right ascensions were obtained. In these observations two clocks were used, as a check upon each other, one by the celebrated Julien Le Roy, and the other by D. Thiout; but the latter did not afford much assistance. These clocks were regulated to show sidereal time. We have already explained La Caille's method of observing with his reticle in § XXVI.

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* Astronomiæ Fundamenta, &c. a Nicolao-Ludovico De la Caille, p 20.

1752 AD CAPUT BONÆ SPEI TEMPORA HOROLOGIO ALTITUDINUM CORRESPONDENTIUM.

Die 9 Januarii 1752.

SOL

Ad Orient	Altit	Ad Occid
h m s.		h m s
16 11 6.5	47° 10′	22 32 22.5
13		16
12 43 5	47 30	30 45 5
50		39
15 57	48 10	27 31 5
16 3 5		25
17 34	48 30	25 54 5
40 5		48
20 48.5	49 10	22 40.5
55		34
22 25 5	49 30	21 3 5
32		20 57
24 3	49 50	19 26
9		20

Meridies medius 19 21 44 5
Æquatio .... + 2 3

Verus meridies 19 21 46 8

α ORIONIS.

2 42 57 5	30 50	8 42 11 5
43 4		5
44 57	31 10	40 12 5
45 4		6
46 57	31 30	38 13 5
47 4		6 5
48 56 5	31 50	36 13 5
49 3 5		6 5
50 57	32 10	34 12
51 4		5
52 58.5	32 30	32 10 5
53 5 5		3 5
55 0 5	32 50	30 8 5
7 5		1 5
57 4 5	33 10	28 0
11 5		27 58 5

Culminatio 5 42 34.8

SYRIUS

3 1 48 5	39 20	10 8 17
54 5		11
3 25.5	39 40	6 40.5
31 5		34
5 3	40 0	5 3
9		4 56 5
6 40 5	40 20	3 25 5
46 5		19 5
8 18	40 40	1 48 5

Eodem Die (9 Januarii)

SYRIUS

Ad Orient.	Altit	Ad Occid
h m s		h m s
3 8 24		10 1 42 5
9 55	41° 0′	0 11
10 1		5
11 32	41 20	9 58 33 5
38 5		27
13 9 5	41 40	56 56 5
16		50 5

Culminatio 6 35 2 95

δ PISCIS VOLANTIS

3 29 7	43 30	11 6 22
19		9 5
30 56	43 40	4 33
31 8		20 5
32 46	43 50	2 42
59		30
34 36	44 0	0 52 5
49		40
36 25	44 10	10 59 4
38		58 51
38 14 5	44 20	57 13 5
27		1 5

Culminatio.. . .. . 7 17 44 3

τ ARGÛS

3 55 53	55 20	9 33 20
56 0 5		12 5
58 2	55 40	31 11
9 5		4
4 0 10	56 0	29 3
18		28 55 5
2 19	56 20	26 54
26 5		46 5

Culminatio 6 41 36 6

α UNICORNU.

4 12 13	38 20	10 48 13 5
19		8
13 54 5	38 40	46 32 5
14 0 5		26 5
15 35 5	39 0	44 51
41 5		45
17 17	39 20	43 10
23		4

Eodem Die (9 Januarii)

α UNICORNU

Ad Orient	Altit	Ad Occid
h m s		h m s.
4 18 58 5	39° 40′	10 41 28 5
19 4 5		22 5
20 40	40 0	39 46 5
46		40.5
22 22 5	40 20	38 5
28 5		37 58.5

Culminatio 7 30 13 5

PROCYON

4 26 58 5	31 50	10 27 16 5
27 6		10
28 56	32 10	25 19 5
27 3		12
30 54	32 30	23 21.5
31 1 5		14 5
32 51 5	32 50	21 24
59		17
34 51	33 10	19 24 5
57 5		17 5
36 50 5	33 30	17 25 5
57 5		18.5
38 50 5	33 50	15 26
58		18 5

Culminatio.. . . ... 7 27 7 9

Die 10 Januarii

SOL

16 0 28 5	44 0	22 51 46
35		39 5
2 5	44 20	50 9 5
12		3
3 42	44 40	48 32.5
48 5		26
5 19 5	45 0	46 55 5
25		49 5
6 56 5	45 20	45 18 5
7 3		12 5
8 33	15 40	43 42
39 5		35 5
10 10	46 0	42 5
16 5		41 58 5

Meridies medius .. 19 26 7.4
Æquatio .. . . + 2.7

Verus meridies . . . 19 26 1 1

Eodem Die (10 Januarii)

SYRIUS

Ad Orient	Altit	Ad Occid
h m s		h m s
2 54 33 5	37° 50′	10 15 36
39		30
56 10 5	38 10	13 58 5
16 .5		52 5
57 47 5	38 30	12 21 5
53 5		15 5
59 24 5	38 50	10 45
30 5		39
3 1 1 5	39 10	9 8
7		2
2 38 5	39 30	7 30 5
44 5		24 5
4 15 5	39 50	5 53 5
21 5		47 5

Culminatio 6 35 4 6

γ ARGÙS LUCIDIOR

Ad Orient	Altit	Ad Occid
5 15 26 5	56 10	10 50 5 5
33 5		49 58
17 23	56 30	48 8 5
30		1 5
19 21	56 50	46 10 5
28		3 5
21 18 5	57 10	44 13
25 5		6 5
23 16 5	57 30	42 16
24		8 5
25 14	57 50	40 17 5
20 5		10 5
27 12	58 10	38 20
19		12 .5
29 10	58 30	36 21 5
17		15

Culminatio 8 2 45 9

ο ARGÛS

Ad Orient	Altit	Ad Occid
5 50 11	55 10	11 17 56
19		48
52 26 5	56 0	15 41
34 5		31
54 42	56 20	13 21 5
49 5		17
56 57 5	56 40	11 9
57 5		1 5
59 14 5	57 0	8 51 5
22 5		43 5
6 1 30 5	57 20	6 35 5
38 5		27 .5
3 48 5	57 40	4 17 .5
56		10

Culminatio 8 34 3 3

Eodem Die (10 Januarii)

β ARGÛS

Ad Orient	Altit	Ad Occid
h m s		h m s
6 10 57	47° 10′	12 11 35
11 13		17
13 4	47 20	9 26
21		9
15 13	47 30	7 17
30		0
17 24	47 40	5 8
41		4 51
19 31	47 50	2 56
51		39
21 47	48 0	0 46
22 5		28

Culminatio... .... 9 11 15 5

λ ARGÛS.

Ad Orient	Altit	Ad Occid
6 28 9	59 20	11 31 21 5
16		15
29 58	59 40	20 33
30 4 5		26
31 46 5	60 0	27 44
53 5		37
33 34	60 20	25 55 5
40 .5		49 5
35 24	60 40	24 6
31		23 59
37 13	61 0	22 18
19 5		11
39 1	61 20	20 28 5
8		22

Culminatio 8 59 45 2

α HYDRÆ

Ad Orient	Altit	Ad Occid
6 44 38 5	46 20	11 47 52 5
45 5		45 5
46 20	46 40	46 2
35 5		45 55 5
48 19 5	47 0	44 12
26 5		5
50 10	47 20	42 22
17		15
52 2	47 40	40 30
9		23 5
53 53	48 0	38 38 5
54 0		32
55 44 5	48 20	36 47 .5
51 5		40 5

Culminatio...... .... 9 16 15 .8

Die 11 Januarii

SOL.

Ad Orient	Altit	Ad Occid.
h. m. s		h m s
15 57 7 5	42° 20′	23 3 53
13 5		47
58 45	42 40	2 15 .5
51		9
16 0 21	43 0	0 39
27		33
1 58	43 20	22 59 2 .5
2 4 5		58 56
3 35	43 40	57 25 5
41 5		19
5 11 5	44 0	55 48 .5
18		42
6 48	44 20	54 12
54 5		6

Meridies medius	10 30 30 1
Æquatio.	+ 2 9
Verus meridies	10 30 33 0

CALCULUS

Ex Syrio revolutio fixarum a 9 ad 10 diem fuit $21^h\ 0^m\ 1^s\ 05$, at ex sole et motu diurno $4^m\ 21^s\ 3$ ea fuit $24^h\ 0^m\ 2^s\ 0$ intermediam $24^h\ 0^m\ 1^s\ 9$ usurpabimus Praeterea ex sole et motu ejus diurno $4^m\ 20^s\ 8$ fuit inter dies 10 et 11 revolutio $21^h\ 0^m\ 2^s\ 1$, adeoque die 9 Januarii inter Syrium

Et Solem meridie	108° 18′ 50″.2
Et α Orionis	13 7 1 5
Et δ Piscis Volantis.	10 40 20 0
Et τ Argûs	2 23 21 0
Et α Unicornu	13 47 30 2
Et Procyonem	13 1 13 .0

Die 10.

Et solem meridie . .	107 13 22 5
Et γ Argûs	21 55 17 7
Et ο Argûs	20 44 38 2
Et β Argûs ..	39 2 40 .5
Et λ Argûs	36 10 5 7
Et α Hydræ	40 17 45 .2

Die 22 Januarii

N B —Hâc vesperâ laminam orichalceam loco movi, eamque paulò supra filum dioptræ fixum reliqui, filum verò micrometri ad unum minutum infrà fixum reduxi prout fuerat ante diem 3 Decemb Adeoque Stellæ minutiores demùm ope laminæ · Stellæ verò clariores ope filorum observabantur de iis quæ ad laminam appulerint monitum fiet

§ LIII ON THE TRANSIT-CLOCK

1. Before we proceed to treat of the transit-instrument, it will be proper to offer some observations on the different constructions of its companion, the clock, on the good performance of which the exact determination of right ascensions of the heavenly bodies mainly depends. It would carry us beyond our prescribed limits, to enter into a minute detail of all the constituent parts of this useful machine, and to show how all the acting portions are mutually connected in the different constructions, and assist each other's action in the most advantageous manner; such explanation would require several plates of reference, and at a time when clock-work has claimed and obtained its share of public attention, may be considered unnecessary It will be sufficient for our purpose to notice, that whatever may be the construction of the clock made choice of for an observatory, the escapement and the pendulum are the two most essential objects of consideration, of each of which mechanical ingenuity has furnished a great variety.

2. Escapements may be divided into five classes, *recoil, dead-beat, isochronal, free,* and *remontoir*, the first of these, and which is most commonly applied to ordinary clocks, impels and retards by turns the alternate motions of the pallets, by its continued action on their faces, occasioned by a force derived through the media of the wheels and pinions from a suspended weight, denominated the maintaining power, while the crutch, on the axis of the pallets, transmits this modified force to the rod of the pendulum, and thus perpetuates the vibrations. The constancy of the escapement-wheel's action with the pallets, in this construction, owing to their cuneiform shape, is so circumstanced, that the force of the returning pendulum makes each pallet in its turn oppose the force derived from the maintaining power, when the wheel is obliged to recede for a moment, and the second's hand inserted on its axis at the same time recoils. The effect of this action is, such alternate pushing and opposing of the pendulum, in different parts of each vibration, that an increase in the maintaining power accelerates the rate of going, and consequently any irregularity in the action of the wheel-work, which deals the force out in small portions, or any foulness from dirt or thickened oil, will have an undue influence on the rate, by varying the arc of vibration, and accordingly this escapement is never adopted in an astronomical clock.

3 To avoid the bad effect of a recoil, the justly celebrated Graham invented that species of escapement which is called *dead-beat*, in which the faces of the pallets are circular as well as concentric, and allow the teeth of the escapement-wheel to rest on them, while the pendulum is completing its vibration, after having received its impulse, this wheel therefore never recedes, and the hand carried by it remains motionless or dead, for a certain portion of each vibration The rubbing of the teeth against the circular portion of the pallet, however, produces some friction that requires a little fine oil to lubricate the parts of action, and an addition given to the maintaining power, by increasing this friction, retards the rate of going Hence the wheel-work is usually made with the greatest care, and pinions of not fewer than eight or ten leaves each, are nicely fitted, hardened and polished, to diminish the irregularities of transmitted force, the pivot-holes also of the two last wheels' arbors, as well as the pallets themselves, are frequently jewelled in the best clocks, to avoid friction as much as possible, and to supersede the necessity

of applying much oil. Escapements of this description have long been held in high estimation where they have had the advantage of being united with a well compensated pendulum. Indeed, till within these few years, no other escapement was put in competition with it; and the observations made in the different public observatories, for half a century back, bear testimony to its competency. The circumstance of its giving the impulse to the pendulum at or near the lowest point of its arc, where its momentum is a maximum, affords it a most important advantage.

4. The *isochronal* escapement partakes of the properties of both the preceding escapements, the faces of its pallets being formed of such excentric curves, as produce only a very small recoil. The intention of such pallets is to make the pendulum perform all arcs of vibration, however different in length, in the same time; and the practical proof of this is, when an addition to the maintaining power, though it increases the arc of vibration, does not alter its duration, as indicated by the clock. The theory supposes, that the alternate acceleration downwards, and retardation upwards of the pendulum's motion, is thus made to coincide with the law of gravity, and to augment it without interfering with its simplicity; but while such a variable property as friction on the excentric curves of the pallets is concerned as an agent, the permanence of the principle cannot be reckoned upon; and though we have made trials of such an escapement, executed by an eminent artist, we cannot recommend its preferable adoption in the present state of horology.

5. In the *free* escapement the pendulum receives but a momentary impulse from the escapement-wheel at each alternate excursion, and moves quite detached from the clock-work at every returning vibration; the position of such escapement may be either, as usual with other escapements, near the pendulum's point of suspension, or below the ball at the inferior extremity of the rod; but in either case a detent, entering the notch between two contiguous teeth of the wheel, detains it until it is unlocked by means of the returning vibration, when the temporary impulse takes place, and after that a second locking, which again detains the wheel urged by the maintaining power, until the pendulum has completed its vibration, and made a second return by the simple force of gravity. This escapement is analogous to the common *detached* escapement of an ordinary chronometer, and may be executed with or without a spring; but is not often applied to a clock.

6. The *remontoir* escapement has been differently formed by clock makers, who have aimed at the same object, which is, to detach the pendulum entirely from the irregular influence of the maintaining power that actuates the wheel-work, and to substitute some other small power, which is barely competent to the purpose of keeping up the vibrations of the pendulum without varying its intensity. This independent or diminished power may be either that of a small ponderous body, or of a spring, giving its regulated impulse to the pendulum, for a longer or shorter time, in some part of each vibration, just sufficient to overcome its tendency to come to repose, occasioned by friction, the rigidity of its spring, or resistance of the air. We will not pursue all the variety of shapes and modifications of action that a remontoir escapement is capable of receiving, and actually has received, but state only what is the principle of them all, and point out what particular construction has received the sanction of experimental accuracy of performance. With respect to the principle of an escapement of this description, the escapement-

wheel is urged by the maintaining power as in other constructions, and moves while under such influence whenever it is not locked, or restrained by some mechanical means, from which restraint it is liable to be liberated at recurring intervals; its office, when in motion, is to raise the small ponderous body, usually carried by an adjustable lever, or to wind up the slender spring, of whatever shape, which in either case is connected with the pendulum, and gives its independent impulse: when this body is raised, or this spring wound up, a detent holds the wheel at rest in its place, till a slight blow given by the moving pendulum, nearly at the point of greatest momentum, disengages it; the impulse having been given, and the ponderous body or spring having returned to its original state, the wheel, being again at liberty to run on, repeats its operation, and becomes again locked, while the pendulum now under a new impulse completes its vibration, and on its return repeats the blow given to the detent. In this way the motion of the pendulum is perpetuated by a secondary power, and the irregularities of transmitted force affect not the pendulum, but only the wheel, which has nothing more to do than prepare the independent force, which is substituted for the ordinary maintaining power. The periodic intervals of raising the weight, or of winding the spring, have been varied by different clock-makers, but the more frequent they are, the more simple will be the operation. In the Greenwich transit-clock made by Hardy, the locking detents and pallets are both placed at the extreme ends of straight springs, which are raised by the maintaining power after every vibration of the pendulum; and the excellent performance of this escapement, which has now been tried with different clocks, leaves little more to be expected in the way of improvement of this very important part of the machine. As the construction of the Greenwich transit-clock is not generally known to astronomers, and is not in the list of the forty escapements which we described in another work*, we shall introduce here a short account of this escapement, of which such representations are given in our Plate XIV, as will enable us to explain its mode of action.

7 *Hardy's Clock-escapement.*—Fig. 5. exhibits a view of the whole escapement, including the wheel, the upper end of the pendulum, the two spring-detents, the two spring-pallets, and the cross-bar or double hammer carried by the pendulum, together with the cock that holds one of the pivots of the wheel's arbor, all which are supposed to be viewed from a point behind the frame that contains the works, when dismounted from the wooden case: but as all the essential parts of action cannot be sufficiently seen from any one point of view, when they are in their proper places, contiguous to, and sometimes covering one another, we have given detached portions in figures 6, 7, 8, and 9, to show how the pallets and detents have their separate offices to perform, as they regard the motion of the wheel, which therefore we have represented on a diminished scale; first, in connexion with the pallets only, and then again, with the detents without the pallets. We will first describe the various parts of the different figures by the same letters of reference, that belong to the corresponding parts in all the figures, and then explain their mode of acting. In fig. 5. *a* is a vertical steel bar, having a cross piece at the upper end which lies in a pair of notches, made on the upper face of the cock of suspension, which is screwed across the plates forming the frame, but which does not appear in the drawing;

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* Cyclopædia, by the late Dr Ab Rees.

this bar holds the suspension-spring *c* connecting it with the pendulum *b b′*, to which spring it is made fast by the screw that is seen near its lower end; the other end of the spring is attached to the pendulum by a plain pin passing through them both in the usual way; *d d′* is a portion of the back plate of the frame, to which the smaller plate *e e′* is made fast by the strong screw seen under *c*, when adjusted to its due position, the upper portion of this small plate *e e′* is formed into a triangular cock, projecting so as to receive the four strong screws seen in the figure, into its plane sides the pallet springs *f f′*, and the detent springs *g g′*, which are all bent above the letter *c*, are made fast to the sides of the triangular cock, and lie side by side, the pallets being next to the plate, require other four screws, two to each, which are not seen in this figure, but which are seen detached in fig 6, where the escapement wheel occupies its proper place between the pallet springs, and has two of its teeth resting on the sloped faces of the respective pallets. One of these springs is turned half round in fig 7, to show its breadth, and also the projecting pin attached to its lower end, which gives the impulse to the pendulum in one direction, and a similar pin, projecting from the inferior end of the other pallet spring, gives the impulse in the contrary direction, as we shall presently explain. The spring-detents *g* and *g′* with their fixing screws, are also seen inclosing the same wheel in fig. 8, and fig. 9 placed near it, shows the breadth of the spring *g* and pin at its lower end, similar to that of fig. 7, already mentioned, the spring *g′* has a similar pin at its lower extremity, and sections of these four pins, represented by four small circles, are seen in fig. 5, contiguous to the sloping ends of the cross-bar or double hammer *i i′*, borne by the upper part of the pendulum. In fig. 9 is seen a small projecting piece *m*, attached to the spring of the detent, which entering between two teeth of the wheel detains it in a quiescent state during a part of each vibration, but when the vibrating hammer *i i′* meets with the projecting pin at the lower end of the spring, it disengages this detent *m*, and the wheel being at liberty to run on, when urged by the maintaining power, meets with the pallet *n* fixed on the contiguous spring, and one of its teeth, sliding on the inclined plane forming its face, puts this spring into a state of tension. The banking screws *h h′*, limit the depth of the detent's position, and the adjustment of the triangular cock by means of a stud moving in a concealed slit, made in the principal plate *d d′*, when rendered permanent by the screw *c*, regulates the quantity of drop of the tooth at each vibration, which in this escapement is equal to one half of a space left between each pair of adjoining teeth. The two small screws entering the bass or rounded thick part of the detent springs, are used to fix the jewelled pieces *m* and *m′* forming the detents, when they are properly adjusted. in fig. 5. the screws may be clearly seen near *m* and *m′*, but the sections of the detents are there represented by small circles. The adjustment of the cross piece *i i′* is effected by means of a squared key turning, by small quantities, a stud made square to fit the key, attached to the pendulum rod, and having a single tooth entering a notch on the edge of the cross, which has its position so regulated by a sliding adjustment, that the excursions of the pendulum may become alike on both sides of the lowest point of the arc of vibration, or, in other words, that the pendulum may be in proper beat, before the central fixing screw makes the position permanent. The large solid piece *k k′* is the cock that holds the back pivot of the escapement wheel, and the two small cocks *l l′*, made fast to it, ascend just far enough to allow the banking screws *h* and *h′* to penetrate them: these cocks all lie between the wheel and the pendulum, and the cross piece receiving the impulse, and in its turn unlocking the detents, is placed far enough from the rod of the pendulum to be in the same vertical line with the centre of oscillation of the clock's pendulum. The arrows show the direction in which the wheel moves when the clock is going

8. After this description of the acting parts of the escapement, we may now proceed to explain the action, on a supposition that the clock is wound up, and all the adjustments complete. By means of the ordinary maintaining power, consisting of a suspended weight, the force of which is diminished as it ascends the train of wheels and pinions, the last or escapement wheel will run on till it meets with some opposing body, which will be one of the two pallets n or n', suppose it to be n, then, because twelve out of the thirty teeth, of which the wheel consists, are included between the two opposite pallets, we will call the tooth that first meets with pallet n 1, the next 2, and the tooth that has last passed over the other pallet n' will be 12, and its following one 13, as we have designated them in fig 6, where they are shown without the detents. When tooth 1 has fallen on the corner of pallet n, its pressure lifts the pallet spring f, and the tooth in question slides along the inclined face of this pallet, till the following tooth 2 falls on the detent, as in the position of the wheel and springs exhibited in our figure 5, in this situation the wheel is detained at rest, as well as the pallet-spring, which now rests in a state of partial tension on the second corner of tooth 1, which is ready to escape whenever the detent may be withdrawn let now the pendulum be put in motion, by raising it a little beyond the quiescent point, that it may move by its own gravity back again; in its return the end i of the cross piece will meet with the projecting pin of the inner spring, which is that to which the detent is attached, and, by giving it a blow, will drive it out, and thereby unlock the wheel, leaving it at liberty to run on; in the next instant the moving cross-piece meets also with the projecting pin of the pallet-spring, and drives it along with the detent-spring outwards from their former state of rest, and tooth 1 of the wheel escapes from the pallet n. In the mean time the detent m' catches the tooth 12, and detains the wheel again, till the pendulum has completed its vibration under the retarding influence of both the springs in its remaining ascent, and under their joint accelerating influence in its descent, till the detent-spring, falling on the end of its banking screw h, stops, and the pallet-spring alone exerts its accelerating force a little further, till its tension is expended, the pendulum has then to descend about half a degree in a free state, before it reaches its lowest point, and to ascend in the same uncontrolled state about half a degree at the other side of the said point, before it meets with the projecting pin of the second detent-spring; the wheel having in the mean time raised the pallet n' by means of its tooth 13 sliding on its inclined face The blow, which the end i' of the cross-piece, now moving in a backward direction, gives to the projecting pin of the detent spring f', drives out the detent m', and sets the wheel at liberty again to move on, and the pallet-spring being further raised by the end i' of the cross-piece, the tooth 13 escapes, the wheel again runs on, till its tooth 2, meeting with pallet n, raises its spring, and the next following tooth becomes locked by the detent n, as in the first instance. the pendulum now finishes its retrograde vibration under the opposing action of both the springs f' and g', and returns accelerated by the same joint influence, till the spring of the detent falls on the end of the banking-screw h', when the spring g' alone urges it till its tension is expended. The pendulum being now left to descend half a degree by the sole force of gravity, renews the operation we have described, by first unlocking the tooth 3 and then suffering tooth 2 to escape by lifting the pallet-spring immediately after the blow has been given to the detent, when the pendulum

is again under the influence of the springs, and will continue to vibrate in this way as long as the escapement-wheel has power to raise the springs of the pallets into a state of tension, *i. e* as long as the clock continues to be wound up at the stated periods.

9 In constructing this escapement the inventor assumed, that the force of an uniform spring applied to a moving pendulum, being analogous to the force of gravity, would so unite with the latter, as not to derange its momentum at any part of the arc, and accordingly he has contrived an adjustment that shall regulate the force of the springs to the weight carried by the pendulum in each pair of screws that fix the four springs to the triangular cock, the upper screw is tapped into the screw itself, and has its point pressing against the cock, but the lower screw passes through a hole in the head of the spring and is tapped into the cock; so that screwing one forwards and the other backwards, will alter the position and tension of the spring. The points of flexure of all the springs, including the pendulum spring, are at the angular point of the cock, and therefore have similar arcs, hence the pins of the springs, acting with the ends of the cross-piece, remain in contact with it as they move without the least friction, and the blow given by the ends, acting as hammers, produces a direct push. Also the detents, having no fang or heel-piece, and being jewelled, are displaced with the least mechanical force, and yet the locking is secure; for the detent takes its position before the tooth which it is destined to lock arrives at it, thereby avoiding all tendency to rebound, to which moving bodies are liable when they come in collision. The substitute for the maintaining power has no connection with the wheelwork, but is simply the *excess* of the accelerating force of the springs, bearing the pallets and detents, over their retarding force, which excess is occasioned by the springs of the pallets being raised a short space by the escapement-wheel at each vibration, so that the arcs of their descent are longer than the arcs of their ascent, as they have reference to their connexion with the cross-piece and pendulum, and as this excess is a constant quantity, so long as the tension of the springs remains unaltered, any variations produced in the maintaining power by irregular transmission, foulness, or wear, will not affect the pendulum or time depending on it, provided that power be competent to raise the pallet-springs through the action of the escapement-wheel.

10. The second requisite of a good clock is an *invariable pendulum;* for on the regularity of its vibrations the indication of time entirely depends, whatever may be the properties of the escapement which perpetuates its motion. No single substance has yet been discovered, that is not liable to have its dimensions altered by changes of temperature, and some of them by different degrees of humidity, hence a simple rod of either metal or wood cannot be depended upon, as the measure of invariable distance between the centre of oscillation and point of suspension of an astronomical pendulum for to the constancy of this distance the undeviating regularity of the pendulum's motion under the same arc of vibration, the same rigidity of the spring of suspension, and the same medium, will be entirely indebted. The only remedy for this natural inconstancy in the dimensions of all materials, which has yet occurred to the philosophical artist, is an *opposition of expansions*, or such an arrangement of the materials used, that while one part of them lengthens the distance of the centre of oscillation from the point of suspension, another part shortens the said distance in the same proportion; and the perfection of the mechanical contrivance, be it what it may, will depend on the exact balance of the opposite motions of the compensating mechanism at all times and under all circumstances. Various attempts have been

made by ingenious men to accomplish the desired object, and different opinions have been entertained respecting the preference due to particular contrivances, but all the constructions of a compensating pendulum are derivable from the original contrivances of Graham or Harrison, to whom, more than to any other persons, horology is indebted for its first improvements The former of these mechanists, having in 1715 ascertained the relative expansions of different metals, and found by his pyrometer, that the difference of the expansions of iron and brass with the same degree of heat, is so small, that there appeared no hope of his being able to oppose them to each other with success, fixed upon mercury, as a substance more liable to variations of its bulk by changes of temperature, and therefore promising a more easy, as well as more simple method of effecting the compensation. The method he adopted for the application of his principle was this; the verge of his pendulum was made of steel, and terminated below with a stirrup of the same metal, holding a cylinder of glass nearly filled with mercury, and the column elongated upwards from the bottom of the containing vessel, to compensate the descending elongation of the verge or rod of steel downwards during an increase of temperature, and vice versâ, and a small quantity of the mercury added or subtracted, as the adjustment for the extremes of heat and cold required after the clock was brought to its rate, completed the compensation without further trouble, except that which attended the obtaining a new rate. A sliding piece of metal was adjustable to any part of the verge, for regulating the rate of going without altering the length of the pendulum when once determined, and the account which we propose to give of the pendulum, attached to the Greenwich transit clock, will show what little improvement has taken place in this construction for more than a century, since 1721, which is the date of Graham's invention. It does not appear that the capacity of the glass vessel was determined by computation, but merely by experiment, after a trial of some months.

11. When John Harrison, the son of a carpenter at Barrow, in Lincolnshire, and himself following the same occupation, had seen Graham's paper on the properties of his new pendulum, published in the Philosophical Transactions of London, Vol. XXXIV 40.*, and about the year 1725 had ascertained that the relative expansions of iron and brass are, in ordinary specimens, as 3 . 5 very nearly, he arranged a set of iron and brass rods according to this proportion, side by side in such way, that the expansion of the iron rods downwards was compensated by the brass rods upwards, and the lenticular ball, which was suspended by the system of rods, was thus kept at the same invariable distance from the cock of suspension. To effect this, it was necessary to have some of the rods intermixed at each side of the bearing rod, to keep it perpendicular; and when the parallel rods had been united alternately at the top and bottom to each next adjoining rod respectively, and braced by cross-bars, the appearance of the nine rods so arranged, resembled that of a *gridiron*, from which therefore the pendulum borrowed its name. This pendulum has been much in use, and Troughton arranged a set of iron rods and brass tubes into a tubular pendulum in such a way, that while the compound verge, contained within the outer tube, has all the appearance of a simple rod, it possesses all the advantages of the gridiron, except that the internal parts are not constantly exposed to the atmosphere. With his new pyrometer the author can adjust the rods and tubes so nicely to each other, that the distance from the point of suspension to the estimated centre of oscillation will not vary by any change

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* Abridged, Vol VI 297

of temperature, thus affording a great saving of time in effecting the compensating adjustment of a new pendulum, which is usually done at intervals of many months. Yet this pendulum has not yet been brought into general use, perhaps has not been sufficiently tried. A description of it is given in the Cyclopædia already referred to, among the seventeen compensation-pendulums which are described in that work, and represented by corresponding engravings by the late Wilson Lowry, who also engraved the figures of the different escapements with great precision. But our present object is to describe the mercurial pendulum of the Greenwich clock, which appears to answer its purpose as well as any pendulum that has hitherto been invented, though it scarcely differs from the pendulum contrived and constructed by Graham.

12. *Mercurial Pendulum.*—This mercurial pendulum is represented on a diminished scale by fig. 10 of Plate XIV, in which *a a'* is the lower end of the steel rod, terminating with a fine but deep screw; the graduated circular plate *b*, which is tapped with a corresponding thread, forms with it a micrometer, by which the effective length of the pendulum is adjusted to measure the proper sidereal time, when the distance between the cock of suspension and centre of oscillation is 39$\frac{1}{8}$ inches, the due regulation will lie within the screw; *c* is the head, *d* the bottom, and *e f* the sides of a steel frame or stirrup, which terminates the pendulum and bears a glass cylinder *g h* of two inches diameter within, and upwards of seven long; the head *c* is perforated to receive the rod well fitted without the least shake, and *i* is an index pointing out the degree to which the length of the pendulum is at any time adjusted; a brass cup is screwed to the bottom piece *d*, to hold the cylinder in its place, the interior sides of which are denoted by the two extreme dotted lines, and the middle line shows the height of the column of mercury, which on trial is found to be about 6.8 inches, namely, a little more than computation gives it; *k* is a brass cap surrounding the upper end of the cylinder, and is kept down by the cross bar carrying the thumb-pieces *l l'*, which take hold of corresponding notches cut in the side-bars *e f*, thus allowing the cap to be easily removed, even while the pendulum is in motion, for increasing or diminishing the quantity of mercury, as the compensation may demand. An ivory scale of degrees is fixed to the back of the clock-case under the pendulum, and an index *m*, descending from the bottom of the frame, points out the arc of vibration, which in Hardy's clocks is generally about two degrees at each side of the lowest point. It is of importance to the good performance of the clock, that the strong frame, holding the cock of suspension, be firmly attached to a pillar or wall, though sometimes it is fixed to the mahogany case made fast to a wall, and also that the beats be equally loud, and the intervals between them exactly equal, which will not be the case unless the clock be in proper beat. The maker generally adjusts his clocks for beat before he delivers them, and levels some part of the case or frame when so adjusted, by a spirit-level, which he delivers with the clock as a guide for the proper position. The same position, however, might be ensured as well, if the scale were adjusted so that the pointer may indicate zero, when the pendulum is at rest, provided the works are carried by the case itself when fastened to a pillar or wall. An inequality in the beat will generally detect the change of position that may arise from a yielding of the wall or pillar, or from shrinking and swelling of the case, and an examination of the two halves of the total arc of vibration will explain to which side the pendulum is most inclined, without stopping the vibrations. For obvious reasons it is desirable that a second clock should be used at no great distance from the transit-clock, not only as a check on its going, but to keep the time during the period

in which any alteration is making in the adjustment for temperature, which necessarily makes a considerable alteration in the rate, as well as to afford the opportunity of having the clock occasionally cleaned. It is hardly necessary to add, that a sidereal clock is always provided with some contrivance for keeping it going during the short intervals in which it is periodically wound up. Without a clock which can be depended on, there is no confidence to be placed in celestial observations, nor comfort in observing, for even with the best clock, temporary deviations in the position and adjustments of the transit-instrument, by which the rate is usually determined, will frequently produce primâ facie evidence of irregularities in the going, which corrections from known data can only rectify. It is not so necessary that a clock should indicate time exactly to a single second or fraction of a second per day, by means of a nice adjustment of the pendulum's length, as it is that the going should be invariable on each successive day, the quantity of the rate is of no importance, provided it be uniform, though it is more convenient to the observer that his clock should gain a little rather than lose.

13. Notwithstanding the simplicity of the mercurial pendulum's construction, and the accuracy of its performance, which may be brought to give the same indication, in the extremes of temperature, within half a second per day, yet it is very little used in the continental observatories. Each nation has adopted some peculiarity of construction, both of the escapement and pendulum, which probably prevails from custom more than from any preference due to the principle, which circumstance shows that the same effect may be produced by different means. In France, Le Roy, Berthoud, Breguet, &c. have contrived a great variety of escapements, one or other of which are now copied as models, but some modification of the gridiron pendulum, with a knife-edge suspension, substituted for the pendulum spring, seems to have gained the preference over the mercurial pendulum, and, as it has been asserted, because the surface of the mercury has been found liable to oxidation at its surface. In Italy, and particularly at Milan, a great variety of the *free* escapements have been constructed and used, and the mercurial pendulum has had a tube and bulb of glass substituted for the steel verge, in which a column of the fluid ascends and descends, with changes of temperature, after the manner of a thermometer, and assists in regulating the compensation. We remember that Troughton had a mercurial pendulum upon the same plan, which he contrived many years ago, and which we understand is still in existence.

14. We have had an opportunity of satisfying ourselves, from our own observations, that the principal deviations in the rate of a Hardy's clock, like the one at Greenwich, are attributable to a want of perfect compensation of the pendulum, rather than to any defect in the escapement, but as we have been favoured with the rates of three clocks of his construction, including the Greenwich transit-clock, observed and registered in different climates, we prefer subjoining them as a testimony of what accuracy of performance clocks of this construction are generally capable. The first series of rates was noted at Greenwich; the second at the Royal Military College at Sandhurst by Mr. Narrien, and the third at the Imperial Observatory at Wilna, by P. Slawinski, determined by observations made with a large transit-instrument, in which series he has favoured us with all the particulars of the stars observed, and temperature corresponding to the different dates, which we have given accordingly in our subjoined Table.

15.

THE RATES

OF THREE DIFFERENT CLOCKS BY HARDY

CLOCK AT GREENWICH

Days	Rates
1820	s
April 22	
25	— 1 4
29	— 0 9
May 4	— 1 1
9	— 1 1
12	— 1 0
15	— 1 0
19	— 0 9
23	— 1 3
27	— 0 9
June 12	— 0 9
25	— 0 9
27	— 0 9
July 1	— 1 3
6	— 1 2
11	— 1 0
17	— 0 9
24	— 1 1
30	— 1 2
Aug 4	— 1 1
8	— 1 4
13	— 1 3
18	— 1 1
20	— 0 9
26	— 1 4
28	— 1 4
Sept 2	— 0 9
7	— 1 2
12	— 1 3
18	— 1 3
20	— 1 0
Oct 3	— 1 0
6	— 0 7
12	— 1 1
17	— 0 8
21	— 0 7
28	— 0 7
30	— 0 7
Nov 2	— 0 9
14	— 0 9
17	— 0 8
Dec 7	— 0 9
28	— 0 8
1821	
Jan. 29	— 0 5

CLOCK AT GREENWICH

Days	Rates
1821	s
Mar 15	— 0 5
April 9	— 0 6

CLOCK AT SANDHURST

Days	Rates
1820	s
Feb 17	+ 0 6
28	+ 0 8
Mar 1	+ 1 0
5	+ 1 1
8	+ 0 9
15	+ 0 6
30	+ 0 5
April 5	+ 0 4
11	+ 0 6
20	+ 0 3
25	+ 0 3
28	+ 0 1
May 5	+ 0 2
11	+ 0 1
21	+ 0 1
29	+ 0 2
June 7	+ 0 1
12	+ 0 2
15	+ 0 2
23	+ 0 1
July 10	+ 0 1
17	+ 0 2
23	+ 0 1
27	+ 0 2
Aug. 1	+ 0 1
10	+ 0 2
Sept 1	+ 0 3
21	+ 0 7
Oct 3	+ 0 7
17	+ 0 8
26	+ 1 1
Nov. 1	+ 0 9
4	+ 1 1
16	+ 0 9
23	+ 0 8
Dec 13	+ 1 0

CLOCK AT WILNA OBSERVATORY

Days	Stars	Times of Mean Passage	Rates.	Réaumur's Thermometer.
1820		H M S		
Nov 27	Arcturus	14 8 2 8		+ 2.25
28		8 2 4	— 0.4	+ 2 00
Dec 1		8 1 2	— 0 4	+ 1.75
3		8 0 4	— 0 4	+ 1 75
15		7 55 2	— 0 43	+ 2 00
20		7 53 2	— 0.4	+ 3 00
Dec 1	Spica Virg.	13 16 18 9		+ 1 75
3		16 17 8	— 0.55	+ 1 75
1821				
Mar 21	α Aquilæ	19 41 45.4		+ 3 00
24		41 44 5	— 0 3	+ 3 25
25		41 44 2	— 0 3	+ 3.00
26		41 43 8	— 0.4	+ 2 75
Clock suffered to go down Wound up again, and the bob raised a very little				
Mar 31	α Aquilæ	19 41 46 2		+ 4.50
April 1		41 46.0	— 0.2	+ 4 75
5		41 44.7	— 0 32	+ 5 50
10		41 43 6	— 0 22	+ 6 00
22		41 41 7	— 0 16	+ 9 00
28		41 39 4	— 0 38	+12.25
1823				
April 14	Sirius	6 30 54 5		+ 4 50
15		54 15	— 0 35	+ 5 00
23		49 23	— 0 61	+ 5 00
27		47 75	— 0 37	+ 6.00
May 3		44 45	— 0 55	+ 6.50
16		39 50	— 0 38	+10 00
19		38.55	— 0 31	+10 50
22		36 05	— 0.83	+12 75
June 3		28 50	— 0.63	+14.50
5		27 40	— 0 55	+15.50
10		25 00	— 0.48	+16.50
26		15 55	— 0 59	+14.25
April 26	α Aquilæ	19 35 35.40		+ 5 75
May 1		33 70	— 0 34	+ 6 00
4		32 20	— 0 50	+ 6.50
6		31.40	— 0 40	+ 5.75
12		29.50	— 0.31	+ 7 00
Oct 29		19 33 35 50	— 0 67	+ 4 25
Nov 10		29 00	— 0.54	+ 4 00
19		25.10	— 0 43	+ 3 50

16. We shall not lengthen this section by any remarks on the going of these different clocks, but leave our readers to examine the changes that respectively took place at the different dates specified in the Table We have not the means of ascertaining how far any changes in the intensity of the detent and pallet springs may have affected the slight variations in the rates, because the arcs of vibration are not stated by the observers, but it is desirable that the total arc should be observed and registered occasionally, for the purpose of affording data for such determination for when pendulums of the same lengths vibrate with different forces, the times will vary inversely as the square roots of the accelerative forces in the same latitude; and the lengths of the pendulums, vibrating in the same times, vary directly as the forces which accelerate them. When the same pendulum is by any cause made to vibrate in different arcs, the daily error, or difference in the daily rates, in seconds of time, will be $\frac{1}{10504}$th of the squares of those arcs, reckoned in degrees and parts Mr. Baily has calculated that if the screw at the lower end of a pendulum-rod has 40 threads in the inch, $\frac{1}{28}$th of a revolution, in raising or lowering the cylinder of mercury, will produce a gain or loss of just one second per day, and a similar computation may be made for a screw of any other number of threads in the inch, which would be convenient for assigning the divisions of the circular nut that regulates the going of the clock. Let n be the number of threads per inch, and x the number of divisions required to be on the circular nut, then we shall have $x = \frac{40 \times 28}{n}$. He has also proposed to apply an adjustable small weight on the verge of the pendulum to bring it to time, as was originally done by the inventor of the mercurial pendulum.

17. Besides two good sidereal clocks, a well furnished observatory ought not to be without a good solar clock, to prevent the trouble of computing the times when the eclipses of Jupiter's satellites are to be observed, and various other phenomena, the occurrence of which is computed in solar time, and, to give notice of the approach of any phenomenon which has been previously computed, a journeyman clock with an alarum would be very serviceable. Likewise when observations are made out of doors, at a distance from the meridian, a good chronometer should always be at hand, to convey the time of an observation to the clock for comparison, or to carry the time gained at one place to a distant station, the longitude of which may be required to be known by comparison with a known meridian.

18. For the assistance of such of our readers as may be induced to prefer a clock similar to the one now in use with the transit-instrument at Greenwich, we will copy the printed directions of the maker, which he sometimes gives to guide the astronomer, and which he has put into our hands for this purpose.

DIRECTIONS FOR FITTING UP THE CLOCK

" The case must be fixed up to a solid stone pier by the four blocks and screws. The screw-holes must be made in the back of the case, where they are marked. The brass plate on which the clock is seated is to be made perfectly horizontal, by placing the spirit level betwixt the pencil lines drawn thereon. The auxiliary springs, which keep the clock going whilst winding it up, must be relaxed by pulling the barrel-line to ease the click, and lifting it up from the ratch wheel, and then again letting go the line Next untie the springs of the escapement, and put beneath them, on each side, the wooden wedges (these wedges keep the springs apart, and prevent their being injured in hanging up the pendulum), then fix up the clock-work and pendulum, lifting it up no higher than to allow the pin to pass on to the top of the bar into its place, taking care, at the same time, not to touch the pins of the escapement; then put the glass tube with the quicksilver into the frame, observing that the cross mark near the bottom of the tube, and that on the cover above, be in front of the pendulum After securing the pendulum at the top by the side screw in the bar, withdraw the wedges from beneath the springs, and then hang on the pulley-weight. In order to put the clock on beat, first put the lever key on the square-headed screw which binds the cross piece to the pendulum, and slacken it a little, and then remove the key to the other square-headed screw, and proceed in like manner, moving the cross piece either way, till the beat is adjusted; then screw it fast again. To fill the glass tube with the quicksilver, first clean it out before a fire with a soft silk handkerchief from dust and damp; then take a sheet of thick writing paper, and roll it up into the form of a cone or funnel, until the small end has a hole not larger than the size of a pin, through which funnel pour the quicksilver into the tube, keeping it below the surface to prevent air-bubbles The height of the quicksilver in the tube measures 6 inches $\frac{0}{10}$ths, as marked on the upper part of the tube; and it takes about 11lbs. or 12lbs. to fill it to that height The bone spoon is to take out or put in more quicksilver into the tube, as the adjustment requires.

"In order to take down the clock to be cleaned, first take off the pulley-weight; then relax the auxiliary springs, and put the wedges beneath the escapement-springs, as before mentioned, loosen the side screw in the bar, take off the pendulum with care, and seat it in the middle of the bracket inside the case, so that it does not touch the pins of the escapement at the top, then take the clock from the case, and remove the escapement entire, by taking out the long-headed screw in the middle of the cock; then carefully clean the faces of the pallets and detents. There must be no oil put to the cross piece on the pendulum, and very little to the pallets of the escapement. The other parts of the clock-work are the same as in common.

"W HARDY"

§ LIV ON THE PROPERTIES OF THE TRANSIT-INSTRUMENT

1 SINCE the time of Dr. Halley's introduction of the transit-instrument into the royal observatory, its use has superseded the former methods of obtaining the right ascension of a star, and La Caille was the only astronomer of celebrity, who afterwards adhered to the method of equal ante-meridian and post-meridian altitudes at the present time, and for more than seventy years past, its adoption is and has been almost universal. The principal condition in the construction of this instrument (which was invented by Roemer) is, that its telescope, which should be as good as can be obtained, shall move in the plane of the meridian of the place of observation, and be furnished with a system of wires, or spider's lines, in the focus of its positive eyepiece, across which the passage of a star at any altitude may be observed at the instant it is on the meridian. To effect this purpose completely, several parts of the instrument must necessa-

rily be connected in such a way, as to be capable of affording adjustments for its position, and for the final adaptation of the optical to the mechanical axis of the telescope, while yet those adjustments may be rendered as permanent as the influence of variable temperature on the metallic parts will allow. The inventor's original idea, which has not since been departed from, was, that the axis of the telescope's motion should lie perfectly horizontal, in a direction due east and west; and to prevent the tendency of such axis to bend by the incumbent weight of the telescope, Dr. Halley at first placed the tube near one of its ends, that the pressure might lie principally on its contiguous pivot, a position, which was more unfavourable to the wear of one pivot than of the other, and was therefore afterwards abandoned. According to Dr Smith, the focal distance of Halley's telescope was 5½ feet, and the length of its horizontal axis of motion about an ell, which was rendered perfectly horizontal in the reversed positions by a spirit-level The system of ocular lines was illuminated by light reflected by a perforated card into the object-end of the telescope, in the way we have already described [§ XXII. 1.], and the pivots of the axis of motion rested in adjustable Ys attached to the bearing pillars, so that little room was left for subsequent improvements, except what had reference to the goodness of the telescope's vision, to the formation of the horizontal axis by an union of two inverted cones, to strengthen it sufficiently for receiving the telescope at the middle, and for allowing it to press alike on both pivots; and to a more convenient method of introducing and modifying the reflected light. In Dr. Smith's time (1738), the telescope was fixed in a frame on the middle of the axis, which braced both at the same time, and an aperture made in the side of the telescope's tube, covered by a thin disc of horn, received the light for illumination, which mode probably suggested the idea to Dr. Usher of Dublin, of transmitting the light through the axis itself, about the year 1787. We have expressed our belief that Troughton first opened a hole in the side of the main tube of a telescope for the admission of light [§ XXII. 3], but have since found that the same thing was done previously.

2 Transit-instruments may be divided into two classes, *portable* and *fixed*, the former of which, when placed truly in the meridian, and well adjusted, may be advantageously used as a stationary instrument in an observatory, if its dimensions be such as to admit of a telescope of 3½ feet focal length; but when the main tube is only from twenty to thirty inches long, with a proportional aperture, it is more suited for a travelling instrument to give the exact time, and when carried on board a ship in a voyage of discovery, may be taken on shore at any convenient place, for determining the solar time of that place, and for correcting the daily rate of the chronometer giving the time at the first meridian, so that the longitude of the place of observation may be obtained from the difference of the observed and indicated times, after the proper corrections have been made Telescopes which have tubes exceeding 3½ feet in length, are most advantageously placed on pillars of masonry, ascending, in an insulated state, from a firm basis below the surface of the ground, at such depth as the subsoil and distance from a public road may require, attention being paid, in the selection of a site, and formation of the building, to such considerations as we have stated in our second section It would not be easy to determine, by theory alone, what dimensions and corresponding strength, in the formation of the best possible transit-instrument ought to be adopted, for insuring the greatest possible accuracy; for the accompanying clock is involved in the consideration, and this is at present probably less perfect, than are our large transit-instruments of modern construction; but it is

not probable that a case will occur, the exigencies of which may not be satisfied by a telescope of ten feet focal length and five inches aperture, like the one now used at Greenwich. That we may afford our readers all the information necessary for understanding the different constructions of this important instrument, we will first describe a large portable one which may be used advantageously in an observatory, and then proceed to give directions for the examination and adjustments of such instrument, after which, we will explain the methods of determining the exact place of a meridian mark, and investigate and apply the corrections due to the position of an instrument out of adjustment, in so many separate sections; we propose subsequently to describe the larger instruments of some other makers, before we proceed finally to show, how a series of observations in right ascension may be practically arranged and computed for the formation of a catalogue, as far as the observations extend. These successive details will of course include, what is always most material to know in every operation of this kind, the means of determining, by approved methods, the existing error of the clock at any moment, when an observation may be made. As the right ascension of a star has reference to an absolute, not synodic, revolution of the earth round its axis, and is represented by its angular distance in arc from the vernal equinoctial point of the ecliptic, where the sun is seen at some moment of the 21st of March, civil time, in each year, it has been found convenient to shorten the solar pendulum till it will indicate *sidereal* hours, minutes and seconds, which require no equation of time, depending on the earth's annual motion, or on the obliquity of the ecliptic. Indeed the sidereal day is the only uniform standard of equable time that we know of, being alike in all the four seasons of the year, so far as the stars themselves are concerned. If the recurrence of a star's transit, over the meridian of any given place, shows slight variations of sidereal time, as indicated by the clock, having a known rate of going, these variations depend not on the true place of the star itself, nor on the want of uniformity in the earth's rotations, but on the time that the transmission of stellar light takes up in its passage to the earth, conjointly with the alteration produced in the position of the earth's axis, and of the equinoctial point, by the separate or joint influences of the sun and moon in different parts of their apparent orbits. These small appreciable variations form the subject of corrections, that reduce the apparent to the mean right ascensions of the stars, and *vice versâ*; and in gaining these small quantities, which may be either additive or subtractive, when taken collectively, consists the trouble of making the necessary reductions of the apparent to the mean places of the stars, as well in declination as in right ascension. In watching the rate of the clock, it will seldom happen, if it be a good one, that it will vary very sensibly from day to day, much less from hour to hour, from its established daily rate, while the temperature is not materially changed; but care must be taken that the compensation be such as will produce the same rate very nearly, in the extreme changes of winter and summer

§ LV A PORTABLE TRANSIT-INSTRUMENT BY T JONES [PLATE XIV]

1. The small portable transit-instrument, with a telescope of twenty inches focal length, mounted on an axis of twelve inches, which is supported by a circular ring of brass forming a tripod, has been so long supplied to the public by its inventor, Troughton, that it must be

considered as generally known; and, for its size, cannot be too much admired as a travelling instrument for when well adjusted and placed exactly in the meridian, it is competent to give the time of the sun's passage to the accuracy of half a second, and the chronometer's rate may be obtained by consecutive passages of the *same* star over any vertical out of the meridian. But, instead of it, we propose to describe an instrument of the portable kind, which is formed on a larger scale, and is capable of being used with advantage in any place where a single pier has been erected for its support.

2. The instrument represented in two different views, by figures 1 and 2 of Plate XIV., was made by T. Jones for our use, and has some peculiarities in its construction, which may be deemed worthy of notice In fig. 1, *a b* is the side of the stone cap of the pier, taken lengthwise, and in fig. 2, *c d* is the end of the same, on both which the frame of the instrument stands, by means of its four adjustable feet-screws resting on as many corresponding pieces of brass cemented to the stone. The frame consists of three principal pieces of cast iron, covered with several coats of paint, the two side-pieces *e f* and *e′ f′*, which support the horizontal axis of the telescope *g g′*, are so shaped as to unite strength with lightness, according to the pattern *e f* seen in fig. 2, and are high enough to admit of a telescope to be mounted upon them of 46 or 48 inches focal length they are screwed fast at right angles to the horizontal bottom piece of open work, near the feet screws, by four corner braces, which are cast as solid parts of the upright side pieces At the superior ends of these side-pieces two brass boxes are made fast, the upper sides of which contain a pair of angular notches, usually called Ys, from their resemblance to this letter, one of the Ys is adjustable horizontally, and the other vertically, by proper screws, turned by a capstan metallic pin, as the adjustments may demand, but cannot be well seen in the plate by reason of the contiguous appendages. The axis of the telescope's motion in elevation, *g g′*, consists of two brass cones connected at their bases, with an intermediate hollow piece, made large and strong enough to admit of the two halves of the telescope's tube being screwed fast into it separately, and as nearly at right angles to the axis of the cones as art can effect The tube has an aperture large enough to admit of an object-glass of 3½ inches in diameter, the length of the axis is two feet, exclusive of the pivots, which are of bell metal, about $\frac{9}{16}$ths of an inch in diameter One of these pivots projects an inch, which is the thickness of the supporting side pieces and brass box; but the other extends outwards far enough to receive on it a graduated circle of 18 inches diameter, and beyond that a revolving six armed piece, on three of which equi-distant arms so many verniers are divided, to read with the divisions of the circle to every 3″, each vernier indicating one hundredth part of 5′. [§ XLVII. 4.].

3 On the two arms, forming a diameter to the circle, at right angles to the bar carrying the first or clamping vernier, called A, two cylindrical metallic pins are inserted, as pivots of suspension for the Ys of the level's end pieces to rest on, which level is seen in fig. 2 denoted by *i i*, and when this level is adjusted, one of the clamps and tangent-screws, made fast to the supporting side piece at *h* or *h′*, accordingly as the face of the circle is turned towards the east or west, will afford the means of adjusting the level to its fixed horizontal position, when its attached vernier A is put to zero of the circle. The circle is divided into four quadrants, and numbered 10, 20, 30, &c. from 0 to 90° in the first quadrant, and again in the same successive order through the other three quadrants; so that the reversed observations of the same star

will be read by its altitude in one of the two positions, and by its zenith distance in the other, the sum of which should always be exactly 90°, when there is no index nor collimation error The ivory scale of this level, which may be called the altitude level, is divided into spaces which indicate single seconds from two zeros, placed at the ends of the bubble, when at a mean temperature. When the telescope is elevated to a given star, so as to have it running along the horizontal spider's line, the verniers may be read, or at least vernier A, and the clamp at h, fig. 1, which holds the side-piece, by means of a fixing screw, with another mouth piece, turning on a double joint, bites the edge of the circle by means of a second fixing screw, and detains the circle, and consequently the telescope also, in their unaltered positions, until the revolving verniers are reversed, along with the reversed level, the fixing screw of the tangent-clamp h being previously released, and then the bubble may be brought to the middle of the scale again by the tangent screw l, fig. 2, which fixes the vernier A to the circle: after which second adjustment of the level, a second set of three readings may be obtained, frequently before the star has departed from the field, by dexterous management, and if not, the verniers may be read subsequently, provided the star has been observed to pass the same spider's line after the reversion of the vernier-bars. When these double readings are obtained on one night with the face of the circle looking towards the east, and on the following night towards the west, a star's declination may be obtained free from any considerable error, except what may arise from the bending of the telescope, at the same time that a correct observation of the transit is obtained. When however a better circular instrument is in use, the circle of a transit-instrument ought to be confined to its proper office, of pointing out the altitude at which a heavenly body may be expected to pass, according to a previous computation of its approximate altitude in a known latitude

4. The circle of the instrument now under our consideration is contrived to perform the office of a finder in two ways; either, when the level is clamped to the fixed tangent clamp h, by reading the computed altitude or zenith distance at the stationary vernier, or by releasing the clamp h, and placing the vernier to the altitude or zenith distance on the circle, previously to elevating the telescope, and then noticing when the bubble stands in its right place, which it will do at the proper elevation. When the verniers are made stationary by the clamp h, the tangent-screw l, of the vernier A, is used as the screw of slow motion in elevation, and the observer must accustom himself to use each tangent-screw for its own purpose. But when a simple transit is taken, it is better to release the clamp h, and allow the circle and vernier to move together without any rubbing, or check upon the elevation; the balance of the opposite ends of the telescope being sufficient to keep it stationary.

5. In fig. 1 is seen a second level, suspended by the Ys of its end pieces, on the two cylindrical pins carried by a pair of cocks, attached to the smaller ends of the cones forming the axis, which is made horizontal by means of this second level. When this level is detached from the axis, it may be suspended from a pair of similar pins carried by a pair of cocks, n and n', screwed to the tube of the telescope, in which situation the value of the run of its bubble may be readily ascertained, by being thus connected with the circle, as we have already explained [§ L 3]. In this level, one division indicates a quantity of inclination as small as $0''.75$. Both the levels are attached and detached by means of grooves cut in the end-pieces, which allow the pins of suspension to slide into a situation to be caught by the Ys, after which

they are secured from falling by small pins passing across the said grooves; they are both of that denomination which we have called *revolving* levels, and consequently perform their office at all times, and in all elevations of the telescope The eye-pieces, even the diagonal ones, which are required in high elevations of the telescope, are all of the positive kind, viewing the system of spider's lines, of which five are vertical and one equatorial, in front of both lenses the diaphragm that holds the lines is adjustable horizontally by a pair of pulling and pushing screws, and by another pair vertically, for adjusting the collimation both in altitude and azimuth and the holder of the eye-pieces has a sliding horizontal motion, which brings the centres of the lenses directly over any one of the vertical spider's lines, that a star may be passing, so that it may be viewed without looking aside, which contrivance prevents the bad effect of oblique vision. A Dollond's lantern [§ XXII *5*], which is seen detached in fig 3, applies to the perforated end of the axis, as shown in fig. 1, with its chimney turned back, and the screw *δ* being within reach of the hand, limits the quantity of light which is admitted to illuminate the spider's lines, while discs of green glass are provided to modify the light by their different shades, when successively inserted at the front opening of the lantern, to suit the magnitude of the stars to be observed.

6. As a check upon the level of the circle, in determining its true zero point, a plumb-line may occasionally be suspended from a cock *o*, near the object-end of the telescope, in zenith observations, which, descending down holes made at opposite sides of the cone *g* of the axis, may be immersed in a vessel of water, standing on the cap of the pier, or on the bottom of the cast iron frame, in which situation, when at rest, it may be viewed by the eye looking through a lens at *p*, opposite to which, at *q*, is an adjustable piece of tube holding a pair of fine silver wires, crossing one another in an acute angle, which the plumb-line may be made to bisect, by means of the screw *l* of slow motion, the verniers being first adjusted to their stationary position, the point of suspension of the plumb line is also adjustable by the screws of its cock, as well as by a sliding piece projecting from the cock, and this line may be rendered visible at the same time with the pair of cross wires. when a star near the zenith has been observed, and its place read on the circle, in the reversed positions of the telescope, on two successive nights, while the plumb line bisects the angles of the cross wires, a mean of all the readings on the circle, so taken, will give the star's zenith distance; and the difference of the eastern and western readings will be double the error of the index and collimation in altitude taken conjointly.

7. If the point of suspension of the plumb line were placed on a cock at a quarter of the tube's circumference from its present situation, and if the lens at *p* and wires at *q* were put to correspond, such plumb-line might be used to level the axis by simple reversion, without the aid of a star; but such application could only be made occasionally, and would not immediately detect a change of inclination in the manner pointed out by a good revolving level, carried constantly by the axis. It has however been sometimes objected, that the position of a revolving level is not so certain as that of a striding level, which rests on the pivots of the axis, since the connexion is not immediate, but if the hanging level will preserve the bubble in the same position, near the middle of the tube, in every degree of the telescope's elevation in both the reversed positions, the level must necessarily be parallel to the axis, as well as the axis itself horizontal. The piece of tube seen in fig. 4. may occasionally screw into the place occupied by

the holder of the eye-piece, when the star is too small to be seen at the same time with the spider's lines; and the diametrical line formed by the straight edge of the small slip of brass seen crossing it, will take the place of the middle line, and show, by either an emersion or immersion of the small star, as it may be adjusted, the moment of its transit over the meridian; but the stars for which such contrivance may be required are so numerous, that we take notice of it rather as matter of curiosity than of practical utility, though Delambre says he frequently used such a slip of metal for the same purpose. The level *r* lies in a detached state on the cap of the pillar to show that it remains level, and the small inverted pendulum invented by Hardy, and known by the name of Hardy's *noddy*, stands surrounded by a glass cover carrying a scale, to show the position in which it is usually placed, its principal use, is to try if the case of the clock be perfectly insulated from the floor of the room, and firmly fixed to the wall, when the least motion is communicated to the case by the heavy vibrating pendulum, the noddy, having its vibrations isochronal to those of the pendulum, will show the effect by commencing its vibrations the screw at the bottom, opposed to the two feet, serves to adjust the pointer of the noddy to the middle of the scale, when it has taken its station at the head of the clock case. The verniers of the circle are usually read by a pair of positive eye-pieces, at the opposite ends of a revolving bar, in front of the verniers, but a pair of compound microscopes may occasionally be substituted, when the circle is used for nice purposes. The bending of an unbraced telescope is easily seen, when directed to a distant mark; for even a twopenny-piece laid on the object-end of the tube will produce a difference of several seconds in the measure of altitude.

§ LVI DIRECTIONS FOR THE EXAMINATION AND ADJUSTMENTS OF A TRANSIT-INSTRUMENT

1. When a transit-instrument is first received from the maker, it will be proper for the observer to make himself acquainted with its properties, by a due examination of all its parts; and to become master of all its adjustments, before he can proceed to make observations with confidence. We will suppose him to have obtained an instrument mounted in a metallic frame, such as we have just described, and that one pillar is sufficient for its support, in this case he may place as many circular pieces of brass about the size of a penny-piece, with conical holes at their centres, as there are feet, upon the covering stone of the pillar, without fixing them at first, for the purpose of receiving the feet-screws which support the frame. At the first opportunity he must direct the telescope to a star of the first or second magnitude, and try if the vision be good, as we have directed in our fourth section, if the star is seen round, and free from irradiations at each side, and equally so through every part of the field of view, the object-glass is adjusted, and the eye piece properly adapted, then if, when the eye piece is pushed too far in, or drawn too much out, the telescope being directed to the moon, the disc is surrounded by a border of green in the first case, and by a boundary of purple in the second, the telescope may be considered achromatic, and there will be reason to be satisfied with the optical portion of the instrument, particularly if the diagonal eye-pieces and higher powers be found also good

near the borders of the field of view. In instruments of a portable construction there is usually a piece of large tube, holding the smaller tube for receiving the eye-pieces and diaphragm, which is adjustable within the main tube, for regulating both the vertical position of the transit spider's-lines, and also their distance from the object-glass, which must be such, that the star shall appear as a small luminous point, at the same time that the dark lines are seen well defined in the illuminated field; in this case the diaphragm containing the lines is in its place, but cannot be finally adjusted till the axis has been levelled. It will be proper also to examine that the feet screws act well, and remain steady in any position given them; and also that all the screws which fix the different parts of the frame together, are turned home, and remain so after being shaken by carriage. The length of the axis of the telescope's motion, from shoulder to shoulder, should be just sufficient to reach from one Y to the other, without either friction or liberty to move endwise; and the lamp should be held by the supports in such way, that illumination may always be insured without further adjustment for position. The equipoise of the telescope should also be such, that it may retain its position without a clamp in any given degree of altitude, when an ordinary eye-piece is applied, and the covering cap removed from the object-end. An useful magnifying power for a telescope from 3½ to 4 feet focal length, will be from 70 to 90 or 100. The screws of adjustment of the diaphragm and Ys, both horizontal and vertical, should finally be examined, to see that they are competent to give security of position to the parts adjusted by them, under all circumstances. The metallic parts should all be free from flaws in the casting; and the pivots of the axis should be of hard bell-metal in preference to steel, which is liable to rust, and thereby to alter their dimensions, independently of the wear by partial attrition in certain altitudes.

2. *Adjustment of the Level.*—After the optical and mechanical parts of the instrument have been carefully examined and approved, the first adjustment necessary to be performed is that of the level itself, whether of the riding or revolving kind, which is intended to insure the horizontality of the axis to which the telescope is fixed. As we have already given a full account of the properties of the spirit-level [§ L.], and of the methods of adjusting and using it for different purposes, we have only to remark here, that the level is in adjustment when the bubble gives the same indication on its scale at both the reversed positions; and that, if it does not, one half of the error must be corrected by the vertical screw of the proper Y of the axis, and the other half by scraping or filing the small Y of the level, at that end to which the bubble runs, till, by thus halving the error, the indication will be the same in both positions.

3. *Horizontality of the Telescope's Axis.*—The same operation which adjusts the riding level will place the axis also horizontal, provided the diameters of its pivots be of the same dimensions; because, when the level is made horizontal, and also parallel to the axis, the axis itself must necessarily be horizontal also. Hence when the level is previously adjusted, and the pivots known to be alike, bringing the bubble to zero of the scale, by the vertical screw of the proper Y, will at once level the axis, when inclined a little; and if any subsequent inclination should take place in it, or alteration in the level, an occasional reversion of the position will immediately detect them. To discover whether the pivots are alike or otherwise, we must refer our readers to what has been already said on the subject [§ L. 7.]. When the revolving level is used, the line joining the pivots of its suspension must be made parallel to the axis of the telescope, by reversing the axis, and correcting one half of the error by the screws that adjust

the pivots of suspension, and the other half by the vertical screw of the Y, after the level itself is adjusted.

4. *The Spider's Lines.*—When the axis of the telescope has been truly levelled, and its vision adjusted, the next object will be to place the parallel lines truly vertical, and to determine the equatorial value of their intervals. The spider's lines, or wires, are usually laid parallel to one another on a circular plate of brass, which is perforated at the centre, and is capable of different adjustments at the eye-end of the telescope: a short piece of tube, just fitting the interior cavity of the main tube, receives the circular plate, the adjusting screws, and the eye piece, and is capable of being pushed inwards for giving distinct vision of a star upon the lines, or of being turned round till the five parallel lines are made truly vertical. When the former has been effected, the latter may be adjusted by a distant mark in or near the horizon thus: turn the frame, standing on the discs of brass, round till the chosen mark is bisected by the middle line, and level the axis, as above described, then elevate the telescope slowly, and observe if the same point of the mark continues to be bisected by the middle line in every part of the field's vertical diameter; if this be the case, the line is already vertical, but if not, the interior piece of tube, containing the lines, must be turned gradually round, till this appearance takes place, and in that situation the tube must be secured by a fixing screw or steady pin taking hold of both the fixed external and adjustable internal tubes. If, instead of a point taken in a distant object, a thick white plumb-line be suspended on a dark ground at a distance in its place, the middle line may be made to coincide with it, to ensure its verticality, and then a motion in altitude given to the telescope will show whether the coincidence continues unaltered by change of elevation, and consequently whether the axis has been properly levelled. When the parallel lines have been rendered truly vertical, and found to be so also in the reversed positions of the axis, the value of each interval may be found in minutes and seconds of a degree, by means of a well graduated scale, with dividing lines strongly marked, placed horizontally at a known distance, and within the field of view, so that the divisions may appear crossing the vertical spider's lines at right angles; then if the telescope's aperture be diminished to an inch or less by a perforated cap, the scale will be seen as distinctly, though not so brightly, as if viewed at a much greater distance with the whole aperture, and the dividing strokes of the scale will thus be visible. The value of any division on the scale, with which the intervals of the lines may now be compared, may be known by reference to the Table given in our twentieth section and its accompanying explanation, as though the intervals formed a micrometrical scale; then if the value in arc be divided by 15, it will give the equatorial value in time. The comparison with the scale will further show whether the intervals are all equal to each other. But the most usual, and perhaps the most satisfactory mode of determining the values of the different intervals in *time* is, by counting the time, in seconds and parts, occupied by the passage of a star without declination, or nearly so, over all the intervals, taken both separately and collectively, by several repetitions at or near the meridian. If the star is not on the equator, and has its apparent declination known at the time, the value of an interval τ', obtained from its passage over the meridian, may be converted into the equatorial value τ, by multiplying the seconds counted by the cosine of the star's declination, or by the sine of its polar distance. But before this method can be used, the instrument must have been placed in the meridian. If all the intervals have not their values

alike, a mean of all the transits will not be the moment of true passage over the middle line, and a table of corrections must be computed from the formula $\frac{a+b-c-d}{5 \cos. \text{dec}}$, where a, b, c, and d are the five unequal equatorial intervals, but now that dividing engines and microscopic transfers of divisions are so competent to insure the equality of divided spaces, the better plan will be to have the parallel lines themselves reformed.

5. *Collimation in Azimuth.*—When the telescope is directed to a visible point in a distant object, and the middle spider's line brought to bisect it, after the preceding adjustments have been made, its axis must be turned end for end, and if after this reversion the same point be again bisected by the same line, this is a proof that a line, passing from the middle spider's line to the optical centre of the object glass, is at right angles with the axis of the telescope's motion, or that the collimation in azimuth is in due adjustment; but if, after reversion of the axis, the visible point chosen for a mark be found at one side of the middle line, it will show that the middle line is not in the optical axis of the object-glass, and the adjustment must be made partly by the horizontal screw, that moves the Y in azimuth, and partly by the screws to the right and left of the circular plate containing the system of lines, the squared ends of which project from the inner piece of adjustable tube, when one half of the deviation has been rectified by the screw for azimuthal motion, and the other half by releasing one of the small screws at the eye-end and screwing up the other, till the mark is again bisected, the axis must be again reversed into its original situation, and if a deviation is still perceptible, either to the right or left, it must be diminished by halving as before, and again reversing, till the two positions are exactly alike, as the middle line regards the chosen mark the adjustment for collimation in azimuth will then be complete. The verification of this adjustment may be proved by the passage of the pole star · when the time has been noted at the wire preceding the middle one, and again at the middle wire, the position of the axis may be reversed, so that the preceding may become the following wire; then noting the time of passage over it a second time, will show whether the position of the middle wire has been altered by the reversion, and how much; namely, by one half the difference of the two intervals before and after the passage over the middle wire This method was spoken of by Carlini, in the *Ephemeridi di Milano*, some years ago, as well known on the continent

6. *Collimation in Altitude.*—Though the transit-instrument, considered simply as such, is never intended to measure altitudes with perfect correctness, yet as it has always a circle or arc graduated to point out, as a finder, the altitude of a given star approximately, it is proper that the horizontal line of the system should cross the parallel vertical lines not only at right angles, but in the optical centre of the field of view also, as this is the line on which a star should be made to pass through the field, when its approximate altitude or zenith distance is indicated on the circle. Indeed if suitable mechanical means were employed to prevent the bending of the telescope, its circle, placed at one end of the axis, as in the instrument before us, and as is usual in Germany, might be made large enough to obtain declinations, at the same time that right ascensions are observed. We will therefore explain how the adjustment may be made for collimation in altitude. When the telescope is directed to the pole star at the time of passing the meridian, or to a well defined distant point by daylight, and the horizontal line is made to bisect it, the verniers of the circle must be read, and a mean of them noted, while the bubble of the

level, carried by the vernier-bar, is at zero, the axis of the telescope must then be reversed, and the horizontal line be brought again to bisect the star, or the same distant point, as the case may be, and when the bubble of the level, which is now also reversed, is made to stand at zero as before, the mean of the readings by the verniers must be again noted, then if altitudes are read in both positions, half their sum will be the true apparent altitude, and half their difference the error of collimation in altitude, provided there has been no bending of the telescope, but this error, like most other instrumental errors, may consist of two parts, the horizontal spider's line may be out of the optical centre of the field of view, and the level, supposing it previously adjusted to reverse properly in position, may not be in its true position as it regards the zero of the circle's divisions, i. e. it may occasion an index error. one half of the error, arising from half the difference of the altitudes, must be corrected by the pair of small screws above and below the outer end of the adjustable tube, in the same way that was done with the pair of side screws, in adjusting for collimation in azimuth, and the other half by the screw that alters the relative position of the level. Then a repetition of the measurements, in the reversed positions of the axis, and a corresponding halving of the error by the respective screws at each time, will at length annihilate the error, or make it so small, that it may be allowed for as a constant correction

7. *Position in the Meridian.*—The last, and most difficult of all the adjustments of the transit-instrument, is that by which it is placed in the true meridian of the place of observation There are many methods, however, of accomplishing this task, by direct and indirect means, but the most convenient, as well as the most correct, and consequently the most practised, are those in which the pole star is employed, in which two circumpolar stars of nearly the same declination, and differing nearly twelve hours in right ascension; or in which two stars differing considerably in altitude, and but little in right ascension, are successively observed, but whichever method be chosen, the clock that gives the times, must have its rate previously determined, which may be done by successive transits of the same star, even before the instrument is truly placed in the meridian, provided it remain firmly in the same position. The approximate position of the instrument may in the first instance be made from the shadow of a plumb-line falling, at apparent noon, as near as can be ascertained by ordinary means, along the tube of the telescope, or parallel to it; or what will be still better, the solar time of the pole star's passage over the meridian on the given day, may be computed, as we have directed in our first volume (p. 334), either for the upper or under culmination; and then the telescope, levelled and pointed to it at the computed time, which may always be known very nearly by some of the common methods, will require but little subsequent adjustment. If the sidereal clock be now put to indicate the right ascension of a known star, as it passes the telescope turned over towards the south, while in this position, subsequent observations of circumpolar, or of high and low stars, will rectify the position gradually, provided all the adjustments which have been described, continue unaltered for a sufficient length of time, and then a meridian mark, that is capable of future adjustment, may be placed at a convenient distance to the north, and another to the south as temporary guides, till their positions be finally ascertained by some of the different methods, which we propose to illustrate in our next section.

§ LVII. TO DETERMINE THE EXACT PLACE FOR A MERIDIAN MARK

1. **When** a mark has been placed nearly in the meridian of an instrument's position, by any of the rough methods, the final adjustment requires both accurate observations, and certain computations depending on the method adopted. We have already specified three modes of bringing a transit-instrument into the meridian of the place of observation, which are generally considered the most desirable, a more particular explanation of which we reserved for this section, and shall now proceed to exemplify them in succession.

2. *A Circumpolar Star.*—The motion of the pole-star is so slow, and its apparent place is now so well known by previous computation, given in the Nautical Almanac, and other ephemerides, that we may safely make use of the intervals in time between its successive upper and lower passages over the middle wire of the transit telescope, to ascertain whether or not this wire bisects the circle of its diurnal motion. If the eastern and western semicircles are not performed in like times, by a clock which is regulated to show correct sidereal time, when its rate is allowed for, we can, from the observed difference of time, determine a, the error in azimuth at the horizon, where it is presumed that the meridian mark is placed, or very nearly so; and then we can adjust for this error by means of the horizontal screw, which moves one of the Ys of the instrument's axis in azimuth, for which purpose it is desirable that its value should be known, or that a scale of seconds in arc should be placed horizontally above or below the said mark, and that the mark itself be adjustable. This mark may be an acute angle formed by the meeting of two strong coloured lines, a circular disc, or vertical parallelogram, painted on any solid material, or illuminated, according to the fancy or convenience of the observer; any one of which may be bisected by the middle wire, after the mark is finally adjusted. When a convex lens is placed in front of the mark, at the exact distance of its solar focus, and at right angles to the line of vision of the telescope, it will be of no importance at what distance the mark be placed, because the rays issuing from it are rendered parallel by their passage through the lens, and the vision of the telescope will be distinct with its eye-piece adjusted to the solar focus. Instead of this contrivance, suggested by Gauss, it has been usual, in adjusting small transit-instruments, to fix an additional plano-convex lens, ground to a radius equal to the distance from the mark, before the object-glass, to answer the same purpose of rendering the mark visible at a short distance, but as the two lenses may not have the same optical centre, an error may be thus introduced, which can only be detected by turning the additional lens round, and trying the effect in different positions. At 95.49 yards from the object end of the telescope, one inch will subtend 60″, and the scale will vary inversely as the distance, as may be seen by our Table computed for distances corresponding to angles subtended by a yard [§ XX.]. As an example of this method of adjusting the mark, let us take two observations of the pole-star's transits as made on July 16, 1826, in latitude 52° 26′ N., the upper one of which took place when the clock indicated $0^h\ 59^m\ 27^s.5$, and the lower one when the time shown was $12^h\ 58^m\ 34^s.5$, the declination of the star being at that time nearly 88° 23′. If we call the time given by the clock t at the upper passage over the middle wire, and t' at the lower passage, then, if we put L for the latitude of the place, δ for the declina-

tion of the star, and *a* for the azimuthal deviation in seconds of time at the horizon, this deviation may easily be found by the following formula.

$$a = \frac{t - t' - 12^{h}}{2. \cos L} . \cot \text{an}\, \delta. \quad (1.)$$

Thus . . $t - t' - 12^{h}$ in this example is 53^{s} . . log.	1 .72427
2 Ar. co. . . . . . constant log.	9 .69897
Cos. L 52° 26′ ar. co. = sec. L	0 .21490
Cotan. δ 88° 23′	8 .45061
a in time = $1^{s}.226$	0 .08875
15 . . . . constant log.	1 .17609
a in arc = 18″.40	1 .26484

Now as the first or western semicircle was passed through by the star in a shorter time than the second or eastern one, this shows that the object-end of the telescope was pointed 18″.4 to the western side of the true mark, or that the eastern end of the axis deviated so much from the east and west line, towards the north, which quantity must therefore be rectified by the screw and scale accordingly; and then the point, where the middle wire then cuts, will be the place of the meridian mark, provided the axis has been perfectly horizontal, and the collimation in azimuth well adjusted, which the work supposes. It is obvious that this method is independent of the right ascension of the star's place, but is not free from any change of rate which may have taken place in the clock's going for twelve hours, it being presumed that the clock does not vary, and this is the principal objection to the method, which however vanishes when the clock is good, since the motion of this star is very slow.

3. *A Pair of Circumpolar Stars.*—The late Mr. Butt proposed a method of rectifying an approximate meridian mark, by means of two stars, of nearly the same declination, but differing twelve hours, more or less, in right ascension: the object of this method is to do away the necessity of placing implicit confidence in the clock's rate for a longer period than the short interval elapsing between the superior, or inferior, passage of the first star, and the opposite passage of the second, at the two periods of observation, which ought to be at nearly twelve hours' distance from one another: if the differences of the two passages, taken at the reversed situations of the two stars, are the same at both periods, it is a proof that the semi-diurnal arcs of the entire circle are alike, and that consequently the transit instrument is properly placed in the meridian, provided it be truly adjusted in other respects; but if the intervals denoted by the differences be dissimilar, then a rectification is necessary, and its quantity must be computed and applied in the proper direction. We have given a catalogue of many pairs of stars which are adapted for this purpose, at pages 275 and 276 of our first volume; and shown at pages 411, 412, and 413, how without computation the position of the transit-instrument may be approximately adjusted, agreeably to the plan suggested by Mr. Butt; but as the polar distances of the stars were not considered, nor the latitude of the place taken into the account, the determinations there given are incorrect approximations, and ought not to be relied on where accuracy is the main object: that method is convenient for temporary observations, to be

taken with a small travelling instrument, but is not sufficiently accurate for a regular observatory furnished with fixed instruments. We will therefore resume the subject in this section, and explain how a meridian mark may be adjusted with the greatest precision by means of two circumpolar stars, differing nearly twelve hours in right ascension, and without having regard to their true right ascensions. If we put as before t for the time of the first star's upper passage, and t' for that of its lower passage, also τ and τ' for the times of the contrary passages of the second star, which is chosen to follow the former at a distance of nearly twelve hours in right ascension, then Biot and Delambre have shown that the proper formula for our purpose will be

$$a = \frac{(t-t'-12^h)-(\tau-\tau'-12^h) \sin\Delta \ \sin\Delta'}{2 \ . \ \cos. L \sin(\Delta'-\Delta)}, \quad (2.)$$

in which expression a is, as above, the azimuthal error in position, as it regards the true meridian, Δ represents the polar distance of the preceding star, Δ' the polar distance of the latter star, and L the latitude of the place. Now it is manifest that to obtain $(t-t'-12^h)$, or $(\tau-\tau'-12^h)$, the clock's going must be depended on for at least twelve hours, and in this case either of the two stars may be separately used, as we have already done the pole-star, and therefore nothing would be gained by this complex formula, more than could be obtained by the preceding more simple one but this formula will retain its value if we modify the terms thus, omitting the twelve hours, viz.

$$a = \frac{(t-\tau)-(t'-\tau') \ \sin\Delta. \ \sin\Delta'}{2. \cos L. \ \sin\ (\Delta'-\Delta)}. \quad (3.)$$

4. According to the notation used in this equivalent formula, it is obvious that the terms $(t-\tau)$ and $(t'-\tau')$ express simply the two short intervals of time taken up respectively between the two consecutive passages, one observed above, and the other soon after below, and taken again, after about twelve hours, in the reversed order, viz the first now below, and the second above; and it is only to indicate the duration of each of these short intervals that the clock is required. This latter formula, therefore, is suitable for giving the deviation a from the observed intervals only, according to Mr Butt's plan, without reference to the absolute right ascensions of the stars, or to the whole 12^h indicated by the clock. The application of the formula will be easily understood from the work of an example. But it must be understood that, when $(t-t'-12^h)$ is a greater interval than $(\tau-\tau'-12^h)$, the horizontal deviation a will be towards the east, and towards the west when $(\tau-\tau'-12^h)$ is greater than $(t-t'-12^h)$, and also that when the second interval $(t'-\tau')$ is greater than the first $(t-\tau)$, the deviation will be to the east, and the contrary.

5 On the 1st of January, 1828, the apparent right ascensions of the pole-star, and of ζ Ursæ Majoris præcedens (Mizar), are given in the Nautical Almanac 0^h 59^m 29^s, and 13^h 16^m 58^s, and their apparent north polar distances 1° 36′ 9″ and 34° 10′ 44″, which may be taken at 1° 36′ and 34° 11′, as arguments, sufficiently near the truth for our purpose. When the clock at an observatory in latitude 52° 26′ N. was regulated properly, and its error and rate allowed for, the times of the four passages taken by the transit-instrument, placed a little out of the meridian, but otherwise well adjusted, were as follow; viz.

Pole-star above	$1^h\ 0^m\ 0^s.55 = t$	Pole-star below	$12^h\ 58^m\ 55^s.47 = t'$
ζ Ursæ Maj below	$1\ 16\ 55\ 46 = \tau$	ζ Ursæ Maj. above	$13\ 16\ 58\ 16 = \tau'$
$t-\tau$.	$-\ 16\ 54.91$	$t'-\tau'$ . . .	$-\ 18\ 2.69$
Subtract .	$-\ 18\ 2\ 69$		
$(t-\tau)-(t'-\tau')$	$+\ 1\ 7.78 = 67^s.78$		

Now as $(t'-\tau')$, the second interval, exceeds $(t-\tau)$ the first, the deviation is towards the east, and the difference of the short intervals is $67^s.78$ let us try what will be the difference between the two longer intervals $(t-t'-12^h)$ and $(\tau-\tau'-12^h)$ thus

t . .	$1^h\ 0^m\ 0^s\ 55$	τ . .	$1^h\ 16^m\ 55^s\ 46$
t' .	$12\ 58\ 55.47$	. . τ' . .	$13\ 16\ 58.16$
$(t-t'-12^h)$	$1\ 5.08 = 65^s.08$	$(\tau-\tau'-12^h)$.	$-\ 2.70$
Subtract . . .	. . $-\ 2.70$		
$(t-t'-12^h)-(\tau-\tau'-12^h)$	67.78 as before,		

but to acquire this latter difference the clock must go truly for the longer interval, or for about $12^h\ 18^m$, whereas to obtain the former difference, the longer of the two intervals is little more than 18 minutes for the clock to perform. In either case the quantity $67^s.78$ is all we want from observations at the transit-instrument, for the work depending on our latter formula (3) will accomplish the rest in the following manner.

Sin Δ 1° 36′	8.44594
Sin Δ' 34° 11′	9.74961
$67^s.78$	1.83110
2 ar. co. . . . . . constant log.	9.69897
Cos L 52° 26′ ar co. = sec L	0.21490
Sin $(\Delta'-\Delta)$ 32° 35′ ar. co. = cosec .	0.26879
$a = 1^s.619$	0 20931
15 . . . . constant log.	1.17609
$a = 24''\ 29$ in arc, towards the east, . . .	1.38540

6 This method of determining the deviation of a transit-instrument from the meridian, requires four observations, and is therefore charged with four errors, which together will probably be greater than the single error of the clock in twelve hours, for a good clock will never vary half a second from its daily rate in this time, if it does, it is not fit to be admitted into a regular observatory According to the data now before us, either of the two stars should give the deviation separately, on a supposition of the times being taken and noted correctly, from our formula (1.), thus.

POLL-STAR.	
$t-t'-12^h = 65^s.08$.	1.81344
2 ar. co . . constant	9 69897
Sec. of lat 52° 26′	0.21490
Cotang. δ 88° 24′	8.44611
a = 1^s.491 or 22″.365 . .	0 17342

ζ URSÆ MAJORIS	
$\tau-\tau'-12^h = 2^s 70$ . .	0.43136
2 ar co. . .	9.69897
Sec. of lat. . .	0 21490
Cotang 55° 49′ . .	9.83225
a = 1^s.505 or 22″ 575	0.17748

The accordance of these two last results show, that the determination of the deviation may be accomplished as well by one circumpolar star as by two, when the clock can be depended upon for an interval of twelve hours, after a proportional part of its daily rate has been applied. The discrepance of one tenth part of the second of time, when both stars are used, arises probably partly out of the errors of observation, and partly from the declinations being taken only to the nearest minute. It will be convenient to unite the constant log of 2 ar. co. = 9 69897 with the log. secant of the latitude for a given observatory, and to make one constant of the two, which in our case becomes 0.91387, for Greenwich it will be 9.90444: then the sum of three logarithms, viz of $t-t'-12^h$, of cotang δ, and of the constant, will be the logarithm of the deviation a, whatever may be the circumpolar star and the only objection to the use of this method is, that the star must necessarily be twice visible within twelve successive hours, and if one of the passages were even visible by day-light, the clouds must be absent at both periods, which will not frequently be the case in variable climates. When the azimuthal deviation is required in arc, the logarithm of 15 may also be united in the constant, which in the last examples would become 1 08996, or 1.09, and the work of the pole-star would be abridged thus:

$t-t'-12^h$ 65^s 08	1 81344
Constant	1 08996
Cotang δ 88° 24′	8.44611
a = 22″.36 as before . . .	1.34951

7. We are now in a situation to show that the times of the transits of β Cephei and θ Ursæ Majoris, as given in the example at page 412 of our first volume, were assumed for the mere purpose of illustration. if we suppose the observations to have been made at Edinburgh (for the latitude of the place must necessarily exceed the polar distance of either star, that they may both be visible below the pole,) our formula ought to give the azimuthal error of the telescope's position the same, whichever star be chosen for affording the computation; but that this will not be the case in that example will appear from the subjoined statement; viz

β CEPHEI.	h	m	s
Superior passage (t) . . .	21^h	26^m	22^s 74
Inferior . . . . (t') . .	9	26	20 70
$t-t'-12^h$			2.04

θ URSÆ MAJORIS.	h	m	s
Inferior passage (τ) . . .	21^h	21^m	1^s 5
Superior . (τ') . . .	9	21	3.5
$\tau-\tau'-12^h$ . . .			− 2.0

Then $t - t' - 12^h = 2\,.04$ . . .	0 .30963	$\tau - \tau' - 12^h = -2\ 0$	0 30103 −
2 ar. co.	0 .69897	2 ar. co.	0 .69897
Sec of lat. $55^\circ\ 56'$	0 25169	Sec. of lat	0 .25169
Cotang $\delta\ 69^\circ\ 47'$ . .	9 .56615	Cotang. $52^\circ\ 29'$ . .	9 88524
$a = 6^s.70$	0 82644	$a = -13^s.71$	1 .13693 −

Hence it is evident that the assumption, of $t - t' - 12^h$, and of $\tau - \tau' - 12^h$, in this example, is different from what actual observation would afford, where the difference of the stars' declinations is so considerable. As a further proof, if we were to compute from our formula (3.), by taking $(t - \tau) - (t' - \tau') = 4^s.04$ from the same assumed data, we should find $a = 25^s.52$, differing from both the foregoing determinations

In the preceding examples the adjustments for the inclination of the axis, and for collimation in AR, are supposed to have been, as usual, correct. When they are not so, their co-efficients b and c, will require to be determined, the former from the formula already explained [§ L. 11.], $b = \frac{h}{60} \cdot (w + w') - e + e')$, and the latter from the annexed formula,

$$c = \tfrac{1}{2}\,(t_{,} - t) \,.\, \cos\delta + \tfrac{1}{2}\,(b_{,} - b) \,.\, \cos(L - \delta)$$

in which t denotes the time of the pole star's culmination, as derived from the wire next to the middle wire; b the inclination at that time, and $t_{,}$, $b_{,}$ the corresponding values of t and b after the axis is reversed. In this case the azimuthal error, a, must be computed by a more complex formula, thus ·

$$a = \frac{t - t' - 12^h}{2\ \cos L} \,.\, \text{cotang.}\,\delta + \frac{b \,.\, \cos(L - \delta) - b' \,.\, \cos(L + \delta) + 2\,c}{2\ \cos L\ \sin\delta} \qquad (4.)$$

where t' and b' belong to the *lower* culmination. But if the axis only is adjusted, then we shall have $c = \frac{1}{2}\,(t_{,} - t) \,.\, \cos\delta$.

8. The third method of determining the deviation, a, is by means of the observed passages over the meridian of two successive stars, having nearly the same right ascension, but differing greatly in declination The principle of this method is, that, if the difference of the times of the observed passages be exactly equal to the difference of the computed apparent right ascensions of the two stars, the instrument will necessarily be in the meridian, supposing the axis truly horizontal, and the collimation adjusted; but if one interval differ from the other, there must be a deviation from the meridian. If we put $(t - t') = dt$ for the difference of the observed times of passage, and $(\text{AR} - \text{AR}') = d\,\text{AR}$, for the difference of the apparent right ascensions of the two given stars, the formula for determining the deviation, a, will in this case be,

$$a = \frac{(dt - d\,\text{AR}) \,.\, \sin\Delta \,.\, \sin\Delta'}{\cos L\ \sin(\Delta' - \Delta)},\ \text{or} = \frac{(dt - d\,\text{AR}) \,.\, \cos\delta \,.\, \cos\delta'}{\cos L \,.\, \sin(\delta' - \delta)}, \qquad (5.)$$

in which expressions Δ and δ' represent the polar distance and declination of the higher star, and Δ' and δ those of the lower. In general the two stars suitable for our present purpose will have opposite declinations, one north and the other south, and ought to be removed from each other not less than forty degrees, we have given a list of such stars, with their subsidiary numbers, at pages 277 and 278 of our first volume but as the Nautical Almanac now gives the computed apparent places of sixty stars for every tenth day, we may frequently avail ourselves of a pair of those, to save the trouble of computing the apparent places For instance, we may take η Ursæ Majoris, or ε Bootis, with α^1 Libræ; α Coronæ Borealis with Antares, or with α^2 Libræ, α Capricorni with α Cygni, or with 61 Cygni, &c when the times of their passages occur conveniently for making the observations When the interval of time denoted by $(dt - d\text{Æ})$ is positive, the horizontal deviation of the instrument will be towards the east of the southern point, in northern latitudes, and the contrary when negative. The advantages of this method are, that the indication of time by the clock is wanted for only one short interval, while the instrument is turned to the south, and that the place requires not a northern aspect, it is however necessary that the apparent right ascensions of both stars be correctly known, for on this knowledge the accuracy of the method depends, as much as on the correctness of the observed intervals As an example to illustrate this method, we will take the computed right ascensions and observed times of passage of β Aurigæ and Sirius, on the 10th of January, 1828, at an observatory in latitude 52° 26′ N., when the transit-instrument had suffered a slight derangement in its position The work will stand thus

β Aurigæ . $5^h\ 46^m\ 51^s.91 = t$, and $5^h\ 46^m\ 53^s\,50 = \text{Æ}$
$6\ \ 37\ \ 25.66 = t'$. $6\ \ 37\ \ 33\ 66 = \text{Æ}'$

dt . . . . . $= -\ 50\ \ 33.75$ $d\text{Æ} = -\ 50\ \ 40.16$
Subtract $d\text{Æ}$. $-\ 50\ \ 40.16$

$dt - d\text{Æ}$ . . $=$ $+\ 6\ 41$

Then 6.41 . . . . log 0.80686
Sin Δ 45° 5′, or cos δ' 44° 55′ 9 85011
Sin Δ' 106° 29′, or cos δ −16° 29′ 9 98177
Sec L 52° 26′ 0 21490
Sin $(\Delta' - \Delta)$, or sin $(\delta' - \delta)$ 61° 24′ ar. co. = cosec 0.05652

$a = 8^s.131$, or 121″ 97 . . . . . 0.91016

This deduction shows that the object-end of the telescope deviated from the south point, towards the east, 8.131 seconds of time, or 121″ 97 of a great circle, at the time of making the observations, and if no meridian mark had previously existed, the determination here made points out the horizontal correction to be made towards the west of the point covered by the middle wire, to obtain the true place for a meridian mark. But a continuation of similar observations on other pairs of stars would render the final determination more complete.

9 As a second example we will take the data given at page 414 of our first volume, where *α Piscis Austrini* and *β Pegasi* have the times of their passages given, without any specified latitude of the place, on the 11th of October 1828, in this case also the example was introduced for the sole purpose of illustrating the use that may be made of the Catalogue of High and Low Stars, without adverting to the formula on which the accuracy of the results depends, and which will now be better understood. The work depending on the data before given, agreeably to our formula (5), will stand thus, if we take the latitude at 52° 26′; viz.

AR of β Pegasi	22^h	54^m	$22^s.26$	Observed time	22^h	54^m	$17^s.8$
AR of α Piscis Austrini	22	47	55.90		22	47	51.5
*d*AR . .		6	26.36	*dt* . . .		6	26.3
				*d*AR		6	26.36
				dt−*d*AR .		− 0	06

Then $-0^s.06$	log.	8.77815 −
Sin Δ′ 120° 33′, or 59° 27′ . .		9.93509
Sin Δ 62° 53′		9.94913
Cos lat. 52° 26′ ar. co	= Sec L	0.21490
Sin (Δ′−Δ) 57 40 ar. co.	= cosec	10.07317
$a = -0^s.089$, or $-1''.335$		8.95074 −

In this example, as *d*AR is greater than *dt*, we have *dt*−*d*AR *negative*, and the deviation in arc at the horizon, = − 1″.335, was therefore to the west of the southern point, and required adjustment accordingly. We shall have occasion to explain how the error in time, depending on the azimuthal deviation in position, may be correctly obtained, when we come to treat of the ERRORS OF THE TRANSIT-INSTRUMENT IN CONNEXION WITH THE CLOCK OR CHRONOMETER, in our subsequent section.

10. The work depending on the formulæ (3.) and (5.) is not capable of abbreviation by the aid of a comprehensive constant logarithm, as is the case when a single circumpolar star is used, and the computation made agreeably to formula (1.), but a general table may be constructed, in which the tabular logarithms may be equivalent to that portion of either formula, which is represented by $\frac{\sin\Delta \ \sin\Delta'}{\sin\Delta'-\Delta}$, for then the sum of three logarithms, viz. of (*dt*−*d*AR), of secant *L*, and of the tabular logarithm, will be the logarithm of *a* in time, or in arc if the logarithm of 15 be combined with the log. sec. of the latitude. Indeed for a given observatory the logarithms of the sec. of lat. and of 15, may be combined with the tabular logarithm, and then the logarithm of (*dt*−*d*AR), the quantity depending on the observations, will be the only number to be united with such tabular logarithm, for giving the azimuthal error in arc without computation. The co-efficient (*dt*−*d*AR) will generally be a small quantity, and therefore it will not be necessary to extend the arguments of the table to fractional parts of a degree. We have computed a general table which, notwithstanding it is contained in two pages, comprehends a sufficient variety of tabular logarithms for the choice of high and low stars to be observed in succession, in any part of the world, and when the constant logarithm adapted to any particular observatory is united with all the contained logarithms successively, the table will become appropriate to that particular observatory. We have preferred using the polar distance as an argument, instead of the declination of a star, because sin (Δ′−Δ) makes no distinction between north and south declination, whereas (δ′−δ), to give the same arc in a circle of declination, demands particular attention to be paid to the signs in subtracting, when one star has north and the other south declination, which will generally be the case. The Table

in question we have denominated A, to denote its use in computations of the *azimuthal* error of position, and to distinguish it from the Table given in our subsequent section, denoted C, as giving the *corrections* in time depending on the azimuthal and other errors of the transit-instrument. The argument at the head of the table is the polar distance (Δ) of the *higher* star, and that at the side is the *difference* of the polar distances ($\Delta'-\Delta$) of the two stars observed, the tabular logarithm, as usual, being at the junction of the two vertical and horizontal columns, belonging to the respective arguments, taken to the nearest whole degrees. The examples already given will suffice to explain the mode of using the Table, and also the degree of accuracy that may be attributed to it. In the case of β Aurigæ and Sirius we have, with

Arg. 45° at the head (Δ), and 61° ($\Delta'-\Delta$) at the side . tab log.	9 .8905
$(dt-d\text{Æ}) = 6^s.41$ as before	0 .8068
Secant of lat. 52° 26′ (constant)	0 .2149
$a = 8^s.133$, or $121''.99$	0 .9122

In the second example of β Pegasi and a Piscis Austrini we have,

With the head arg. 63° (Δ), and at the side ($\Delta'-\Delta$) 48° tab. log.	9 .9546
$(dt-d\text{Æ}) - 0^s.06$	8 .7782 −
Secant lat 52° 26′ (constant)	0 .2149
$a = - 0^s.089$, or $- 1''.335$	8 .9477 −

11. In France, a circular disc, illuminated by a large reflector, and placed at a great distance, is sometimes successfully used as a meridian mark, which may be distinctly seen by night; and Mr. Pond, we understand, has lately proposed to use the cross wires of a small transit-instrument, at a short distance, for the same purpose, for when the small transit-telescope is directed under the tube of the large one, to view the northern distant meridian mark, its position as a southern one may be properly adjusted, and the northern mark may be observed by it from time to time, as a test of the due adjustment of the small transit-instrument's position. Such contrivance may prove very convenient by night, for placing the mark at an accessible distance, and for illuminating it, when proper pillars have been erected for its firm support. That no doubt may remain, with respect to both transit-instruments being in the true meridian, the Astronomer Royal proposes to make observations on the passages of the same stars occasionally, by both the instruments used at the same moments for as the smaller telescope has an object glass of four feet focal length, it is quite competent to afford accurate observations, and for this purpose has recently been fitted up in the southern opening of the observatory. The principle, on which the cross lines of the small telescope become visible through the large one, is the same, as when the collimator is used, which will be described hereafter, i. e. the rays of light, coming from the cross lines of the small telescope, become *parallel* after their passage through its object-glass, which is turned towards the large telescope, when used as a meridian mark.

TABLE A

AZIMUTHAL ERROR OF THE TRANSIT-INSTRUMENT

Arguments = Polar Distance of the Higher Star at the Top or Bottom, and Δ'—Δ at the Side

Δ'—Δ	74°	73°	72°	71°	70°	69°	68°	67°	66°	65°	64°	63°	62°	61°	60°
40°	0 1354	0 1365	0 1373	0 1378	0 1379	0 1378	0 1373	0 1365	0 1354	0 1341	0 1326	0 1305	0 1282	0 1256	0 1228
41	1232	1244	1253	1260	1263	1263	1260	1253	1244	1232	1217	1199	1177	1153	1125
42	1110	1124	1131	1142	1147	1149	1147	1141	1131	1124	1110	1093	1073	1050	1024
43	0989	.1005	1017	1026	1032	1036	1036	1032	1026	1017	1005	0989	0970	0949	0924
44	0869	0886	0900	0912	0919	0921	0926	0924	0919	0912	0901	0887	0869	0849	0826
45	0751	0770	0786	0799	0808	0815	0817	0817	0814	0808	0799	0786	0770	0752	.0729
46	0 0634	0 0651	0 0672	0 0687	0 0698	0 0706	0 0710	0 0711	0 0710	0 0706	0 0698	0 0687	0 0672	0 0655	0 0634
47	0519	0540	0559	0576	0588	0598	0601	0606	0601	0604	0598	0598	0575	0559	0540
48	0403	0426	0446	0461	0478	0490	0498	0502	0503	0502	0498	0490	0478	0464	0447
49	0386	0312	0335	0351	0370	0383	0392	0398	0402	0402	0399	0393	0383	0370	0354
50	0268	0199	0221	0216	0262	0277	0288	0296	0301	0303	0301	0296	0288	0277	0262
51	0057	0087	0113	0137	0155	0172	0184	0194	.0201	0205	0205	0201	0194	0186	0170
52	9 9912	9 9975	0 0002	0 0028	0 0049	0 0068	0 0082	0 0092	0 0101	0 .0107	0 0109	0 0107	0 0101	0 0093	0 0082
53	9828	9863	9 9893	9 9920	9 9943	9 9964	9 9980	9 9992	0002	0009	0013	0013	0009	0002	9 .9992
54	9711	9751	9783	9812	9837	.9858	9878	9892	9 9902	9 9911	9 9916	9 9918	9 9916	9 9911	9902
55	9599	9637	9671	9703	9730	.9753	9774	9792	9804	9814	9821	9824	.9824	9821	.9814
56	9484	9525	9559	9591	9624	9650	9670	.9690	9705	9718	9726	9731	9732	.9731	.9726
57	9370	9413	9451	9486	9517	9546	9570	.9590	9607	9621	9632	9638	9641	9641	9638
58	9 9255	9 9300	9 9341	9 9378	9 9411	9 9441	9 9468	9 9490	9 9509	9 9525	9 9537	9 9546	9 9549	9 9551	9 9550
59	9138	9186	9229	9269	9304	9336	9365	9389	9410	9428	9442	9454	9459	9462	9462
60	9022	9072	9117	9160	9197	9232	9262	9289	9312	9332	9348	9360	9369	9374	9374
61	8905	8957	9005	9050	9090	9127	9158	9188	9212	9236	9253	9266	9277	9285	9288
62	8787	8842	8892	8939	8981	9021	9056	9086	9113	9137	9158	9171	9185	9195	9200
63	8667	8725	8778	8827	8872	8914	8951	8984	9013	9039	9061	9080	9091	9105	9112
64	9 8546	9 8608	9 8663	9 8715	9 8762	9 8806	9 8846	9 8882	9 8913	9 8941	9 8965	9 8985	9 9002	9 9015	9 9024
65	8421	8488	8547	8602	8652	8698	8740	8778	8812	8843	8869	8890	8909	8925	8936
66	8302	8368	8430	8488	8541	8590	8634	8674	8711	.8744	8773	8797	8817	.8831	8848
67	8177	8247	8311	8372	8428	8480	8527	8569	8608	8644	8675	8702	8724	8743	8758
68	8049	8123	8191	8255	8313	8368	8418	8463	8504	8542	8576	8605	8630	8651	8668
69	7921	7997	8069	8136	8198	8255	8308	8356	8400	8440	8476	8508	8535	8559	.8578
70	9 7790	9 7870	9 7946	9 8016	9 8081	9 8141	9 8196	9 8246	9 8295	9 8338	9 8376	9 8410	9 8440	9 8466	9 8488
71	7657	7741	7820	7893	7961	8026	8081	8139	8188	8231	8275	8312	8343	.8372	8396
72	7522	7610	7692	7769	7841	7909	7970	8027	8080	8129	8173	8212	8246	8277	8303
73	7383	7476	7562	7643	7718	7790	7855	7915	7970	8022	8069	8112	8148	8182	8210
74	7242	7330	7429	7515	7594	7669	7737	7801	7860	7914	7964	8009	8049	8085	.8116
75	7097	7199	7291	7381	7467	7546	7618	7684	7747	7805	7857	7905	7948	7986	8021
76	9 6949	9 7054	9 7151	9 7240	9 7337	9 7410	9 7495	9 7566	9 7631	9 7693	9 7749	9 7799	9 7847	9 .7887	9 7925
77	6797	6909	7013	7112	7205	7289	7371	7444	7515	7579	7639	7693	7741	7786	.7826
78	6640	6759	6868	6971	7068	7159	7244	7322	7395	7461	7526	7581	7635	7683	7726
79	6479	6603	6719	6828	6928	7024	7114	7197	7274	7346	7413	7473	7529	7579	7625
80	6312	6442	6561	6679	6786	6886	6979	7069	7149	7225	7295	7360	7423	7473	7522
81	6141	6278	6406	6527	6640	6746	6844	6936	7022	7103	7177	7245	.7308	.7365	7418
82	5963	6107	6242	6369	6488	6600	6704	6800	6894	6976	7055	.7127	.7193	7255	7310
	74°	73°	72°	71°	70°	69°	68°	67°	66°	65°	64°	63°	62°	61°	60°

The Azimuthal Error a = tab log. + log $(dt - d\,Ж)$ + log Sec of Lat

TABLE A

AZIMUTHAL ERROR OF THE TRANSIT-INSTRUMENT [CONCLUDED]

Arguments = Polar Distance of the Higher Star at the Top and Bottom, and Δ − Δ at the Side

Δ − Δ	59°	58°	57°	56°	55°	54°	53°	52°	51°	50°	49°	48°	47°	46°	45°
40°	0 1196	0 1161	0 1123	0 1081	0 1036	0 0988	0 0936	0 0881	0 0823	0 0762	0 0696	0 0627	0 0554	0 0477	0 0397
41	1096	1061	1025	0985	0941	0891	0843	0790	0733	0673	0609	0541	0469	0391	0316
42	0996	0963	0928	0890	0847	0802	0751	0700	0641	0585	0522	0456	0385	0311	0234
43	0899	0865	0832	0796	0754	0710	0661	0610	0556	0499	0437	0372	0303	0230	0154
44	0800	0770	0738	0702	0663	0620	0573	0523	0470	0414	0353	0290	0222	0151	0076
45	0701	0677	0645	0610	0573	0531	0486	0438	0386	0331	0272	0210	0144	0071	0000
46	0 0619	0 0584	0 0554	0 0520	0 0484	0 0444	0 0400	0 0351	0 0301	0 0250	0 0190	0 0131	0 0066	9 9999	9 9925
47	0523	0492	0461	0432	0397	0358	0316	0270	0222	0170	0113	0053	9 9989	9923	9850
48	0426	0402	0375	0344	0311	0273	0232	0188	0140	0090	0036	9 9976	9913	9847	9778
49	0335	0312	0286	0257	0225	0189	0149	0106	0060	0011	9 9958	9900	9839	9774	9706
50	0245	0223	0199	0171	0140	0106	0068	0026	9 9981	9 9934	9882	9826	9766	9702	9635
51	0157	0136	0113	0086	0057	0024	9 9987	9 9947	9903	9857	9807	9752	9693	9631	9566
52	0 0068	0 0049	0 0027	0 0003	9 9975	9 9943	9 9907	9 9868	9 9827	9 9782	9 9733	9 9680	9 9622	9 9562	9 9498
53	9 9980	9 9963	9 9943	9 9920	9893	9863	9828	9791	9751	9707	9659	9608	9551	9493	9430
54	9892	9877	9859	9836	9811	9782	9750	9714	9674	9632	9585	9535	9480	9423	9362
55	9804	9790	9771	9753	9730	9703	9671	9637	9599	9558	9513	9464	9411	9351	9295
56	9718	9706	9690	9671	9649	9624	9593	9561	9525	9485	9441	9394	9342	9287	9228
57	9632	9621	9607	9590	9569	9545	9517	9486	9451	9412	9370	9321	9274	9220	9163
58	9 9546	9 9537	9 9525	9 9509	9 9490	9 9468	9 9441	9 9411	9 9377	9 9341	9 9299	9 9255	9 9206	9 9154	9 9098
59	9460	9453	9443	9429	9411	9391	9364	9336	9304	9270	9229	9186	9138	9088	9033
60	9374	9368	9360	9348	9332	9312	9288	9262	9231	9198	9159	9118	9072	9022	8968
61	9288	9284	9277	9268	9253	9235	9213	9187	9159	9126	9090	9050	9006	8956	8905
62	9202	9200	9194	9187	9174	9158	9136	9113	9086	9056	9020	8982	8939	8892	8841
63	9116	9116	9112	9105	9095	9080	9061	9039	9013	8985	8951	8915	8872	8827	8778
64	9 9030	9 9031	9 9030	9 9024	9 9015	9 9002	9 8986	9 8965	9 8940	9 8913	9 8882	9 8846	9 8806	9 8762	9 8715
65	8944	8947	8948	8941	8936	8921	8909	8891	8869	8843	8812	8778	8740	8697	8652
66	8858	8863	8865	8864	8857	8848	8833	8817	8798	8773	8743	8711	8674	8634	8590
67	8771	8778	8782	8782	8778	8771	8758	8743	8724	8702	8675	8644	8608	8570	8527
68	8683	8692	8698	8700	8698	8692	8682	8669	8650	8630	8605	8576	8542	8504	8464
69	8594	8605	8614	8618	8618	8613	8605	8594	8578	8558	8535	8508	8476	8439	8400
70	9 8505	9 8518	9 8529	9 8536	9 8538	9 8536	9 8528	9 8519	9 8506	9 8488	9 8465	9 8440	9 8410	9 8376	9 8337
71	8416	8432	8444	8453	8457	8458	8452	8444	8432	8418	8396	8372	8344	8312	8275
72	8327	8345	8359	8369	8375	8378	8375	8369	8356	8345	8327	8301	8278	8247	8213
73	8236	8256	8273	8285	8293	8298	8297	8293	8285	8272	8256	8236	8210	8181	8148
74	8144	8167	8186	8200	8210	8217	8218	8217	8211	8201	8185	8167	8142	8116	8083
75	8050	8076	8098	8115	8128	8136	8139	8139	8136	8128	8115	8098	8074	8051	8021
76	9 7956	9 7984	9 8009	9 8028	9 8044	9 8054	9 8060	9 8061	9 8060	9 8054	9 8043	9 8028	9 8008	9 7984	9 7957
77	7862	7892	7918	7940	7958	7971	7979	7983	7983	7980	7971	7958	7940	7917	7892
78	7765	7798	7827	7851	7871	7887	7899	7904	7906	7904	7898	7887	7872	7851	7827
79	7667	7703	7735	7761	7784	7801	7815	7824	7829	7828	7821	7815	7802	7783	7762
80	7566	7605	7640	7670	7695	7715	7730	7742	7749	7752	7749	7742	7732	7715	7695
81	7466	7507	7545	7578	7606	7629	7646	7660	7670	7675	7675	7670	7660	7646	7628
82	7362	7407	7447	7483	7514	7540	7560	7576	7588	7596	7598	7596	7588	7576	7560
	59°	58°	57°	56°	55°	54°	53°	52°	51°	50°	49°	48°	47°	46°	45°

The Azimuthal Error a = tab log + log $(dt - d\,Æ)$ + log Sec. of Lat.

§ LVIII. ON THE ERRORS OF THE TRANSIT-INSTRUMENT, IN CONNEXION WITH A CLOCK OR CHRONOMETER.

1. When a transit-instrument has been placed in the meridian, and carefully adjusted by any of the methods above explained, the permanency of its position and adjustments cannot be confidently reckoned upon in any climate on the globe, for any number of months, or even weeks, but as the ordinary observations will generally furnish data for computing the corrections due to any derangement, of which the nature and quantity are ascertainable, it becomes an important consideration with the observer to be able to render all his observations available, notwithstanding that some of them may have been taken by his instrument not perfectly adjusted. We will therefore devote this section to the investigation of such corrections, as may be made applicable to observations which must otherwise have been rejected, or which would have vitiated a regular series, if admitted.

2. When the axis of a transit-instrument is placed horizontally in the true direction of east and west, it may be considered as situated in a plane parallel to the equator: and when there is an error in this position, it may arise from either of two causes; the axis may not be perfectly parallel to the equator, or its direction may be out of the line that is truly east and west, as it regards those points in the horizon. Let the inclination of the axis of the instrument to the equator, measured upon an horary circle, be denoted by the character η, and let the angle which its projection on the equator makes with the east and west line be called θ, as measured on the equator; let the first angle η, be taken positive, when the axis rises above the equator on the eastern side, and negative on the western; also let the second angle θ, be taken positive, when the projection of the eastern end of the axis on the equator falls towards the south, and negative when it falls towards the north: then if we call t the sidereal time of a star's passage over the middle wire, or spider's line, of the instrument, the apparent right ascension, Æ, of such star, on account of the two errors, will be expressed by the formula

$$\text{Æ} = t - \theta - \eta \operatorname{tang} \delta,$$

where δ represents the star's declination.

3. Another error may exist in the observation, arising from a want of perfect collimation in azimuth: suppose the line of collimation to make an angle of $90° + c$ with the eastern side of the instrument's axis, the passage of the star over the middle wire will be retarded on account of this error, by the quantity $c \sec. \delta$, and the apparent right ascension in this case will be given by the formula,

$$\text{Æ} = t - \theta - \eta \operatorname{tang} \delta - c \sec \delta.$$

This formula however supposes that we know the exact sidereal time of the passage, which is seldom the case when the instrument is not in due adjustment: if therefore we put t for the observed time by the clock, and represent its error $(t - \text{Æ})$ by e, taken positively when the clock is fast, then we must substitute $(t - e)$ for t, and the complete formula, which will give the apparent right ascension of the star, will be

$$\text{Æ} = t - e - \theta - \eta \operatorname{tang} \delta - c \sec \delta. \qquad (1.)$$

When this equation is applied to the *lower* passage of a circumpolar star, let t be diminished by twelve hours, and for δ let the supplement of the declination be taken. If we suppose the quantities e, θ, η, and c to be unknown, we may determine them by the observed transits, over the middle wire, of three different stars of known right ascensions; but in this case $(-e-\theta)$ constitutes a single unknown quantity, hence it is impossible to separate, by observations alone, the error of the projection of the axis on the equator, from the error of the clock. With respect to the error θ, the observations of the successive stars may be compared to such as are made by a telescope mounted on an equatorial axis in due adjustment, and making an horary angle θ towards the west: all the observations would in that case be mutually comparable among themselves, if we were to employ $(e+\theta)$ as the error of the clock; but the true horary angle, under which they were made, would still remain unknown.

4. The process for determining the three quantities $(e+\theta)$, η, and c, in the preceding formula (1.), by means of the observations of three stars of known right ascension, will be understood from an inspection of the following example. On the evening of the 15th of July, 1826, the passages of the three subjoined stars over the transit-instrument were observed as stated in the subjoined register, viz.

Names of the Stars	Times of their Transits	Apparent Right Ascensions	Apparent Declinations
η Ophiuchi	$17^h\ 1^m\ 2^s.08$	$17^h\ 0^m 29^s.82$	− 15° 30′
α Herculis	17 7 19 84	17 6 46 44	+ 14 36
γ Draconis	17 53 10 .52	17 52 37 .33	+ 51 31

In these observations the rate of the clock is supposed to remain unaltered for an hour. By substituting these values, or the quantities deduced from them successively in the preceding formula, we shall have

$$17^h\ 0^m 29^s.82 = 17^h\ 1^m\ 2^s.08 - e - \theta + 0.277\ \eta' - 1.038\ c$$
$$17\ \ 6\ 46.44 = 17\ \ 7\ 19.84 - e - \theta - 0.260\ \eta' - 1.033\ c$$
$$17\ 52\ 37.33 = 17\ 53\ 10.52 - e - \theta - 1.258\ \eta' - 1.607\ c$$

If now we subtract these equations one from another respectively, we obtain

$$6^m 16^s.62 = 6^m 17^s.76 - 0.537\ \eta' + 0.005\ c$$
$$45\ 50.89 = 45\ 50.68 - 0.998\ \eta' - 0.574\ c$$

or,

$$0.537\ \eta' - 0.005\ c = 1^s.14$$
$$0.998\ \eta' + 0.574\ c = -0.21$$

Let us multiply the first of these equations by 0.574, and the second by 0.005, and add the products together, when the sum will be

$$0.313\ \eta' = 0^s.653$$

which gives

$$\eta' = \frac{0.653}{0.313} = 2^{s}.09$$

This value of η, being substituted in the second of the two preceding equations, affords

$$0.574\, c = -0.21 - 2.09 = -2.30$$

from which we deduce

$$c = -\frac{2.30}{0.574} = 4^{s}.01.$$

Put now these determined values of η and c in the first of the above three equations, and we shall have

$$17^{h}\,0^{m}\,29^{s}.82 = 17^{h}\,1^{m}\,2^{s}.08 - e - \theta + 0^{s}.57 + 4^{s}.16,$$

which equation gives $e + \theta = 36^{s}.99$ for the error of the clock, including the error from the deviation of the projection of the instrument's axis on the equator. Now by substituting the determined values of $(e + \theta)$, η, and c in our formula (1.), we have at length

$$\text{AR} = t - 36^{s}.99 - 2^{s}.09 \operatorname{tang} \delta + 4^{s}.01 \sec \delta.$$

This formula will give the apparent right ascension of any observed star corresponding to the time, t, of the clock, and to the approximate declination of the said star, as read on the circle attached to the transit-instrument, as a finder; and, as we have said, all the observations made under the same circumstances of incorrect adjustments, will be comparable among themselves; but yet we know not hitherto the true error of the clock taken separately from the error θ, depending on a devious projection of the axis over the equator.

5. Instead of employing the corrections given by the preceding formula, it will be more convenient in practice, to adjust the position of the instrument as often as may be found necessary, which indeed is usual in England. If we represent the bias or inclination of the axis to the horizon, as determined by a good level, by b, and make it positive when the eastern end is elevated, and negative when the same end is depressed; and if we put a for the azimuth of the projection of the axis on the horizon, counted from the eastern point, and make it positive when it lies to the south, and negative when to the north, these errors, b and a, may be expressed, in function of the former terms η and θ, by the two annexed equations, viz.

$$b = \eta \sin L + \theta \cos L$$
$$a = \theta \sin L - \eta \cos L$$

in which L represents the latitude of the place of observation. These two equations will determine the errors b and a, when the errors η and θ are previously known. And in the inverse case we deduce from them

$$\eta = b \sin L - a \cos L$$
$$\theta = b \cos L + a \sin L,$$

and these expressions, being substituted in our formula (1.) by easy reductions, give

$$\text{AR} = t - e - b \cos (L - \delta).\sec \delta - a \sin (L - \delta).\sec \delta - c \sec \delta. \qquad (2.)$$

6. When the axis of the instrument has been rendered truly horizontal by the proper application of its level, the term multiplied by b will disappear in the last formula, as will also

the term in which c is involved after the adjustment for collimation has been carefully made, as we have before directed [§ LVI], the formula will then become

$$Æ = t - e - a \sin (L - \delta) . \sec \delta \qquad (3.)$$

We have explained in our last section the different methods of determining the deviation of the telescope, designated by a, first by the upper and lower passages of any circumpolar star, secondly by means of two circumpolar stars, differing nearly twelve hours in right ascension; and thirdly by means of a high and low star having nearly the same right ascension

7. When the transit-instrument is adjusted for collimation and properly levelled, and when we know the exact deviation in time of the transit-telescope's position, we may compute the error e', or effect that it will have on the times of the passages of any circumpolar or other star by the formula

$$e' = a \frac{\cos (L \pm \Delta)}{\pm \sin \Delta} = a \cos \text{alt} \times \sec \text{dec} \qquad (4.)$$

in the former of which expressions the positive sign belongs to the upper culmination of a circumpolar star, and the negative sign to the lower passage. The altitude and declination of the star may be taken near enough from the observation at the instrument, when the latitude is known Let it be required to ascertain what were the errors in the times of both passages of Polaris and ζ Ursæ Majoris, in the example given in our last section, taking the deviation from the meridian towards the east at 1ˢ.5?

UPPER PASSAGE OF POLARIS

$a = 1^s 5$ ... . 0 .17609
Cos $(L + \Delta)$ 54° 2′. .. 9 76887 = cos alt
Sin Δ Ar Co 1° 36 .... 1 .55406 = sec dec
$e' = 31^s 55$ 1 49902

LOWER PASSAGE OF POLARIS

$a = 1^s.5$ ... . 0 .17609
Cos $(L - \Delta)$ 50° 50′ (or cos alt) 9 .80013 -
Sin Δ Ar co (or sec dec). .. 1 .55406
$e' = - 33^s.93$ . . ... 1 .53058 -

Æ of Polaris by Naut. Alm. Jan. 1, 1827	0ʰ 59ᵐ 29ˢ
The error arising from the deviation	+ 31 55
Observed time of the upper passage	1 0 0 55
The error at the lower passage	— 33 93
Apparent time of lower meridian passage by Naut Alm . .	12 59 29 .4 nearly
Observed time of the lower passage	12 58 55 .47

UPPER PASSAGE OF MIZAR.

$a = 1^s 5$ 0 17609
Cos $(L + \Delta)$ 86° 37′ . 8 .77097 = cos alt
Sin Δ 34° 11′ Ar Co. . 0 .25039
$e' = + 0\ 157$ ...... 9 .19745

LOWER PASSAGE OF MIZAR

$a = 1^s 5$ 0 17609
Cos $(L - \Delta)$ 18° 15′ .. 9 97758 —
Sin Δ Ar Co 0 25039
$e' = - 2^s.535$ .... 0 .40406 —

Æ of Mizar, Jan. 1, 1827, by Naut Alm	$13^h 16^m 58^s$
Error c' at the upper passage.	+ 0 .16
Observed time of passage above	13 16 58 16
Error c' at the lower passage	— 2 .54
Observed time of passage below	1 16 55 .46

These all accord with the observed times given in the last section

8. When the instrument is truly placed in the meridian, and properly adjusted for collimation and horizontality of its axis, the simple difference between the observed time of a star's passage and its apparent right ascension, computed from the best tables, $(t - \text{Æ})$ is the *error*, *e*, of the clock, and will be *plus* or *minus* accordingly as the observed time exceeds or falls short of the apparent right ascension. In the preceding example we have supposed the clock to have no such error, viz $(t - \text{Æ}) = 0$, or that it is allowed for in the observations, but otherwise the error arising from the going of the clock, computed from the rate up to the moment of the star's passage must be increased or diminished by the error arising out of the position, as given by our formula (4.), and the whole error denoted by ε will be, in this case,

$$\varepsilon = (t - \text{Æ}) \pm c', \text{ and its correction } = (\text{Æ} - t) \mp c'\cdot$$

for it must be remembered, that every *error*, whether arising from the going of the clock, or from a deviation in the position of the instrument, and its corresponding *correction* must have opposite signs. The *rate* of the clock is its daily gain or loss, which is always equal to the difference of its errors on any two successive days, or to the difference of its errors taken on two distant days, when divided by the number of intervening days, in the former case the rate may be expressed by $(t - \text{Æ}) - (t' - \text{Æ}')$, where t' and $\text{Æ}'$ belong to the second day, but in the latter, the rate will be $\frac{(t - \text{Æ}) - (t' - \text{Æ}')}{n}$ where t and Æ belong to the first day, t' and $\text{Æ}'$ as before to the last, and n denotes the number of intervening days. The rate thus expressed does not require the instrument to be placed in the meridian, when the observations made to determine it are confined to one star, because the uniformity of the earth's rotation will bring the star to any permanent vertical line at the termination of every twenty four sidereal hours, but if the days of observation do not immediately follow one another, the apparent place of the star will alter a little by reason of precession, aberration, and nutation, and therefore meridianal observations of the same star, compared with its computed apparent right ascension, for each successive day, will be preferable. In using a solar clock, the same star should return to the meridian $3^m 55^s.908$ sooner on each successive night. When the error of the clock is known by observation at a given hour on any day, and also its rate, or daily variation of its error, it will be easy to compute the existing error due to any moment, so far as the going of the clock is concerned; but the three modifications of such error, depending on the telescope's deviation, on the inclination of its axis, and on the want of due adjustment of the line of collimation in azimuth, must be determined by our preceding formulæ respectively, and applied as corrections of the clock's error $(t - \text{Æ})$, or c, as often as the causes on which they depend are found to exist, otherwise the secondary errors, derived from the state of the transit-instrument, not being corrected, will affect the right ascensions derived immediately from the simple or primary error of the clock. Bessel, Struve, and Littrow strongly advocate the practice of detecting the errors of the transit-instrument as often as may be convenient, and of registering them along with the observations, that the reduction may afterwards take cognizance of them, in gaining the mean right ascensions As we have called $(t-\text{Æ})$ the error of the clock c, and the error in time derived from the telescope's deviation e', we will denominate the error arising from b, the inclination of the axis, e'', and the error due to c, the want of collimation, e''', then $c+e'+e''+e'''=\varepsilon$ will be the amount of all the corrections that can take place, except for the errors of observation, and, when these are applied, we shall have $\text{Æ} = t + c + e' + e'' + e'''$, or $\text{Æ} = t + \varepsilon$. The different terms contained in our formula (2.) point out the processes by which these various corrections may be computed, when the coefficients b, a, and c, are known from examination, but to facilitate the computations, we have constructed a table of twelve pages, subjoined to this section, that will enable the practical astronomer to correct his observations, made by a transit-instrument out of due adjustment, without much trouble or risk of committing mistakes, provided a proper pair of high and low principal stars should be found in his evening's list

9 The table which we have computed is denominated C, because it gives the *correcting* tabular logarithms for the three separate corrections, viz. log cos $(L-\delta)$. sec δ, to be added to the logarithm of the coefficient b, log sin $(L-\delta)$. sec δ, to be added to the logarithm of the azimuthal error a, found by means of our table A, in the last section, and log sec δ to be added to the logarithm of the coefficient c. The last of these terms is contained in the second column of the two first pages of the table, the side argument is the *declination* of the star in every case, and the argument at the head may be either the sine or the cosine of $(L-\delta)$ which term denotes the zen dist.; or of $(L+\Delta)$, which is the altitude, or its supplement. The differences in each single degree at the right hand side of each page, are common to all the horizontal adjacent lines of the same page, and the differences at the bottom are common to all the adjacent numbers in the vertical columns the first series of differences applies to the declinations, and the second to the arguments at the head, but when the coefficients are small quantities, the differences may in general be disregarded, and in no case will it be necessary to attend to second differences, in gaining proportional sexagesimal parts. The stars for which this table is adapted are supposed to be observed to the south in north latitudes, or to the north in south latitudes, in the latter of which cases the sign of the azimuthal error a will be changed, but it will answer also for circumpolar stars, provided they be at a considerable distance from the pole, so as not to require second differences to be regarded in taking out the tabular logarithms Indeed if both the stars were near the pole, though at opposite sides, the difference of their meridian altitudes would not be large enough, to allow of their being considered *high* and *low* stars of the description required.

10 To explain more fully the connexion that this section has with the preceding one, and to give a more connected detail of the successive operations that the young astronomer will have to encounter, in his first attempt to take an accurate observation of the transit of a star, we will conclude this section with an example of the application of a high and low star which we had lately occasion to observe, and which will afford us an opportunity of conducting him step by step to the practical application of the directions, which we have attempted to afford him, as well with regard to the adjustments of the transit-instrument, as to the errors derived

from its devious position, and the corrections of time arising out of such errors. On the evening of the 25th of May, 1827, at our occasional residence in the parish of Islington, in latitude 51° 33′ nearly, we had a friend who was desirous of witnessing the process of placing in the meridian, and of adjusting a new portable transit-instrument, on a pier that had been previously erected in the garden, which commanded both a northern and southern aspect, with dwelling-houses at a convenient distance both to the north and south. The focal length of the telescope was only about 32 inches, its aperture 2 .5, and the length of its axis 18 , its frame was supported by only three feet, forming an isosceles triangle, and it had five vertical parallel spider's-lines, and a striding level for the axis, as well as a short adjustable level made fast to the vernier-bar, that carried two opposite verniers, reading to the accuracy of single minutes, on a circle of 7 .5 inches diameter, fixed on one of the pivots of the axis. The instrument was well made, and had all the requisite appendages for completing the adjustments. A pocket chronometer, beating quarters of a second, was put nearly to the time given by the church clock, which seldom differs more than from one to two minutes from Greenwich time. The pole-star's lower passage, according to the Nautical Almanac, was computed to be at $12^h\ 59^m\ 43^s$ of sidereal time, which was equal to $8^h\ 49^m\ 5^s.7$ solar time (Vol. I. p. 334.), the sun's complement to the equinox being then $19^h\ 54^m\ 18^s.6$, and the equation of time $-\ 3^m\ 28^s.7$.

11. The instrument was carried and placed on the pier more than an hour before the expected time of the pole star's passage, to afford time enough for preparation. Three small clock-wheels without arbors, but having their teeth cut, and a hole in the centre of each, were placed on the pier to receive the feet of the stand, which was of cast iron, similar to the one we have described, except as to dimensions. The cross lines at the eye-piece were found to be 0 .2 of an inch out of the focal point of the object-glass, and the Ys of the level were so formed, that the place of the bubble was at the same end of the axis before and after reversion, and therefore required adjustment, by scraping with the back of a pen knife, after several trials, but at length it was rendered capable of being reversed without altering the station of the bubble, and the pivots of the axis were also found to be alike. The same process made the axis horizontal at the same time, by means of the third or single foot-screw, which was altered a little at each reversion, after the proper Y had been scraped. The telescope was now pointed to an accidental small mark in a brick of the house to the north, which bisected the middle line, the bubble of the vernier having been previously put to its zero, after this an elevation given to the telescope showed that the parallel lines were already truly vertical, by the middle line continuing to bisect the chosen mark; the horizontal line was then brought to bisect the same mark, the telescope having now a diminished aperture to render the mark clearly visible, and the exact altitude was read on both the opposite verniers, while the circle's bubble remained unaltered by means of its clamp. the axis of the telescope was then reversed, end for end, and the telescope again directed to the said mark, which was now seen on one side of the middle line, which appearance proved that the collimation wanted adjustment; the mark, however, was again bisected by the horizontal line, by means of the screw of slow motion, attached to the circle's clamp, and the verniers gave an altitude differing by some minutes from the former reading, one half of the difference, as nearly as could be estimated, was allowed for by the screw of the circle's level, and the other half by the proper screws at the eye end of the telescope the middle vertical line was also now brought towards the mark,

one half of its deviation by the horizontal screw of the proper Y, and the other half by the pair of side screws near the eye-end of the telescope, appropriated to this purpose. The telescope was then returned to its original position, and found so nearly in adjustment in all the respects that have been mentioned, that slight alterations and repeated adjustments, in the positions a second time reversed, completed the preparatory work, and put the instrument into a state fit for being used

12. By this time the pole-star had become visible to the naked eye, and the telescope was therefore elevated to $49^\circ\ 53'$, and the whole frame turned round a little, by sliding the two indented small plates under the two feet at one end, till the star was in the field of view, when the axis was again levelled As the star was now in the interval next to the middle line, and apparently to the right of it, it gradually approached this line in its lower passage, and the horizontal screw of the Y, that alters the azimuth, had range enough to keep the star on this line till the chronometer indicated $8^h\ 49^m\ 6^s$, when the instrument was suffered to remain, on a supposition that it stood nearly in the meridian, but as the chronometer did not give the time correctly, it was expected that there would be a certain deviation, which remained to be determined by subsequent observations. Two of the supporting small wheels, or plates, were now cemented to the stone cap of the pier, by a composition of pitch, bees wax, and fine brick-dust, by means of a heated rod of iron, which made the liquified substance fill the vacant spaces between the teeth, and form a border all round their circumferences, that prevented all dislocation of the instrument till further observations had been made

13. After the instrument had been fixed in the meridian approximately, the sky became partially obscured by passing clouds, but after waiting upwards of an hour, we had an opportunity of observing ε Bootis and α^2 Libræ, which were seen faintly through the thin clouds, and on turning the object end of the telescope over towards the south, the former was observed to pass, on taking a mean of the five lines, at $10^h\ 24^m\ 11^s$ 65 and the latter at $10^h\ 28^m\ 7^s.5$ solar time by the chronometer. These times were converted into sidereal times by the usual process, by means of our table above referred to, in the following manner, viz

ε Bootis	$10^h\ 24^m\ 11^s$ 65	α^2 Libræ	$10^h\ 28^m\ 7^s.50$
Equation in this case .	+ 3 28.70		+ 3 28.70
Apparent time . . .	10 27 40 35		10 31 36.20
10^h accel. = $1^m\ 38^s$ 56 27^m = 4 44 40.35 = 0.11	+ 1 43 11	10^h = $1^m\ 38^s$ 56 31^m = 5 09 36.2 = 10	+ 1 43.75
	10 29 23.46		10 33 19 95
Comp. to Equinox . . sub	19 54 18.60	sub.	19 54 18.60
Sidereal time t .	= 14 35 4.86	t'	= 14 39 1.35
		t	= 14 35 4.86
		$d\,t = t - t'$	− 3 56.49

		h	m	s
ε Bootis Æ by Naut. Alm		=14	37	28.91
α^2 Libræ Æ′		14	41	23.04
Æ − Æ′ or d Æ	sub.	—	3	54.13
$t-t'$ or dt.		—	3	56.49 (brought over).
$dt-d$ Æ		—		2.36

If the quantities dt, and d Æ thus obtained had been alike, the instrument would have been found properly placed in the meridian, supposing the axis horizontal, and collimation perfect, but as they were not alike, we must now determine the azimuthal error of position, and thence the error e' depending on it, according to our formula exemplified in the last section, thus,

$dt-d$ Æ $= -2^s.36$	0.37291
Sin Δ 62° 12′ ε Bootis	9.94674
Sin Δ′ 105 19 α^2 Libræ (Comp.) .	9.98429
Sec L, 51° 33′	10.20633
Cosec (Δ′−Δ) 43° 7′	10.16527
Azimuthal error − $4^s.737$	0.67554 towards the west.

Then to gain e' the error in time due to each star, we have (formula 4.)

ε Bootis $a = -4.737$.	0.67554 −	α^2 Libræ a	0.67554 −
Cos $(L+\Delta)$ 113° 45′ = 66° 15′	9.60503	Cos $(L+\Delta')$ 156° 52′ = 23° 8′ .	9.96360
Cosec 62° 12′ (Δ) .	0.05326	Cosec 74° 41′ (Δ′)	10.01571
$e' = -2.157$	0.33383 −	$e' = -4.517$	0.65485 −

Now for ε Bootis we have $t-$Æ$=e$ . .	$= -2^m\ 24^s.050$,	α^2 Libræ	$= -2^m\ 21^s.690$
e' . . .	$=\quad -\ 2.157$		$=\quad -\ 4.517$
And the whole error $\varepsilon = e + e'$.	$= -2\quad 26.207$		$= -2\quad 26.207$

This last quantity, ε, is the error of the chronometer in sidereal time, and as it comes out the same for both stars, it proves that our computation is correct. The chronometer was, therefore, put forwards just two minutes. Let us now see what our tables A and C will give, to save us the trouble of rigid computation by the formulæ In table A, with argument sin 62° at the head, and 43° δ at the side, we have,

The tabular number . .	0.0970		
Then add log cosec L 51° 33′	0.1061		
Also log $-2^s.36 = dt-d$ Æ . .	0.3729 −		
To obtain the deviation, $a = -4^s.737$	0.6755 −	. . . .	0.6755 −
Table C, cos 66° at the head $(L+\Delta)$ At the side 28° (δ) for ε Bootis . . }	9.6633	α^2 Libræ 23° $(L+\Delta')$ . . 15 (δ′) . . . . . }	9.9791
Nearly as above, viz. $e' = -2^s.18$ . .	0.3388 −,	and $e' = -4^s.515$	0.6546 −

14. With respect to fixing a meridian mark from the azimuthal error, which we have now obtained, we had no micrometer at the eye-piece of the telescope before us, nor scale at either of the walls to the north and south; but we had certain intervals between the vertical spider's lines, to which we had the means of assigning a value, and then we obtained a fractional part of such value, which, by the aid of estimation of the eye, was substituted for a scale. We have not here given the exact times of the two passages over each of the five spider's lines; but on reference to our register, we find that the mean value of the four intervals by ε Bootis was 30ˢ.35; and as α^2 Libræ was seen only at the two last lines, and had the passage at the middle line inferred, this mean interval must, for the present, be considered as the true one: then to find the fractional part of the equatorial interval, we have

30ˢ.35	1.48216
Cos. δ of ε Bootis 27° 48′ .	9.94674
−4ˢ.737, ar co	9.32450−
−5.668	0.75340−

Hence it appears that the azimuthal deviation towards the west was between $\frac{1}{5}$th and $\frac{1}{6}$th of an interval, which fractional part was afterwards adjusted by estimation on a brick of the south wall, by means of the horizontal screw of the proper Y: and in this way a portable instrument, carried on shore from a vessel, may be placed on a rock, or other solid support, and be nearly adjusted at the first subsequent passage of the pole-star, and then more correctly by a pair of following high and low stars, so as to gain from them the correct time at the place of observation. If the azimuthal error alone had been the object for determination, the solar times of passage, of the high and low stars, would have been sufficient for the purpose, without converting them both into the corresponding sidereal times for the given day, by simply applying the *acceleration* to the *difference* of the solar times; which would at once give $t-t'$, though we should not have the absolute sidereal times t and t' themselves; in this case we had

	h	m	s
ε Bootis passing at	10	24	11.65
α^2 Libræ, at	10	28	7.50
Diff.		3	55.85
Acceleration . . .			0.64
$d\,t = (t-t')$, exactly as before .	=	3	56.49

but in this way we could not have obtained the chief error $t-Æ$, or $t'-Æ'$, because neither t nor t' would have been known, without computation.

15. If, when the observations of ε Bootis and α Libræ were made, the level had detected an error in the inclination of the axis towards the west, the error in the time would have been an additional quantity for each star, which quantity may be obtained either from the term $b\,.\cos(L-\delta)\,.\sec\delta$, or from the table. Suppose the level had shown the inclination to be 1ˢ.75, arising from the formula before explained [§ L. 11] $\frac{k}{60}\;(w+w')-(e+e')$, obtained from the level having, both before and after reversion, its bubble rising to the east, in the observations

of these stars the times would have been affected by the error e'' thus arising; for according to the formula, we shall have

For ε Bootis.		For α^2 Libræ.	
$b = -1.75$	0.24304	$b = -1.75$	0.24304
Cos $(L - \delta)$ 23° 45′. . .	9.96157	Cos $(L - \delta)$ 66° 52′. . . .	9.59425
Secant δ — 27° 48′ . . .	10.05326	Secant δ — 15° 19′. . . .	10.01370
$-1^s.811 = e''$.	0.25787	$e'' = 0^s.709$.	9.85099

It is evident from these deductions, that the effect of an inclination of the axis is greater on the time of passage of a high star than of a low one, indeed so much so, that the intervals $t - t'$, $t -$ Æ, and $t' -$ Æ′, must be visibly affected by it at the moments of taking the observations. In using Table C for obtaining this error, we must enter it thus, viz.

For ε Bootis.		For α^2 Libræ.	
Cos 24° at the head and 28° at the side } give	0.0148	Cos 67° at the head and 15° at the side } give . .	9.6070
-1.75	0.2430 —	-1.75.	0.2430
$-1^s.811 = e''$	0.2578 —	$-0^s.708 = e''$.	9.8500

Also if the error of collimation, c, had been towards the east or west, a given quantity, say 12″ or $0^s.8$, the value of e''', the effect produced on the time of ε Bootis, would have been $0.8 . \sec \delta$, viz.

$$\left.\begin{array}{l} 0.8 \;.\;\;.\;.\; 9.90309 \\ \text{Sec } 27^\circ\, 48' \;.\;.\;.\;.\;.\; 0.05326 \end{array}\right\} = \pm 9.95635 = \pm\, 0^s.904$$

which error e''' will be $0^s.906$ from using 28° as an argument in the second column of the first page of Table C.

16 The errors e' and e'', arising from the deviation of the telescope, and inclination of its axis, may also be obtained in natural numbers from our long Table, XVIII, at pages 47—91 of our first volume, if the *zenith distance* be substituted for the altitude as an argument, in taking out e', and if the *altitude* be made the argument in the latter case for gaining e'', for the table is composed of multiples of the sines only, but may be used for cosines with the complements of the given angles. The columns numbered 1, 2, 3, &c. at the head of each page, must be used for the numbers belonging to the azimuthal error a, as the second argument, in the way that the table is used for giving the aberration or nutation, by means of the known constant. This application of the table extends its utility, and will be understood by a repetition of the examples we have already given, where our new Table C was used. The difference in the construction of these tables is, that the Table C, contained in this section, includes the secant of the star's declination in the tabular logarithm, and requires the logarithm of the deviation a to be added to it, whereas Table XVIII, in the other Volume, includes the natural number denoted by a, but not the natural secant of the declination, which must therefore at all times be applied as an additional factor, to obtain the required product. In the example of ε Bootis, where a is

given = $4^{s}.737$, the argument for the zenith distance is 23° 45′, or in the table $\overset{s}{\mathrm{O}}$ 43° 45′, which gives

4^{s}	1 .611
.7	282
04	.16
	1 .909 × 1 13 (nat sec 27° 48′) = $2^{s}.157 = e'$

For α^2 Libræ, with zenith distance $\overset{s}{\mathrm{II}}$ 6° 52′ we have

4^{s}	3 679
7	644
.04	37
Sec 15° 19′	1 .036 × 4 .360 = $4^{s}.517 = e'$.

Also for e'' of ϵ Bootis

Arg. $\overset{s}{\mathrm{II}}$ 6° 45 = alt, and b = 1 .75

$1^{s}.7$	1 .557
.05	46

Natsecδ27°48′(=1.13) × 1 .603 = $1^{s}.811 = e''$.

And of α^2 Libræ

Arg. $\overset{s}{\mathrm{O}}$ 23° 8′ = alt.

$1^{s}7$	6 .679
.05	19

Sec 15° 19′ (=1 .036) × 6 .698 = $0^{s}.6939 = e''$.

17. On the 2d of June, the evening being free from clouds, after we had turned the object end of the same telescope to bisect the point on the wall, which now formed the meridian mark towards the south, we again observed the same stars, and deduced the following results, which may be put down in an abridged form thus:

	SOLAR TIME	SID. TIME	Æ				
ϵ Bootis	9 50 41 0	14 39 2 13	14 37 28 86	$dt-d$Æ	a=+5 .781	e or (t− Æ) = +$1^{m}33^{s}$ 27	e'=+2^{s} 63
α^2 Libræ	10 0 32 8	14 42 53 44	14 41 23 05	=+2 88	a=+5 781	e or (t'− Æ′) = +1 30 39	e'=+5 51

Hence we have ϵ = + $1^{m}35^{s}.9$ as the whole error of the chronometer, whether we take it = (t — Æ) + e', or = (t' — Æ′) + e', as they regard the respective stars. The accordance of the two is another proof of the correctness of the method. If we compare the error of the chronometer, now obtained from these last observations, with that derived from the observations of the same stars, taken on the 25th of May, we may determine the rate of our chronometer on a mean of eight days, supposing its performance to have been uniform during the whole interval.

On May 25 the chronometer was slow by	$2^{m}26^{s}.207$
On June 2 it was fast by	1 35 .900
The whole gain, if the indication had not been altered	4 2 .107
Deduct two minutes for the clock being put forwards	2 0 .0
	8)2 2 .107
One eighth part of the gain in eight sidereal days is	+ 15 .26
Or in mean solar time (by Tab. at p. 110, Vol. I.)	+ 15 .22

We have preferred giving the detail of our observations made by a portable transit-instrument, in the different positions, with their corresponding errors, in connexion with a chronometer rather than with a sidereal clock, because such instrument will generally require to be treated in the way we have here described, when placed in a temporary situation, or used as a travelling instrument. We shall however add an example, in which the observations of a high and low star may be applied to determine, by similar computations, the gain or loss of a sidereal clock in a given interval, and consequently its rate derived from that interval. We have seen that the chronometer made use of was fast by $1^m 35^s.9$ on the evening of the 2d of June; a clock, which had been suffered to go down, was wound up and put to the said chronometer, after deducting one minute, i. e. it was put in motion and made to indicate sidereal time too fast by $35^s.9$. On the 10th of June an opportunity occurred of observing η Ursæ Majoris and Spica Virginis in succession with the same portable transit-instrument, directed to the meridian-mark of the 2d of June, and adjusted for the horizontality of the axis, as well as for collimation, the chronometer being put on this evening exactly to the solar time corresponding to the sidereal time of the clock, and being used to transmit the time from the pillar out of doors to the clock within, at the time of each observation: the following deductions were obtained from two observations made under these circumstances, which prove that the meridian mark was very nearly in its place, by the small value of a, which together with c', the corresponding error of position in time for each star, were computed by the proper formulæ, or may be obtained from the Tables A and C, as we have directed; viz.

	SID. TIME.	Æ					
Spica Virg.	$13^h 16^m 29^s.91$	$13^h 16^m 8^s.35$	$dt - d$Æ	$a = -1^s.083$	c or ($t -$ Æ) $= 21^s.56$	$c' = -0^s.97$	$\varepsilon = +20^s.59$
η Ursæ Min.	13 41 6.04	13 40 45.41	$= -0^s.93$	$a = -1^s.083$	c or ($t' -$ Æ$'$) $= 20.63$	$c' = -0.04$	$\varepsilon = +20.59$

Then we have June 2, whole error of the clock $\varepsilon = +$ . . . . . . $35^s.90$

10, Ditto $\varepsilon = +$ . . . . 20 .59

Therefore the clock lost in 8 sidereal days . . . . 15 .31

One eighth part of which is the daily rate = ... — 1 .913

TABLE C.

Arguments { For c' = sin Zen dist, or cos alt / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	Log c + log sec dec = log c‴	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	Sine.
		89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	Cosin
																	Com Diff
0°	0 0000	8 2419	8 5428	8 7188	8 8436	8 9408	9 0192	9 0859	9 1436	9 1943	9 2397	9 2806	9 3179	9 3521	9 3837	9 4130	1
1	0001	2420	5429	7189	8437	9404	0193	0860	1437	1944	2398	2807	3180	3522	3838	4131	2
2	0003	2422	5431	7191	8439	9406	0195	0862	1439	1946	2400	2809	3182	3524	3840	4133	3
3	0006	2425	5434	7194	8442	9409	0198	0865	1442	1949	2403	2812	3185	3527	3843	4136	5
4	0011	2430	5439	7199	8447	9414	0203	0870	1447	1954	2408	2817	3190	3532	3848	4141	6
5	0017	2436	5445	7205	8453	9420	0209	0876	1453	1960	2414	2823	3196	3538	3854	4147	7
6	0 0024	8 2443	8 5452	8 7212	8 8460	8 9427	9 0216	9 0883	9 1460	9 1967	9 2421	9 2830	9 3203	9 3545	9 3861	9 4154	8
7	0032	2451	5460	7220	8468	9435	0224	0891	1468	1975	2429	2838	3211	3553	3869	4162	10
8	0042	2461	5470	7230	8478	9445	0234	0901	1478	1985	2439	2848	3221	3563	3879	4172	12
9	0054	2473	5482	7242	8490	9457	0246	0913	1490	1997	2451	2860	3233	3575	3891	4184	13
10	0067	2486	5495	7255	8503	9470	0259	0926	1503	2010	2464	2873	3246	3588	3904	4197	14
11	0 0081	8 2500	8 5509	8 7269	8 8517	8 9484	9 0273	9 0940	9 1517	9 2024	9 2478	9 2887	9 3260	9 3602	9 3918	9 4211	15
12	0096	2515	5524	7284	8532	9499	0288	0955	1532	2039	2493	2902	3275	3617	3933	4226	17
13	0113	2532	5541	7301	8549	9516	0305	0972	1549	2056	2510	2919	3292	3634	3950	4243	18
14	0131	2550	5559	7319	8567	9534	0323	0990	1567	2074	2528	2937	3310	3652	3968	4261	20
15	0151	2570	5579	7339	8587	9554	0343	1010	1587	2094	2548	2957	3330	3672	3988	4281	21
16	0 0172	8 2591	8 5600	8 7360	8 8608	8 9575	9 0364	9 1031	9 1608	9 2115	9 2569	9 2978	9 3351	9 3693	9 4009	9 4302	22
17	0194	2613	5622	7382	8630	9597	0386	1053	1630	2137	2591	3000	3373	3715	4031	4324	24
18	0218	2637	5646	7406	8654	9621	0410	1077	1654	2161	2615	3024	3397	3739	4055	4348	25
19	0243	2662	5671	7431	8679	9646	0435	1102	1679	2186	2640	3049	3422	3764	4080	4373	27
20	0270	2689	5698	7458	8706	9673	0462	1129	1706	2213	2667	3076	3449	3791	4107	4400	28
21	0 0298	8 2717	8 5726	8 7486	8 8734	8 9701	9 0490	9 1157	9 1734	9 2241	9 2695	9 3104	9 3477	9 3819	9 4135	9 4428	30
22	0328	2747	5756	7516	8764	9731	0520	1187	1764	2271	2725	3134	3507	3849	4165	4458	32
23	0360	2779	5788	7548	8796	9763	0552	1219	1796	2303	2757	3166	3539	3881	4197	4490	33
24	0393	2812	5821	7581	8829	9796	0585	1252	1829	2336	2790	3199	3572	3914	4230	4523	34
25	0427	2846	5855	7615	8863	9830	0619	1286	1863	2370	2824	3233	3606	3948	4264	4557	36
26	0 0463	8 2882	8 5891	8 7651	8 8899	8 9866	9 0655	9 1322	9 1899	9 2406	9 2860	9 3269	9 3642	9 3984	9 4300	9 4593	38
27	0501	2920	5929	7689	8937	9904	0693	1360	1937	2444	2898	3307	3680	4022	4338	4631	39
28	0540	2959	5968	7728	8976	9943	0732	1399	1976	2483	2937	3346	3719	4061	4377	4670	42
29	0582	3001	6010	7770	9018	9985	0774	1441	2018	2525	2979	3388	3761	4103	4419	4712	43
30	0625	3044	6053	7813	9061	9 0028	0817	1484	2061	2568	3022	3431	3804	4146	4462	4755	44
31	0 0669	8 3088	8 6097	8 7857	8 9105	9 0072	9 0861	9 1528	9 2105	9 2612	9 3066	9 3475	9 3848	9 4190	9 4506	9 4799	46
32	0715	3134	6143	7903	9151	0118	0907	1574	2151	2658	3112	3521	3894	4236	4552	4845	48
33	0764	3183	6192	7952	9200	0167	0956	1623	2200	2707	3161	3570	3943	4285	4601	4894	50
34	0814	3233	6242	8002	9250	0217	1006	1673	2250	2757	3211	3620	3993	4335	4651	4944	52
35	0866	3285	6294	8054	9302	0269	1058	1725	2302	2809	3263	3672	4045	4387	4703	4996	54
36	0 0920	8 3339	8 6348	8 8108	8 9356	9 0323	9 1112	9 1779	9 2356	9 2863	9 3317	9 3726	9 4099	9 4441	9 4757	9 5050	57
37	0977	3396	6405	8165	9413	0380	1169	1836	2413	2920	3374	3783	4156	4498	4814	5107	58
38	1035	3454	6463	8223	9471	0438	1227	1894	2471	2978	3432	3841	4214	4556	4872	5165	60
39	1095	3514	6523	8283	9531	0498	1287	1954	2531	3038	3492	3901	4274	4616	4932	5225	62
40	1157	3576	6585	8345	9593	0560	1349	2016	2593	3100	3554	3963	4336	4678	4994	5287	65
41	0 1222	8 3641	8 6650	8 8410	8 9658	9 0625	9 1414	9 2081	9 2658	9 3165	9 3619	9 4028	9 4401	9 4743	9 5059	9 5352	67
42	1289	3708	6717	8477	9725	0692	1481	2148	2725	3232	3686	4095	4468	4810	5126	5419	70
43	1359	3778	6787	8547	9795	0762	1551	2218	2795	3302	3756	4165	4538	4880	5196	5489	72
44	1431	3850	6859	8619	9867	0834	1623	2290	2867	3371	3828	4237	4610	4952	5268	5561	74
45	1505	3924	6933	8693	9941	0908	1697	2361	2941	3448	3902	4311	4684	5026	5342	5635	77
Common Diff		3009	1760	1248	967	789	667	577	507	454	409	373	342	316	293	273	

TABLE C (CONTINUED).

Arguments { For c′ = sin Zen dist, or cos alt / For c″ = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	Log c + log sec dec = log ε‴	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	Sine
		89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	Cosin Com. Diff
46°	0 1582	8 4001	8 7010	8 8770	9 0018	9 0985	9 1774	9 2441	9 3018	9 3525	9 3979	9 4388	9 4761	9 .5103	9 5419	9 5712	
47	1662	4081	7090	8850	0098	1065	1854	2521	3098	3605	4059	4468	4841	.5183	5499	5792	80
48	1746	4164	7173	8933	0181	1148	1937	2604	3181	3688	4142	4551	4924	5266	.5582	5875	83
49	1831	4250	7259	9019	0267	1234	2023	2690	3267	3774	4228	4637	5010	.5352	5668	5961	86
50	1919	4338	7347	9107	0355	1322	2111	2778	3355	3862	4316	4725	5098	5440	5756	6049	88
51	0 2011	8 4430	8 7439	8 9199	9 0447	9 1414	9 2203	9 2870	9 3447	9 3954	9 4408	9 4817	9 5190	9 .5532	9 5848	9 6141	92
52	2107	4526	7535	9295	0543	1510	2299	2966	3543	4050	4504	4913	5286	.5628	5944	.6237	96
53	2206	4624	7633	9393	0641	1608	2397	3064	3641	4148	4602	5011	5384	5726	6042	6335	98
54	.2305	4727	7736	9496	0744	1711	2500	3167	3744	4251	4705	5114	5487	5829	6145	6438	103
55	2414	4833	7842	9602	0850	1817	2606	3273	3850	4357	4811	5220	5593	5935	6251	6544	106
56	0 2524	8 4943	8 7952	8 9712	9 0960	9 1927	9 2716	9 3383	9 3960	9 .4467	9 4921	9 5330	9 5703	9 6045	9 6361	9 .6654	110
57	.2639	5058	8067	9827	1075	2042	2831	3498	4075	4582	5036	5445	5818	.6160	6476	6769	115
58	2756	5177	8186	9946	1194	2161	2950	3617	4194	4701	5155	5564	5937	.6279	.6595	6888	119
59	2882	5301	8310	9 0070	1318	2285	3074	3741	4318	.4825	5279	5688	6061	6403	6719	.7012	124
60	3010	5429	8438	0198	1446	2413	3202	3869	4446	4953	.5407	5816	6189	.6531	6847	.7140	128
61	0 .3144	8 5563	8 8572	9 0332	9 1580	9 2547	9 3336	9 .4003	9 .4580	9 5087	9 5541	9 5950	9 6323	9 6665	9 6981	9 7274	134
62	.3284	5703	8712	0472	1720	2687	3476	4143	4720	5227	.5681	6090	.6463	.6805	7121	7414	140
63	.3430	5849	8858	0618	1866	2833	3622	4289	4866	.5373	5827	.6236	6609	.6951	7267	.7560	146
64	3582	6001	9010	0770	2018	2985	3774	4441	5018	5525	5979	6388	6761	7103	7419	7712	152
65	3741	6160	9169	0929	2177	3144	3933	4600	5177	5684	6138	6547	6920	7262	7578	7871	159
66	0 3907	8 6326	8 9335	9 1095	9 2343	9 3310	9 4099	9 4766	9 5343	9 5850	9 6304	9 6713	9 7086	9 7428	9 7744	9 8037	166
67	4081	6500	9509	1269	2517	3484	4273	4940	5517	6024	6478	6887	7260	7602	7918	8211	174
68	4264	.6683	9692	1452	2700	3667	4456	5123	.5700	6207	6661	7070	7443	.7785	.8101	8394	183
69	.4457	.6876	9885	1645	2893	3860	4649	5316	5893	6400	6854	7263	7636	7978	8294	8587	193
70	4660	7079	9 0088	1848	3096	4063	4852	5519	6096	6603	7057	7466	7839	8181	8497	8790	203
71	0 4874	8 7293	9 0302	9 2062	9 3310	9 .4277	9 5066	9 5733	9 6310	9 6817	9 7271	9 7680	9 8053	9 .8395	9 8711	9 9004	214
72	5100	7519	0528	2288	3536	.4503	5292	5959	6536	7043	7497	7906	8279	8621	.8937	.9230	226
73	5341	7760	0769	2529	3777	4744	5533	6200	6777	7284	7738	8147	8520	8862	.9178	9471	241
74	5597	8016	1025	2785	4033	5000	5789	6456	.7033	7540	7994	8403	8776	9118	.9434	9727	256
75	5870	.8289	1298	3058	4306	5273	6062	6729	7306	.7813	.8267	8676	9049	.9391	9707	0 0000	273
76	0 6163	8 8582	9 1591	9 3351	9 4599	9 5566	9 6355	9 7022	9 7599	9 8106	9 8560	9 8969	9 9342	9 9684	0 .0000	0 0293	293
77	6479	8898	1907	3667	.4915	5882	6671	7338	7915	8422	8876	9285	9658	0 0000	0316	0609	316
78	6821	9240	2249	4009	.5257	6224	7013	7680	8257	8764	9218	.0027	0 0000	0342	0658	.0951	342
79	7194	9613	2622	4382	.5630	6597	7386	8053	8630	9137	9591	0 0000	0373	0715	1031	.1324	373
80	7603	9 0022	.3031	4791	6039	7006	7795	8462	9039	9546	0 0000	0409	0782	.1124	.1440	1733	409
81	0 8057	9 0476	9 3485	9 5245	9 6493	9 7460	9 8249	9 8916	9 9493	0 0000	0 0454	0 0863	0 .1236	0 1578	0 1894	0 2187	454
82	8564	0983	3992	5752	7000	7967	8756	9423	0 0000	0507	0961	.1370	1743	2085	2401	2694	507
83	.9141	1560	4569	6329	7577	8544	9333	0 0000	0577	1084	1538	1947	2320	.2662	.2978	3271	577
84	9808	2227	5236	6996	8244	9211	0 0000	0667	1244	.1751	2205	2614	2987	3329	.3645	3938	667
85	1 0597	3016	6025	7785	9033	0 0000	0789	1456	2033	2540	2994	3403	3776	4118	4434	4727	789
86	1 1564	9 3983	9 6992	9 8752	0 0000	0 0967	0 1756	0 2423	0 3000	0 3507	0 3961	0 .4370	0 4743	0 5085	0 5401	0 5694	967
87	2812	5231	8240	0 0000	1248	2215	3004	3671	4248	4755	5209	5618	.5991	.6333	6649	6942	1248
88	.4572	6991	0 0000	1760	3008	3975	4764	.5431	6008	6515	.6969	.7378	.7751	.8093	8409	.8702	1760
89	7581	0 0000	3009	4769	6017	.6984	7773	8440	9017	9524	9978	1 0387	1 0760	1 1021	1 1418	1 1711	3009
Common Diff.		3009	1760	1248	967	789	667	577	507	454	409	373	342	316	293	273	

TABLE C (CONTINUED).

Arguments { For c' = sin Zen dist, or cos alt. / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *.	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	Sine.
	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	Cosin.
																Com Diff
0°	9 4403	9 4659	9 4900	9 5126	9 5340	9 5543	9 5736	9 5919	9 6093	9 6259	9 6418	9 6570	9 6716	9 6856	9 6990	1
1	4404	4660	4901	5127	5341	5544	5737	5920	6094	6260	6419	6571	6717	6857	6991	2
2	4406	4662	4903	5129	5343	5546	5739	5922	6096	6262	6421	6573	6719	6859	6993	3
3	4409	4665	4906	5132	5346	5549	5742	5925	6099	6265	6424	6576	6722	6862	6996	5
4	4414	4670	4911	5137	5351	5554	5747	5930	6104	6270	6429	6581	6727	6867	7001	6
5	4420	4676	4917	5143	5357	5560	5753	5936	6110	6276	6435	6587	6733	6873	7007	7
6	9 4427	9 4683	9 4924	9 5150	9 5364	9 5567	9 5760	9 5943	9 6117	9 6283	9 6442	9 6594	9 6740	9 6880	9 7014	8
7	4435	4691	.4932	5158	5372	5575	5768	5951	6125	6291	6450	6602	6748	6888	7022	10
8	4445	.4701	.4942	5168	5382	5585	5778	5961	6135	6301	6460	6612	6758	6898	7032	12
9	4457	4713	4954	5180	5394	5597	5790	5973	6147	6313	6472	6624	.6770	6910	7044	13
10	4470	4726	4967	5193	5407	5610	5803	5986	6160	6326	.6485	6637	6783	6923	7057	14
11	9 4484	9 4740	9 .4981	9 5207	9 5421	9 5624	9 5817	9 6000	9 .6174	9 6340	9 6499	9 6651	9 6797	9 6937	9 7071	16
12	4499	4755	4996	5222	5436	5639	5832	6015	6189	.6355	6514	6666	.6812	6952	.7086	17
13	4516	4772	5013	5239	5453	5656	5849	6032	6206	6372	6531	6683	6829	6969	.7103	18
14	4534	4790	5031	5257	5471	5674	5867	6050	6224	.6390	6549	6701	6847	6987	7121	20
15	4554	4810	5051	5277	5491	5694	5887	6070	6244	6410	6569	6721	6867	7007	7141	21
16	9 4575	9 4831	9 5072	9 5298	9 5512	9 5715	9 5908	9 6091	9 6265	9 6431	9 6590	9 6742	9 6888	9 7028	9 .7162	22
17	4597	4853	.5094	5320	5534	5737	.5930	6113	6287	6453	6612	6764	6910	7050	7184	24
18	4621	.4877	.5118	5344	5558	5761	5954	6137	6311	6477	6636	6788	6934	7074	7208	25
19	4646	.4902	5143	5369	5583	5786	5979	6162	6336	6502	6661	6813	6959	7099	7233	27
20	.4673	.4929	5170	5396	5610	5813	6006	6189	6363	6529	6688	.6840	6986	7126	7260	28
21	9 4701	9 .4957	9 .5198	9 5424	9 5638	9 5841	9 6034	9 6217	9 6391	9 6557	9 6716	9 6868	9 7014	9 7154	9 .7288	30
22	4731	4987	.5228	5454	5668	5871	6064	6247	.6421	6587	6746	6898	7044	7184	7318	32
23	4763	5019	.5260	5486	5700	.5903	6096	6279	6453	.6619	6778	6930	7076	7216	7350	33
24	.4796	5052	.5293	5519	5733	5936	6129	6312	6486	6652	6811	6963	7109	.7249	7383	34
25	4830	.5086	5327	5553	.5767	5970	6163	6346	6520	6686	6845	.6997	7143	.7283	7417	36
26	9 4866	9 5122	9 5363	9 5589	9 .5803	9 6006	9 6199	9 6382	9 6556	9 6722	9 6881	9 .7033	9 7179	9 7319	9 .7453	38
27	4904	5160	5401	5627	.5841	6044	6237	6420	6594	6760	6919	7071	7217	7357	7491	39
28	4943	5199	.5440	5666	5880	6083	6276	6459	6633	6799	6958	.7110	7256	7396	7530	42
29	4985	5241	5482	5708	5922	6125	6318	6501	6675	6841	7000	7152	7298	7438	7572	43
30	5028	5284	5525	5751	5965	6168	6361	6544	6718	6884	7043	7195	7341	7481	7615	44
31	9 5072	9 5328	9 5569	9 5795	9 6009	9 6212	9 6405	9 6588	9 6762	9 6928	9 7087	9 7239	9 7385	9 7525	9 7659	46
32	5118	5374	5616	5841	6055	6258	6451	6634	6808	6974	7133	7285	7431	7571	7705	49
33	5167	5423	5664	5890	6104	6307	6500	6683	6857	7023	7182	7334	7480	7620	7754	50
34	5217	5473	5714	5940	6154	6357	6550	6733	6907	7073	7232	7384	7530	7670	7804	52
35	5269	5525	5766	.5992	6206	6409	6602	6785	6959	7125	7284	7436	7582	7722	7856	54
36	9 5323	9 5579	9 5820	9 6046	9 6260	9 6463	9 6656	9 6839	9 7013	9 7179	9 7338	9 7490	9 7636	9 7776	9 7910	57
37	5380	5636	5877	6103	6317	6520	6713	6896	7070	7236	.7395	7547	7693	7833	7967	58
38	5438	5694	5935	6161	6375	6578	6771	6954	7128	7294	7453	7605	7751	.7891	.8025	60
39	5498	5754	5995	6221	6435	6638	6831	7014	7188	7354	7513	7665	7811	7951	8085	62
40	5560	5816	6057	6283	6497	.6700	6893	7076	7250	7416	7575	7727	7873	8013	.8147	65
41	9 5625	9 5881	9 6122	9 6348	9 6562	9 6765	9 6958	9 7141	9 7316	9 7481	9 7640	9 7792	9 .7938	9 8078	9 8212	67
42	5692	5948	6189	6415	6629	6832	7025	7208	7382	7548	7707	7859	8005	8145	8279	70
43	5762	6018	6259	6485	6699	6902	7095	7278	7452	7618	7777	.7929	8075	8215	8349	72
44	5834	6090	6331	6557	6771	6974	7167	7350	7524	7690	7849	.8001	8147	8287	8421	74
45	5908	6164	6405	6631	6845	7048	7241	7424	7598	7764	7923	.8075	8221	8361	8495	77
Com Diff	273	256	241	226	214	203	193	183	174	166	159	152	146	140	134	128

TABLE C (CONTINUED).

Arguments { For c′ = sin Zen dist, or cos alt / For c″ = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	Sine
	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	Cosh
																Com Diff
46°	9 5985	9 6241	9 6482	9 6708	9 6922	9 7125	9 7318	9 7501	9 7675	9 7841	9 8000	9 8152	9 8298	9 8438	9 8572	8
47	6065	6321	6562	6788	7002	7205	7398	7581	7755	7921	8080	8232	8378	8518	8652	83
48	6148	6404	6645	6871	7085	7288	7481	7664	7838	8004	8163	8315	8461	8601	8735	86
49	6234	6490	6731	6957	7171	7374	7567	7750	7924	8090	8249	8401	8547	8687	8821	88
50	6322	6578	6819	7045	7259	7462	7655	7838	8012	8178	8337	8489	8635	8775	8909	92
51	9 6414	9 6670	9 6911	9 7137	9 7351	9 7554	9 7747	9 7930	9 8104	9 8270	9 8429	9 8581	9 8727	9 8867	9 9001	96
52	6510	6766	7007	7233	7447	7650	7843	8026	8200	8366	8525	8677	8823	8963	9097	98
53	6608	6864	7105	7331	7545	7748	7941	8124	8298	8464	8623	8775	8921	9061	9195	103
54	6711	6967	7208	7434	7648	7851	8044	8227	8401	8567	8726	8878	9024	9164	9298	106
55	6817	7073	7314	7540	7754	7957	8150	8333	8507	,8673	8832	8984	9130	9270	9404	110
56	9 6927	9 7183	9 7424	9 7650	9 7864	9 8067	9 8260	9 8443	9 8617	9 8783	9 8942	9 9094	9 9240	9 9380	9 9514	115
57	7042	7298	7539	7765	7979	8182	8375	8558	8732	8898	9057	9209	9355	9495	9629	119
58	7161	7417	7658	7884	8098	8301	8494	8677	8851	9017	9176	9328	9474	9614	9748	124
59	7285	7541	7782	,8008	8222	8425	8618	8801	8975	9141	9300	9452	,9598	9738	9872	128
60	7413	7669	7910	8136	8350	8553	8746	8929	9103	9269	9428	9580	9726	9866	0 0000	134
61	9 ,7547	9 7803	9 8044	9 8270	9 8484	9 8687	9 8880	9 9063	9 9237	9 9403	9 9562	9 9714	9 9860	0 0000	0 0134	140
62	7687	7943	8184	8410	8624	8827	9020	9203	9377	,9543	,9702	9854	0 0000	0140	0274	146
63	7833	8089	8330	8556	8770	8973	9166	9349	9523	9689	9848	0 0000	0146	,0286	0420	152
64	7985	8241	8482	8708	8922	9125	9318	9501	9675	9841	0 0000	0152	0298	0438	0572	159
65	8144	8400	8641	8867	9081	9284	9477	9660	9834	0 0000	0159	0311	0457	0597	0731	166
66	9 8310	9 8566	9 8807	9 9033	9 9247	9 9450	9 9643	9 9826	0 0000	0 0166	0 0325	0 0477	0 0623	0 0763	0 0897	174
67	8484	8740	8981	9207	9421	9624	9817	0 0000	0174	0340	0499	0651	0797	0937	1071	183
68	8667	8923	9164	9390	9604	9807	0 0000	0183	0357	0523	0682	0834	0980	1120	1254	193
69	8860	9116	9357	9583	9797	0 0000	0193	0376	0550	0716	0875	1027	1173	1313	1447	203
70	9063	9319	9560	9786	0 0000	0203	0396	0579	0753	0919	1078	1230	1376	1516	,1650	214
71	9 9277	9 9533	9 9774	0 0000	0 0214	0 0417	0 0610	0 0793	0 0967	0 1133	0 1292	0 1444	0 1590	0 1730	0 ,1864	226
72	9503	9759	0 0000	0226	0440	0643	0836	1019	1193	1359	1518	1670	1816	1956	,2090	241
73	9744	0 0000	0241	0467	0681	0884	1077	1260	1434	1600	1759	1911	2057	2197	2331	256
74	0 0000	0256	0497	0723	0937	1140	1333	1516	1690	1856	2015	2167	2313	2453	2587	273
75	0273	,0529	0770	0996	1210	1413	1606	1789	,1963	2129	2288	2440	2586	2726	2860	293
76	0 0566	0 0822	0 1063	0 1289	0 1503	0 1706	0 1899	0 2082	0 2256	0 2422	0 2581	0 2733	0 2879	0 ,3019	0 3153	316
77	0882	1138	1379	1605	1819	2022	2215	2398	2572	2738	2897	3049	3195	3335	3469	342
78	1224	1480	1721	1947	2161	2364	2557	,2740	2914	3080	3239	3391	3537	,3677	3811	373
79	1597	1853	2094	2320	2534	2737	2930	3113	3287	3453	3612	3764	3910	,4050	4184	409
80	2006	2262	2503	2729	2943	3146	3339	3522	,3696	3862	4021	,4173	4319	4459	4593	454
81	0 2460	0 2716	0 2957	0 3183	0 3397	0 3600	0 3793	0 3976	0 4150	0 4316	0 4475	0 4627	0 4773	0 4913	0 5047	507
82	2967	3223	3464	3690	3904	4107	4300	4483	4657	4823	4982	5134	5280	5420	5554	577
83	3544	3800	4041	4267	4481	4684	4877	5060	5234	5400	5559	5711	5857	5997	6131	667
84	,4211	4467	4708	4934	5148	5351	5544	5727	5901	6067	6226	6378	6524	6664	6798	789
85	5000	5256	5497	,5723	5937	6140	6333	6516	6690	6856	7015	7167	,7313	7453	7587	969
86	0 5969	0 6223	0 6464	0 6690	0 6904	0 7107	0 7300	0 7483	0 7657	0 7823	0 7982	0 8134	0 8280	0 8420	0 8554	1246
87	7215	7471	7712	7938	8152	8355	8548	8731	8905	9071	9230	9382	9528	,9668	9802	1760
88	8975	9231	9472	9698	9912	1 0115	1 0308	1 0491	1 0665	1 0831	1 0990	1 1142	1 1288	1 1428	1 1562	3009
89	1 1984	1 2240	1 2481	1 2707	1 2921	3124	3317	3500	3674	3840	3999	,4151	,4297	4437	4571	
Com Diff	273	256	241	226	214	203	193	183	174	166	159	152	146	140	134	128

TABLE C (CONTINUED)

Arguments { For c' = sin Zen dist, or cos alt / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	Sine
	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	Cosine Com. Diff.
0°	9 7118	9 7242	9 7361	9 7476	9 7586	9 7692	9 7791	9 7893	9 7989	9 8081	9 8169	9 8255	9 8338	9 8418	9 8495	1
1	.7119	7243	7362	7477	7587	7693	7795	.7894	7990	8082	8170	8256	8339	8419	.8496	2
2	7121	7245	7364	7479	7589	7695	7797	7896	7992	8084	8172	8258	8341	8421	8498	3
3	7124	7248	7367	7482	7592	7698	7800	7899	7995	8087	8175	8261	.8344	8424	8501	5
4	.7129	7253	7372	7487	7597	7703	7805	7904	8000	8092	8180	8266	8349	8429	8506	6
5	7135	7259	7378	7493	7603	7709	7811	7910	8006	8098	8186	8272	8355	8435	8512	7
6	9 7142	9 7266	9 7385	9 7500	9 7610	9 7716	9 7818	9 7917	9 8013	9 8105	9 8193	9 8279	9 8362	9 8442	9 8519	8
7	7150	7274	7393	7508	7618	7724	7826	7925	8021	8113	8201	8287	8370	8450	8527	10
8	7160	7284	7403	7518	7628	7734	7836	7935	8031	8123	8211	8297	8380	8460	8537	12
9	7172	7296	7415	7530	7640	7746	7848	7947	8043	8135	8223	8309	8392	.8472	8549	13
10	7185	7309	7428	7543	7653	7759	7861	7960	8056	8148	8236	8322	8405	8485	8562	14
11	9 7199	9.7323	9 7442	9 7557	9 7667	9 7773	9 7875	9 7974	9 8070	9 8162	9 8250	9 8336	9 8419	9 8499	9 8576	15
12	7214	7338	7457	7572	7682	7788	7890	7989	8085	8177	8265	8351	8434	8514	8591	17
13	7231	7355	7474	7589	7699	7805	7907	8006	8102	8194	8282	8368	8451	8531	8608	18
14	7249	7373	7492	7607	7717	7823	7925	8024	8120	8212	8300	8386	8469	8549	8626	20
15	7269	7393	7512	7627	7737	7843	7945	8044	8140	8232	.8320	8406	8489	8569	8646	21
16	9 7290	9 7414	9 7533	9 7648	9 7758	9 7864	9 7966	9 8065	9 8161	9 8253	9 8341	9 8427	9 8510	9 8590	9 8667	22
17	7312	7436	7555	7670	.7780	7886	7988	8087	8183	8275	8363	8449	8532	8612	8689	24
18	.7336	7460	7579	7694	7804	.7910	8012	8111	8207	8299	8387	8473	8556	8636	8713	25
19	7361	7485	7604	7719	7829	.7935	8037	8136	8232	8324	8412	8498	8581	8661	.8738	27
20	7388	7512	7631	7746	7856	7962	8064	8163	8259	8351	8439	8525	8608	8688	8765	28
21	9.7416	9.7540	9 7659	9 7774	9 7884	9 7990	9 8092	9 8191	9 8287	9 8379	9 8467	9 8553	9 8636	9 8716	9 8793	30
22	7446	7570	7689	.7804	7914	.8020	8122	8221	8317	8409	8497	8583	8666	8746	8823	32
23	7478	7602	.7721	7836	7946	8052	8154	8253	8349	8441	8529	8615	8698	8778	8855	33
24	7511	7635	7754	7869	7979	8085	8187	8286	8382	8474	8562	8648	8731	8811	8888	34
25	7546	7669	7788	7903	8013	8119	8221	8320	8416	8508	8596	8682	.8765	8845	8922	36
26	9 7581	9 7705	9.7824	9 7939	9 8049	9 8155	9 8257	9 8356	9 8452	9 8544	9 8632	9 8718	9 8801	9 8881	9 8958	38
27	7619	7743	7862	7977	8087	8193	8295	8394	8490	8582	8670	8756	8839	8919	8996	39
28	7658	7782	7901	8016	8126	8232	8334	8433	8529	8621	8709	8795	8878	8958	9035	42
29	7700	7824	7943	8058	8168	8274	8376	8475	8571	8663	8751	8837	8920	9000	9077	43
30	7743	7867	7986	8101	8211	8317	8419	8518	.8614	8706	8794	8880	8963	9043	9120	44
31	9.7787	9 7911	9 8030	9 8145	9 8255	9 8361	9 8463	9 8562	9 8658	9 8750	9 8838	9 8924	9 9007	9 9087	9 9164	46
32	7833	7957	8076	8191	8301	8407	8509	.8608	8704	8796	8884	8970	9053	9133	.9210	49
33	7882	8006	8125	8240	8350	8456	8558	.8657	8753	8845	8933	9019	9102	9182	9259	50
34	7932	8056	8175	8290	8400	8506	8608	8707	8803	8895	8983	9069	9152	9232	9309	52
35	7984	8108	8227	8342	8452	8558	8660	8759	8855	8947	9035	9121	9204	9284	9361	54
36	9 8038	9 8162	9 8281	9 8396	9 8506	9 8612	9 8714	9 8813	9 8909	9 9001	9 9089	9 9175	9 9258	9 9338	9 9415	57
37	8095	8219	8338	8453	8563	8669	8771	8870	8966	9058	9146	9232	9315	9395	9472	58
38	.8153	8277	8396	8511	8621	8727	8829	8928	9024	9116	9204	9290	9373	9453	9530	60
39	8213	8337	8456	8571	8681	8787	8889	8988	9084	9176	9264	9350	9433	9513	9590	62
40	8275	8399	8518	8633	8743	8849	8951	9050	9146	.9238	9326	9412	9495	9575	9652	65
41	9.8340	9 8464	9 8583	9 8698	9 8808	9 8914	9 9016	9 9115	9 9211	9 9303	9 9391	9 9477	9 9560	9 9640	9.9717	67
42	.8407	.8531	8650	.8765	8875	8981	9083	9182	9278	.9370	9458	9544	9627	9707	.9784	70
43	.8477	8601	8720	.8835	8945	9051	9153	9252	9348	9440	9528	9614	9697	9777	9851	72
44	8549	.8673	8792	8907	.9017	9123	9225	9324	9420	9512	9600	9686	9769	.9849	9926	74
45	8623	8747	.8866	8981	9091	9197	9299	9398	.9494	9586	9674	9760	9843	9923	0 0000	77
Com Diff	123	124	119	115	110	106	103	99	96	92	88	86	83	80	77	74

TABLE C (CONTINUED).

Arguments { For c' = sin Zen dist, or cos alt / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	Sine
	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	Cosin
																Com Dif
46°	9 8700	9 8824	9 8943	9 9058	9 9168	9 9274	9 9376	9 9475	9 9571	9 9663	9 9751	9 9837	9 9920	0 0000	0 0077	80
47	8780	8904	9023	9138	9248	9354	9456	9555	9651	9743	9831	9917	0 0000	0080	0157	83
48	8863	8987	9106	9221	9331	9437	9539	9638	9734	9826	9914	0 0000	0083	0163	0240	86
49	8949	9073	9192	9307	9417	9523	9625	9724	9820	9912	0 0000	0086	0169	0249	0326	88
50	9037	9161	9280	9395	9505	9611	9713	9812	9908	0 0000	0088	0174	0257	0337	0414	92
51	9 9129	9 9253	9 9372	9 9487	9 9597	9 9703	9 9805	9 9904	0 0000	0 0092	0 0180	0 0266	0 0349	0 0429	0 0506	96
52	9225	9349	9468	9583	9693	9799	9901	0 0000	0096	0188	0276	0362	0445	0525	0602	98
53	9323	9447	9566	9681	9791	9897	0 0000	0098	0194	0286	0374	0460	0543	0624	0700	103
54	9426	9550	9669	9784	9894	0 0000	0102	0201	0297	0389	0477	0563	0646	0726	0803	106
55	9532	9656	9775	9890	0 0000	0106	0208	0307	0403	0495	0583	0669	0752	0832	0909	110
56	9 9642	9 9766	9 9885	0 0000	0 0110	0 0216	0 0318	0 0417	0 0513	0 0605	0 0693	0 0779	0 0862	0 0942	0 1019	115
57	9757	9881	0 0000	0115	0225	0331	0433	0532	0628	0720	0808	0894	0977	1057	1134	119
58	9876	0 0000	0119	0234	0344	0450	0552	0651	0747	0839	0927	1013	1096	1176	1253	124
59	0 0000	0124	0243	0358	0468	0574	0676	0775	0871	0963	1051	1137	1220	1300	1377	128
60	0128	0252	0371	0486	0596	0702	0804	0903	0999	1091	1179	1265	1348	1428	1505	134
61	0 0262	0 0386	0 0505	0 0620	0 0730	0 0836	0 0938	0 1037	0 1133	0 1225	0 1313	0 1399	0 1482	0 1562	0 1639	140
62	0402	0526	0645	0760	0870	0976	1078	1177	1273	1365	1453	1539	1622	1702	1779	146
63	0548	0672	0791	0906	1016	1122	1224	1323	1419	1511	1599	1685	1768	1848	1925	152
64	0700	0824	0943	1058	1168	1274	1376	1475	1571	1663	1751	1837	1920	2000	2077	159
65	0859	0983	1102	1217	1327	1433	1535	1634	1730	1822	1910	1996	2079	2159	2236	166
66	0 1025	0 1149	0 1268	0 1383	0 1493	0 1599	0 1701	0 1800	0 1896	0 1988	0 2076	0 2162	0 2245	0 2325	0 2402	174
67	1199	1323	1442	1557	1667	1773	1875	1974	2070	2162	2250	2336	2419	2499	2576	183
68	1382	1506	1625	1740	1850	1956	2058	2157	2253	2345	2433	2519	2602	2682	2759	193
69	1575	1699	1818	1933	2043	2149	2251	2350	2446	2538	2626	2712	2795	2875	2952	203
70	1778	1902	2021	2136	2246	2352	2454	2553	2649	2741	2829	2915	2998	3078	3155	214
71	0 1992	0 2116	0 2235	0 2350	0 2460	0 2566	0 2668	0 2767	0 2863	0 2955	0 3043	0 3129	0 3212	0 3292	0 3369	226
72	2218	2342	2461	2576	2686	2792	2894	2993	3089	3181	3269	3355	3438	3518	3595	241
73	2459	2583	2702	2817	2927	3033	3135	3234	3330	3422	3510	3596	3679	3759	3836	256
74	2715	2839	2958	3073	3183	3289	3391	3490	3586	3678	3766	3852	3935	4015	4092	273
75	2988	3112	3231	3346	3456	3562	3664	3763	3859	3951	4039	4125	4208	4288	4365	293
76	0 3281	0 3405	0 3524	0 3639	0 3749	0 3855	0 3957	0 4056	0 4152	0 4244	0 4332	0 4418	0 4501	0 4581	0 4658	316
77	3597	3721	3840	3955	4065	4171	4273	4372	4468	4560	4648	4734	4817	4897	4974	342
78	3939	4063	4182	4297	4407	4513	4616	4714	4810	4902	4990	5076	5159	5239	5316	373
79	4312	4436	4555	4670	4780	4886	4988	5087	5183	5275	5363	5449	5532	5612	5689	409
80	4721	4845	4964	5079	5189	5295	5397	5496	5592	5684	5772	5858	5941	6021	6098	454
81	0 5175	0 5299	0 5418	0 5533	0 5643	0 5749	0 5851	0 5950	0 6046	0 6138	0 6226	0 6312	0 6395	0 6475	0 6552	507
82	5682	5806	5925	6040	6150	6256	6358	6457	6553	6645	6733	6819	6902	6982	7059	577
83	6259	6383	6502	6617	6727	6833	6935	7034	7130	7222	7310	7396	7479	7559	7636	667
84	6926	7050	7169	7284	7394	7500	7602	7701	7797	7889	7977	8063	8146	8226	8303	789
85	7715	7839	7958	8073	8183	8289	8391	8490	8586	8678	8766	8852	8935	9015	9092	967
86	0 8682	0 8806	0 8925	0 9040	0 9150	0 9256	0 9358	0 9457	0 9553	0 9645	0 9733	0 9819	0 9902	0 9982	1 0059	1248
87	9930	1 0054	1 0173	1 0288	1 0398	1 0504	1 0606	1 0705	1 0801	1 0893	1 0981	1 1067	1 1150	1 1230	1307	1760
88	1 1690	1814	1933	2048	2158	2264	2366	2465	2561	2653	2741	2827	2910	2990	3067	3009
89	4699	4823	4942	5057	5167	5273	5375	5474	5570	5662	5750	5836	5919	5999	6076	
Com Dif	128	124	119	115	110	106	102	99	96	92	88	86	83	80	77	74

TABLE C (CONTINUED).

Arguments { For c' = sin Zen dist, or cos alt / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec. of *	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	Sine
	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	Cosin
																Com. Diff.
0°	9 8569	9 8641	9 8711	9 8778	9 8843	9 8905	9 8965	9 9023	9 9079	9 9134	9 9186	9 9236	9 9284	9 9330	9 9375	1
1	8570	8642	8712	8779	8844	8906	8966	9024	9080	9135	9187	9237	,9285	9331	9376	2
2	8572	8644	8714	8781	8846	8908	8968	9026	9082	9137	9189	9239	9287	,9333	9378	3
3	8575	8647	8717	8784	8849	,8911	8971	9029	9085	9140	9192	9242	9290	9336	,9381	5
4	8580	8652	8722	8789	,8854	8916	8976	9034	9090	9145	9197	9247	9295	,9341	,9386	6
5	8586	8658	8728	8795	,8860	8922	8982	9040	9096	9151	9203	9253	9301	9347	,9392	7
6	9 8593	9 8665	9 8735	9 8802	9 8867	9 8929	9 8989	9 9047	9 9103	9 9158	9 9210	9 9260	9 9308	9 9354	9 9399	8
7	8601	8673	8743	8810	8875	8937	8997	9055	9111	9166	9218	9268	9316	9362	9407	10
8	8611	8683	8753	8820	8885	8947	9007	9065	9121	9176	9228	9278	9326	9372	9417	12
9	8623	8695	8765	8832	8897	,8959	9019	9077	9133	9188	9240	,9290	9338	9384	9429	13
10	8636	8708	8778	8845	8910	,8972	9032	9090	9146	9201	9253	9303	9351	9397	9442	14
11	9 ,8650	9 8722	9 8792	9 8859	9 8924	9 8986	9 9046	9 9104	9 9160	9 9215	9 9267	9 9317	9 9365	9 9411	9 9456	15
12	,8665	8737	8807	8874	8939	9001	9061	9119	9175	9230	9282	9332	9380	9426	9471	17
13	8682	8754	8824	8891	8956	9018	9078	9136	9192	9247	9299	9349	9397	,9443	9488	18
14	8700	8772	8842	8909	8974	9036	9096	9154	9210	9265	9317	,9367	9415	9461	9506	20
15	8720	8792	8862	8929	8994	9056	9116	9174	9230	9285	9337	9387	9435	,9481	9526	21
16	9 8741	9 8813	9 8883	9 8950	9 9015	9 9077	9 9137	9 9195	9 9251	9 9306	9 9358	9 9408	9 9456	9 9502	9 9547	22
17	8763	8835	8905	,8972	,9037	9099	9159	9217	9273	9328	9380	9430	9478	9524	9569	24
18	8787	8859	8929	8996	9061	9123	9183	9241	9297	9352	9404	9454	9502	9548	,9593	25
19	,8812	8884	,8954	9021	,9086	9148	9208	9266	,9322	9377	9429	9479	9527	9573	9618	27
20	8839	,8911	8981	9048	,9113	9175	9235	9293	9349	9404	9456	9506	9554	9600	,9645	28
21	9 8867	9 8939	9 9009	9 9076	9 9141	9 ,9203	9 9263	9 9321	9 9377	9 9432	9 9484	9 9534	9 ,9582	9 9628	9 9673	30
22	,8897	,8969	9039	9106	9171	9233	9293	9351	9407	9462	9514	9564	9612	9658	9703	32
23	,8929	9001	,9071	9138	9203	9265	9325	9383	9439	9494	9546	9596	9644	9690	,9735	33
24	8962	9034	,9104	9171	9236	9298	9358	9416	9472	9527	9579	9629	9677	9723	9768	34
25	8996	9068	9138	9205	9270	9332	9392	9450	9506	,9561	9613	9663	9711	9757	9802	36
26	9 9032	9 9104	9 9174	9 9241	9 9306	9 9368	9 9428	9 9486	9 9542	9 9597	9 9649	9 9699	9 9747	9 9793	9 9838	38
27	,9070	9142	9212	9279	9344	9406	9466	9524	9580	9635	9687	9737	9785	9831	,9876	39
28	9109	9181	9251	9318	9383	9445	9505	9563	9620	9674	9726	9776	9824	9870	9915	42
29	9151	9223	9293	9360	9425	9487	9547	9605	9661	9716	9768	9818	9866	9912	9957	43
30	,9194	9266	9336	9403	9468	,9530	9590	9648	9704	9759	9811	9861	9909	9955	0 0000	44
31	9 9238	9 9310	9 9380	9 9447	9 9512	9 9574	9 9634	9 9692	9 9748	9 9803	9 9855	9 9905	9 9953	0 0000	0 0044	46
32	9284	9356	,9426	9493	9558	9620	9680	9738	9794	9849	9901	9951	0 0000	0046	0090	49
33	9333	9405	,9475	9542	9607	9669	9729	9787	,9843	,9898	9950	0 0000	0048	0094	0139	50
34	9383	9455	9525	9592	,9657	9719	9779	9837	9893	9948	0 0000	0050	0098	0144	0189	52
35	9435	9507	9577	9644	9709	9771	9831	9889	9945	0 0000	0052	,0102	0150	0196	0241	54
36	9 9489	9 9561	9 ,9631	9 9698	9 9763	9 9825	9 9885	9 9943	0 0000	0 0054	0 ,0106	0 0156	0 0204	0 0250	0 0295	57
37	9546	9618	9688	9755	9820	9882	9942	0 0000	0056	0111	0163	0213	0261	0307	0352	58
38	9604	9676	,9746	9813	,9878	9940	0 0000	0058	0114	0169	0221	0271	0319	0365	0410	60
39	,9664	9736	9806	,9873	9938	0 0000	0060	0118	0174	0229	0281	0331	,0379	0425	,0470	62
40	,9726	9798	9868	9935	0 0000	0062	0122	0180	0236	0291	0343	0393	0441	0487	0532	65
41	9 ,9791	9 9863	9 9933	0 0000	0 0065	0 0127	0 0187	0 0245	0 0301	0 ,0356	0 0408	0 ,0458	0 0506	0 0552	0 0597	67
42	9858	9930	0 0000	0067	0132	,0194	0254	,0312	0368	0423	0475	0525	,0573	,0619	0664	70
43	,9928	0 0000	0070	0137	0202	0264	,0324	0382	0438	0493	,0545	0595	,0643	0689	0734	72
44	0 ,0000	0072	0142	0209	0274	0336	,0396	0454	0510	0565	0617	0667	0715	0761	0806	74
45	0074	0146	0216	0283	0348	0410	0470	0528	0584	0639	0691	0741	0789	0835	0880	77
Com Diff	74	72	70	67	65	62	60	58	56	55	52	50	48	46	45	43

TABLE C (CONTINUED)

Arguments { For c' = sin Zen. dist, or cos alt / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	Sine
	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	Cosin
46°	0 0151	0 0223	0 0293	0.0360	0 0425	0 0487	0 0547	0 0605	0 0661	0.0716	0 0768	0 0818	0 0866	0 0912	0 0957	Com Diff
47	0231	0303	0373	0440	0505	0567	0627	0685	0741	0796	0848	0898	0946	0992	1037	80
48	.0314	.0386	.0456	0523	0588	0650	0710	0768	0824	0879	0931	0981	1029	1075	1120	83
49	0400	0472	0542	0609	0674	0736	0796	0854	0910	0965	1017	1067	1115	1161	1206	86
50	.0488	0560	0630	0697	0762	0824	0884	0942	0998	1053	1105	1155	1203	1249	1294	88
51	0.0580	0 0652	0 0722	0 0789	0 0854	0 0916	0 0976	0 1034	0.1090	0 1145	0 1197	0 1247	0 1295	0 1341	0 1386	92
52	0676	0748	0818	0885	0950	1012	1072	1130	1186	1241	1293	1343	1391	1437	1482	96
53	.0774	.0846	0916	0983	1048	1110	1170	1228	1284	1339	1391	1441	1489	1535	1580	98
54	0877	0949	1019	1086	1151	1213	1273	1331	1387	1442	1494	1544	1592	1638	1683	103
55	0983	1055	1125	1192	1257	1319	1379	1437	1493	.1548	1600	1650	1698	1744	1789	106
56	0.1093	0 1165	0 1235	0.1302	0 1367	0 1429	0 1489	0 1547	0 1603	0 1658	0 1710	0 1760	0 1808	0 1854	0 1899	110
57	.1208	1280	1350	1417	1482	1544	1604	1662	1718	1773	1825	1875	.1923	1969	2014	115
58	.1327	1399	1469	1536	1601	.1663	1723	1781	1837	1892	1944	1994	2042	2088	2133	119
59	.1451	1523	1593	1660	1725	1787	1847	.1905	1961	2016	2068	2118	2166	2212	2257	124
60	1579	1651	1721	1788	1853	1915	1975	2033	2089	.2144	.2196	2246	.2294	2340	2385	128
61	0 1713	0 1785	0 1855	0 1922	0 1987	0 2049	0 2109	0 2167	0 2223	0 2278	0 2330	0 2380	0 2428	0 2474	0 2519	134
62	.1853	1925	1995	2062	2127	2189	2249	2307	2363	2418	.2470	2520	2568	2614	2659	140
63	1999	2071	.2141	2208	2273	2335	2395	2453	2509	2564	2616	2666	2714	2760	2805	146
64	2151	2223	2293	.2360	2425	2487	2547	2605	2661	2716	.2768	2818	2866	2912	2957	152
65	2310	2382	2452	2519	2584	2646	2706	2764	2820	2875	2927	2977	3025	3071	3116	159
66	0 2476	0 2548	0 2618	0 2685	0 2750	0 2812	0 2872	0 2930	0 2986	0 3041	0 3093	0 3143	0 3191	0 3237	0 3282	166
67	.2650	2722	2792	2859	2924	2986	3046	.3104	3160	.3215	3267	3317	3365	3411	3456	174
68	2833	2905	2975	3042	3107	3169	3229	3287	3343	3398	3450	3500	3548	3594	3639	183
69	3026	3098	3168	3235	3300	3362	3422	3480	3536	3591	3643	3693	3741	3787	3832	193
70	3229	.3301	3371	3438	3503	3565	3625	3683	3739	3794	3846	3896	3944	3990	4035	203
71	0 3443	0 3515	0 3585	0 3652	0 3717	0 3779	0 3839	0 3897	0 3953	0 4008	0.4060	0 4110	0 4158	0 4204	0 4249	214
72	3669	3741	3811	3878	3943	4005	4065	4123	4179	4234	4286	4336	4384	4430	4475	226
73	3910	3982	4052	4119	4184	4246	4306	4364	.4420	4475	4527	4577	4625	4671	.4716	241
74	4166	4238	4308	.4375	4440	4502	4562	4620	4676	4731	.4783	4833	4881	4927	4972	256
75	4439	4511	4581	4648	4713	4775	4835	4893	4949	5004	5056	5106	5154	5200	5245	273
76	0.4732	0 4804	0 4874	0 4941	0 5006	0 5068	0 5128	0 5186	0 5242	0 5297	0 5349	0 5399	0 5447	0 5493	0 5538	293
77	.5048	5120	5190	5257	5322	.5384	.5444	5502	5558	5613	5665	5715	.5763	5809	5854	316
78	5390	5462	5532	5599	5664	5726	5786	5844	.5900	5955	6007	6057	6105	6151	6196	342
79	5763	5835	5905	5972	6037	6099	6159	6217	6273	6328	6380	6430	6478	6524	6569	373
80	6172	6244	6314	6381	6446	6508	6568	6626	6682	6737	6789	6839	6887	6933	6978	409
81	0 6626	0.6698	0 6768	0 6835	0 6900	0 6962	0 7022	0 7080	0 7136	0 7191	0 7243	0 7293	0 7341	0 7387	0.7432	454
82	7133	7205	7275	7342	7407	7469	7529	.7587	7643	7698	7750	7800	7848	7894	.7939	507
83	.7710	7782	7852	7919	7984	8046	8106	8164	8220	8275	8327	8377	8425	8471	8516	577
84	.8377	8449	8519	8586	.8651	8713	8773	8831	8887	8942	8994	9044	9092	9138	.9183	667
85	9166	9238	9308	9375	9440	9502	9562	9620	9676	9731	9783	9833	9881	9927	.9972	789
86	1 0133	1 0205	1 0275	1 0342	1 0407	1 0469	1 0529	1 0587	1 0643	1 0698	1 0750	1 0800	1 0848	1 0894	1 0939	967
87	1381	1453	1523	1590	1655	1717	1777	1835	1891	1946	1998	2048	2096	2142	2187	1248
88	3141	3213	3283	3350	3415	3477	3537	.3595	.3651	3706	3758	3808	3856	3902	3947	1760
89	6150	6222	6292	6359	6424	6486	6546	6604	6660	6715	6767	6817	6865	6911	6956	3009

Com Diff 74 72 70 67 65 62 60 58 56 55 52 50 48 46 45 43

TABLE C (CONTINUED.)

Arguments { For c = sin Zen dist, or cos alt
For c' = cos Zen dist, or sin alt } at the Top, and Declination at the Side.

Dec of *	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	Sine
	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	Cosin
																Com. Diff.
0°	9 9418	9 9459	9 9499	9 9537	9 9573	9 9607	9 9640	9 9672	9 9701	9 9730	9 9757	9 9782	9 9806	9 9828	9 9849	1
1	9419	9460	9500	9538	9574	9608	9641	9673	9702	9731	9758	9783	9807	9829	9850	2
2	9421	9462	9502	9540	9576	9610	9643	9675	9704	9733	9760	9785	9809	9831	9852	3
3	9424	9465	9505	9543	9579	9613	9646	9678	9707	9736	9763	9788	9812	9834	9855	5
4	9429	9470	9510	9548	9584	9618	9651	9683	9712	9741	9768	9793	9817	9839	9860	6
5	9435	9476	9516	9554	9590	9624	9657	9689	9718	9747	9774	9799	9823	9845	9866	7
6	9 9442	9 9483	9 9523	9 9561	9 9597	9 9631	9 9664	9 9696	9 9725	9 9754	9 9781	9 9806	9 9830	9 9852	9 9873	8
7	9450	9491	9531	9569	9605	9639	9672	9704	9733	9762	9789	9814	9838	9860	9881	10
8	9460	9501	9541	9579	9615	9649	9682	9714	9743	9772	9799	9824	9848	9870	9891	12
9	9472	9513	9553	9591	9627	9661	9694	9726	9755	9784	9811	9836	9860	9882	9903	13
10	9485	9526	9566	9604	9640	9674	9707	9739	9768	9797	9824	9849	9873	9895	9916	14
11	9 9499	9 9540	9 9580	9 9618	9 9654	9 9688	9 9721	9 9753	9 9782	9 9811	9 9838	9 9863	9 9887	9 9909	9 9930	15
12	9514	9555	9595	9633	9669	9703	9736	9768	9797	9826	9853	9878	9902	9924	9945	17
13	9531	9572	9612	9650	9686	9720	9753	9785	9814	9843	9870	9895	9919	9941	9962	18
14	9549	9590	9630	9668	9704	9738	9771	9803	9832	9861	9888	9913	9937	9959	9980	20
15	9569	9610	9650	9688	9724	9758	9791	9823	9852	9881	9908	9933	9957	9979	0 0000	21
16	9 9590	9 9631	9 9671	9 9709	9 9745	9 9779	9 9812	9 9844	9 9873	9 9902	9 9929	9 9954	9 9978	0 0000	0 0021	22
17	9612	9653	9693	9731	9767	9801	9834	9866	9895	9924	9951	9976	0 0000	0022	0043	24
18	9636	9677	9717	9755	9791	9825	9858	9890	9919	9948	9975	0 0000	0024	0046	0067	25
19	9661	9702	9742	9780	9816	9850	9883	9915	9944	9973	0 0000	0025	0049	0071	0092	27
20	9688	9729	9769	9807	9843	9877	9910	9942	9971	0 0000	0027	0052	0076	0098	0119	28
21	9 9716	9 9757	9 9797	9 9835	9 9871	9 9905	9 9938	9 9970	0 0000	0 0028	0 0055	0 0080	0 0104	0 0126	0 0147	30
22	9746	9787	9827	9865	9901	9935	9968	0 0000	0029	0058	0085	0110	0134	0156	0177	32
23	9778	9819	9859	9897	9933	9967	0 0000	0032	0061	0090	0117	0142	0166	0188	0209	33
24	9811	9852	9892	9930	9966	0 0000	0033	0065	0094	0123	0150	0175	0199	0221	0242	34
25	9845	9886	9926	9964	0 0000	0034	0067	0099	0128	0157	0184	0209	0233	0255	0276	36
26	9 9881	9 9922	9 9962	0 0000	0 0036	0 0070	0 0103	0 0135	0 0164	0 0193	0 0220	0 0245	0 0269	0 0291	0 0312	38
27	9919	9960	0 0000	0038	0074	0108	0141	0173	0202	0231	0258	0283	0307	0329	0350	39
28	9958	0 0000	0039	0077	0113	0147	0180	0212	0241	0270	0297	0322	0346	0368	0389	42
29	0 0000	0041	0081	0119	0155	0189	0222	0254	0283	0312	0339	0364	0388	0410	0431	43
30	0043	0084	0124	0162	0198	0232	0265	0297	0326	0355	0382	0407	0431	0453	0474	44
31	0 0087	0 0128	0 0168	0 0206	0 0242	0 0276	0 0309	0 0341	0 0370	0 0399	0 0426	0 0451	0 0475	0 0497	0 0518	46
32	0133	0174	0214	0252	0288	0322	0355	0387	0416	0445	0472	0497	0521	0543	0564	49
33	0182	0223	0263	0301	0337	0371	0404	0436	0465	0494	0521	0546	0570	0592	0613	50
34	0232	0273	0313	0351	0387	0421	0454	0486	0515	0544	0571	0596	0620	0642	0663	52
35	0284	0325	0365	0403	0439	0473	0506	0538	0567	0596	0623	0648	0672	0694	0715	54
36	0 0338	0 0379	0 0419	0 0457	0 0493	0 0527	0 0560	0 0592	0 0621	0 0650	0 0677	0 0702	0 0726	0 0748	0 0769	57
37	0395	0436	0476	0514	0550	0584	0617	0649	0678	0707	0734	0759	0783	0805	0826	58
38	0453	0494	0534	0572	0608	0642	0675	0707	0736	0765	0792	0817	0841	0863	0884	60
39	0513	0554	0594	0632	0668	0702	0735	0767	0796	0825	0852	0877	0901	0923	0944	62
40	0575	0616	0656	0694	0730	0764	0797	0829	0858	0887	0914	0939	0963	0985	1006	65
41	0 0640	0 0681	0 0721	0 0759	0 0795	0 0829	0 0862	0 0894	0 0923	0 0952	0 0979	0 1004	0 1028	0 1050	0 1071	67
42	0707	0748	0788	0826	0862	0896	0929	0961	0990	1019	1046	1071	1095	1117	1138	70
43	0777	0818	0858	0896	0932	0966	0999	1031	1060	1089	1116	1141	1165	1187	1208	72
44	0849	0890	0930	0968	1004	1038	1071	1103	1132	1161	1188	1213	1237	1259	1280	74
45	0923	0964	1004	1042	1078	1112	1145	1177	1206	1235	1262	1287	1311	1333	1354	77
Com Diff	43	41	40	38	36	34	33	32	29	29	27	25	24	22	21	20

TABLE C (CONTINUED.)

Arguments { For c′ = sin Zen dist, or cos alt / For c″ = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	Sine
	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	Cosin
46°	0 1000	0 1041	0 1081	0 1119	0 1155	0 1189	0 1222	0 1251	0 1283	0 1312	0 1339	0 1364	0 1388	0 1410	0 1431	Com Diff
47	1080	1121	1161	1199	1235	.1269	1302	1331	1363	1392	1419	1444	1468	.1490	1511	80
48	.1163	1204	1244	1282	1318	1352	1385	1417	1446	1475	1502	1527	1551	1573	1594	83
49	1249	.1290	1330	1368	1404	1438	1471	1503	1532	1561	1588	1613	1637	1659	1680	86
50	1337	.1378	1418	1456	1492	1526	1559	1591	1620	1649	1676	1701	1725	1747	1768	88
51	0 1429	0 1470	0 1510	0 1548	0 1584	0 1618	0 1651	0 1683	0 1712	0 1741	0 1768	0 1793	0 1817	0 1839	0 1860	92
52	1525	1566	1606	1644	1680	1714	1747	1779	1808	1837	1864	1889	1913	1935	1956	96
53	1623	1664	1704	1742	.1778	1812	1845	1877	1906	1935	1962	1987	2011	2033	2054	98
54	1726	1767	1807	1845	1881	1915	1948	1980	2009	2038	2065	2090	2114	2136	2157	103
55	1832	1873	1913	1951	1987	2021	2054	2086	2115	2144	2171	2196	2220	2242	2263	106
56	0 1942	0 1983	0 2023	0 2061	0 2097	0 2131	0 2164	0 2196	0 2225	0 2254	0 2281	0 2306	0 2330	0 2352	0 2373	110
57	2057	.2098	2138	2176	2212	.2246	2279	2311	2340	2369	2396	2421	2445	2467	2488	115
58	2176	.2217	2257	2295	2331	2365	2398	2430	2459	2488	2515	2540	2564	2586	2607	119
59	2300	2341	2381	2419	2455	2489	2522	2554	2583	2612	2639	2664	2688	2710	2731	124
60	.2428	2469	2509	2547	2583	2617	2650	2682	2711	2740	2767	2792	2816	.2838	2860	128
61	0 2562	0 2603	0 2643	0 2681	0 2717	0 2751	0 2784	0 2816	0 2845	0 2874	0 2901	0 2926	0 2950	0 2972	0 2993	134
62	2702	2743	2783	2821	2857	2891	2924	2956	2985	3014	3041	3066	3090	.3112	3133	140
63	2848	2889	2929	2967	3003	3037	3070	3102	3131	3160	3187	3212	3236	3258	3279	146
64	3000	3041	3081	3119	3155	3189	3222	3254	3283	3312	3339	3364	3388	3410	3431	152
65	3159	3200	3240	3278	3314	3348	3381	3413	3442	3471	3498	3523	3547	3569	3590	159
66	0 3325	0 3366	0 3406	0 3444	0 3480	0 3514	0 3547	0 3579	0 3608	0 3637	0 3664	0 3689	0 3713	0 3735	0 3756	166
67	3499	3540	3580	.3618	3654	3688	3721	3753	3782	3811	3838	3863	3887	3909	3930	174
68	3682	.3723	3763	3801	3837	3871	.3904	3936	3965	3994	4021	4046	4070	4092	4113	183
69	3875	3916	3956	.3994	4030	4064	4097	4129	4158	4187	4214	4239	4263	4285	4306	193
70	.4078	4119	4159	4197	4233	4267	4300	4332	4361	4390	4417	4442	4466	4488	4509	203
71	0 4292	0 4333	0 4373	0 4411	0 4447	0 4481	0 4514	0 4546	0 4575	0 4604	0 4631	0 4656	0 4680	0 4702	0 4723	214
72	.4518	4559	.4599	4637	4673	4707	4740	4772	4801	4830	4857	4882	4906	4928	4949	226
73	4760	4800	4840	4878	4914	4948	4981	5013	5042	5071	5098	5123	5147	5169	5190	241
74	5015	5056	5096	5134	5170	5204	5237	5269	5298	5327	5354	5379	.5403	5425	.5446	256
75	5288	5329	5369	5407	5443	5477	5510	5542	5571	5600	5627	5652	5676	5698	5719	273
76	0 5581	0 5622	0 5662	0 5700	0 5736	0 5770	0 5803	0 5835	0 5864	0 5893	0 .5920	0 5945	0 5969	0 5991	0 6012	293
77	5897	5938	5978	6016	6052	6086	6119	6151	6180	6209	6236	6261	6285	6307	6328	316
78	6239	6280	6320	6358	6394	6428	6461	6493	6522	6551	6578	6603	.6627	6649	6670	342
79	6612	6653	6693	6731	6767	6801	6834	6866	6895	6924	6951	6976	7000	.7022	.7043	373
80	.7021	7062	7103	7140	.7176	7210	7243	7275	7304	7333	7360	7385	7409	7431	7452	409
81	0 7475	0 7516	0 7556	0 7594	0 7630	0 7664	0 7697	0 7729	0 7758	0 7787	0 7814	0 7839	0 7863	0 7885	0 7906	454
82	7982	8023	8063	8101	8137	8171	8204	8236	8265	8294	8321	8346	.8370	8392	.8413	507
83	8559	8600	8640	8678	8714	.8748	8781	8813	8842	8871	8898	8923	8947	8969	8990	577
84	9226	9267	9307	9345	9381	9415	9448	9480	9509	9538	9565	9590	9614	9636	9657	667
85	1 0015	1 0056	1 0096	1 0134	1 0170	1 0204	1 0237	1 0269	1 0298	1 0327	1 0354	1 0379	1 0403	1 0425	1 .0446	789
86	1 0982	1 1023	1 1063	1 1101	1 1137	1 1171	1 .1204	1 1236	1 1265	1 .1294	1 1321	1 .1346	1 1370	1 1392	1 1413	967
87	2230	2271	2311	2349	2385	2419	2452	2484	2513	2542	2569	.2594	.2618	.2640	2661	1248
88	3990	4031	4071	4109	4145	4179	.4212	4244	4273	4302	4329	4354	4378	4400	4421	1760
89	6999	7040	7080	7118	7154	7188	7221	7253	7282	7311	7338	7363	7387	7409	7430	3009
Com Dif	48	41	40	38	36	34	33	32	29	29	27	25	24	22	21	20

TABLE C (CONTINUED).

Arguments { For e' = sin Zen dist, or cos alt / For e'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *.	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	Sine
	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	Cosin Com. Diff.
0°	9 9869	9 9887	9 9904	9 9919	9 9933	9 9946	9 9957	9 9967	9 9976	9 9983	9 9989	9 9994	9 9997	9 9999	0 0000	1
1	9870	9888	9905	9920	9934	9947	9958	9968	9977	9984	9990	9995	9998	0 0000	0001	2
2	9872	9890	9907	9922	9936	9949	9960	9970	9979	9986	9992	9997	0 0000	0002	0003	3
3	9875	9893	9910	.9925	9939	9952	9963	9973	9982	9989	9995	0 0000	0003	0005	0006	5
4	9880	9898	9915	9930	9944	9957	9968	9978	9987	9994	0 0000	0005	0008	0010	0011	6
5	9886	9901	9921	9936	9950	9963	9974	9984	9993	0 0000	0006	0011	0014	0016	.0017	7
6	9 9893	9 9911	9 9928	9 9943	9 9957	9 9970	9 9981	9 9991	0 0000	0 0007	0 0013	0 0018	0 0021	0 0023	0 0024	8
7	9901	9919	9936	9951	9965	9978	9989	0 0000	0008	0015	0021	0026	0029	0031	.0032	10
8	9911	9929	9946	9961	9975	9988	0 0000	0009	0018	0025	0031	0036	0039	.0041	0042	12
9	9923	9941	9958	9973	9987	0 0000	0011	0021	0030	0037	0043	0048	0051	0053	0054	13
10	9936	9954	9971	9986	0 .0000	0013	0024	0034	0043	0050	0056	0061	0064	0066	0067	14
11	9 9950	9 9968	9 9985	0 0000	0 0014	0 0027	0 0038	0 0048	0 0057	0 0064	0 0070	0 0075	0 0078	0 0080	0 0081	15
12	9965	9983	0 .0000	0015	0029	0042	.0053	0063	0072	0079	0085	0090	0093	0095	0096	17
13	9982	0 0000	0017	0032	0046	0059	0070	0080	0089	0096	0102	0107	0110	.0112	0113	18
14	0 0000	0018	0035	0050	0064	0077	0088	0098	0107	0114	0120	0125	0128	0130	0131	20
15	0020	0038	0055	0070	0084	0097	0108	0118	0127	0134	0140	0145	0148	0150	0151	21
16	0 0041	0 0059	0 0076	0 0091	0 0105	0 0118	0 0129	0 0139	0 0148	0 0155	0 0161	0 0166	0 0169	0 .0171	0 0172	22
17	0063	0081	0098	.0113	0127	0140	0151	0161	0170	0177	0183	0188	0191	0193	0194	24
18	0087	0105	0122	0137	0151	0164	0175	0185	0194	0201	0207	0212	0215	0217	0218	25
19	0112	0130	0147	0162	0176	0189	0200	0210	0219	0226	0232	0237	0240	0242	0243	27
20	0139	.0157	0174	0189	0203	0216	0227	0237	0246	0253	0259	0264	0267	0269	0270	28
21	0 .0167	0 0185	0 0202	0 0217	0 0231	0 0244	0 0255	0 0265	0 0274	0 0281	0 0287	0 0292	0 0295	0 0297	0 0298	30
22	0197	.0215	0232	0247	0261	0274	0285	0295	0304	0311	0317	0322	0325	0327	.0328	32
23	0229	0247	.0264	.0279	0293	.0306	0317	0327	0336	0343	0349	0354	0357	0359	0360	33
24	0262	0280	0297	0312	0326	0339	0350	0360	0369	0376	0382	0387	0390	0392	0393	34
25	0296	0314	0331	0346	0360	0373	0384	0394	0403	0410	0416	0421	0424	0426	0427	36
26	0 0332	0 0350	0 0367	0 0382	0 0396	0 0409	0 0420	0 0430	0 0439	0 0446	0 0452	0 0457	0 0460	0 0462	0 0463	38
27	0370	0388	0405	0420	0434	0447	0458	0468	0477	0484	0490	0495	0498	0500	0501	39
28	0409	0427	0444	0459	0473	0486	0497	0507	0516	0523	0529	0534	0537	0539	0540	42
29	.0451	0469	0486	0501	0515	0528	0539	0549	0558	0565	0571	0576	0579	0581	0582	43
30	.0494	0512	0529	0544	0558	0571	0582	0592	0601	0608	0614	0619	0622	0624	0625	44
31	0 0538	0 0556	0 0573	0 0588	0 0602	0 0615	0 0626	0 0636	0 0645	0 0652	0 0658	0 0663	0 0666	0 0668	0 0669	46
32	0584	0602	0619	0634	0648	0661	0672	0682	0691	0698	0704	0709	0712	0714	0715	49
33	0633	0651	0668	0683	0697	0710	0721	0731	0740	0747	0753	0758	0761	0763	0764	50
34	0683	0701	0718	0733	0747	0760	0771	0781	0790	0797	0803	0808	0811	0813	0814	52
35	0735	0753	0770	0785	0799	0812	0823	0833	.0842	0849	0855	0860	0863	0865	0866	54
36	0 0789	0 0807	0 .0824	0 0839	0 0853	0 0866	0 0877	0 0887	0 0896	0 0903	0 0909	0 0914	0 0917	0 0919	0 0920	57
37	0846	0864	0881	0896	0910	0923	0934	0944	.0953	0960	0966	0971	0974	0976	0977	58
38	0904	0922	0939	0954	0968	.0981	0992	1002	1011	1018	1024	1029	1032	1034	1035	60
39	0964	0982	0999	1014	1028	1041	1052	1062	1071	1078	1084	1089	1092	1094	.1095	62
40	1026	1044	1061	1076	1090	1103	1114	1124	1133	1140	1146	1151	1154	1156	1157	65
41	0 .1091	0 .1109	0 1126	0 1141	0 1155	0 1168	0 1179	0 1189	0 1198	0 1205	0 1211	0 1216	0 1219	0 1221	0 1222	67
42	1158	1176	1193	1208	1222	1235	1246	1256	1265	1272	1278	1283	1286	1288	1289	70
43	1228	1246	1263	1278	1292	.1305	1316	1326	1335	1342	1348	1353	1356	1358	1359	72
44	.1300	1318	1335	1350	1364	1377	1388	1398	1407	1414	1420	1425	1428	1430	1431	74
45	.1374	.1392	1409	1424	1438	1451	1462	1472	1481	1488	1494	1499	1502	1504	1505	77
Com Diff	20	18	17	15	14	13	11	10	9	7	6	5	3	2	1	0

TABLE C (CONCLUDED).

Arguments { For c' = sin Zen dist, or cos alt. / For c'' = cos Zen dist, or sin alt } at the Top, and Declination at the Side

Dec of *	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	Sine.
	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	Cosin
46°	0 1451	0 1469	0 1486	0 1501	0 1515	0 1528	0 1539	0 1549	0 .1558	0 1565	0 .1571	0 1576	0 1579	0 1581	0 1582	Com Diff
47	1531	1549	1566	1581	1595	1608	1619	1629	1638	1645	1651	1656	1659	1661	1662	80
48	1614	1632	1649	1664	1678	1691	.1702	1712	1721	1728	.1734	1739	1742	1744	1745	83
49	.1700	1718	1735	1750	1764	1777	1788	1798	1807	1814	1820	1825	1828	1830	1831	86
50	1788	1806	1823	1838	1852	1865	1876	1886	1895	1902	1908	1913	1916	1918	1919	88
51	0 1880	0 1898	0 1915	0 1930	0 1944	0 1957	0 1968	0 1978	0 1987	0 1994	0 2000	0 2005	0 2008	0 2010	0 2011	92
52	1976	1994	2011	2026	2040	2053	2064	2074	2083	2090	2096	2101	2104	2106	2107	96
53	2074	2092	2109	2124	2138	2151	2162	2172	2181	2188	2194	2199	2202	2204	2205	98
54	2177	2195	2212	2227	2241	2254	2265	2275	2284	2291	2297	2302	2305	2307	2308	103
55	2283	2301	2318	2333	2347	2360	2371	2381	2390	2397	2403	2408	2411	2413	2414	106
56	0 2393	0 2411	0 2428	0 2443	0 2457	0 2470	0 2481	0 2491	0 2500	0 .2507	0 2513	0 2518	0 2521	0 2523	0 2524	110
57	2508	2526	2543	2558	2572	2585	2596	.2606	2615	.2622	2628	2633	2636	2638	2639	115
58	2627	2645	2662	.2677	2691	2704	2715	2725	2734	2741	.2747	2752	2755	2757	2758	119
59	2751	2769	2786	2801	2815	2828	2839	2849	2858	2865	2871	2876	2879	2881	2882	124
60	2879	2897	2914	2929	2943	.2956	2967	.2977	2986	2993	2999	3004	.3007	3009	.3010	128
61	0 3013	0 3031	0 3048	0 3063	0 3077	0 3090	0 3101	0 3111	0 3120	0 3127	0 .3133	0 3138	0 3141	0 3143	0 3144	134
62	3153	3171	.3188	3203	3217	3230	.3241	3251	3260	3267	.3273	3278	3281	3283	3284	140
63	3299	3317	3334	3349	3363	3376	3387	3397	3406	3413	3419	.3424	3427	3429	3430	146
64	3451	3469	3486	3501	3515	3528	3539	3549	3558	.3565	3571	3576	3579	3581	.3582	152
65	3610	.3628	3645	3660	3674	3687	3698	3708	3717	3724	3730	3735	3738	3740	3741	159
66	0 .3776	0 .3794	0 3811	0 .3826	0 3840	0 3853	0 3864	0 3874	0 3883	0 3890	0 3896	0 3901	0 .3904	0 3906	0 3907	166
67	3950	3968	3985	4000	4014	4027	4038	4048	4057	4064	4070	4075	4078	4080	4081	174
68	4133	4151	4168	4183	4197	4210	4221	4231	.4240	4247	4253	4258	4261	4263	.4264	183
69	.4326	4344	4361	4376	4390	4403	4414	4424	4433	.4440	4446	4451	.4454	4456	4457	193
70	.4529	4547	4564	4579	4593	4606	.4617	4627	4636	.4643	4649	4654	4657	4659	4660	203
71	0 4743	0 4761	0 4778	0 4793	0 4807	0 4820	0 4831	0 4841	0 4850	0 4857	0 4863	0 .4868	0 4871	0 4873	0 4874	214
72	4969	4987	5004	5019	5033	5046	5057	5067	5076	5083	5089	.5094	5097	5099	5100	226
73	5210	5228	5245	5260	5274	5287	5298	5308	5317	5324	5330	5335	5338	5340	5341	241
74	5466	5484	5501	5516	5530	5543	5554	5564	5573	5580	5586	5591	5594	5596	5597	256
75	5739	5757	5774	5789	5803	5816	5827	5837	5846	5853	.5859	5864	5867	5869	5870	273
76	0 6032	0 6050	0 6067	0 6082	0 6096	0 6109	0 6120	0 6130	0 6139	0 6146	0 6152	0 6157	0 6160	0 6162	0 .6163	293
77	6348	6366	6383	6398	6412	6425	6436	6446	6455	6462	6468	6473	6476	6478	.6479	316
78	6690	6708	6725	6740	6754	6767	6778	6788	6797	6804	6810	6815	6818	6820	6821	342
79	7063	7081	7098	7113	7127	7140	7151	7161	7170	7177	7183	7188	7191	7193	.7194	373
80	7472	7490	7507	7522	7536	7549	7560	7570	7579	7586	7592	7597	7600	7602	7603	409
81	0 7926	0 7944	0 7961	0 7976	0 7990	0 8003	0 8014	0 8024	0 8033	0 8040	0 8046	0 8051	0 8054	0 .8056	0 8057	451
82	8433	8451	8468	8483	8497	8510	8521	8531	.8540	8547	8553	8558	8561	8563	8564	507
83	9010	9028	9045	9060	9074	9087	9098	9108	9117	9124	9130	9135	9138	9140	9141	577
84	9677	9695	9712	9727	9741	9754	9765	9775	9784	9791	9797	9802	9805	9807	9808	667
85	1 0466	1 0484	1 0501	1 0516	1 0530	1 0543	1 0554	1 0564	1 0573	1 .0580	1 0586	1 .0591	1 0594	1 0596	1 0597	789
86	1 1433	1 1451	1 1468	1 1483	1 1497	1 1510	1 1521	1 1531	1 1540	1 1547	1 1553	1 1558	1 1561	1 1563	1 1564	967
87	2681	2699	2716	2731	2745	2758	2769	2779	2788	2795	2801	2806	2809	2811	2812	1248
88	4441	4459	4476	4491	4505	4518	4529	4539	4548	4555	4561	4566	.4569	4571	4572	1760
89	7450	7468	7485	7500	.7514	7527	7538	7548	7557	7564	.7570	7575	7578	7580	7581	3009
Com Diff	20	18	17	15	14	13	11	10	9	7	6	5	3	2	1	

Though we have given a Table (*A*) in our last section, for the sole purpose of facilitating the computation of the azimuthal error a, yet our Table C, which was computed purposely for giving the corrections e', e'', and e''', will serve moreover the same purpose as Table A, when the *natural* numbers belonging to the tabular logarithms are made use of in the computation

If we put for the upper star $(t - \text{Æ}) = e + a \sin(L - \delta) \sec\delta$

and for the lower star .... $(t' - \text{Æ}') = e + a \sin(L - \delta') \sec\delta'$

we shall have by subtraction $(t' - \text{Æ}') - (t - \text{Æ}) = a \sin(L - \delta')\ \sec\delta' - \sin(L - \delta) . \sec\delta$

and consequently

$$a = \frac{(t' - \text{Æ}') - (t - \text{Æ})}{\sin(L - \delta') . \sec\delta' - (L - \delta) . \sec\delta} \qquad (5.)$$

Now as $(L - \delta')$ and $(L - \delta)$ are the zenith distances of the respective stars, and the tabular logarithms are the sines of those distances united with the log-secants of the declinations δ' and δ, we can take out the separate tabular logarithms belonging to $\sin(L - \delta') . \sec\delta'$, and $\sin(L - \delta) . \sec\delta$, and subtract their natural numbers, the latter from the former; and the remainder will become the proper divisor for the dividend $(t' - \text{Æ}') - (t - \text{Æ})$, which quantity is always given in natural numbers. The example of η Ursæ Minoris, and of Spica Virginis, which we have already computed, will serve to explain the application of our present formula, and also of Table C, for determining the azimuthal error, thus,

For Spica Virg. $\delta = -10° 16'$, in lat 51° 33′ zen. dist. . . . . 61° 49′ $= (L - \delta)$
η Ursæ Majoris $\delta' = 51° 11'$ . . . . 1 22 $= (L - \delta')$
And $(t' - \text{Æ}') - (t - \text{Æ})$, as given in the example, is $= -$ 0ˢ.93

BY CALCULATION.

Sin 61° 49′.	9 .94519
Sec 10 16	10 .00701
Nat. Num. 0 .8958 . .	9 .95220
Sin 1 .22	8 .37750
Sec 50 .11	10 19359
Nat. Num. 0 .0372	8 .57109

Diff. 0ˢ.8586 and $\frac{-\ 93}{.8586} = -$ 1ˢ.083 $= a$.

BY THE TABLE C.

Sin 62° at top Sec 10° at side	tab. log 9 .9526 = 0 .8966
Sin 1° at top Sec 50° at side	tab. log 8 .4338 = 0 0272 Diff. 8694

And $\frac{-\ .93}{8694} = -$ 1ˢ.081 $= a$, nearly as before.

If proportional parts had been taken, the agreement would have been still nearer.

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§ LIX. THE MOSCOW TRANSIT-INSTRUMENT, BY CARY [Plate XV.]

1. A TRANSIT-INSTRUMENT made by the late W. Cary, and sent in the year 1805, for the observatory at Moscow, is represented in Plate XV. together with its appendages, and is that

which Bonaparte ordered to be preserved when that ill-fated city was nearly destroyed by the French army. The construction of this instrument is so simple, that an outline engraving will exhibit all its parts with sufficient precision, and explain a method of adjusting the horizontal position of the axis, which does not seem to have been applied by any other maker, except Ramsden, though it seems well adapted for insuring its proposed purpose, when the instrument is on a large scale, and permanently fixed. The pillars *AB* and *CD* are of solid masonry, ascending from a good foundation in the ground, and constitute the supports of all the mechanism that is required for forming the instrument, and for holding it in a proper state for use. The brass frame *EFGH* is made fast to the pillar *A B*, and sustains different parts of the instrument, as will be presently explained. The telescope *I K* has an object-glass of six feet focal length, and four inches diameter, with various eye-pieces of different powers, and seven wires in the focus. The tube tapers a little from the horizontal axis *L M*, towards both ends, but not so much as transit-telescopes of long focal distance and large aperture have since been made by Troughton and others; the axis, which is four feet and a half long, also tapers towards both ends in a smaller degree than is customary at this time, but the maker assured us, that no complaint had been made respecting flexure of either the tube or its axis.

2 Near the end *M* of the axis a vernier bar is made fast, which indicates the elevation of the telescope on an adjustable semicircle attached to the pillar *CD*, and carrying the Y, with the usual apparatus for adjusting the axis for azimuthal motion, while the pivot at the end *L* rests on a Y which has the adjustment for varying the inclination. The vertical bars *R S* and *R′ S′*, are suspended from a pair of similar frames, *P* and *P′*, made fast to the summits of their respective pillars by the extreme ends *R* and *R′* of the double levers *R Q* and *R′ Q′*, the fulcra of which are horizontal bars reaching across the frames, the remote end of each of those levers has a joint, *Q* and *Q′*, loaded, from which are suspended a pair of vertical rods, carrying counterpoises, descending into cavities made in the pillars near *A* and *C* respectively, when the lever is horizontal. The bars *R S* and *R′ S′* terminate above with a screw of steel each, passing through the ends of the short arms of the levers, and a tapped nut with a milled head takes hold of each, and connects them with the counterpoises in the pillars, so that when the cranked ends, or Ys, at their inferior ends are applied to the pivots of the telescope's horizontal axis, the screws at *R* and *R′* may be made to bear any portion of the telescope's weight, by regulating the effect to be produced by the counterpoises, while the joints at *S* and *S′* allow the Ys to adapt themselves to the surface of the cylindrical pivots. When this adjustment is made for taking off a part of the weight from the adjustable Ys, in which the pivots revolve, the friction will be so diminished that those extreme portions of the pivots on which the axis revolves will not be altered by wear.

3. As it is of the utmost importance that the line of collimation of the transit-telescope should move in a circle that is truly vertical, this instrument has two methods of levelling the axis, which may be applied at the same time, one of which may be referred to occasionally, and the other may remain as a constant watch on the permanency of the adjustment, when once finished. When a transit-telescope is properly fixed to its axis of motion by the maker, it is said to be at right angles thereto by construction; and therefore any contrivance that will secure the horizontal position of the axis, is supposed to insure also the vertical motion of the telescope in altitude, but here is a separate contrivance for making each adjustment. The

swinging level, which is seen above the pillar *C D* in the Plate, is made to hang on the pivots of the axis, beyond *L* and *M*, by its Ys contained in the cranked ends, in either of the reversed positions, and when both the level and axis are properly adjusted, the former may remain suspended from the latter in all degrees of elevation, and so long as the bubble remains stationary in every position, the axis may be considered as perfectly horizontal, provided the bubble give the same indication in the reversed positions.

4. The second contrivance is not so well known, but may be understood from the representation in the figure, by the aid of a short explanation. The letters *N O* stand at the opposite ends of a vertical brass tube, firmly attached to the frame *E F G H*, before mentioned as made fast to the pillar *A B*, a plumb line, suspended from the superior end of the said frame, descends through this tube, till its load is immersed in a small vessel of water below *N*, which is seen resting on an adjustable stand beneath it, made fast to the bottom of the frame, and which can be raised or lowered at pleasure, by a thumb-screw acting with concealed rack-work. Near the upper end of this tube is a stage or bracket-piece, *T*, screwed to the frame under *G*, and a similar one, *U*, is screwed to the lower part of the frame, above *H*, for receiving the ghost-apparatus to be applied for the examination of the plumb line This apparatus is seen in a detached state in the Plate, under the eye-piece of the telescope, the construction of which may be understood by simple inspection. When this piece of mechanism is placed horizontally on the bracket *T*, as represented below, at bracket *U*, its forked end embraces the vertical tube *N O*, and the compound microscope, one-half of which projects from each prong, is so adjusted as to bisect the plumb line, and then the milled head of the screw *Z*, clamped below the object-end of the telescope, is turned round carefully, till its plane comes just in contact with the plane projecting end of the piece *U*, without putting the microscope out of a state of bisection; when this is effected by delicate management, the telescope is reversed in position, with respect to the eye-end, which is now made to take the place of the object-end in a vertical line, and the microscopic piece *U* is removed to the lower bracket, and adjusted as before, to bisect the plumb line near its lower extremity, then if the head of the screw *Z* is found to touch the projecting-end of the piece *U*, exactly as before in the upper situation, the plane end of this screw will move in a plane parallel to the plumb-line as it passes through a circle of altitude, which it would not do, if the axis of the telescope's motion were not truly horizontal. These alternate contacts of the screw, above and below, will seldom be found perfect at the first trial, and therefore it will be necessary to adjust one-half of the error by the said screw, moved inwards or outwards as the case may be, and the other half by the screw which elevates or depresses the pivot of the telescope's axis; a few trials and corresponding adjustments by the method of halving the remaining error, will bring the axis exactly to right angles with the plumb-line, and consequently into a truly horizontal position In the meantime the level may continue to hang on the pivots of the axis by its inverted Ys, and the places where the ends of the bubble rest may be noted; then if its position be reversed, end for end, and the places be again noted where the ends of the bubble rest, it will be seen whether or not the level is in adjustment, and also whether any error yet remains in the horizontal position of the axis, a repetition of the final adjustments of both the level and plumb-line apparatus till they agree, will determine when the tube of the telescope moves in a truly vertical plane, provided it is fixed at right angles to the axis of motion.

5. It seldom however happens that the line joining the true optical centre of the object-lens, and middle vertical line, in the focus of the eye-piece, is at first quite parallel to the plumb-line, or, which is the same thing, exactly perpendicular to the axis, to make this adjustment, the axis itself must be reversed, end for end, after a distant point, in or near the horizon, has been bisected by the central vertical wire, and if after reversion the same point is found equally bisected, when the telescope is directed to it, the line of collimation will be right; but if otherwise, one half of the error, as before directed [§ LVI. 5.], must be corrected by the two screws at each side of the eye end of the telescope, and the other half by the horizontal screw that gives a small azimuthal motion to the graduated semicircle, and consequently to the end *M* of the telescope's axis; and by thus halving the error in alternate reversed positions of the axis, the line of collimation will at last become perpendicular to a line passing through the centres of its pivots, provided they have equal diameters; and this adjustment may be completed without deranging the former adjustment for horizontality. When these two principal adjustments are finished, the latter will remain permanent only while the object-glass is suffered to remain in the same position with respect to the tube which holds it, and the former only while the pillars and parts attached to them are unaltered by accident or change of temperature

6. The lantern *V* contains a lamp, and is placed in a hole cut in the pillar, opposite the pivot, at the end *L* of the axis, where a perforation through the pivot admits the light to pass to a diagonal reflector placed in the middle of the telescope, which illuminates the wires, as usual, in the common focus of the object glass and eye-pieces. This telescope is on a scale of magnitude to admit of an observing-chair being placed under it, and therefore, when so used, requires no diagonal eye-piece. We are not aware that any observations made with this instrument have been transmitted to this country, and therefore we must leave the reader to form his own opinion of the eligibility of its construction and means of adjustment We must not however dismiss our description of this instrument, without mentioning a contrivance applied to it for limiting the light admitted for the illumination of the wires in the focus of the glasses, which is done by a thin wedge of green glass, made to slide over the end of the axis between the lantern and the aperture that admits the light into the axis; this gives any required shade or tinge of green colour to the field of view, that is found necessary for rendering the wires just visible, without admitting so much light as may occasion the small stars to disappear. Ramsden, whose pupil Cary was, had previously introduced a similar contrivance into Piazzi's circle. The method of applying the wedge is thus, a counterpoise *X*, is fixed on a lever, which moves on an axis, as seen in the figure, and at the opposite end of the lever, concealed from sight, an upright bar ascends, which terminates with a fork embracing and guided by the pivot of the telescope's axis; before this fork the wedge is placed, but so that it may pass across the extreme end of the said axis, and intercept the bright light coming from the lantern, and modify it to suit the purpose of the observer, who, taking hold of the handle *Y*, passing through a tube clamped to the axis of lever *X*, elevates or depresses the wedge as occasion may require, without quitting his place at the telescope. The wedge tapers from the thickness of a halfpenny to that of strong writing paper, and is six inches long and about an inch broad. When the most suitable part of the green wedge has been brought to intercept the light of the lamp, the counterpoise will sustain it in that state of elevation, though the handle be relinquished. The instrument may be brought into the meridian by any of the methods explained in our two preceding sections.

§ LX CONSTRUCTION OF THE GREENWICH TRANSIT-INSTRUMENT

1 Soon after Mr. Pond's appointment to the Royal Observatory, a new transit-instrument was required, for it was known to many persons, that the old instrument, slender and feeble in its construction, did not describe an accurate vertical in the meridian. this was proved by observing stars in the reversed positions of the axis, when it was found that the lines described by the telescope, in those positions, cut each other at two certain points. That instrument was constructed by Bird, before achromatic object-glasses were used for astronomical purposes, one of these was indeed afterwards applied, but so small was the tube of the telescope, that a greater aperture than 2 8 inches could not be admitted. The telescope, with this achromatic object-glass, commanded extreme distinctness, on which account it was at first intended to apply it to the new instrument about to be constructed. There had been for several years in the Royal Observatory, a ten feet object-glass, with an aperture of full five inches, this was mounted in a very ordinary manner as a spy-glass, and had never been of the least use to astronomy. it was from the hands of the late Messrs. P. and J. Dollond, and deservedly possessed a very high character. Mr. Troughton was fixed upon to make the new instrument, having casually said to the astronomer royal, that the above-mentioned object glass would mark out the dimensions of a transit instrument that would be a noble appendage to our national astronomical establishment, this idea was finally adopted, and the instrument was ordered and made accordingly, This transit instrument being required to be made so much larger than any that had preceded it, the maker saw the necessity of forming and connecting many of the most essential parts differently from what had been done before. To describe those differences seem now to be all that is required; for the reader, from what has been said before, has, it is hoped, gained a general knowledge of the instrument; but as there are many figures in the plate appropriated to the description of the Greenwich construction, we will explain their connexion and relative uses in the formation of this important piece of mechanism, and nearly in the words of the eminent artist himself, who has favoured us with a particular account from his own pen.

2. The instrument, as placed upon its piers, is represented by fig. 1, of Plate XVI. with the telescope elevated, and of course foreshortened. The piers are the same which supported the former transit they are two feet square, and were six feet two inches high, but on account of the telescope of the new instrument being two feet longer than the old one, it became necessary to augment their height one foot, in order that the eye-end of this new telescope, when pointed to the zenith, might be at the original distance above the floor of the Observatory. For this purpose a semi-cylindrical stone is fastened upon each of the piers, which are alike two feet in diameter, and two feet three inches long. the reason why the stones are longer than the thickness of the piers, is, that it was determined the axis of the present transit (three feet six inches) should not be so long as the former by six inches. On the semi circular faces of these stones, which project inwards, are fixed the side plates, also of a semi-circular figure; the Ys or angles, on which the pivots of the axis rest, are formed in the side plates, in one of which is the adjustment for the horizontality of the axis, and in the other that for placing the telescope in the meridian. The manner in which these adjustable parts are formed, is not shown in any of

the figures, indeed, they are quite concealed from view. It is, however, easy to comprehend, that the adjustment to the meridian is effected by antagonist screws, that draw or push the moveable Y; so that, in performing the adjustment, one screw is first eased, and then the other brought to bear against it The adjustment for levelling the axis is performed by a differential screw; that is, a screw, having at one end a coarse thread, working in the fixed part, and at the other end a finer thread, working in the piece carrying the moveable Y All these screws are of the capstan kind, and are acted on through slits in the covering plates, by a pin or lever. Thus, in these most important adjustments, not a single milled head presents itself to the curious fingers of any ignorant person, who may happen to have access to the instrument.

3 The former instrument at Greenwich, like all others of large dimensions hitherto constructed, had always varied much, and frequently in these two adjustments; and it was the opinion of many, that the ground upon which the instrument stood, caused those variations, but Mr. Troughton was inclined to attribute them to the errors of art, rather than the deviations of nature, he therefore employed all his skill to render those adjustments permanent, rather than obvious or convenient. The adjustment for levelling, indeed, is quite convenient, for the differential screw may be acted on when the level is full in sight, but the antagonist screws, which adjust for position in the meridian, cannot, in the Greenwich instrument, be got at with the eye remaining at the telescope.

4. To Mr. South's seven-feet instrument, which was made since, and which is nearly a model of the Greenwich one, there has been added a system of pinion work, which, acting on both of the antagonist screws, at the same time screws one and unscrews the other, by equal quantities, and, by means of a long handle, the effect produced is seen by the telescope. It is believed that the artist has succeeded to his wishes, in the steadiness of these adjustments, for there is evidence that both the Greenwich transit-instrument, and the one which has been casually mentioned, as made since, keep their positions unvaried for months, better than the former ones had done for hours.

5. The tube of the telescope is conical; and from a broad base, where it joins the axis, tapers towards each end, where its diameter is equal to that of the object-glass. Each half of the tube consists of three pieces, screwed together at the places marked by rings in the figures. this was done, partly, for the more easy means of making large tubes, but principally in order that the two parts which join the axis, should be finally fixed to it, previously to the pivots of the axis being turned, for the lathe would not admit of the whole length of the telescope, and even if it could, would have been otherwise objectionable. The centre of the axis is a *hollow* sphere, of about 14 inches diameter. there are four large apertures in it, two in the *direction* of the axis, of four inches each, and other two in the direction of the telescope, of each *three* inches diameter, these holes are required in order that the sphere should be turned at the inside, which can only be done through them; for it was required that the sphere should throughout be of the same thickness and strength. the diameters of the holes above stated were required for forming the inside of the sphere, as well as for admitting the cone of rays from the object-glass, to pass without interruption to the eye; and in the other direction, that the rays of the lamp should pass equally uninterrupted to the illuminator of the wires. Had the apertures in the sphere been made as large as the bases of the conical tubes which join it, the sphere would have been weakened, and rendered unable to perform its duty. Fig. 2 gives a view of the in-

strument as seen from the east, the eastern pier being removed to show the parts; in this figure the telescope is seen in its full length, as well as several other parts which will be described presently. Figs. 3 and 4 are designed to show the manner of connecting the four branches of the cross to the central sphere; they are drawn to a scale of one-tenth of the real dimensions, but do not exhibit very obviously the parts they were intended to do, for the draftsman had a very confined view of them, and the instrument could not be taken in pieces for his use; we must therefore try to supply the defect by verbal explanation. Mr Troughton calls the pieces which bind the four branches together *tension-bars*; six of these connect the opposite parts of the axis with the sphere, and four the opposite ends of the telescope with the same. Fig. 3 is a section along the axis of the telescope, and shows the places of the four bars belonging to it, which project towards the eye perpendicularly, and the whole length of four of the six bars which belong to the axis, and lie horizontally; but in fig. 4, which is a section across the centre of the axis, its six bars are shown by small circles, from which they project perpendicularly, and two of the four belonging to the telescope are seen in a horizontal position. These bars at their ends are formed into screws; at one end the screw is coarse, at the other fine, the screw at the finer end being about twice as long as at the other: now, to explain the manner of their action, let us suppose that the finer one is screwed home into any piece of metal, and that another piece is offered to the coarse end; then unscrewing the finer will make the coarser enter, and bring the two pieces towards each other, with a power equal to the difference of the two screws.

6. The four branches of the cross, forming the body of the instrument, receive as many solid pieces that join the sphere, which reach full three inches into the cavities of the respective cones, within which they are soldered and pinned; the parts of these pieces which extend much into the cones are thin, compared with that part which comes in contact with the sphere, for there they are substantial and concave, so as to bed fairly on the sphere; they are rabbeted into the apertures of the latter. This expedient assures the similar parts to be placed in a right line with each other, and the dissimilar at right angles. Now let the ends of the tension-bars, having the fine screws, be screwed home into that end of the axis to which they are adapted, and let the other end of the axis be brought in contact with the ends of the bars having the coarse screws; then unscrewing the fine screws will make the coarse ones penetrate, and bring up the other part of the axis, as above described. The two branches of the telescope are in the same manner made to bear upon the sphere, which, like those of the axis, are joined to it by a prodigious pressure. Nearly four-sixths of the surface of the sphere are covered by the cones of the axis and telescope, and the other two-sixths are perforated, so as just to admit the hand; for it is only through these perforations that the tension-bars can be acted on, in bringing the parts together. The sphere, the cones, and the tension-bars form so important a part of the instrument, that too much, rather than too little, should be said of them. In former large transit instruments the centre-piece was a cube, and of course the four branches were placed on platforms, whereas, since that centre-piece has had the form of a sphere given to it, those parts have been supported by arches. In the former the branches were screwed to the cube by *flanches* exterior to the tubes; in the latter they are interior, and give to the cones the greatest possible base. The tension-bars have, not unaptly, been called *bones*, for they are concealed within the *skin*, and give to the combined parts the chief part of their strength;

but when the cubes were employed, as described above, it was altogether a cuticular adhesion

7. In the engineering part of this instrument, the maker, from its great weight and magnitude, apprehended some difficulty, he had had ample experience that small ones, in proportion to their powers, performed their duty better than the larger ones. To render the new instrument (which must ever be considered the most important apparatus in an observatory) equal to the improved state of practical astronomy, not only were the tension-bars applied, but also four tubular braces, as represented in figs 1 and 2 These bars are fastened to the branches of the telescope and axis, at places rendered strong by internal rings driven hard into the respective parts. These braces have been very much criticised, and, the maker thinks, by those who were not practically qualified to appreciate their use. Both the telescope and axis of a transit instrument will bend, let the artist do what he may, but if that bending is in the direction of gravity, all will be well, for the adjusted line of collimation will, notwithstanding, describe a great circle In an instrument for measuring altitudes bending can do no mischief, provided that both ends bend alike, and to this consideration astronomical instrument makers, or at least those who have a right to be called so, have always been very attentive. In a transit instrument, when the telescope is vertical, no tendency to bending takes place, but when horizontal, that tendency is the greatest, and it was to prevent any part of the bending from taking place laterally that the tubular braces were applied. These braces, therefore, act as props to prevent lateral deviation, in every degree of altitude, except at the zenith, where as props they are neutral, but in all positions whatever they bind the four branches of the instrument more firmly together, and especially where those branches join the central sphere. It has been the opinion of some astronomers, that the sun's rays falling irregularly upon the braces would derange the position of the telescope by causing expansion, and in order to set this matter at rest, Mr. South, whose instrument has been before alluded to, instituted a series of judicious experiments, detailed in the Phil. Trans. for 1826 (Part III.), which produced no injurious effect. those experiments were, indeed, confined altogether to the heating rays of the sun, he did not like to torture his instrument by wrapping the opposite braces in heated flannel, and exposing the others naked to the cold air.

8. The Greenwich instrument is different from all others previously constructed, with respect also to the means provided for pointing the telescope to an object in the heavens; in this the apparatus is placed at the eye-end of the telescope, as seen edgewise in fig. 1, and flatwise in fig 2, but much better, on an enlarged scale, in fig. 5. It consists of two semicircles, placed on opposite sides of the telescope, and screwed fast to the latter; each semicircle has an index, which moves round on its centre, carrying a good spirit-level, and a vernier, which subdivides the divisions of the arc to single minutes; each index has also a clamp and screw for quick and slow motion. The semicircles are divided and numbered, so as to show polar distances, and there are two of them, not only for the sake of uniformity, but that the transits may be taken with the same facility in reversed positions of the axis; moreover, they are very convenient finders, in cases where two stars come to the meridian within a short space of time, for one index being set to the place of the first star, and the other to that of the second, the telescope is pointed to either by merely turning it up or down till the proper bubble stands in the middle of its tube, without having again recourse to the divisions. There

is a great advantage in this method of pointing the telescope, for if by accident it should be moved during an observation, it may readily be set to its place again without rising from the observing-chair, whereas by the old method, in such a case the observation was always lost. A great oversight, however, was committed in the construction of the part last described; the semicircles ought to have been entire circles, for to verify the adjustments, and to prove the sufficiency of the instrument in the most satisfactory manner, it is necessary to have recourse to observations by reflection Now when the telescope is depressed below the horizon, so as to look at the image of a star reflected from a basin of mercury, the semicircles become of no use. To have rectified this oversight in the best manner, would have been to apply entire circles, but this would have discontinued the observations for some time, a plan which Mr. Pond would not submit to, and, at his suggestion, a two feet circle was applied to one end of the axis, which is shown edgewise in fig 1, and flatwise in fig 2, the same is better shown in fig. 6, which exhibits also a part of the pier, the side-plate, and position of the reading microscope, which, being affixed to the pier, performs the office of index to this circle. This contrivance for finding a star is not a bad one, but it is subject to some of the inconveniences of the old method, and is contrary to the whole scheme of the instrument, as well as unsightly.

9. There is a contrivance for relieving the pivots and Ys from a great part of the weight of the instrument, and for preventing the former from being worn by great pressure; no part of this apparatus is seen in any of the figures, except in fig. 1, and there only the counterpoising weight; it consists of a strong beam, carrying at the end next the axis a Y, which takes hold of the pivot beyond the part which rests on the side plate. At one-third of the length of the beam from the pivot, the former opens into a circle, the inside diameter of which admits the illuminating lantern; near the exterior border of the circle, and at right angles to the beam, the points of two steel screws rest upon steel plates fixed in the piers, and thus form a fulcrum for the whole to rest on. The beams extend outwards a little beyond the exterior surface of the piers, where the cylindrical weights are attached to them, as shown in the figure referred to, to the east and west of the piers.

10. The illumination of the wires in the field is performed as usual through the axis; the position of the lantern being as already described, the illuminating plate is fixed in the centre of the sphere; there is, however, one thing that is peculiar to this instrument, namely, a kind of *iris* placed in the tube of the telescope, at about eighteen inches from the eye; with this, which opens or shuts by means of a pinion carrying a milled head, the field of view may be illuminated, from almost total darkness to the brightness of broad day-light, according to what different stars may require, without intercepting a single ray that comes from the object-glass.

11. This instrument has no plumb-line; the axis is adjusted by a fine ground spirit level, with a strong deal frame, which, when in use, stands above the axis upon the pivots, it was made many years ago by Troughton for the old transit, and, having undergone a few alterations, now serves the new one. Its accuracy has been proved by observations taken by reflection from the surface of mercury.

12. There are seven vertical wires in the focus of the Greenwich telescope, which are placed at an equatorial distance of 18 .3 seconds each; there are also two horizontal ones, each of which is placed at about a minute and a half of a degree from the centre of the field,

between which it is intended a star should pass during an observation There is besides attached to the eye-piece a fine micrometer, which carries a single vertical wire through a large range, this was applied on the discovery of a small star extremely near the pole, for by a table computed for this star and this micrometer, the state of the telescope respecting the meridian may be examined at any hour of the night The micrometer also serves a still more important purpose, in observing Polaris, and other slowly moving stars, it is tedious to wait the time they take in passing from wire to wire, but with the micrometer adjusted, and set to one or more entire revolutions of its screw, which are terminated by a *tick*, the observer may, on each side of the middle wire, make as many observations as he pleases, and with the same accuracy as if the fixed wires had been used.

13. Figures 7 and 8 represent the observing-chair or sofa, in two different points of view, the basis of which is supported by castors, and the back may be made to recline in any required angle, to suit the elevation of the star which is the object of observation. This variation of position is effected by a handle within reach of the left hand, that turns a pair of bevelled pinions, the latter of which is put on an axis formed into a long screw, which acts with a tail piece descending from the back, and regulates the reclination. It was said before, that the two vacant sides of the sphere were perforated, so that the hand might be admitted for screwing the parts together, it remains to be mentioned, that those openings were finally closed by brass plates covered with platina: on one side is engraved the maker's name and date, 1816, and on the other the following inscription: "To the President and Council of the Royal Society, this and the Mural Circle, being his greatest and best works, are dedicated by the Maker."

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§ LXI ON THE METHODS OF OBSERVING AND REGISTERING TRANSITS

1. In our preceding sections we have supposed our readers acquainted with the operation of observing the passage of a star, or other heavenly body, over the wires or spider's lines of the transit-instrument, and with some plan of registering them, as well as with the method of reducing the times, shown by the clock at the separate wires, to the time of passage at the middle wire. This supposition enabled us to treat of the errors and corresponding corrections of the clock, considered in connexion with the transit-instrument, separately from the skill that is necessary for managing the instrumental observations. The mode of observing the passage of a heavenly body is the same, whatever may be the size of the instrument, or the number of its vertical wires, but the methods of registering, and of reducing the observations to the middle wire, depend partly on the number of those wires, and partly on the polar distances of the stars that are chosen as the objects of observation. We will first advert to the mode of observing, and then explain the different methods of registering the observations, and of reducing them to the middle wire.

2 In general the observer knows previously the approximate right ascension and declination, or zenith distance, of the body he is preparing to observe, by the assistance of which information, and of the approximate latitude of his place of observation, he can set the

vernier of his small circle, carried by the instrument, to either the altitude or zenith distance, sufficiently near the truth to bring the object into the field of the telescope, when the elevation is made such that the bubble of the level, attached to the vernier, stands nearly in the middle of its tube, for in that situation of the telescope the star will be seen passing the wires at the time of its right ascension, provided the clock indicate true sidereal time, or nearly so, and provided the adjustments of the transit-instrument have been previously attended to, agreeably to the directions that have been offered in section LVI When the star is viewed in the southern part of the visible hemisphere, in a northern latitude, or under the pole in the northern part, it will enter the field of view of the telescope, having a celestial eye-piece, at the right-hand edge, and, passing from right to left, will depart at the left-hand edge; but, on looking towards the north, the stars above the pole, extending the arc as far as to the zenith, will enter at the left-hand side, and depart at the right, which relative appearances are entirely occasioned by the earth's rotation on its axis, and the observer's position The business of the practical astronomer, when he is satisfied that the proper object is passing the field, is to count and mark down the exact second and fractional part of the second indicated by the clock, having its dial illuminated, and its beat audible, at the instants when such object is bisected by each successive vertical wire. The seconds will require to be counted tacitly from some given second which the clock is indicating before the star arrives at the first wire, suppose it to be at 10^s beyond some minute, then going on 11^s, 12^s, 13^s, &c. as the beats are heard to proceed, the number last counted *before* the first bisection takes place (which will be mostly instantaneous) will be the second to be noted in the proper column of the journal, and the fractional part will best be obtained by the eye, in comparing the small intervals short of and beyond the said vertical line, at the instants of the preceding and following beats of the clock, as the star's place regards the observed wire at those instants, when such intervals before and after the passage are judged to be equal, the fractional part will be ½ or .5 of a second; if the preceding interval exceeds the following, it may be .6, .7, .8, or .9, as the eye may judge, or, on the contrary, if it is less, it may be .1, .2, .3, or .4 it will, however, require considerable practice to discriminate to the nicety of .1, or even of .2 of a second at each wire, and in this discrimination consists the excellence of the observation.

3 When the intervals between the lines are equidistant, which in the present state of dividing they ought always to be, the sum of all the times noted, divided by the number of the lines passed over, provided none be omitted in the observation, will give the time of passage over the middle wire, or what is called the *reduction* to the middle wire. If one of the wires, preceding the middle one, should by any accident or impediment be omitted, its corresponding following one must also be omitted, for in taking a mean, the pairs that are observed before and after the middle wire should be equidistant from it, or the reduction that arises out of a mean of the whole will be improper When a mean of all the equidistant odd number of observations is taken, its near agreement with the time of the single passage, taken at the middle wire, will be a proof that the transit has been carefully and skilfully observed; and when this is not the case, the observation ought to be rejected, unless, by a comparison of the differences of the times belonging to the respective intervals, it appears evidently where the error lies, and what is its cause and amount. When the weather is very hot or very cold, or the changes of temperature sudden, the state of the instrument should be examined two

or three times every evening, to see what reliance can be placed on the permanence of its adjustments, and that notice may be taken in the journal of such deviations, as may occasion errors in the time indicated by the clock, in order that the corrections due to them, as explained in our last section, may be applied, when the apparent comes to be reduced to the mean right ascension for a given epoch. The different methods of registering the observations will be most clearly understood from an inspection of the journals containing transit observations of different public observers, with specimens of which we propose to present our readers, as models for their selection, accordingly as their respective instruments may have three, five, or seven wires, or other lines, in the common foci of the object-glasses and positive eye-pieces.

4. Several of the same principal stars are usually observed, as often as opportunity occurs, at each public observatory, from night to night, because their places, being well ascertained, afford so many fixed points in the heavens with which the places of other stars may be compared, and by means of which the time shown by the clock may be frequently appreciated. Indeed the difference of the times indicated at two successive passages of the same star, in the same state of adjustment of the instrument, will at once discover the daily *rate* of the clock without computation, and such rate is frequently recorded, particularly at Greenwich. But at some other observatories the time is noted according to which the clock is *fast* or *slow*, from the differences of which errors the rate may at any time be derived, but not so conveniently, nor perhaps so accurately, as when it is derived from entire rotations of the earth to the same star again, and when the comparison is made after an interval of two or three entire sidereal days. It will contribute to accuracy, as well as convenience in correcting a catalogue, to fix on some star of the first magnitude, near the equator, such as may be easily found in the day-time, and as has a quick apparent motion, for the *standard point* of departure, and to assume the known right ascension of that star, as regulating the time of the clock, in preference to any other; in which case the error derived from it will be common to all the stars. Dr. Maskelyne chose α Aquilæ, and others have used α Pegasi for this purpose, their true distances in time from the vernal equinoctial point being respectively determined from a comparison of their observed places with the observed places of the sun, when near the equinoxes, the method of doing which will be explained hereafter.

5. When five equidistant wires are used in the observations, Dr. Maskelyne's method of abridging the numerical reduction to the middle wire will be found very convenient: the common arithmetical process is, to divide the sum of the five times of passage by the figure *5*, after the hours, minutes, and seconds are all added up; but multiplying by the decimal *.2* is equivalent to dividing by the integer *5*, and when such factor is substituted, it will only be necessary to add together the *seconds*, without any reference to the minutes, provided one-fifth part of a minute, or 12', be as many times added to or subtracted from the product, as will convert it into the same number of seconds that were indicated at the middle wire; for the fractional portion of the second, to be expressed by a decimal, is all that, in a good observation, is wanted to complete the reduction. The annexed example will render this mode of abridging the computation quite intelligible. Let the star be *α Aquarii*, as observed at Greenwich on the 30th of September 1826, agreeably to the following observation: viz.

1826	I	II	III	Mean wire IV	V	VI	VII	Reduction of wires	
♄ Sept. 30	. . .	$44^s.9$	$3^s.3$	$21^h\ 57^m\ 21^s.6$	$39^s.9$	$58^s.1$	. .	$21^s.56$	α Aquarii.

COMMON METHOD OF REDUCTION.

	21^h	56^m	$44^s.9$
	21	57	3 .3
	21	57	21 .6
	21	57	39 .9
	21	57	58 .1
	5)109	46	47 .8
Time reduced =	21	57	21 .56

ABRIDGED METHOD.

	$44^s.9$
	3 .3
	21 .6
	39 .9
	58 .1
167 .8 × 2 =	33 .56
Subtract	12 .00
Seconds reduced .	= 21 .56

In like manner, when three equidistant wires only have been observed, one-third of the sum of the seconds, increased or diminished by 20^s (one-third of a minute) as often as may be necessary, will give the reduction with equal facility. for instance, in the example before us, if we take the third, fourth, and fifth wires only, we shall have the sum of the seconds = $64^s.8$, and one-third of this sum = $21^s.6$ without addition or diminution, because these three passages were all taken in the same minute; but if we take the sum of the times given at the second, fourth, and sixth wires, it will be $124^s.6$, one-third of which is $41^s.533$, which must therefore be diminished by 20^s to give $21^s.533$, the reduction arising from these three wires. In this observation the intervals derived from the differences of the times of passage are respectively 18 .4, 18 .3, 18 .3, and 18 .2, giving the mean = $18^s.3$, which is the determined equatorial interval already mentioned in our account of the Greenwich transit-instrument, the star in question being only 1° 10′ out of the equator.

6 When the sun is the object observed, each limb must be considered as a distinct object, and the lines are usually so arranged that the first limb will have passed them all before the second limb has arrived at number I, when this is the case, it will be convenient to register the passages of the second limb in a retrograde order, beginning with the last column and ending with the first, and then the sum of each equidistant pair will be alike in each column, within the fraction of a second, and a mean of the separate sums may readily be obtained by the abridged mode of reduction to the middle wire. If, for instance, we take the Greenwich observation of the sun on September the 23d, 1826, when his declination was only 2′ 36″ N, we find the journal recording the passage of the separate limbs in succession, as though they were distinct objects, thus: viz.

Sept.	I	II	III	Merid wire IV	V	VI.	VII	Reduction	Days	Daily rate	Names or Characters
♄ 23.	.	$20^{s}.4$ 28 .7	$38^{s}.7$ 47 .2	11^{h} 58^{m} $57^{s}.0$ 12 1 5 7	15^{s} 5 24 .0	$33^{s}.7$ 42 .3	. . . . .	1 .32	1	+0 .30	☉ 1 L. ☉ 2 L.

Now, according to this mode of registering, we are obliged to reduce each limb of the sun to the middle wire separately, and to take the mean of the two reduced times as the time of the centre's passage over the same, thus:

First Limb

$20^{s}.4$
38 .7
11^{h} 58^{m} 57 .0
15 .5
33 .7

165 .3 × .2 = $33^{s}.06$
Add here . 24 .00

Time reduced . . . . 11 58 57 .06
Second Limb . . . . 12 1 5 .58

Difference . . . . = 2 8 .42
Half diff. or semi-duration . = 1 4 .21
By the Nautical Almanac . 1 3 .9

Second Limb.

$28^{s}.7$
47 .2
12^{h} 1^{m} 5 .7
24 .0
42 .3

147 .9 × .2 = 29 .58
Subtract . . 24 .00

. . . . . . 12 1 5 .58
11 58 57 .06 first limb.

24 0 2 .64 = the sum.

App. AR by the clock } 12 0 1 .32 = half the sum
By Naut. Alm. 11 59 36 .10

Clock fast . . . . 25 .22

This appears to have been the method by which the reduction stated in the journal has been made, but the work may be shortened by registering the passages of the second limb in an inverted order, as we have already proposed, and as we have always been accustomed to do in taking down solar transits, agreeably to the following plan:

1826. Sept 23.	$20^{s}.4$ 42 .3	$38^{s}.7$ 24 .0	11^{h} 58^{m} 57^{s} 0 12 1 5 .7	$15^{s}.5$ 47 .2	$33^{s}.7$ 28 .7	☉ 1 Limb. ☉ 2 Limb.
	2 .7	2 .7	24 0 2 7	2 .7	2 .4	the sum = $13^{s}.2$

This amount of 13 .2 seconds requires to be divided, first by 2, to reduce it into the mean of both limbs, or centre, and again by 5, to obtain one-fifth of the said mean arising from the five wires, but these two operations by our method become unnecessary; for if we remove the decimal point one degree to the left, or, which is the same thing, if we suppose the number first divided by the unit 2, and then multiplied by the decimal .2, the figures are not altered, but instead of $13^s.2$ we get 1^s 32 for the reduced seconds, as was done by the common circuitous way Than this method nothing can be more simple, when, as is usual, both limbs are observed at five successive and equidistant wires. When a planet is observed, which is generally done with a high power, the first limb may be observed at the first and last wires, and the second limb at the intermediate ones, and then a mean of the two reductions will give the time of passage of the centre, and half their difference the duration of the semidiameter's passage. If we examine the solar observation of September 23, we shall find that the equatorial intervals determined by the first limb were successively 18 .3, 18 .3, 18 5, 18 2, and by the second limb 18 .5, 18 5, 18 .3, 18 .3 , from which it is obvious that the discrepances were errors of observation, rather than of the intervals of the instrument, which can never be well determined from the tremulous motion of the sun In the present instance it is evident from *our mode* of registering, that the principal errors were committed at wires VI. and II. which attach to the two observations standing in our last column, of which one was the last of the first limb, and the other the first of the last limb the other passages appear to have been well observed, and would probably give a better result if the seconds noted in our last column were omitted.

7 In taking transits of the moon exactly at full, which, indeed, will seldom happen on the meridian, the edges may both be observed, like the sun's limbs, but at all other times the luminous edge alone can usually be observed, from which the passage of the centre must necessarily be inferred by the aid of our LUNAR TABLES 10 and 12, given at pages 200 and 202 of our first volume. On the 17th of August 1826, the moon was full very nearly at her meridian passage, and therefore had both her limbs observed at Greenwich, thus

Aug.	I	II.	III	IV	V	VI.	VII	Reduction		
♃ 17.	. . . . . .	$17^s.3$ 27 .4	$36^s.4$ 46 .3	21^h 52^m $55^s.3$ 21 55 5 .3	14^s 4 24 2	33^s 3 43 2	. . . . . .	0 .31	. . . .	☽ 1 L ☽ 2 L

Or, by our method, thus

Aug. 17.		17^s 3 43 2	$63^s.4$ 24 .2	21^h 52^m $55^s.3$ 21 55 5 .3	14^s 4 46 .3	$33^s.3$ 27 .4		☽ 1 L. ☽ 2 L.
		0 .5	0 .6	0 .6	0 .7	0 .7	$=0^s.31$	

Passage of centre, 21^h 54^m $0^s.31$

8. When the intervals between the contiguous wires are discovered to be *unequal* to one another, and circumstances will not admit of their rectification, then equatorial values must be respectively found from observation, or experiment, and the accurate observer will be under the necessity of separately reducing the time noted at each wire to the central wire; and the

reductions will require as many distinct computations for each star, according to its declination. As an instance we may give the case recorded by Professor Struve, who, on the 18th of October, 1818, having put seven wires to the eye-piece of his transit-instrument, which before had only five, found that the equatorial values of the intervals counted from the central wire IV, were these, viz. $49^s.681$ . . . $25^s.257$ . . . $8^s.231$ . . . . $8^s.637$ . . . . $25^s.329$ . . . $50^s.191$

$$\left.\begin{array}{lr}\text{Whence the mean correction of the whole was} & =\pm 0^s.141\\ \text{From III. and V.} \ldots\ldots\ldots\ldots\ldots & =\pm 0.203\\ \text{From II. and VI} \ldots\ldots\ldots\ldots\ldots & =\pm 0.036\\ \text{From I. and VII.} \ldots\ldots\ldots\ldots\ldots & =\pm 0.255\end{array}\right\}\text{. sec dec.}\quad\begin{array}{l}-\text{ above the pole.}\\ \\ \\ +\text{ below the pole.}\end{array}$$

But on the 19th of May, 1819, when, on account of the expansion of the metallic tube, it was found necessary to bring the object-glass of the telescope nearer to the wires, for the sake of distinct vision, the preceding values were a little altered, and the following were necessarily substituted for the remainder of summer, viz. $49^s.684$ . . . . $25^s.261$ . . . . $8^s.216$ . . . . $8^s.633$ . . . $25^s.375$ . . . $50^s.245$. Whence the mean corrections became for summer

$$\left.\begin{array}{lr}\text{From the whole} \ldots & \pm 0^s.156\\ \text{From III. and V.} \ldots & \pm 0.208\\ \text{From II. and VI} \ldots & \pm 0\ 057\\ \text{From I. and VII.} \ldots & \pm 0.280\end{array}\right\}\text{. sec dec.}\quad\begin{array}{l}-\text{ above the pole.}\\ \\ \\ +\text{ below the pole.}\end{array}$$

In the preface to the first volume of Dr. Maskelyne's Observations we find the following notice of his equatorial intervals;

From May 7 to June 4, 1765	36″.20	36″.45	36″.57	36″.26
From June 4 to May, 1768	36 .02	36 34	36 34	35 .48
From May, 1768, to August 20, 1769	35 .78	36 .07	36 .07	35 48
From August 20, 1769, to July 14, 1772	35 .65	36 05	36 .05	35 .68
From July 14, 1772, to August 1, 1772	40 .85	41 .13	41 .13	40 .65
From August 1, 1772	30 .40	30 .54	30 .36	30 .55

9. It will sometimes happen from the state of the weather, from want of time, from inadvertence, or from other circumstances, that a star, or other heavenly body, may not have been observed at more than one wire, and in such case, if its declination be known, the observation need not be lost; for when the value of the equatorial interval in seconds of time has been previously ascertained, this value multiplied by the secant of the star's declination will give the time due to the interval, to be applied as the reduction with + or −, accordingly as the observed wire was before or behind the central wire. When the star is at a considerable distance from the pole, its apparent path will be very nearly a straight line, and near the equator may be considered exactly so, and in such cases, if we call the value of the equatorial interval in seconds V, and the reduction R, we shall have $R = V \sec \delta$. (1.)

10. But if the star be near the pole, its apparent path through the field of view will be the *arc* of a circle, that is greater than the *sine*, or straight line; and the star will be longer in passing the extreme intervals than in transiting either of the intervals bounded at one side by the central wire, in this case we shall have the reduction

$$R' = \frac{\text{arc sin } (15\ V \sec \delta)}{15} \qquad (2.)$$

and as R' is greater than R, their difference $(R'-R)$ will be

$$= \frac{\text{arc} \sin (15\, V \sec \delta)}{15} - V \sec \delta$$

$$= \frac{15\, V \sin \delta + \frac{1}{6} (15\, V \sec \delta)^3 + \ldots}{15} - V \sec \delta$$

or, neglecting the higher powers,

$$= \frac{1}{90} (15\, V \sec \delta)$$

For instance, in the eye-piece of Professor Struve's transit instrument, which we have mentioned, the values of the extreme intervals, 1 and 6, are stated to be very nearly 50ˢ each, and $15\, V$ therefore $= 12' \, 30'' = 0.003636$, also

$$(R' - R) = \frac{0.003636 \sec \delta^3}{90 \,.\, \text{arc } 1''} = 0.0001102 \sec \delta^3;$$

which in a great circle at the star makes

$$\frac{15\, (R' - R)}{\sec \delta} = 0.001653 \sec \delta^2.$$

Now if we assume, that the greatest allowable error should not exceed $0''.05$ in the reduction, and if we wish to determine the exact point of declination where R may begin to be used in receding from the pole, and R' in approaching it, we have the natural $\sec \delta = \frac{0.05}{0.001653} = 5500$, viz. $\delta = 79^\circ \, 32'$. For the intermediate intervals 2 and 4, V is given nearly $= 25^s$, and we have $(R' - R) = 0.0000138 \sec \delta^3$, or

$$\frac{15\, (R' - R)}{\sec \delta} = 0.000207 \sec \delta^2,$$

and when this is $= 0.05$, we have $\delta = 1556$, or nat. sec of $86^\circ \, 19'$. Hence, for the reduction from the wires I. and V., formula (2) must be used, when the declination exceeds $79^\circ \, 32'$; and for the wires II. and IV., when it exceeds $86^\circ \, 19'$, in order that the error in the reduction may not exceed $0''.05$ The error in time increases as the cube of the distance from the pole, but the error in arc as the square of the same. When two equidistant wires are observed, even near the pole, the mean time arising from the two, provided they be at opposite sides of the central wire, will however give the reduced time due to the middle, or intermediate wire, without reference to the formulæ; but near the pole it is seldom convenient to wait the whole time necessary for obtaining the observed passages at all the three or five wires.

11. When the reduction is made to the middle wire from an observation at a single wire by the preceding formula (2.), it will be necessary to know the declination of the star observed to the accuracy of a few seconds, for if we put the declination $= (\delta \mp d'')$, the error, when using an extreme wire, will be $e = 50^s (\sec \delta \mp d) - \sec \delta$

$$= 50^s \, \frac{\cos \delta - (\cos \delta \cos d + \sin \delta \sin d)}{\cos (\delta + d) \cos \delta},$$

or, if $\cos(\delta \mp \cos d)$ in the denominator be nearly $= \cos\delta$, the error in time may be taken $e = \mp 50^{s} \dfrac{\sin\delta \sin d}{\cos\delta^2}$, or for space $e = 750'' \sin d \operatorname{tang}\delta$; and if the error ought not to exceed 0″.05, the maximum value of d, deduced from the formula, $750'' \sin d \operatorname{tang}\delta = 0''.05$, may be taken

$$d = \frac{1}{15000} \operatorname{cotan}\delta,$$

or, if Δ be put for the polar distance,

$$d = \frac{1}{15000} \operatorname{tang}\Delta.$$

For the wires II. and IV. we may put

$$d = \frac{1}{7500} \operatorname{cotang}\delta, \text{ or } \frac{1}{7500} \operatorname{tang}\Delta.$$

12 On the contrary, when the declination of any star is not correctly known, or is given erroneously in any catalogue, by comparing the observed with the computed intervals, the correction may be determined by the formula

$$\frac{V \sin\delta \sin d}{\cos(\delta + d)\cos\delta} = e;$$

for if x represent the true, and x' the observed interval, we shall have $\dfrac{V}{\cos(\delta + d)} = x'$, and

$$\frac{e}{x'} \cdot \operatorname{cotang}\delta = d.$$

In this way Struve determined the errors in the declination of various stars, one of which is 4 Ursæ Minoris, which he found to be = 89° 3′ 56″.8, when in Bode's Uranographia it is given = 89° 5′ 13″.4

13. The following specimens of a transit-journal will explain the different methods of arranging the observations in several of the different public observatories, which differ from one another chiefly in the arrangement of the columns. It will be seen that at the three first observatories, the observations are made with the transit-instrument adjusted, but that at the three last the errors of the instrument are registered, for the purpose of giving the corrections to each observation. At Greenwich, Paris, and Madras, the clock's rate is given without its error in time; at Königsberg, and Dorpat, neither the error of the clock's time, nor its rate is separately given, but the stars' observed passages and corrections of the instrument at the respective times, from which the true times and mean right ascensions may be computed afterwards, and at Vienna the clock's error and data for computing the error of the instrument are separately given, where the symbol ω appears to represent the azimuthal error, and a the right ascension of the star.

OBSERVED TRANSITS OF FIXED STARS AND PLANETS OVER THE MERIDIAN

(AT GREENWICH)

In the year M DCCC XXVI

Day of the Month	I	II	III.	Merid Wire IV.	V	VI	VII	Reduction of Wires	No. of Days	Daily rate of Clock	Names or Characters of the Stars and Planets	
June ♄ 24		40^s 0	0^s 2	6^h 10^m 20^s 3	40^s 1	59^s 5		28^s 86	1	−0^s 00	☉ 1 L	
		58 0	17 6	6 12 37 6	57 6	17 7					☉ 2 L	
		42 0	2 1	8 10 22 0	41 9	1 6		21 92			♀ 1 L	
		47 1	6 1	11 41 25 2	44 3	3 3		25 18			β Leonis	
		38^m 17 0	49^m 4 5	12 59 51 5	10^m 40 5	21^m 29 0		54 11			Polaris S P	
		57 9	26 6	13 41 55 1	23 8	52 4		55 16			η Ursæ Maj	
		20 0	30 5	14 8 58 9	18 6	38 1		59 02	2	+0 20	Arcturus	
			1 1	14 20 31 2	1 3			31 20			θ Bootis	
		11 0	30 0	14 33 49 0	8 0	26 9		48 98			π	
		42 6	1 6	14 42 20 7	39 8	58 8		20 70	1	+0 30	α¹ Libræ.	
		54 6	13.3	14 42 32 1	51 0	9 8		32 16	1	+0 26	α²	
			23 4	14 52 33 5	43 6			33 50			β Ursæ Min	
		18 2	36 6	15 8 55 2	13 8	32 8		55 22			β Libræ	
		53 6	11 2	15 28 34 9	55 1	16 2		31 86	1	+0 20	α Cor Bor	
			39 8	15 36 57 9	16 2			57 97	1	+0 20	α Serpentis	
		3 2	23 2	18 8 48 2	3 2	23 8		43 22			Pallas.	
		7 2	28 0	18 41 48 6	9 2	29 8		48 56			Ceres	R.
		10 0	38 1	22 40 57 1	16 0	34 8		57 00			☽ 2 L	
				0 59 54 0				51 00			Polaris	
			23 4	2 52 33 7	43.6			33 57			β Ur Mi S P	
		15 1	43 2	3 13 11 3	39 5	7.5		11 32			α Persei	

OBSERVED TRANSITS OF FIXED STARS AND PLANETS OVER THE MERIDIAN,

AT THE OBSERVATORY AT MADRAS (BY J GOLDINGHAM)

In the year M DCCC XXII

	I. Wire	Meridian	III Wire	Reduction of Wires	No of Days	Rate of Clock	Zenith distance	Names or Characters of the Stars or Planets
	Wound up the Clock			Arc of Vibration 1° 37′				
Octob 18	27^m 49^s 0	13^h 20^m 0^s 3	30^m 14^s 0	5^s 88				☉ 1 L
	29 59 0	13 31 11 2	32 24 2					☉ 2 L
	6 39 2	20 7 50 0	9 2 0	50 00			25½° S	1 α Capricorni
	7 1 0	20 8 14 0	9 27 6	13 85			25½ S	2 α Capricorni
	14 22 0	20 15 54 0	17 27 6	54 01	1	+1 73	27½ N	γ Cygni
	33 46 5	20 35 25 3	37 6 4	25 44	1	+1 40	32½ N	α Cygni.
	37 40 2	20 39 4 3	40 30 2	4 38	1	+1 03	20½ N	ι Cygni
	11 54 0	21 14 23 3	16 55.0	23 25			18½ N	α Cephei
	21 4 0	21 22 15 0	23 27 8	14 90			19½ S	β Aquarii
		21 37 17.0	38 32 2	17 08			30½ S	γ Capricorni
	41 42 5	21 43 12 2	41 43 3	12 16			52½ S	γ Gruis
	56 18 0	21 57 3 0	58 50 2	3 10			61½ S.	α Gruis
19	31^m 35^s 2	13^h 32^m 47 0	34^m 0^s 0					☉ 1 L
	33 46.0	13 34 59 3	36 12 4					☉ 2 L
	Western end of the axis 0″ 77 too high				P.M lengthened the pendulum a little			

OBSERVATIONS

FAITES A LA LUNETTE MERIDIENNE

A L'OBSERVATOIRE ROYAL DE PARIS

Août, 1819

Jours du Mois	I	II	Fil Méridien III	IV	V	Passages conclus	Intervalles	Mouvement diurne de la Pendule	Noms et Caractères des Astres
♂ 10	46^{m} 39^{s} 3	46^{m} 50^{s} 3	22^{h} 47^{m} 19^{s} 7	47^{m} 30^{s} 7	47^{m} 50^{s} 8	47^{m} 19^{s} 56	2	—2^{s} 01	Fomalhaut.
	54 50 2	55 8 1	22 55 26 0	55 43 8	56 1 7	55 25 06	2	—1 07	α de Pegase.
	2 9 1	2 34 0	5 2 58 9	3 23 8	3 18 6	2 58 02	.		La Chèvre
	4 51 3	5 12 0	5 5 20 4	5 47 0	6 4 5	5 29 41	. .	. ..	Rigel
	41 26 3	44 43 8	5 45 1 3	45 18 8	45 30 3	45 1 30	.	... ...	α d'Orion
	15 33 9	15 52 0	9 16 10 0	16 28 1	16 46 0	17 15 66	1	—2 27	○ Premier Bord
	17 45 4	18 3 3	9 18 21 3	18 39 3	18 57.3				☉ Deuxième Bord
	36 43 5	37 1.2	19 37 18 8	37 36 4	37 54 0	37 18.78	.	. ..	γ de l'Aigle
	41 1 5	41 19 0	19 41 36 5	41 51 0	42 11 5	41 36 5	2	—1 91	α de l'Aigle
	45 29 5	45 47 0	19 46 4 4	46 21 9	46 39 3	46 4 42	2	—2.14	β de l'Aigle
	7 4 3	7 22 2	20 7 40 3	7 58 2	8 16 2	7 40 20	2	—1 90	2 α du Capricorne
	21 46.0	22 11 0	20 22 41 0	23 8 0	23 36 0	22 41 00	...	... .	Comete, très-faible, 23^{h} 8' 46" 0 t m
	56 8.0	56 26 2	20 56 41 5	57 2 7	57 21 0	56 41 48	...	....	♃ centre, 23^{h} 42' 44" 0 t m

KONIGSBERG TRANSIT OBSERVATIONS.

(BY BESSEL)

October, 1822.

(n = —0 466, c = —0 030, m = —0 38)

Days	Names,	I	II,	III	IV	V.	Mean at the middle	Correction of Inst.	Correction of Time.
26	Sonne	50' 27" 7	42" 7	57" 9	0' 12" 7	....	13^{h} 50' 57" 83	+0 07	
		.... ..	. .. .	2' 10.3	25.4	39" 3	14 2 10.19	+0 07	
	α Bootis ..	7 3 4	19 0	34 9	50 6	7' 6.4	7 31 82	—0.20	—0 22
	γ Aquilæ .	37 21.1	36 0	51 0	38 5 8	20.2	19 37 50.97	—0 13	—0 24
	α ——— ..	41 39 3	54 2	42 9 1	23.8	38 2	42 9.07	—0 11	—0 24
	β ——— ...	46 7 7	22 4	37.4	52.1	47 6 2	46 37.31	—0 09	—0 10
	α Cygni	34 43 2	35' 4 2	25 0	45 3	36 6 3	20 35 24.81	—0.50	—0 30
	3 Piscium .	51 5 3	19 8	34 6	49 1	52 3 4	22 51 34 59	—0 02	
	Mond I R,	58 47 6	59 2 8	17 8	32 7	47.4	59 17 82	0.00	
	XXIII 17.	4 32.1	46 6	5 1 3	15 7	30 4	23 5 1.37	0 00	
	96 Aquarii	9 44 7	59 7	10 14 5	29 2	43.6	10 14 47	+0 02	
	Saturn .	21 38 7	.... .	22 8 7	. ... .	38.2	2 22 8.75	—0 12	
		....	21 57 3	..... .	22 27 2	....	22 12 32	—0 12	
	α Ceti. . .	52 34 3	49.4	53 4 0	18 5	32 8	53 3 05	—0 06	—0 19
	α Urs Min	41 4 0	49 25 0	57 56 0	6 29 0	15 8.5	12 57 55 10		

OBSERVATIONES ASTRONOMICÆ DORPATENSES.

(BY PROFESSOR STRUVE)

1818
5 Nov

5 Novembris

Instrumentum propius in meridianum et justum ad horizontem situm admotum est Scilicet axem orientalem, qui, ut anteà, 4″ altior occidentali inventus est, correxi, et deviationem instrumenti a meridiano, quæ circiter 2″ 2 temporis erat a puncto australi horizontis ad occidentem, penitus tollere conabar In collimatione nil mutatum est

	I	II	III	IV	V.	VI.	VII	Nomen	Med pro filo IV.	Correc
	6 Novembris									$m = -0$ 88 $n = +0$ 49 $c = -0$ 275
679	. . .	. ...		59′ 0 0	31 75	11 8	10 95	α Draconis ..	13^h 59^m 11^s 17	+0 46
680	50 6	23 8	29.6	51 1 0	34 0	38 4	13 8	β Urs Min.	4 51 1 06	+0 70
681	55.6	33.3	58 9	11 12 2	24 8	50 7	28 35	s α Persei .	15 11 12.18	−0 15
682	1 5	22 0	19 3	20 46 6	15 0	10 8	33 35	γ Urs Min .	20 46 55	+0 61
683	. .	28 7	52.8	3 5 0	17 0	41 3	16 4	s Capella ..	17 3 5 11	−0 11
684	49 35	24 25	48 0	46 0 3	11 6	35 75	10 5	s β Aurigæ .	46 0 16	−0.10
685	. ...	20 1	51 55	50 6.7	22 45	. ...	..	ξ Draconis . ..	50 6 57	+0 25
686	... ..	26.5	53.8	52 7 0	20 9	47 75	27 65	γ Draconis .	52 7 03	+0 18
687	... ..	......	....	12 33.7	26 7	21.24	.....	s Camelop. 120	18 12 33 60	−4 51
688	... ..	23.8	52 6	....	32 34	15 0	12.5	δ Urs Min	30 10 52	+3 59
689	26.1	59 3	21 25	30 31.65	42 85	4 15	35 8	α Lyræ....	30 31 70	+0 04

ANNALEN DER K. K. STERNWARTE IN WIEN,

BEOBACHTUNGEN AU DEM MITTAGSROHRE

(BY PROFESSOR LITTROW)

1820	Gestirn	Nro	I	II	III	IV	V.	II	Anmerkungen
July 4	ι Delphini ...	467	23′ 3″0	23′ 18″8	23′ 34″0	23′ 48″.2	24′ 3″8	20	
	ζ Delphini ...	468	25 19 0	25 35.0	25 50 2	26 5.0	26 20 8	20	
	β Delphini .	469	27 32 0	27 48 0	28 3 1	28 18 0	28 33 9	20	4 July
	α Delphini .	470	29 42 2	29 58 0	30 13 5	30 28 5	30 44.0	20	
	α Cygni	472	33 31 5	33 53 0	34 14 0	34 34 0	34 55.8	20	α Ursæ . .. .. α Piscis Austr.
	μ Aquarii	476	41 22 8	41 38 8	41 53 5	42 8.0	42 23.8	20	$\omega = -0''$ 58. .
	q Vulpis .	477	. . ..	45 33.0	45 50 0	46 6 0	46 23.0	20	γ Aquilæ .. +1′ 6′24 α Aquilæ ... +1 6 00
	χ Vulpis	478	48 37.3	48 54 0	49 10 0	49 25 5	49 42.0	20	β Aquilæ . +1 6 08 α Piscis Aus. +1 5.73
	η Capricorni	479	52 34.0	52 50.5	53 6 0	53 21 8	53 38 0	20	20^h 27′ $x = +1$ 6 01
	ϑ Capricorni .	480	....	54 31 0	54 46 5	5 1 5	55 17 8	20	

$$a = t + b + \omega \frac{\sin(\phi-\delta)}{\cos \delta.}$$

§ LXII REDUCTION OF OBSERVED TRANSITS INTO MEAN RIGHT ASCENSIONS

1. When a practical astronomer undertakes a regular series of observations, with an intention of promoting the science, his choice of objects usually depends on the powers and variety of his instruments, on the local situation of his observatory, on the time he means to devote to them, or on what he conceives to constitute the wants of astronomy in the existing state of the science. Hence we find that different observers employ their labours on different objects, or pursue different means of attaining the same end. At Greenwich a certain number of principal stars have long occupied the attention of successive astronomers royal, almost to the exclusion of other objects, in order to determine so many fixed points in the celestial regions, to which the situation of other bodies may be referred with confidence. The 36 stars, now generally known by the appellation of "the Greenwich stars", or "Dr. Maskelyne's stars", have been watched from year to year with such assiduous care, that they have become unerring guides to the mariner, and tests of accuracy in the work of an observatory, where time is concerned, and will ultimately contribute to fix the constants of precession, aberration, and nutation, solar and lunar. This list has lately (1827) been extended to 60, the apparent places of which are now annually computed, and given as a valuable appendage to our Nautical Almanac, which addition greatly facilitates the acquirement of correct time, to which the sidereal clock requires to be adjusted. A comparison of the observed places of other bodies with the known stations of these fundamental stars becomes the business of the astronomer, in the formation of every new catalogue, in which the *apparent* are reduced to the *mean* places of those objects, in whatever part of the heavens they may be situated. Indeed, astronomers in different countries have now checked each other's determinations of these primary stations so rigidly, that little more is left for future astronomers to do, but to multiply their number, and to assign to each body that is observed its appropriate place. The first step is, to determine by observation the apparent place of a heavenly body, and thence to infer its mean place by suitable corrections, which must depend on circumstances, not however of an arbitrary nature. The beauty of practical astronomy is, that regular and certain results accrue from the application of variable corrections, whenever the observations are accurately obtained. Our present consideration is confined to right ascensions to be deduced from recorded transits, and the modes of converting one into the other admit of various means or processes.

2. But it would not be doing justice to our national observatory, to affirm that the attention of its present numerous observers is confined exclusively to a limited number of fundamental stars various other stars, as well as the sun, moon, and planets, including the lately discovered small ones, and also comets, are regularly observed, as often as opportunities occur, and the observations now annually published afford ample testimony, at the same time, of the excellence of the instruments, and of the skilful use that is regularly made of them. The resources of art have been so well applied, in the construction and fixing of the Greenwich instruments, that the few errors of position to which they may be occasionally liable, are usually corrected by the proper adjustments, instead of being suffered to remain as subjects of computation, to increase the number of reductions, which are always troublesome. Accord-

ingly we find remarks interspersed between certain days' observations in the journal of the transits, stating, that the inclination of the axis was examined, and corrected when necessary, and that the collimation was from time to time treated in like manner, but the deviations from accuracy are rarely so considerable as to require computed corrections, and with respect to the meridian, it is easily discovered, by a comparison of the superior and inferior passages of the pole-star, of β, or of δ Ursæ Minoris, which are observed as often as possible, that the instrument retains its position in this respect also. For instance, in the specimen that we have given of the journal of transists taken at Greenwich on the 24th of June, 1826, which was not selected for any particular purpose, we find Polaris passing S. P. (sub polo)

At	12^h	59^m	$54^s.14$
And above the pole at	0	59	54 00
Also β Ursæ Minoris above at .	14	52	33 .50
Ditto, below, at	2	52	33 57.

Whence it is evident, that the optical axis of the telescope must have been as nearly in the meridian as observation could place it, particularly as we see, moreover, that the rate of the clock did not exceed +0 .30 in the twenty four hours, by any of the known stars that were observed on the same evening Neither could the axis be sensibly out of a state of horizontality at the times when these circumpolar stars were observed, passing as they did at their due times at different altitudes; and as the time was indicated correctly by stars both to the north and south, as well as by those that were high and low, the collimation could not be perceptibly out of adjustment *. Under these favourable circumstances, the application of the error of the clock, of precession, aberration, and nutation, solar and lunar, taken with their proper signs, to the time of (the superior) passage at the middle wire, as reduced from a mean of the whole number of wires, will give the mean right ascensions of the observed stars due to the beginning of the year, according to the application of the signs, by any of the methods explained in our first volume. As the sidereal clock, when corrected, indicates the time elapsed since the vernal equinoctial point passed the meridian, which is the point of departure from which right ascension is counted, the time thus shown, when any star is passing the middle wire of the instrument in due adjustment, is the apparent right ascension of that star at the said moment, provided the clock has no error, and the reductions convert this into the mean right ascension due to the epoch corresponding to such reduction. A single result, depending on the observations of one day, in the present state of practical astronomy, would not be deemed sufficiently accurate to be inserted in a catalogue, until it has been confirmed, or corrected, by antecedent or subsequent observations, or both, in an average of all which the errors of observation may be presumed to merge, as well as other accidental or constant errors, that may not have been singly detected. But when a mean of many results, corroborating each other, has been obtained from a superior instrument, the remaining error will be reduced to a very small fraction of a second of time.

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* Professor Littrow has remarked, that if the values of a and b should accidentally be such, that the tangent of the latitude of the place should be equal to $\frac{a}{b}$, and $c = 0$, all the stars would give the same correction of the clock

["Memoirs of the Astronomical Society", Vol I p 277]

3. When a transit-instrument is kept constantly in a state of perfect adjustment, it becomes the arbiter of the clock's rate from day to day, but before it can be ascertained how much the clock is *fast* or *slow* at any given instant, the *apparent* right ascension of the object observed must be known correctly, for in this case $t - AR = e$ will be the whole error of the clock, and $AR - t$, or e with a contrary sign, its correction, and when such correction is applied, we shall have $AR = t \pm e$. To illustrate this mode of proceeding practically, we will take half a dozen principal stars from the Greenwich journal of June 24, 1826, already referred to, and first compute their corrections for converting their mean into apparent right ascensions, thus:

FROM THE TABLE OF "CORRECTIONS OF 48 PRINCIPAL STARS"—Vol I p 113, &c

Arguments							Corrections				
Stars	A + ☉	Max.	L + ☊	Max	A + 2☉	Max.	Precess	Aberr	☾ Nut.	☉ Nut	Sum
	s ° ′		s ° ′		s ° ′						
Arcturus	4 28 7	1^s 35	1 10 50	1^s 03	11 0 27	0^s 056	+0^s 00	+0^s 71	+1^s 77	−0^s 03	+3^s 44
α^2 Libræ . ..	1 19 32	1 33	2 7 30	1 20	0 9 57	0 006	+1 00	+1.03	+1 11	+0 12	3 80
β Urs Min ...	4 17 4	4 94	10 27 54	1 75	0 0 41	0 070	−0 16	+3 37	− 0 91	−0.07	+2 21
α Cor Bor ..	4 8 6	1 47	1 18 26	0 93	11 24 20	0 051	+1 22	+1 10	+ 0 71	−0 05	+3 04
α Serp	4 6 4	1 32	1 28 39	1 06	0 2 40	0 059	+1 41	+1 07	+ 0 86	+0 03	+ 3.37
Polaris	11 10 27	44 17	4 17 25	22 70	2 17 56	1 036	+7 21	−9 38	+15 36	+0 98	+14.17

When the sums of the four corrections contained in the last column have been respectively applied to the known mean right ascensions of the stars in question for January, 1826, the amounts will be the *apparent* right ascensions reduced to the day of observation, and the differences between these and the observed times of passage, reduced to the middle wire, will be e, the principal error of the clock; and when, as at Greenwich, the transit-instrument is kept in proper adjustment, it will be the only error to be noticed in the computations; the errors, e', e'', and e''', depending on the deviation from the meridian, the inclination of the axis, and the want of collimation in azimuth, being eliminated by the usual adjustments. The remaining portion of the work for determining the error of the clock may be thus arranged: viz.

Stars observed*	Mean AR 1826, by Greenwich Cat	Sum of Corrections	Apparent AR computed.	Times by Clock $= t$.	$t - AR$ $= e$	By a mean of five stars.
Arcturus.	14^h 7^m 43^s 79	+ 3^s 44	14^h 7^m 47^s 23	14 8 59 02	+1^m 11^s 70	$e = +1^m\ 12^s.22$
α^2 Libræ .	14 41 16 30	+ 3 86	14 41 20 16	14 42 32 16	+1 12.00	
β Ursæ Min	14 51 18.54	+ 2 21	14 51 20.75	14 52 33 50	+1 12.75	
α Cor Bor .	15 27 19 52	+ 3 04	15 27 22 56	15 28 31 86	+1 12 30	
α Serpentis	15 35 42 35	+ 3 37	15 35 45.72	15 36 57.97	+1 12 25	
Polaris . .	0 58 31 43	+14.17	0 58 45.60	0 59 54 00	+1 8.40	

We now know, that the clock was fast by 1^m 12^s.22, taken from a mean of five principal stars, and as the rate amounted to no sensible quantity in an hour and a half, that is worth notice, we may apply this quantity, with the sign changed, to each of the observed times, in order to gain the *correct* times at which the several stars passed the middle wire, and these times *diminished* by the sums of the corrections, now called *reductions* when the sign is changed, will give the *mean* right ascensions reduced from June 24 to Jan. 0, 1826, in the following manner:

	Observed Æ, or $t+(Æ-t)$	Sum of Reductions	Mean Æ for 1826 from the observations	Diff from the Greenwich Catalogue	Diff from the Catal of Astron Soc
Arcturus .	14^h 7^m $46^s.80$	− 3.44	14^h 7^m $43^s.36$	$-0^s.43$	$0^s.00$
α^2 Libræ . .	14 41 19.94	− 3.86	14 41 16.08	−0.22	0.00
β Ursæ Min.	14 51 20.28	− 2.21	14 51 18.07	−0.47	−0.37
α Cor. Bor. .	15 27 22.64	− 3.04	15 27 19.60	+0.08	+0.61
α Serpentis .	15 35 45.75	− 3.37	15 35 42.38	+0.03	+0.22
Polaris	0 58 41.78	−14.17	0 58 27.61	−3.82	−1.24

4. When we consider that the mean places of the stars, as given in the best catalogues, are derived, in the way we have here explained, from the mean of several reduced observations, we must allow that the differences inserted in the two last columns, except in the instance of Polaris, are not greater than might be expected from single observations, though made under the most favourable circumstances; for they may arise from errors of observation; from differences in the tabular constants of reduction; from slight unobserved inaccuracies in the adjustments of the instruments, or from the assumed mean right ascensions contained in the existing catalogues; or, what is most probable, in part from each separate source. We will therefore proceed to compute the reductions from the tables and catalogue of the Astronomical Society of London, and see what the columns of differences will become, when the mean right ascensions for Jan 1826 are deduced from them

CORRECTIONS DERIVED FROM THE TABLES AND CATALOGUE OF THE ASTRONOMICAL SOCIETY

Stars	Mean Æ for 1826 from Astr Soc. Cat	Sum of corrections	Apparent Æ computed	$t-Æ$ $=c$	Mean c from 6 stars
Arcturus . .	14^h 7^m $43^s.36$	+ $2^s.75$	14^h 7^m $46^s.11$	$+1^m$ $12^s.91$	$+1^m$ $12^s.64$
α^2 Libræ . .	14 41 16.08	+ 3.38	14 41 19.46	+1 12.70	
β Ursæ Min.	14 51 18.44	+ 2.24	14 51 20.68	+1 12.82	
α Cor. Bor .	15 27 18.99	+ 3.03	15 27 22.02	+1 12.84	
α Serpentis .	15 35 42.16	+ 3.35	15 35 45.51	+1 12.48	
Polaris . . .	0 58 28.85	+13.08	0 58 41.93	+1 12.07	

	Observed App Æ, $=t+(\text{Æ}-t)$	Sum of reductions	Mean Æ for 1826, from observation	Diff from Greenwich	Diff from Astron Soc Catalogue
Arcturus . .	14^{h} 7^{m} $46^{s}.38$	− 2.75	14^{h} 7^{m} $43^{s}.63$	$-0^{s}.16$	+0.27
α^2 Libræ . .	14 41 19 52	− 3.38	14 41 16.14	−0 16	+0 06
β Ursæ Min.	14 51 20.86	− 2 24	14 51 18.62	+0.08	+0.18
α Cor. Bor. .	15 27 22.22	− 3.03	15 27 19.19	−0.33	+0.20
α Serpentis .	15 35 45.33	− 3 35	15 35 41.98	−0.37	−0.18
Polaris . . .	0 58 41.93	−13.08	0 58 28.85	−2.58	0.00

5. From the differences which we have thus obtained, the error of the clock appears to have been $0^{s}.42$ greater than we obtained from the Greenwich Catalogue of 1823, in conjunction with the reductions computed from our own tables, without making any alteration in their original constants, and the differences in the two last columns are not only diminished in amount, but also rendered more equal to one another, than they were from our first series of reductions. But the most remarkable instance is that of Polaris, which, in our last computation, gives the error of the clock within nearly half a second of the mean of the whole; whereas, in the preceding process, the error arising from the reduction of this star differed 3.82 seconds from the mean of the other five, and was not therefore taken into the account. It becomes a matter of considerable interest to ascertain how this unexpected difference arises in the two reductions of the pole star. On examination we find, that Mr. Pond has given the mean right ascension of this star for the Nautical Almanacs a little differently in different years In the year 1826 we find the mean Æ of this star $=0^{h}\ 57^{m}\ 46^{s}.4$ for the epoch 1823, with an annual variation of $+15^{s}.01$, and from this we obtained by the addition of $45^{s}.03$ for three years, the right ascension which we have used above, viz. $0^{h}\ 58^{m}\ 31^{s}.43$. In the body of the Astronomical Society's Catalogue we find the pole star's mean Æ for 1830 $=0^{h}\ 59^{m}\ 19^{s}.64$, with an annual precession of $+15^{s}.430$; but as the author has subjoined a small table to the end of the Catalogue, giving the right ascension of the star in question for four epochs at intervals of ten years each, and differing upwards of 10^{s} from the former Æ, we judged that his intention was to substitute this for the one given in the catalogue, particularly as the annual precession is given also a little differently, viz. $+15^{s}.478$; we therefore substracted $1^{m}\ 1^{s}.91$, the amount of four years' precession, from $0^{h}\ 59^{m}\ 30^{s}.76$, and thus obtained $0^{h}\ 58^{m}\ 28^{s}.85$, which is less than Mr. Pond's right ascension by $2^{s}.58$, and which accounts for so much of the discrepancy In the Nautical Almanac of 1827 the mean Æ of Polaris is stated, at the bottom of page 154, to be taken at $0^{h}\ 58^{m}\ 47^{s}.26$ for Jan. 0, 1827, which, by using the annual variation $15^{s}.01$, would make the seconds for 1826 $32^{s}.25$, even greater than before; but in the years 1828 and 1829, the same almanac gives the mean Æ $=0^{h}\ 58^{m}\ 16^{s}.7$ for 1825, with an annual variation of $15^{s}.19$ from which we derive $0^{h}\ 58^{m}\ 31^{s}.89$ as the mean Æ for 1826; this is also greater than the one we have taken from the Almanac of 1826, and if substituted would increase the great difference already existing. On reference to our own table of the mean places of the pole star, printed at page 274 of our first volume, and formed on the basis of the latest Greenwich observations, we find that the mean right ascension for the epoch 1826 is

given $0^h\ 58^m\ 29^s.942$, with an annual precession of $14^s.997$, which Æ is somewhat less than a mean between the other two; but as there is a difference of a second and upwards between the sums of the corrections arising from the different tables, it is not yet certain, whether the error lies wholly in the mean Æ, or partly in the corrections. One fact however is certain, that, taking the two causes as operating jointly, the computation from the data, afforded by the volume of the Astronomical Society, exactly accords with the present observation of this star, and produces but very slight differences from the mean right ascensions of all the other five.

6. As the Catalogue of the Astronomical Society has been published since the appearance of our first volume, it may be acceptable to some of our readers, who may not have purchased that valuable work, to insert the operations at full length by which the corrections used above were separately computed, which insertion will illustrate the method of using the auxiliary numbers incorporated in the catalogue and preceding tables.

CORRECTIONS SEPARATELY OBTAINED

Arcturus . . . $a = -8.7797$, $b = -8.5750$; $c = +0.4364$; $d = -8.3152$

$A = +9.9523$; $B = -1.3083$; $C = +9.8939$, $D = +0.6954$

$aA = -8.7320$; $bB = +9.8833$, $cC = +0.3303$, $dD = -9.0106$ Sums

Nat. Nos. -0.054; $+0.764$; $+2.139$; $-0.102 = +2^s.747$

α^2 Libræ . . . $a = -8.7215$; $b = -8.6510$; $c = +0.5192$; $d = +8.1437$

$A = +9.9523$; $B = -1.3083$, $C = +9.8939$, $D = -0.6954$

$aA = -8.6738$; $bB = +9.9593$; $cC = +0.4131$; $dD = -8.8391$

Nat. Nos. -0.047; $+0.911$; $+2.589$; $-0.069 = +3.384$

β Ursæ Min. $a = -9.2720$; $b = -9.2390$, $c = -9.4567$, $d = -9.2567$

$A = +9.9523$; $B = -1.3083$, $C = +9.8939$, $D = +0.6954$

$aA = -9.2243$; $bB = +0.5473$, $cC = -9.3506$, $dD = -9.9521$

Nat. Nos -0.167; $+3.526$, -0.224, $-0.896 = +2.239$

α Cor. Bor. . $a = -8.6658$, $b = -8.7709$, $c = +0.4024$; $d = -8.3271$

$A = +9.9523$; $B = -1.3083$, $C = +9.8939$, $D = +0.6954$

$aA = -8.6181$; $bB = +0.0792$, $cC = +0.2963$, $dD = -9.0225$

Nat. Nos. -0.041, $+1.200$, $+1.979$, $-0.105 = +3.033$

α Serpentis . . $a = -8.5966$, $b = -8.7349$, $c = +0.4677$, $d = -7.6805$

$A = +9.9523$; $B = -1.3083$; $C = +9.8939$, $D = +0.6954$

$aA = -8.5489$, $bB = +0.0432$, $cC = +0.3616$, $dD = -8.3759$

Nat Nos. -0.035, $+1.105$; $+2.300$, $-0.024 = +3.346$

Polaris . . . . $a = +0.3584$; $b = +9.7749$; $c = +1.1789$, $d = +0.3583$

$A = +9.9523$; $B = -1.3083$, $C = +9.8939$, $D = +0.6954$

$aA = +0.3107$; $bB = -1.0832$; $cC = +1.0728$, $dD = +1.0537$

Nat. Nos. $+2.045$; -12.110; $+11.825$; $+11.317 = +13.077$

The logarithms contained in the first line of each star's correction are taken from the catalogue; those of the second line from Tables I. and II. as they stand opposite June 24 in the former, and by taking proportional parts from June 20 and 30 in the latter, for the first five stars; but for the pole-star proportional parts for *a*, *b*, *c*, and *d*, were taken also for the year, as being variable quantities for this star, on account of its proximity to the pole.

7. Some one of the foregoing methods of determining, from the apparent right ascensions of the stars, their mean right ascensions due to the beginning of the current year, are such as will be necessary in separately reducing all stellar observations made with an instrument in perfect adjustment; but the error of the clock may be deduced from any of those 60 principal stars now inserted in the Nautical Almanac, as their apparent right ascensions are computed for every ten days of the year; of which number 45 are also computed under the direction of Professor Schumacher, and published in his *Tables Auxiliares*, formerly called *Astronomische Hulfstafeln*. let us now see how our computed apparent right ascensions agree with those included in the publications above mentioned, the computations in which will frequently save much trouble to the practical astronomer, particularly in gaining *e*, the principal error of his clock.

June 24, 1826.	App. Æ by Schumacher	*e*	App Æ by Naut Alm	*e*
Arcturus	14^h 7^m $46^s.40$	$+1^m$ $12^s.62$	14^h 7^m $46^s.60$	$+1^m$ $12^s.42$
α^2 Libræ	14 41 19 .55	+1 12 .61	. . . .	. . . . . .
β Ursæ Min. . .	14 51 20 .97	+1 12 .53	14 51 19 .11	+1 14 .39
α Cor. Bor. . . .	15 27 22 .38	+1 12 .48	. . . .	. . . .
α Serpentis . . .	15 35 45 .54	+1 12 .43	. . . .	. . . . . .
Polaris	0 58 42 .51	+1 11 .49	0 58 44 .28	+1 9 .72
	Mean =	+1 12 .36	Mean =	+1 12 .17

The computations contained in Schumacher's publication give a mean right ascension which is nearly an average between what results from our constants, and from those of the Astronomical Society's catalogue, and, except in the case of Polaris, the computed errors accord with one another very satisfactorily; they are used in many of the continental observatories, for the purpose of regulating the time of their clocks, and save the observers much computation, by giving the apparent places by inspection. The examples which the Nautical Almanac of 1826 contain, are not only limited in their number, but, being computed from Dr. Maskelyne's old tables, are not alike correct: it therefore became time to adopt more accurate tables than had been previously used, and accordingly Dr. Young, from this time, has very judiciously not only had recourse to other more recent Tables, but has extended the number of stars to 60, which in future will greatly contribute to supply the wants of the practical astronomer.

8. There is, however, still a considerable difference in the computed right ascensions of the *Tables Auxiliares* and of the *Nautical Almanac*, as they regard the pole star, which difference is partly derived from the difference of the mean right ascension of this star assumed by the different computers, and partly from the other elements of computation; and as the

deviation of the transit-instrument's position is frequently derived from the times of passage of this star, it is of importance to practical astronomy that this difference should be eliminated. To show what is the varying amount of the difference we have noticed, we will subjoin a comparative statement for different months of the year 1827, as the right ascensions appear in the said works.

AR of Polaris for 1827.

		Nautical Almanac			Tables Auxiliaires			Diff			Nautical Almanac			Tables Auxiliaires			Diff
January	0	0^h	59^m	$10^s.50$	0^h	59^m	$9^s.03$	$1^s\,47$	June	29	0^h	59^m	7.86	0^h	59^m	$6^s.81$	$1^s.05$
	20		58	56.09		58	53.82	2.27	July	19			23.74			22 45	1.29
February	9			42 92			40.09	2.83	August	13			42.12			39.41	2.71
March	1			32.73			28 79	3.94	Sept.	2			53.77			50 41	3.36
	21			26 90			23.06	3.84		22	1	0	1.39			57 27	4.12
April	10			26.07			22.32	3.75	Oct.	12			4.14	1	0	0.58	3.56
	30			30 58			28.20	2.38	Nov.	1			1 70	0	59	58.47	3.23
May	20			39 79			37.98	1.81		21	0	59	54.44			52.48	2 96
June	9			52 67			51.91	0.76	Dec.	11			43.21			41.44	1.77
	29	0	59	7.86	0	59	6.81	1.05		31			29.41			28.06	1.35

9. Sometimes the error of the clock is derived from the observed transit of the sun, by comparing the time shown by the sidereal clock with his computed right ascension for the given day, as inserted in the Nautical Almanac, or *Connaissance des Tems*, but as the evaporation, which is always going on when the sun shines, is unfavourable to the steadiness of this luminary's apparent motion, and as there is supposed to be a small error in the solar computations, observations of the sun should not be depended on for giving the time correctly. On the 24th of June, 1826, for instance, the observed time of passage of the sun's centre, as given in our specimen of the Greenwich journal, was at $6^h\ 11^h\ 28^s\ 86$, and his computed right ascension, as contained in the Nautical Almanac, for the same moment, is $6^h\ 10^m\ 15^s.8$, so that the difference $(t - \mathrm{AR})$ is $1^m\ 13^s\ 06$, or larger than the average error obtained from the stars, by any of the methods of computation, by from four to eight tenths of a second.

10 We come now to consider the more complex case of reducing the observed times into mean right ascensions, when the observations have been made with the transit-instrument a little out of adjustment with respect to both horizontality and collimation, as well as out of the meridian; in this case not only does the clock show the time of the star's meridian passage erroneously, but the instrument observes the star out of the meridian from its want of the proper adjustments. The first computation therefore must be that which gives the whole correction, *e*, of the clock, the second that which converts the corrected time into apparent right ascension, by means of the corrections of the instrument, and the third that which, by means of the reductions, finally turns the apparent into the mean right ascension due to the beginning of the current year. Professor Littrow has shown, in the first volume of the Memoirs of the Astronomical Society of London, but by a different notation, that the correction for the whole error of the clock may be taken thus,

$$e = \mathrm{AR} - t - e' - e'' - e''',$$

as determined by our formula (2.) [§ LVIII. 5.], or as taken from our tables A and C, already exemplified, secondly, this correction of the clock must be applied in conjunction with the errors of the instrument, e', e'', e''', retaining their proper signs, to the time (t) of the observation, to obtain the apparent right ascension of the star; viz. $Æ=t+e+e'+e''+e'''$; and lastly, the precession, aberration, and nutation, lunar and solar, must be applied to the apparent to convert it into the mean right ascension, due to the beginning of the current year. We will take, as our first example, the observations made with a portable transit-instrument of ε Bootis, α² Libræ, Spica Virginis, and η Ursæ Majoris, as related in the section last referred to, when the instrument was out of the meridian, but was otherwise adjusted, so that e', the error arising from a, the azimuthal deviation, was the only one that affected the instrument, though the clock's correction was $Æ-t-e'$, which in this instance constitutes the whole error e.

EXAMPLE I—FOR THE APPARENT RIGHT ASCENSIONS OF FOUR STARS.

	May 25, 1827.		June 2, 1827		June 10, 1827	
	ε Bootis	α² Libræ	ε Bootis.	α² Libræ.	Spica Virg.	η Ursæ Maj
Æ.	14^h 37^m 28^s 91	14^h 41^m 23^s.04	14^h 37^m 28^s 86	14^h 41^m 23^s 05	13^h 16^m 0^s.35	13^h 40^m 45^s.41
t	14 35 4.86	14 39 1 35	14 39 2 13	14 42 53 44	13 16 20 91	13 41 6.04
($Æ-t$)	+ 2 24 05	+ 2 21 69	− 1 33.27	− 1 30.39	− 0 21 56	− 0 20.63
$-e'$	+ 2 16	+ 4 52	− 2 63	− 5.51	+ 0 97	+0 04
$-e''$						
$-e'''$						
e	+ 2 26 21	+ 2 26 21	− 1 35 90	− 1 35 90	− 20 59	− 20 59
e'	− 2 16	− 4 52	+ 2 63	+ 5 51	− 0 97	− 0 04
$t+e+e'=$ App Æ	14 37 28 91	14 41 23 01	14 37 28 86	14 41 23 05	13 16 0 35	13 40 45 41
Sum of the Reduc	− 3 53	− 4 10	− 3.18	− 3 62	− 3 02	− 2 59
Mean Æ, 1827 .	14 37 25 38	14 41 18 94	14 37 25.68	14 41 19 43	13 16 5 33	13 40 43 82
By the Naut Alm	14 37 28 12	14 41 19 07	14 37 26 12	14 41 19.67	13 16 5 50	13 40 43.10

11. From an inspection of the preceding computations it will appear, that when the whole correction of the clock, denoted by e, is determined from any one well known star, the same correction may be applied to the *observed time* of any other star taken within the same hour, provided the clock's rate be inconsiderable; which application will greatly abridge the determination of the apparent right ascension of such second star; for by applying in our case the correction e, gained from ε Bootis, to the observed time of passage of α² Libræ, together with e' due to the latter, we have the apparent right ascension of α² Libræ without further trouble: and the same may be said of Spica Virginis and η Ursæ Majoris; or of any other two, three, or more stars observed successively on the same evening. But it may be satisfactory to compute the right ascension of the last star, when the interval is more than an hour, or otherwise take a star for the last, the right ascension of which is already computed, and given in the Nautical Almanac, or *Tables Auxiliares*, which will operate as a check on the determined right ascensions of the other stars. We have here computed the sums of the pre-

cession, aberration, and nutation, solar and lunar, from the Tables published with the Catalogue of the Astronomical Society of London, as being particularly convenient for the reduction of single observations, but for obtaining daily apparent places, or those for every successive ten days, we consider our own Tables, now used by the computers of the Nautical Almanac, much more serviceable. When it is recollected under what circumstances the observations here reduced were made, and with a portable instrument too, the differences from the Greenwich mean right ascensions for Jan. 0, 1827 come out much smaller than we expected, and prove how useful such a small transit-instrument may be made, by taking a mean of several observations with it, when duly adjusted and carefully used.

12. For our second example we will take a case still more complex, where the instrument was not only out of the true meridian, but had its axis not perfectly level, and its collimation also out of adjustment, when the observations were taken Professor Littrow has supplied us with a case of this description, and has recorded the observations made at Buda in Hungary with a six-feet transit instrument made by Reichenbach, of which we will now avail ourselves, making only such alterations in the processes as arise from a difference of the notation. In this case the azimuthal error a, the inclination b, and defect in the collimation c, are not previously given, but require to be determined from the observations themselves, which were as follow. On May 18, 1822, the pole-star was observed at both the upper and lower culminations, for the purposes of determining both the error of collimation, and the azimuthal error of position; the eye-piece had seven vertical wires, and when the star had passed three of them in the usual position, at the upper culmination, the time of passage at the fourth or middle wire, $=t$, was computed from the known values of the intervals, and the state of the axis examined by the level, both which were found thus, viz.,

$$t \text{ reduced} = 0^h\ 57^m\ 18^s.3 \left\{ \begin{array}{ll} e=37.0 & \quad e'=32.5 \\ w=34.5 & \quad w'=39.0 \end{array} \right.$$

The axis was then reversed, when the three wires, that had been observed before the central wire, were now found at the opposite side of the field of view, and were again transited by the star, when the time reduced to the middle wire, and also the indication of the level became

$$t \text{ reduced} = 0^h\ 57^m\ 13^s.4 \left\{ \begin{array}{ll} e=41.0 & \quad e'=38.3 \\ w=30.5 & \quad w'=33.2 \end{array} \right.$$

the value of h, or of one division of the level, being $=0''.71$ –

On May 19, the following observations were made in the usual position of the telescope, and reduced to the middle wire viz

Pole star . . .	0^h	57^m	$13^s.50$	$e=38.9$ $w=34.2$	$e'=39.0$ $w'=34.5$
Sirius	6	37	55.97	$e=40.2$ $w=33.1$	$e'=37.7$ $w'=35.4$
Procyon . .	7	30	37.01		
α Hydræ . : .	9	19	29.01		
Regulus . , .	9	59	32.11		
Pole star S.P .	12	57	37.97	$e=36.0$ $w=36.0$	$e'=33.0$ $w'=39.0$

the daily rate of the clock being $+0^s.151$ and $\frac{h}{60} = 0.01183$;

13. Now if we call the inclination at the first or upper culmination b, and at the second or lower after reversion $b_{,}$, then by our formula

$$0^{s}.01183 \times (w + w') - (e + e')$$

we shall have $b = 0.01183 \times 4 = 0^{s}.04732$, and $b_{,} = 0.01183 \times -15.6 = -0^{s}.18455$ on May 18, on which day we have for the pole star $\delta = 88^\circ\ 21'\ 32''$, L, or latitude of Buda, $= 47^\circ\ 29'\ 12''$, and $\cos(L-\delta).\ \sec\delta = -26.4$: then by our formula given at page 331, we have the error of collimation

$$c = \tfrac{1}{2}(t_{,}-t).\cos\delta + \tfrac{1}{2}(b_{,}-b)\cos(L-\delta), \text{ thus, viz.}$$

$\frac{1}{2}(t'_{,}-t) = -2^{s}.45$	log.	-0.38917	
$\cos\delta = 88^\circ\ 21'\ 32''$		8.45695	
		$-8.84612 =$	-0.07015
$\frac{1}{2}(b_{,}-b) = .0685$	log.	8.83569	
$\cos(L-\delta)\ 40^\circ\ 52'\ 20''$		-9.87862	
		$-8.71431 =$	-0.05180
		$c =$	-0.12195

14. Having determined the inclination of the axis at both culminations of the pole star, and ascertained the error of collimation due to the 18th of May of the given year, we must in the next place determine, from the times of the transits above and below the pole taken on May 19, the azimuthal error, which, as well as the error c, may be taken as applicable to the observations of the four other stars observed on that day; for the error c is less variable than the error in the horizontality of the axis. If the axis had been levelled, and the collimation correct at the time in question, the error a would have been found from our circumpolar formula (1), viz. $a = \frac{t - t' - 12^{h}}{2\ \cos L}.\ \text{cotang}\ \delta$; but as none of the adjustments were correct we must add to the quantity thus obtained $+ \frac{b.\cos(L-\delta) - b'.\cos(L+\delta) + 2c}{2.\cos L.\sin\delta}$; and then the sum of the two will be true a. [§ LVII. 7.] But we must first know b and b', the inclinations of the axis on the 19th, before and after an interval of twelve hours, as being due to the times of the two culminations of the pole star, because these are the quantities we now want in our supplemental computation. We will keep the parts separate. At the upper culmination of the pole star our formula for finding the inclination from the states of the level, in the reversed positions, gives us $b = -0.109$, and $b' = +0.071$.

Then for the first part we have

$t - t' - 12^{h} = -24^{s}.47$	log.	-1.38863
2 Arith. Comp.		9.69897
Sec of latitude $47^\circ\ 29'\ 12''$		0.17021
Cotang δ. . . 88 21 32		8.45713
First part -0.51872		-9.71494

For the numerator of the second part,

$b = -0.109$ . . . . −9.03743

Cos $(L-\delta)$ 40° 52′ 20″ . . . . −9.87862

$b.\cos(L-\delta)$ . . . . +8.91605 = +0.08242

$b' = +0.071$ 8.85126

Cos $(L+\delta)$ 135° 50′ 44″, or 44° 9′ 16″ . . . . −9.85580

$b'.\cos(L+\delta)$ to be subtracted . . . . −8.70706 = −0.05094

Remainder . . . . +0.13336

Add $2c$ or 2×-0.12195 . . . . −0.24390

Sum of $b.\cos(L-\delta) - b'\cos(L+\delta) + 2c$ . . . . −0.11054

Sum of the numerator −0.11054 . . . . log. −9.04391

2 . . . . log. 0.30103 ⎫ Subtract the sum of

Cos L 47° 29′ 12″ . . . . . . 9.82979 ⎬ the denominators 0.13064

Sin δ 88 21 32 . . . . 9.99982 ⎭ logarithms.

Second part, Nat. No. . . . . . 0.08190 . . . . −8.91327

First part add . . . . −0.51872

Azimuthal error $a =$ . . . . . . −0.60062

15. We have now for the four other stars $a = -0.60062$, $c = -0.12195$, and $b = -9.4 \times .01183 = -0.11120$, all which are very small quantities, to determine the sum of the corrections, e, of the clock, depending on the deviation and imperfect adjustments of the transit-instrument, which may be thus arranged.

FOR THE CORRECTION OF THE CLOCK.

	Sirius	Procyon.	α Hydræ	Regulus
Æ	6^h 37^m $18^s.49$	7^h 29^m $59^s.77$	9^h 18^m 51.61	9^h 58^m $54^s.82$
t	6 37 55.97	7 30 37.01	9 19 29.01	9 59 32.11
Æ−t . . .	−37.48	−37.24	−37.40	−37.29
$-e'$ . . .	0.56	0.40	0.50	0.35
$-e''$ . . .	0.05	0.06	0.06	0.09
$-e'''$ . . .	0.13	0.12	0.12	0.12
Whole cor. $= e$	−36.74	−36.66	−36.72	−36.73

In this type the quantities e', e'', and e''', which may be either computed, or taken from our logarithmic Table C, contained in § LVIII. when considered as *errors* have each a negative sign, but, when put as *corrections* of the clock, are made positive, in order to be *subtracted*. We have then the mean of the whole correction taken from the four stars, $e = \text{Æ} - t - e' - e'' - e''' = -36^s.7125$, which horary correction applied to the observed time of any other star's passage over the middle wire, together with the errors of the transit instrument $e' + e'' + e'''$ will give the respective apparent right ascensions of such stars, supposing them to be observed with the instrument unaltered in all respects, provided that a proportional part of the rate for the interval between the observations be added to or subtracted from e, when that interval and rate are considerable. In our example the mean of the times corresponding to the mean correction of the clock was $8^h\ 22^m$ of sidereal time.

16. If the apparent right ascensions of the three last stars had not been known from computation, we could have easily obtained them by applying the true correction of the clock (e), as derived from Sirius only, in conjunction with the errors of the transit-instrument respectively due to the several stars in the following manner;

FOR THE RIGHT ASCENSIONS.

	Procyon	α Hydræ	Regulus
t . . .	$7^h\ 30^m\ 37^s\ 01$	$9^h\ 19^m\ 29^s.01$	$9^h\ 59^m\ 32^s.11$
$+e$ . . .	−36 .74	−36 .74	−36 .74
$+e'$ . . .	− 0 .40	− 0 .50	− 0 .35
$+e''$ . .	− 0 .06	− 0 .06	− 0 .09
$+e'''$ . . .	− 0 .12	− 0 .12	− 0 .12
App. Æ . .	7 29 59 .69	9 18 51 59	9 58 54 .81
Precession . .	− 1 .20	− 1 .12	− 1 .22
Aber. . .	+ 0 .80	+ 0 .24	+ 0 .01
☾ Nut. . .	− 0 .75	− 0 .62	− 0 .84
☉ Nut. . .	+ 0 .06	+ 0 .05	+ 0 .06
Mean Æ	7 29 0 .60	9 18 50 .14	9 58 52 .82
Greenw^h. Cat.	7 29 59 .03	9 18 50 .55	9 58 53 .09

We have here taken the reductions from the apparent to the mean right ascensions for the epoch Jan. 0, 1822, out of our own TABLE of the CORRECTIONS for forty-eight stars by inspection, as they stand there, but with contrary signs. The mean difference between the mean right ascensions here determined from single observations, with the instrument deranged, and the Greenwich last Catalogue, is $+0^s.42$, and the mean right ascensions of all the stars were increased by $0^s.31$, the supposed error of the equinoctial point, in the year 1823, in the

Catalogue of 1820, which addition has probably been made after too small a number of equinoctial* observations of the sun

17. When several stars have been observed on the same night, and the correction of the clock has been determined carefully from two known stars at the beginning and end of the observations, the reduction of the observed times into apparent and mean right ascensions may be arranged in the right ascension book in one Table, according to the subjoined form viz.

Stars.	t	Correction of Clock e	Corrections of the Instrument. e'	e''	e'''	Apparent Æ	Precession	Aberration	Lunar Nutation	Solar Nutation	Mean Æ for 1822.
Sirius	6h 37m 55s 97	−36s 74	−0s 56	−0s 05	−0s 13	6h 37m 18s 49	−1s 00	+1s 06	−0s 59	+0s 05	6h 37m 18s 01
Procyon	7 30 37 0	−36 74	−0 40	−0 06	−0 12	7 29 59 69	−1 21	+0 63	−0 75	+0 06	7 29 58 .43
α Hydræ	9 19 29 01	−36 74	−0 50	−0 06	−0 12	9 18 51 59	−1 12	+0 24	−0 62	+0 05	9 18 50 14
Regulus	9 59 32 11	−36 .74	−0 35	−0 09	−0 12	9 58 54 81	−1 22	+0 01	−0 84	+0 06	9 58 52 82
&c.	&c.	&c	&c	&c	&c	&c.	&c	&c	&c	&c	&c.

In the third column we have taken the error of the clock as given by Sirius, the first star, for all the other stars; because the rate was too small to be of any importance, the proportion for the whole interval of 3h 20m, between the first and last stars, being only $+0^s.02$, but generally it may be necessary to apply the proportional part of the rate, with its contrary sign, as an additional portion of the horological correction e.

18. Professor Littrow made $\sin(L-\delta) \cdot \sec\delta = m$, and $\cos(L-\delta) \cdot \sec\delta = n$, which are the tabular quantities belonging to our logarithms in Table C, so that instead of e' he puts $a\ m$, and for e'' he uses $a\ n$, otherwise his method of proceeding accords with ours exactly The following specimen taken from one of the volumes of Littrow's Observations will further explain his mode of reduction, which we have adopted.

γ Leonis

Date of the Observations.	Mean reduced to mid. wire	Correction of Clock	Correc of Inst	Aberration	Nutation	Precession.	Mean Æ for 1821
1821 Dec 26	10h 12m 33s 70	−143s 71	−0s 06	−0 79	−0 76	−3 .23	10h 10m 5s 16
1822 Jan 1	12 44 10	−153 62	−0 40	−0 86	−0 77	−3 50	5 13
Feb 20	10 2 40	+ 8 80	+0 24	−1 35	−0 87	−3 77	5 53
&c	&c	&c	&c	&c	&c	&c	&c

Then the mean of the whole number of reduced observations will give the mean right ascension suitable to be inserted in a catalogue, provided the clock's corrected time be true sidereal time, as it regards the vernal equinoctial point.

19 In the other continental observatories, particularly in Bessel's and Struve's, the letters m and n are used to denote expressions different from Littrow's, and as they appear in company with the letter c at the head of every day's observations in the journal of right ascensions, it will be proper to point out the signification of those terms, and the mode in which they are

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* Vide *Connaissance des Tems*, 1829, pp 318—321

applied in effecting the corrections. In Bessel's *Fundamenta Astronomiæ*, the author says that, if we put

$$m = a \sin L + b \cos L$$
$$n = -a \cos L + b \sin L$$

the formula which we have adopted, as the basis of our corrections, first given by him, viz.,

$$\mathit{AR} = t + \tau + a.\frac{\sin (L-\delta)}{\cos \delta} + b.\frac{\cos (L-\delta)}{\cos \delta} + \frac{c}{\cos \delta}$$

may be converted into $\mathit{AR} = t + \tau + m + n\,.\,\text{tang}\,\delta + c\,.\,\sec\,\delta$, where τ is the correction of the clock. When none of the errors a, b, c, is known, it will be necessary to observe three stars, as we have directed [§ LVIII. 4.] by means of which we may determine $(\tau + m)$, n, and c; but if c be known or $=0$, then two will be sufficient, but τ and m cannot be separated, nor can the exact time of a star's passage be determined, but only the differences of the times, which are competent to give the relative right ascensions. But if either a or b be known, we have

$$m = n \text{ cotang } L + a \text{ cosec } L = -n \text{ tang } L + b\,.\,\sec L.$$

Hence it appears that Bessel's τ is our e, his m our θ, and his n our η. We have seen in some of the preceding specimens of a journal of transits the values of m, n, and c prefixed to an evening's work, where m applies, as a correction of time, alike to all the stars, and the last column contains $\pm n\,.\,\text{tang}\,\delta \pm c\,.\,\sec\,\delta$ as a separate correction to be united with m for each individual star, and when the true time of the clock $(t+\tau)$ is known, the application of $m + n\,.\,\text{tang}\,\delta + c\,.\,\sec\,\delta$ will convert the said time into right ascension; and in this way the reduction of the observed times into apparent right ascensions may likewise be effected.

20. We will conclude this section by giving an example of the Greenwich plan of reducing the observed times of the star's meridian passages, to corresponding mean right ascensions, due to the beginning of the year, as practised by Dr. Maskelyne, who, like Mr. Pond, kept his transit-instrument in the meridian, and in proper adjustment. Considering the mean right ascension of α Aquilæ to be known, or very nearly so, this excellent astronomer computed the apparent right ascension of the said star for the day of observation from his Tables, and compared it with the observed time of passage by his clock; the difference between the computed AR and the observed time was called the *reduction* of *α Aquilæ*, and might have been called *the error of the clock* as deduced from this star, since his transit-instrument was considered free from errors. The rate of the clock was always known by comparative observations of α Aquilæ on successive days, so that a proportional part of it could be taken and applied to the reduction of α Aquilæ, to gain the same for any other star, preceding or following; and when this altered reduction was applied to the observed time of the said preceding or following star, accordingly as the quantity required to be augmented or diminished, it gave the apparent right ascension of such star without further computation · after which this apparent was reduced into mean right ascension in the usual way, by the application of the tabular reductions to fix the epoch. The following scheme, taken from a Greenwich manuscript, will explain the process for five stars, observed September 26th, 1806, on which day the rate of the clock, as given in the volume of that year, was $\text{R} = -0^s.68$, the assumed AR of α Aquilæ, for the beginning of the year, being $19^h\ 41^m\ 18^s.32$.

September 26, 1806.

Stars	Transits by Clock	Reductions	Apparent Æ	Tab XVII	Tab XVIII	Sum	Mean Æ 1806	p p of daily rate
Arcturus .	14^h 6^m 41^s 06	+9 25	14^h 6^m 50^s 31	-0^s 87	-0^s 96	-1^s 83	14^h 6^m 48^s 48	-1^s 16
α Herculis	17 5 41 12	+9 34	17 5 50 46	−1 65	−0 96	−2 61	17 5 47 75	−0 07
α Aquilæ . .	19 41 12.54	+9 41	19 41 21 95	+2 51	+1 12	+3 63	19 41 18 32	
α Cygni ..	20 34 42 07	+9 43	20 31 50 00	−2 47	−0 77	−3 24	20 34 48 76	+0 02
α Pegasi	22 53 6 78	+9 50	20 55 10 58	−3 37	−1 07	−4 44	22 55 5 84	+0 09

According to this mode of reduction all the stars are affected with the same common error, whatever that may be. Table XVII of Dr. Maskelyne contains in one sum, the precession from the beginning of the year, the aberration, and solar inequality of precession, together with the proper motions of Arcturus and of α Aquilæ, viz., $-0^s.093$ for the former, and $+0^s.038$ for the latter, and Table XVIII. comprises the equation of the equinoxes and deviation or lunar nutation but his coefficients are no longer in use. A similar method of reduction is, we believe, still practised at the English Royal Observatory, and a mean of many days' observations, separately reduced from modern Tables, supplies the mean right ascension of the Catalogue.

§ LXIII —ON THE CORRECTION OF RIGHT ASCENSION COMMON TO ALL THE STARS

1. When a number of stars have had their mean right ascensions determined by any of the methods explained in our last section, where a *fundamental* star has its mean right ascension *assumed*, to which all the others are referred by their differences, they will all be charged with the error in time, which may happen to belong to the said fundamental star; and it is an object of the utmost importance to the practical astronomer, to ascertain the existence and quantity of such common error, before he constructs his catalogue. The principal difficulty lies in determining the vernal equinoctial point, commonly called the first point of aries, for any given day; for as this is the zero point from which the right ascensions of all the stars are counted, and yet is a variable point in absolute space, some method of determining it by observation is essential to the accuracy of all measures of distance taken from such invisible zero. When we select a principal star out of any catalogue, and reduce its mean right ascension, belonging to the epoch, to the apparent right ascension of the evening of observation, by means of the tabular corrections, we may indeed put the sidereal clock of an observatory in motion, when set to such right ascension, at the instant when the said star is crossing the middle wire of a transit-instrument, in a proper position and state of adjustment, and by knowing the rate of the clock, we may determine the instant when the point $0^h\ 0^m\ 0^s$ is passing the exact meridian; but if the catalogue has any error in right ascension, or if the constants of precession, aberration, and nutation be not perfectly correct, the point thus passing the instrument will not be the *true zero* of right ascension at the existing moment; the point wanted is that in

which the ecliptic crossed the equator at the time of the star's passage. This point might be computed from the solar Tables, but the result would be objectionable, as partaking of the errors of those Tables, that, after all their rectifications, may still exist.

2. The most satisfactory means to adopt will be, first to find the sun's mean right ascension, by comparing the transit of his centre with the transit of the fundamental star, or with the transits of several principal stars, related to it by known differences; and secondly by computing, from his declination deduced from his observed zenith distance on the same day, from the obliquity of the ecliptic at the time, and the known latitude of the place, the right ascension due to the moment of the meridian passage. These operations however must be performed on several days, near both the vernal and autumnal equinoxes, which may afterwards be so paired, that the sun may have nearly the same declination on each day of every pair; and it is important that his daily change of declination be not less than 20′ on any of the days chosen. When the right ascensions derived from a comparison with the stars near the first equinox are put down in one column, and those derived from the observed declinations are put down in another, as belonging to the same days, written in the first column, the differences may be placed in the same horizontal line in the fourth column with their proper sign, + or −, accordingly as the sun's Æ from his declination is the greater or less; then, when the same arrangement is made for the corresponding days of observation at the latter equinox, it will be seen, provided there be an error in the assumption of the fundamental star's right ascension, that the sign in the column of vernal differences will be of one denomination, and that of the column of autumnal differences the contrary, and half the sum of each pair of corresponding differences will be the correction due to any individual pair of vernal and autumnal observations, and consequently the mean of the whole of these half sums, will be the mean correction, with its proper sign, to be applied to the assumed right ascension of the fundamental star, and to the right ascensions of all its descendants.

3. We were favoured with the copy of a Greenwich manuscript, a few years ago, which records ten pairs of corresponding observations of *α* Aquilæ in an unknown year, and as many pairs of solar observations ready computed, which will supply us with a suitable example for illustrating the process we have proposed, as well suited to practice, for correcting assumed right ascensions.

EXAMPLE

Vernal Equinox					Autumnal Equinox						
Days		Sun's Æ by Stars	Sun's Æ by Declin	Diff	Days		Sun's Æ by Stars.	Sun's Æ by Declin.	Diff.	Sums of diff.	½ Sums
February	17	330° 23′ 24″ 0	330° 24′ 46″ 5	+ 22′ 5	October	26	210° 12 15″ 0	210° 11′ 55″ 3	−19″.7	+2″8	+1′ 10
	22	335 11 26 2	335 11 44 9	+18 7		20	201 20 52 5	201 20 31 7	−20 8	−2 1	−1 05
March	5	345 31 40 9	345 32 1 7	+20 8		11	196 6 31 9	196 6 13 9	−18 3	+2 5	+1 25
	7	347 22 48 6	347 22 58 1	+ 9 5		6	191 31 20 2	191 31 12 6	− 7 6	+1 9	+0 95
	13	352 54 0.0	352 54 12 9	+12 9		1	186 58 41 7	186 58 26 8	−14 9	−2 0	−1 00
	15	354 43 44 4	354 43 59 5	+15 1	September	27	183 21 54 6	183 21 45 3	− 9 3	+5 8	+2 90
	21	0 11 56 7	0 12 6 3	+ 9 6		25	181 33 48 4	181 33 31 8	−16 6	−7 0	−3 50
April	5	13 49 36 3	13 49 47 0	+ 10 7		7	165 22 56 4	165 22 43 9	−12 5	−1 8	−0 90
	6	14 44 21 1	14 44 30 8	+ 9 7		6	164 28 46 6	164 28 38 2	− 8 4	+1 3	+0 65
	7	15 39 0 9	15 39 11 6	+10 7		5	163 31 26 5	163 34 24 6	− 1.9	+8 8	+4 40
										Correction for *α* Aquilæ	+0 51

4. According to this method, if there should be any error in the tables of refraction, or of the sun's parallax, when his declination is computed from an observed zenith distance, or if the obliquity of the ecliptic and latitude of the place should be assumed erroneously, the errors both in the declinations and right ascensions will have contrary signs on each side of the equinox, and when the observations are taken equidistant from either equinox, and equal in number before and after the points of no declination, those errors will be equal, and, having contrary signs, will annihilate one another. If we call the correction in right ascension to be found x, and put R and S for the right ascensions of the sun, computed from a comparison of his passage over the meridian, with the transits of the stars having an assumed right ascension, the former before, and the latter after the equinox; then his true right ascensions will be denoted by $(R+x)$ and $(S+x)$: also let the sum of the errors arising from wrong data in the refraction, parallax, obliquity, and latitude be called y, then the right ascensions inferred from the declinations will be respectively $(R+x+y)$, and $(S+x-y)$, from each subtract R and S, the right ascensions of the sun inferred by the transits from the assumed places of the stars, and there will remain $x+y$ and $x-y$, half the sum of which is $=x$, the correction to be applied to the assumed right ascension of α Aquilæ, and of the other stars connected with it, that were compared with the sun. Also half the difference between $x+y$ and $x-y$ will be equal to y, the error arising from any erroneous computation of the declinations; and the constant errors in the declinations, may be found by the following analogy, viz.

As ☉'s daily motion in Æ : to his daily motion in dec. :: y : const error in dec.

Hence the sun's apparent zenith distance may be found when he is in the equator, or when he has any given declination, independently of the latitude, refraction, and parallax.

5. The methods of determining the zenith distance of the sun or of any other heavenly body, and of deducing the declination from it, will be explained hereafter, when we come to describe the circular instruments, but it will be proper to show here, how the sun's right ascension is deduced from his declination at any time when the obliquity of the ecliptic is known; which may now be safely taken from the Tables. For this purpose we have

Rad × sin Æ = cotang obliq × tang declination;

and therefore when any three of these are given, the fourth may be readily found. For instance, if on September 17th, 1826, we take the obliquity $=23^\circ\ 27'\ 39''$ and the sun's declination $=2^\circ\ 22'\ 35''$ N, as given in the Nautical Almanac of that year, the short operation will be

Radius	. . . .	log. 10.000000
Cotang	23° 27′ 39″	10.3625105
Tang	2 22 35	8 6180440
Sine	5 29 13	8 9805545

Now $21^m\ 56^s.87$, the time corresponding to the arc $5^\circ\ 29'\ 13''$, being subtracted from 12^h, the sun's right ascension at the autumnal equinox, leaves $11^h\ 38^m\ 3^s.13$ for the sun's right ascension in time at Greenwich noon on September 17th of the given year; or subtracting the arc found from 180° we shall have the sun's Æ $=174^\circ\ 30'\ 47''$ as deduced from his declination. Then for his right ascension to be deduced from his transit we have

From Naut. Alm. apparent Æ of α Aquilæ, 17 Sep. 1826, ... =	19^h	42^m	$21^s.47$
From the Greenwich observations at the middle wire . . .	19	42	44 .18
Clock fast			22 .71 Sub.
Time of the sun's transit on the same day	11	38	26 .93
Sun's right ascension by α Aquilæ	11	38	4 .22
The same in arc	174°	31′	3″.3
The same deduced from his declination	174	30	47 .0
Difference arising from this single observation			+16 .3

By the same mode of proceeding all the columns of the table were filled up, when the declinations had been deduced from the observed zenith distances, as will be explained hereafter.

6. The theory on which the computation of the star's correction in right ascension is founded, is explained by Delambre * and Professor Woodhouse † at considerable length, to whose works we beg leave to refer our readers. It was in this way that the addition of $0^s.31$ was made to the right ascensions of the Greenwich Catalogue of 1820 by the present Astronomer Royal, in the year 1823, at which time eight equinoxes only had been observed by the new transit-instrument, and, as the author observes, this correction of the equinoctial point is still capable of improvement by continued observation. In the instance of September 17, 1826, we have taken it for granted that the transit-instrument was in the meridian, and also nearly in adjustment, because we find, prefixed to the observations of September 16, the following note, viz.

> "Examined the axis, and found the east end 2″.3 too high.
> Adjusted it, and let it remain 0″.32 too high.
> The meridian marks at Blackwall and Chingford appeared to be bisected by the central wire."

Hence we considered $(Æ-t)$ as the only correction of the observation of α Aquilæ that was necessary; and as the daily rate of the clock is stated to be only $+0^s.28$, one fourth part of this, which was due to the elapsed interval between the star's and sun's passages, is too inconsiderable to be noticed in our case, but which, had the rate been much greater, must have been applied to the error at the time of the star's passage, with its contrary sign, as a correction of the time at the instant of the sun's preceding passage. If the instrument had been incorrectly placed, or out of adjustment, the sum of the corrections, arising from all the causes, must of course have been applied, as we have directed in a former section.

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* *Astronomie Théorique et Pratique* Tome I Chap XVII, 4to, Paris

† A *Treatise on Astronomy*, Vol I Chap VII. New edition

§ LXIV.—TRANSIT CIRCLE BY TROUGHTON. [PLATE XVII.]

1. AFTER having described various transit-instruments, and detailed the different adjustments, and errors arising from devious position, and imperfect rectifications, we proceed to the description of an instrument that is designed to give both right ascensions and zenith distances, or declinations, at the same time. The first instrument of this kind was probably the meridian circle of Horrebow, which had its divisions read by microscopes previously to the year 1735; we know but little of the accuracy of this instrument, nor was its existence known to Troughton, who in the year 1806 contrived his transit-circle, which in the hands of Mr. Groombridge has performed great service to astronomy, in furnishing observations for a circumpolar catalogue of stars, which the Honourable Board of Longitude have liberally undertaken to reduce. Mural quadrants had begun to sink in public estimation, and circles had been proposed to supersede their use, towards the latter part of the past century, as being more capable of being accurately divided, and of having their index errors appreciated, as well as the errors of excentricity corrected by opposite readings. The Rev. Francis Wollaston had contrived a transit-circle so long ago as in the year 1793, which was carefully made by the late excellent workman Cary, but the axis being made too slender, and the circle too small, rendered the observations taken with it less perfect than they might otherwise have been.

2. Troughton seeing the imperfection of Wollaston's instrument, avoided its objections by such an union of skill and strength of materials, as enabled him first to construct an instrument bordering as nearly on perfection as human art could effect, and then to divide it by a new optical method peculiar to himself, which has since been rewarded with the Copley medal by the Royal Society of London, and has lately been copied with success by younger artists, who may continue to practise the method, when its ingenious inventor may no longer be known, but by his well-earned reputation. The instrument under our notice is represented in perspective in our Plate XVII., where all the principal parts of the fabric are exposed to view, and will therefore require but few letters of reference for their explanation. The circular portion is composed of two parallel flat rings and two sets of radial hollow cones, forming together two wheels that are united together by various bars, crossing one another so as to form rhomboidal figures, as well as by the perpendicular rods exhibited in the figure: each of these united circular plates is four feet in diameter, and divided separately all round, by spaces of 5′, into four quadrants. Each end of the horizontal axis is formed into a strong cone, and the middle receives the conical spokes; the whole length of the axis is three feet. Each of the circular plates has an edge bar surrounding its back surface, to give them strength without adding materially to the weight of the structure, and a continuation of tubular braces connects the radial cones all round in a way that is better seen in the plate than verbally described. The axis is supported by a pair of stone piers about five feet four inches high, of a prismatic shape, which have their perpendicular and parallel faces separated about twenty-seven inches: the metallic Ys for holding the pivots, are made fast to the superior ends of the piers, and have the adjusting screws for vertical and azimuthal motion on the separate piers, as is the case with the transit-instruments which we have described, a pair of strong brass tubes are made fast to

the interior faces of the stones forming the piers, and contain each a strong spiral spring, bearing each a roller, that presses against a pair of circular edge-bars surrounding and made fast to the cones of the axis, in such way that the principal part of the instrument's weight is supported by the spiral springs, while the remainder is borne by the pivots, resting in the rectangular Ys, round which the instrument turns in altitude. The telescope, which has about five feet focal length, and three inches and a half aperture, passes between the two circular limbs and through the central portion of the axis, which is cylindrical, so as to form more than a diameter, by projecting about six inches at each side, it is held firmly at both ends by the rhomboidal bars, that embrace it, so that no flexure can possibly take place at any degree of elevation. Another tube, resembling that of the telescope, but smaller in bore, passes diametrically across the double circle at right angles to the telescope, and receives the plumb-line, which we shall revert to presently, at either of its ends.

3. To the interior face of each pier, just below their summits, is a strong horizontal bar of brass made fast, the extreme ends of which are turned upwards, by being curved so as to hold each a pair of microscopes for reading the divisions, at the opposite ends of the circle's horizontal diameter, to which they are capable of being exactly adjusted: and if we call the two microscopes fixed to the eastern pier *A* and *B*, while those made fast to the western pier are denominated *C* and *D*, it is evident from the figure that a zenith distance, or altitude, may be read by each pair in a separate circle, one on the eastern face, and the other on the western, as though there were two distinct instruments, each indicating to the accuracy of single seconds, at the same time that a transit may be taken in the usual way by the system of wires, or rather spider's-lines, that are fixed in the common focal point of the object-glass and eye-piece. Hence it is obvious that, by the proper use of this instrument, a star or other heavenly body may have its place referred both to a circle of declination and to the equator at the same instant; for the time shown by the clock may be counted, while the body passes along the horizontal line, from one vertical line to another, and the divisions on both circles may afterwards be both read in succession, while the instrument retains its position. The fifth microscope, passing through one of the pillars, is considered more firmly placed than the others, and is introduced for the purpose of detecting any change that may take place in the position of the pillar, or shape of the circle, from natural or accidental causes. The clamping apparatus for giving slow motion is carried by a brass frame, made fast to the inner face of one of the piers, and is so contrived, that it applies great resistance in the revolving direction of the circle, but yields laterally to any slight deviation that its face may be liable to, when the axis is not perfectly horizontal. When the hand of the observer cannot reach the milled head of the screw, that directs the elevation, a handle with a Hooke's joint may be conveniently applied; and when the axis is reversed, the clamping mechanism acts with the second circle with the same convenience. The tripod seen under the double circle holds the water vessel, in which the perforated box holding the shot is immersed, when it is suspended by the plumb-line, and a screw, for elevating or depressing this vessel, affords the ready and safe means of suspending the weight at a proper depth in the water.

4. When the telescope is brought into the horizontal position, the tube that is destined to contain the plumb-line becomes vertical, and in that situation the fine silver wire is suspended from an apparatus at its upper end, and descending down the tube, is viewed in two directions

by each pair of microscopes, that stand at right angles to each other, in the manner of Ramsden's ghosts, [§ XLIX 2.] nearly at both the extremities of the tube. These microscopes are so adjusted, that, by reversing the axis of the instrument, one pair of them will adjust the index errors of the circles, or at least detect their values, and the other pair will adjust the axis to a horizontal position But as the plumb line cannot remain in its vertical position while an observation is made, a revolving spirit-level is constantly suspended parallel to the axis, one half of its length, by means of which the error in its horizontality, if any, may at all times be detected. The small hooks, that connect the box with the silver wire at both ends, remain when the load is removed, and stretch the wire just enough to take it through the tube, where it is fixed ready for use, by the cap carrying a bolt for this purpose. A second spirit-level is suspended parallel to the optical axis of the telescope, near the eye-end, by means of which the positions of the microscopes may be adjusted, and examined occasionally, more readily than by the plumb-line, though not so accurately.

5. The eye-pieces, which of course are of the positive kind, may be made to move horizontally across the field of view, by means of a lever connected with a pinion, so as to stand directly in front of any of the vertical lines, to prevent obliquity of vision; and the mechanism is so contrived, that the horizontal may instantly be changed into a vertical motion, while the upper and lower limbs of the sun, or moon, are brought into a state of contact with the equatorial wires to effect this, at about half the mean diameter of the sun from the central horizontal wire, is a fixed wire on one side, and at the other a moveable one, all parallel to each other, and the distance between the fixed and moveable lines can be measured by the micrometer's screw that separates them, and will measure in this way as much as 40′, when a small power is applied. The micrometer's moveable line is known from the fixed ones by inspection at any time, because it crosses the central wire only by a small quantity, when at right angles to each other. The axis is perforated, like that of a transit-instrument, and admits the light of a lamp necessary for illumination, in the usual way, by reflection within the telescope's tube, while the quantity and quality are regulated by passing through glasses of different colours, or having different shades of the same colour

6. The four microscopes *A*, *B*, *C*, and *D*, which have illuminating plates, with universal motion, for throwing light on the divisions of the limbs, were all that the instrument was supplied with in its original state, and during the several years of its being used by Mr. Groombridge, but since it became the property of Mr South, four additional microscopes have been added, by the maker, at each divided face, which it is expected will render the average of the readings more accurate, by dispersing them all round the circumference, and correcting for temperature in all states of the atmosphere. This instrument is too bulky to be easily reversed in position without the assistance of straps, or cords and pulleys; but has the same advantage gained by reversion, that is obtained by instruments that carry their microscopes along with them, and correct for collimation in altitude Since the microscopes are fixed, and the circle only is reversed, a new index error takes place in the second position, which remains constant, for the greatest care of the maker could not insure the zero points of the two faces to be identical in taking measurements The plumb-line well applied must be relied upon for determining the constant difference of the two means of the readings of each face, taken in the reversed positions, and one half of this difference must be applied with the sign + in one position, and

— in the other as a correction, or, when this difference is greatly diminished by the alternate adjustments for collimation in altitude, and for the index errors, in the reversed positions, a mean may be taken of two series of observations of any given number of stars, taken successively by both faces in the two positions, which we understand was the method employed by Mr. Groombridge, who always found a difference of several seconds in the different readings, and made the sum of the errors merge in the mean of all the observations. The polar point might indeed be found on either face of the circle, by circumpolar stars, independently of the plumb-line, or level, and the observations might be referred to such point, on either face, while the position remains the same, and the microscopes continue unaltered. but we do not learn that this instrument was ever adjusted, or the observed places registered in such manner, though the principal observations hitherto made with it have been of circumpolar stars.

7 *Adjustments of the Transit Circle.*—First, place the instrument nearly in the meridian, and adjust it for horizontality of the axis, in doing which the plumb-line must be used instead of the level, by viewing it through that pair of microscopes carried by the vertical tube, that lie parallel to the telescope, when that is horizontal, which it must necessarily be, when the plumb-line is suspended down the vertical tube, standing at right angles to it, then when the luminous discs are both bisected, take off the box of shot, and, securing the hooks of the line, invert the tube and make it again vertical; replace the weight, after the suspending apparatus is changed to the top, and if, when it comes to rest, the same discs are again bisected by the plumb-line, the axis is level, but if not, one half of the error must be corrected by turning the disc most remote from the point of suspension, and the other half by the vertical screw, that elevates and depresses the pivot of the axis, and after a repetition of this operation of inverting and halving the remaining error, the axis will at length become horizontal, and the revolving level may be adjusted to correspond.

Secondly, adjust for collimation in azimuth exactly as explained in Section LVI., considering the instrument destined for observing transits.

Thirdly, place the telescope nearly horizontal by the level that is carried by its eye-end, and adjust this level, till it will reverse in position, end for end, when displaced from the pivots of the cocks that bear it in altitude; this may be effected partly by the screw of the level, and partly by the circle's screw of slow motion, in doing which the telescope will also be placed horizontal, in consequence of its parallelism with the level, when the latter has been made to reverse.

Fourthly, in this position of the telescope adjust all the four microscopes to their respective zeroes, and while in this state of approximation, if there be no well defined mark in the direction of the meridian already existing, let one be fixed, as near the horizon as can be guessed, and at such distance, that it may be distinctly visible, when the aperture of the telescope is diminished, so as to be capable of being bisected by the horizontal wire: when this bisection has taken place, while the microscopes remain at zero, the axis of the circle must be reversed, end for end, and the horizontal wire of the telescope again brought to bisect the said horizontal mark, when the bubble of the level ought to stand at the middle of its tube; but this can hardly be expected to take place at the first trial, nor is it probable, that the microscopes will be found again exactly at their zeroes; but as the four quadrants of the circle are numbered from 0° to 90° four times over, in the same order successively, altitudes will be read in one position of the axis, and zenith distances in the other, so that it will always appear, whether the sum of the two means of each pair of readings exceed or fall short of 90°, then half the amount of the excess or deficiency will be the whole error of the observation, so far as the level alone is concerned: the distance from zero is double the error, which may now be measured by the microscopes, move the circle and telescope, by the tangent screw of slow motion, over one half of the error, and raise or lower the horizontal mark the other half, till it is again bisected; and when the operation is repeated, it will be seen what further adjustment is necessary to reduce the error to its smallest quantity, and to render the mark a truly horizontal point of departure for both faces either jointly, or separately, when the known correction is properly applied. In pursuing these measures for effecting the adjustment of the circle's error, it is taken for granted, that the horizontal wire bisects the field of view across the middle of the optical axis, and is therefore not deranged. The level may be applied for the same purpose by another method thus, when the telescope has been directed to cover the mark, and the microscopes put to their zeroes, the axis must be reversed as before, and the telescope brought again to the mark, then one half of the error may be corrected by moving the mark in the proper direction, and the other half by altering that pivot of the level's suspension, on which the vertical screws act: the former method has the advantage of measuring a moiety of the error by the micrometer, and the latter avoids the necessity of re-adjusting the microscopes. But, in the maker's opinion, the plumb-line is less liable to derangement by changes of temperature than the level, and therefore ought to be had recourse to. It will not be necessary to go through the same steps, since the use of the level and of the plumb-line is the same in principle. Let us suppose the telescope, as at first, directed so, as to bisect the mark already spoken of; the tube containing the plumb line will then be very nearly vertical, and the line being suspended and loaded, will take a perpendicular direction, adjust the point of suspension by the proper screws, and by turning the excentric discs, that form the objects of the superior and inferior microscopes used for this adjustment, till they are both bisected; in the next place, turn the circle half round by its divisions, read by its microscopes, and, fixing it, suspend the plumb-line from the contrary end of its inverted tube the plumb line must then be made to bisect the upper disc by means of the circle's tangent screw, and if the lower end is not now bisected also, the error, or half-difference, must be ascribed to the uncertainty of the preceding operations, and must be thus corrected, turn the circle round slowly till the lower disc is bisected, and measure by the microscopes the small arc so moved over, put the circle back till half that quantity only is measured, and turn the lower disc till, by means of its excentricity, it is bisected by the plumb-line, as well as the upper disc. Since the plumb-line is capable of inversion, it is not necessary to reverse the circle's axis, but for the sake of checking the former operations, it may be deemed desirable to reverse it, and to repeat the observations of the mark carefully in this position also. The plumb-line apparatus, being finally adjusted, regulates the zero point, to which the microscopes must be ultimately fixed this part is perhaps less liable to vary, than any other portion of the instrument, and even if it should be found to vary to a sensible extent, the mode of adjustment, we have last described, will with certainty restore it to its original state The error we have here treated of is compounded of the error of collimation, and of the index error arising out of the difference of the positions of the zero points on the two faces, which, notwithstanding all the care and skill of the most eminent maker, amounts to a constant arc of 7″ or 8″. Mr. Groombridge, we understand, used this circle a long time in the same position, and on every change adjusted all the four microscopes to their respective zeroes, having first brought the circle into position, by making the plumb-line bisect both the lucid discs in the reversed positions. This however was done for the sake of appearance, rather than of real accuracy, for if he had adjusted the microscopes to the mean fixed error, the differences in the readings of the various microscopes would have been greater than was agreeable to the eye. We have proposed an horizontal mark to be erected to facilitate the approximate adjustment, which will be near enough for all ordinary purposes; and particularly for explanation of the process; but the accurate observer will prefer making his final adjustment, and determination of the remaining error, to be applied to single observations, by having recourse to the stars. Indeed after all, as the natural tendency of every part of a large instrument is, to settle into a state of rest, it seems more likely to give accurate results, after every part has found its own bearing, subsequently to an approximate adjustment; and the error finally determined by observations of many stars in both positions may be ultimately applied, in the form of a correction, with perfect satisfaction. It is scarcely necessary to add, that this construction of a circular instrument admits of its error being determined by reflection, or by the collimator not yet described, as well as any other to which they have one or both been applied.

8. With respect to the method of observing a star, in measuring its zenith distance, it seems scarcely necessary to give any particular directions; when the young observer has learnt how to accomplish all the adjustments, he will have confidence in his instrument, and will soon gain that presence of mind and coolness, which prevent his being hurried, when the moment of the observation is approaching, and when he once knows what time must be expended, in computing the altitude or zenith distance of his object, and setting his telescope to the proper elevation, he will be ready to meet the ingress of the little traveller, and will have much pleasure in stringing it on the horizontal wire, by the assistance of the tangent screw of slow motion, which is always ready to obey his commands. When once the star is bisected by the horizontal wire, the instrument must not be again touched, till the transits over all the wires have been noted down, in seconds and parts, as already directed [LXI. 2.], and until all the microscopes have been read, and their values registered to the accuracy of a single second each, as well as the hanging level of the axis examined. In general it will be advisable to leave the instrument in its position undisturbed, till a second observation is required to be made, that the microscopes may be re-examined, in case any cause for suspicion, about the accuracy of any of the readings, should occur when the observation has been registered, and the different readings compared together. The method of adjusting and reading the microscopes has been sufficiently explained in the forty-eighth section. When the elevation of the telescope is too great to allow of a convenient application of the eye, a diagonal eye-piece may be used, or an observing couch may be placed on the floor under the eye-end, within sight of the sidereal clock, which is as much a companion of this instrument as of a regular transit-instrument, and requires the same attention.

9. The plan of registering the observations made by the transit-circle will depend on the size of the paper that forms the journal; but whether one page or two be used for containing the columns, it will be proper to keep the transits distinct from the zenith distances; and as

the state of the barometer and thermometer will form a part of the latter, to modify the refraction, in applying the correction to the observed zenith distances, it seems more convenient to use two pages, with the names of the stars repeated in parallel horizontal lines, in the second page. The specimens of transit-observations, which we have already given [§ LXI.], will supersede the necessity of giving any further direction on that part of the observations, which must be made, corrected, and reduced in precisely the same manner as if the instrument had been destined for transits only; consequently when the position is a little out of the meridian, as examined either by a circumpolar star, or by a pair of high and low stars, the deviation from the meridian must be noted, if not immediately corrected, and also the errors in the adjustments, if any shall be found to have existed during any of the observations, so that the corrections may be afterwards applied when the observations come to be reduced to the corresponding right ascensions.

10. As the manuscript observations of Mr. Groombridge, to the number of 30,000, are now at the command of the Board of Longitude, who have kindly undertaken their reduction, we cannot at present give an exact fac-simile of a day's work, as registered by him; but the annexed plan is submitted to the practical astronomer, of arranging the columns, and of writing down the observations, that may be taken by a circle having four reading microscopes placed on opposite faces, according to the construction of the instrument we have described in the preceding part of this section. In the following example the mean zenith distances accord with those taken on the same days by one of the Greenwich quadrants, but are modified to suit our purpose of illustration, on a supposition that both faces show zenith distances.

ZENITH DISTANCES MODIFIED FOR A TRANSIT-CIRCLE, IN LAT. 51° 28′ 39″.5

DIRECT POSITION OF THE CIRCLE

Date	Barometer	Thermometer		Magnitude.	Names of Stars and Planets	Reading Microscopes										Mean of the Zen. dist.		
		In	Out			A.			B			C		D				
1811 June ☽ 10	29 85	60	61		Polaris, S P	40°	11′	21″4		11′	32′3	11′	20″6	11′	33″4	40°	11′	26′9
					Arcturus …	31	17	50 6		18	1 0	17	49.5	18	2 1	31	17	55 8
		58	60		α Cor. Bor.	21	6	47 5		6	57 9	6	40 4	6	59 0	24	6	52 7
					α Serpentis ..	14	26	1 9		26	12 3	26	0 8	26	13 4	44	26	7 1
					Vesta	64	23	50 4		24	0 8	23	49 3	24	1 9	64	23	55 6
		55	59		Antares .	77	24	32 2		24	42 6	24	31 1	24	43 7	77	24	37 4
					α Herculis	36	51	1 2		51	11 6	51	0 1	51	12 7	36	51	6 4
		54	58		α Ophiuchi	38	35	28 5		35	38 9	35	27 4	35	40 0	38	35	33.7
					♄ Centre .	73	23	56 9		24	7 3	23	55 5	24	8 4	73	24	2.1
REVERSED POSITION OF THE CIRCLE.																		
☿ 12					α Serpentis .	44	25	40 1		25	51 7	25	39 4	25	52 4	44	25	45.9
					Vesta . . .	64	28	45 0		28	56 6	28	54 3	28	57.3	64	28	50 8
					Antares .	77	24	5 6		24	17 2	24	11 9	24	17 9	77	24	11 4
	29 93	50	55		♄ Centre . …	23	23	14 0		23	25 6	23	13 3	23	26 3	73	23	19 8
	29 99	55	55		α Andromedæ	23	25	14 3		25	25 9	25	13 6	25	26 6	23	25	20.1
					Polaris . . .	36	47	18 4		47	30 0	47	17 7	47	30 7	36	47	24 2
	30 02	60	56		α Arietis . .	28	54	13 7	28	54	25 3	54	13 0	54	26 0	28	54	19 5

11. On looking over the two days' observations above exhibited, it will be seen, that α Serpentis and Antares are the only two stars that were observed on both days, and from these we must therefore deduce the amount of the index error and of the collimation in zenith distance, taken together, since the observations do not afford the means of separating them, supposing them to have been actually made by a transit-circle that will reverse in position. With respect to α Serpentis, half the sum of the two observations is = 44° 25′ 56″.5, the apparent zenith distance on the intermediate day, and half the difference will be found to be = 10″.6 for the error, to be subtracted from the first day's observation, and added to that of the second: the two reversed observations of Antares give the half sum 77° 24′ 24″.4 for the apparent zenith distance due to the 11th of June, and 13″ for the half difference, or error, including the diurnal change, then 11″.8, the mean of these two errors, may be taken as the negative correction of the first day's observations of all the southern stars, and also as the positive correction for these stars observed on the second day, which correction may be taken as accurate enough for single observations, to be subsequently checked by other results, when the same stars have been observed after a short interval in both positions. In taking the zenith distances of stars viewed towards the north, in the direct position of the instrument, the same sign must be prefixed to the correction, as is due to the reversed position, when looking towards the south, and the contrary, for when the telescope is turned over from the south to the north, the position of the eye piece and of the horizontal wire is reversed, without the reversion of the instrument's axis. Hence Polaris, in the preceding register, requires the sign of its correction to be contrary to that of the other stars, on each day. The apparent zenith distance of the planet Vesta on the 11th was 64° 26′ 23″.2, supposing its change uniform, but as it had a daily motion in zenith distance of 2′ 27″.6 + 11″.8 = 2′ 39″.4 increasing towards the south, the correction of the instrument could not be detected by any observations of its zenith distance. on the contrary, Saturn's zenith distance on the intermediate day was 73° 23′ 40″.95, arising from the half sum, and was diminishing by a daily change of 21″.15 − 11″.8 = 9″.35; it consequently was also an improper object for giving the error of the instrument.

12. When Mr. Groombridge disposed of his transit circle, it was supposed, from the observed differences in the readings of the microscopes, that an alteration had taken place in the figure of the divided limbs, or in the pivots of the axis; and the maker was applied to for his opinion, who on examination recommended an additional number of microscopes, that might diminish any errors that may have been occasioned by wear, or changes of temperature, in the figure of any of the parts, and the present proprietor has accordingly added double the number of the original microscopes; and the instrument has now six guards stationed at each side, to watch the changes that may hereafter take place, and to ensure accuracy by multiplied readings. Professor Robinson, of Armagh, has lately considered the subject of micrometrical readings with more attention than had been previously paid to them, and his paper "On Correcting Errors of the Astronomical Circle by Opposite Readings", which was read before the Royal Irish Academy on the 5th of December 1825, embraces many considerations that interest both the instrument maker and the astronomer. The learned author assumes, that, since the improved method of graduating a circle by microscopic vision began to be practised, there is little fear of errors of division existing to any considerable extent, but he shows that errors of excentricity may be of two sorts, *fixed* and *variable;* the former takes place

when the line connecting the centres of the pivots, on which the axis revolves, does not pass through the centre of the circle's graduations, and the latter exists, when there is a defect in the figure of the pivot. With respect to the former, or fixed error of excentricity, opposite readings will correct them; and with respect to the latter, or variable error, as the deviation from a circular figure is always minute with careful turning in the lathe, the section of the pivot may be supposed to be an ellipse of small excentricity now if we conceive this to roll in a rectangular *Y*, as is usual in astronomical instruments, its centre will describe a circular arc, of which the radius is $r \times \sqrt{2 - e^2}$, and its sine $\frac{1}{4} e^2 \times \sin 2\theta$, r being the greatest semi diameter of the pivot, θ the angle made by it with the vertical, and e the excentricity. Hence, says the author, "we see that the horizontality of an axis, supported by such a pivot, is not changed during its revolution; but that its extremity moves in azimuth through the space $0.35\, r \times e^2 \sin 2\theta$. In consequence of this the circle revolves round its vertical diameter, and the deviation of its telescope from the meridian, at an altitude α, is as $\cos \alpha \times \sin (2\alpha - \alpha')$, α' being the altitude at which the deviation in altitude vanishes." The same reasoning applies to the pivots of the transit-instrument, and the errors arising from this cause, the author observes, may be easily ascertained and corrected. "Besides this," says he, "the centre of the circle is also transported horizontally through a space bearing a constant ratio to the motion of the pivot, and this constitutes what is called *variable* excentricity, different in no respect, as to its influence on the measurement of angles, from the permanent kind." The author then proceeds to explain the nature and value of the errors in question, by mathematical reasoning too abstruse for our purpose; and shows that two microscopes will correct for excentricity, or for any other error varying by a similar law; as also for all errors which are as odd powers of the sine or cosine; but that three microscopes are still better, failing only where the number expressing the order of the error is divisible by three. In general the excentricity disappears from a mean of any number of microscopes, when their distance from each other is equal, viz. the quotient of the circumference of the circle divided by the number used. The unequal expansion of the radial arms of the circle may deform its figure, and that of the limb may disturb the equality of its divisions, but here, concludes the author, "the greatest masters of analysis can scarcely see their way." It is, however, important in practical astronomy, when several microscopes are used, that their relative positions be frequently examined, and rendered as permanent as possible, though permanency of position is a desideratum in nature scarcely to be accomplished with variable materials.

§ LXV REDUCTION OF THE APPARENT ZENITH DISTANCE, OR DECLINATION, TO THE MEAN POLAR DISTANCE.

1. Before we proceed to describe other circular instruments and their uses, it will be proper to explain how the zenith distances observed by any of them may be converted into mean polar distances, and consequently how apparent altitudes may be turned into mean declinations,

which quantities are only the complements of the former; indeed it depends on the figuring of the graduations in any of the instruments, whether zenith distances or altitudes are indicated; or whether each position has a separate mode of giving the readings, one showing zenith distances and the other altitudes. This last is a very convenient mode of denoting the observations; because it will always appear by inspection, whether the sum of any two corresponding reversed observations exceed or fall short of 90°, and therefore whether the error arising from one half of the difference, must have the sign + or − applied in this case to *both* the observations, which, having their quantities registered of different denominations, will require the *same sign* in the correction.

2. When the instrument is truly placed in the meridian, and properly adjusted for taking meridional zenith distances, and when a mean of all the microscopes, taken at one position, is obtained and corrected for the amount of the index and collimation errors, five reductions are necessary for converting the apparent zenith distances into mean polar distances, besides a knowledge of the latitude of the place. In the first place the observed zenith distance, corrected and added to the co-latitude, will give the observed polar distance in a northern latitude, when the star is towards the south, but if towards the north, and above the pole, the zenith distance subtracted from the co-latitude will give the same. when the star passes below the pole, the zenith distance diminished by the co latitude will be the polar distance, and its complement the declination. Whenever the polar distance is less than 90°, its declination is of the same name as the elevated pole, but when it exceeds 90°, it has a contrary name. In all the European observatories the declination above the equator is north, and has the sign + attached to it, but below the equator it is south, and marked with the sign −, in the various trigonometrical computations. If we substitute the latitude for the co-latitude in the preceding directions, the resulting numbers will be declinations at once, or, if we substitute altitudes for zenith distances, when we use the co-latitudes, the numbers obtained in this case will also be declinations. Secondly, when the observed or apparent polar distances are obtained by any of the means above explained, the refraction, modified by the factors for the barometer and thermometer, must be applied with a negative sign, together with the precession, aberration, and nutation, lunar and solar, in north polar distance, with their *signs changed*, and when the sum of all these five quantities have been algebraically united with the apparent polar distance, the amount will be the reduced polar distance belonging to the beginning of the year in which the observations were made

3. The mean place of a star is usually deduced from several careful observations made on different evenings, or on different days, if its magnitude be such as to render it visible by daylight, and when these evenings, or days, follow in immediate succession, the reduction of an average of the measures may be made for the middle of the period, particularly if the observed zenith distances are nearly alike, but in a variable climate such a succession of observations will rarely be obtained, and a separate reduction of each observation not only becomes necessary, but is besides a more accurate mode of proceeding, since the five quantities are all variable We will take for an example the zenith distances of α Coronæ Borealis, as observed by the south quadrant at Greenwich in the year 1811, by using a mean of the readings on the exterior and interior arcs, that we may compare the resulting north polar distance with that of the Greenwich catalogue derived from the new circles.

α CORONÆ BOREALIS 1811 [Co-LAT = 38° 31′ 20″ 5]

	Bar	Int. Ther	Obs Zen Dist	App Polar Dist	Refrac	Process.	Aber	☾ Nut	⊙ Nut	Reduced N P D.
May 29	29 80	57	21° 6′ 42″0	62° 38′ 2′5	+0′ 25″3	−4″99	− 5″22	+8″08	+0″ 12	62° 38′ 26″09
June 2	29 43	58	43 9	4 4	0 24 9	−5 16	− 4 28	+8 10	+0 42	28 38
9	30 10	59	40 7	1 2	0 25 4	−5 39	− 2 82	+8 12	+0 42	26 93
15	29 88	60	40 5	1 0	0 25 1	−5 59	− 1 08	+8 14	+0 39	26 90
22	29 80	55	38 1	62 37 58 6	0 25 4	−5 83	+ 0 63	+8 16	+0 34	27 30
July 9	29 93	62	30 8	57 3	0 25 0	−6 40	+ 4 82	+8 21	+0 16	20 09
17	29 82	64	35 8	56 3	0 24 8	−6 67	+ 6 67	+8 25	+0 04	29 39
27	29 75	63	35 0	55 5	0 24 8	−7 01	+ 8 80	+8 28	−0 09	30 28
Aug 7	29 60	62	34 5	55 0	0 24 7	−7 38	+10 89	+8 31	−0 24	31 28
18	30 02	62	32 0	52 5	0 25 1	−7 75	+12 61	+8 31	−0 34	30 30
30	30 14	63	32 5	53 0	0 25 1	−8 15	+13 98	+8 36	−0 41	31 78
Sept. 10	30 09	64	30 3	59 8	0 24 9	−8 53	+14 81	+8 38	−0 42	38 91
22	29 80	61	36 1	56 6	0 25 0	−8 93	+15 00	+8 41	−0 36	35 72
Oct. 5	29 56	60	36 4	56 9	0 24 8	−9 37	+14 57	+8 45	−0 22	35 13

The mean N. P. D for Jan 1811 from an average of fourteen observations .. .. = 62° 38′ 30″ 54
From Greenwich Cat. of 1823 (62° 41′ 0″ 8) subtract (12″ 42 × 11 years) 2 16″ 62 = 62 38 44.18

It is not stated in this series of observations what was the index error of the quadrant, which appears here to have been about 14″. In general this error was determined by the zenith sector, and on reference to the page of observations made with that instrument in the months of July and August, we find,

July 29, 1811.	Sector on the western wall, γ Draconis had zen. dist.	= 2° 46′.90
Aug. 5.	Sector on the eastern wall, ditto	= 2 34 .10
	Consequently the mean or apparent zen. dist. . . .	= 2 40 . 5

and, in the register of zenith distances, we have

July 29.	γ Draconis by the southern quadrant	2 35 . 0
Aug. 2	Ditto . . 2 36 .0 } Mean for Aug. 5. . . .	. 2 37 . 5
8.	Ditto . . 2 39 .0 }	

Hence the index-error derived from the sector did not exceed 4″.25 at the time specified, but the indications of the *northern* quadrant were at the same dates only 1′ 25″, 1′ 26″.6, and 1′ 30″ respectively.

4. With all the care that could be employed, the observations made by a mural quadrant could never be considered as quite correct, and until circles were substituted, there appear to have been inaccuracies of from 20″ to 30″ in the determination of several of the continental observatories. We have heard Troughton affirm, that he would depend on a well-divided circle of a single foot in diameter, more than on a fixed quadrant of the largest construction; and on this account we are disposed to omit a particular description of the mural quadrant, as being an instrument no longer to be depended upon in the present state of practical astronomy. The

corrections contained in the example of α Coronæ Borealis were taken by inspection from pages 131 and 132 of our first volume, but may be computed from the proper formulæ, or taken from any other tables constructed for the same purpose. If the tables of the Astronomical Society be used, the amount of precession, aberration, and nutation, lunar and solar, in declination, will be given at one computation by means of the tabular quantities; but when these computations are applied, the observed zenith distances must first be turned into declinations, for which the tables are adapted, and all the signs must be changed when the reduction is from the time of observation to the beginning of the year. If the stars observed are not found in any of the tables of corrections, the general or universal tables of Delambre, Gauss, Groombridge, or Fallows may any of them be consulted, though their constants of aberration are now supposed to be all a little in defect, and their constants of lunar nutation, the two former in defect, and the two latter in excess by small fractions of a second.

§ LXVI RAMSDEN'S ALTITUDE AND AZIMUTH CIRCULAR INSTRUMENT. [PLATE XVIII.]

1. THE first astronomical circular instrument that was made by Ramsden, was that with which the late eminent astronomer, Piazzi of Palermo, took those series of observations, from which the declinations in his much esteemed catalogue were computed and published, first in 1803, and then in an improved state in 1814, the latter of which editions contains 7646 stars. A particular account is given of this instrument in Piazzi's folio work, entitled *Della Specola Astronomica, de' Regi studi di Palermo,* in two volumes; the former containing four books published in 1792, and the latter comprising the fifth in 1794. The figure, contained in our Plate XVIII. is copied from an engraving given in the said work, and being shaded will be sufficient to convey a general idea of the construction, without the detached figures given in the original, to assist the detail of the description. Piazzi informs us, that Ramsden twice undertook the construction of this instrument, and as often abandoned it, but at length in January of the year 1788, he entered upon the work in earnest, and finished it in August 1789.

2. The vertical axis of the instrument is composed of various parts, which, being firmly united together, constitute a frame that revolves in one piece on two pivots at the extreme ends; the cone, that terminates the lower extremity, tapers from a diameter of 14 .2 inches to 5, where it is made fast to the horizontal circle, on which the azimuthal angles are measured, of three feet diameter. This circle has ten tubular radii of a conical form, and is divided into two semicircles, figured into 180°, each of which is subdivided into spaces of 6'; it revolves with the frame composing the vertical axis. The base of the inverted cone is firmly attached to an oblong plate of metal, which may be called the lower stage, and into which four long vertical tubes are fastened, as so many pillars, to support the upper stage, of the same dimensions as the lower one, namely 25 .3 by 16 .8 inches. These four pillars are each $6\frac{1}{2}$ feet long, and $3\frac{1}{2}$ inches in diameter, and, together with the two stages, constitute the frame holding the vertical circle. This circle is five feet in diameter, divided into four successive quadrants, in which

each degree is figured with an Arabic numeral, and each tenth degree with larger numerals of the Roman characters, and, as the reading microscopes have small fields of view, the subdividing strokes, including spaces of 6′, are known by single, double, triple, &c points made contiguous to them on the graduated face. The circumscribing portion of the vertical circle is composed of two flat rings, standing parallel to and concentric with each other, by means of cross bars that unite them, like so many rounds of a ladder, which plan gives strength, without adding materially to the weight of the structure. The graduated face is that which is presented to the eye in the perspective figure.

3 The horizontal axis of the vertical circle consists of three pieces, a central cylindrical hollow piece, and two inverted hollow cones, all of brass, compactly fixed together, at the extreme ends of which two steel cylindrical pivots are made fast, one to the apex of each cone, which bear a portion of the circle's weight. The eight conical radiating tubes of brass are fixed to the central part of the axis at their bases, and at the remote ends to the middle of certain cross bars, connecting the two rings of this circle the telescope passes through the said central portion, instead of two more radii, and is made fast at both ends between the rings of the double circle, so as to prevent any tendency to flexure in its tube. The focal distance of the telescope is equal to the diameter of the circle, and five direct eye pieces are supplied, magnifying the linear dimensions respectively 50, 75, 100, 130, and 170 times; besides which there is a prismatic eye-piece of the description mentioned in the sixth paragraph of our fifth section, which performs the office of a reflecting diagonal eye-piece. This eye-piece has two powers, one of 75, and the other of 130, and is principally used for viewing stars near the zenith. The illumination is effected by transmitting the light of a small lamp through the hollow axis, the inclined reflector, in the middle of which, is exactly similar to that of a transit instrument: the light, however, is limited to suit the object viewed by a parallelopiped composed of three pieces of glass; of which the middle one is white, and the two extreme ones green, they are contained in a frame that has an adjustable motion by means of pulleys, visible at the remote end of the axis, behind the back pillars. The reason of two green wedges being used is, that the lines in the focus of the eye piece may not appear double, by passing through two glasses of unequal refracting powers, the second green glass being made to correct the refraction occasioned by the first. The pulleys are acted upon by a long handle terminating with a Hooke's joint The whole length of the vertical circle's axis, including the pivots, is about two feet. Besides the four long pillars already noticed, there are two shorter ones, ascending, at the distance of eleven inches from each other, from the face of the lower stage, up to the middle of the inverted cones of the horizontal axis; these hollow pillars are each three feet three inches high, and are braced near their upper ends to their adjoining long pillars, a strong rod passes through each pillar, and supports a small frame, containing a pair of rollers, side by side, one pair of which may be seen under the left hand cone of the axis Each cone has a circular edge-bar made fast round it, that rests on the Y formed between the pair of rollers, and a pair of adjusting screws acting on the vertical rods, bearing the rollers, force them against the circular rings of the cones, and support any required portion of the circle's weight, to relieve the pressure on the steel pivots. One of these rods has its adjusting screw under the lower stage, but the other is acted on through a small frame interposed between the two halves of the tube, for some reason that is not obvious. The lower stage is strengthened by several brackets de-

scending from its lower surface to the inferior end of the inverted vertical cone, to which they are made fast, some of which are seen in the engraved figure.

4. The two large metallic pillars, that ascend from the two opposite corners of the square, composed of a marble floor, are each seven feet high, and four inches in diameter, and are matched by a similar pair, that do not appear, but which ascend from the other two opposite corners of the same square floor. These two pillars, with their arched tops of brass, are omitted in the plate, that they might not conceal any portion of the vertical axis. The arch that connects the first pair of pillars will explain the structure of the second, which is exactly similar, and is placed at right angles to the other. A cross of four straight bars connect the arched portions, that rest on the superior ends of the four pillars, to which they are made fast; and a circular hole, at the place of crossing, receives the tubular pivot at the superior end of the vertical axis, and is firmly fixed to the middle of the oblong opening, that nearly severs the upper stage. This opening allows the telescope to view stars near the zenith without obstruction, the bars, connecting the two halves of the stage, being thinner than the diameter of the object-glass.

5. The lower support of the vertical axis consists of three concentric circular plates of iron, laid over one another with attached rollers under the second and third; the uppermost circle bears the conical pivot, on which the axis turns, and the other two have their respective adjustments at right angles to each other, one being moved in the direction of east and west, and the other of north and south. each motion is produced by a horizontal screw, by means of a handle with an universal joint, of which one is seen standing up rather obliquely, to the right of the three circles. The manner of each screw's action may be easily understood, if we conceive one of them made fast to one of the lower circles and the other made fast to the other, with their tapped ends entering the sides of the uppermost, which may thus be pushed forwards or drawn back in either of the assigned directions. A ring of mahogany, three feet two inches in diameter, and three inches thick is laid over, and made fast to the uppermost circular plate; and forms the basis of a balustrade, having a ring of metal above and another below, connected by twenty cylindrical rods, or small pillars of brass, each thirteen inches long. This balustrade preserves the azimuthal circle from injury, and supplies the means of clamping the vertical axis with a tangent screw of slow motion, which is turned by the handle seen above the balustrade towards the left · it also holds the reading microscope in its proper place, over the divisions of the graduated horizontal circle, that the position of the telescope, and of the vertical circle, may at any time be indicated by it, when once adjusted so, that zero will show the meridional position. The microscope is seen in the place of a connecting pillar, towards the right hand side of the balustrade, and carries an inclined circle of silver at the object-end, to throw light on the divided face of the circle. It is furnished with a micrometer not essentially differing from that of the reading microscope, which we have described, in our forty eighth section; except that the lines are fine wires; they are capable of the same adjustments that we have there explained.

6. The vertical circle has its divisions indicated by two reading microscopes placed diametrically opposite each other, to correct for any excentricity that may exist in any of the positions of this circle, as it regards the telescope's elevation; the construction and adjustments of these microscopes are similar to those of the microscope that reads the azimuthal angles, but, though

we know that the vertical circle is graduated into four successive quadrants, it is not quite clear from the original account, whether altitudes or zenith distances are read, or both in the reversed positions, though it is probable that zenith distances were generally indicated, since this is the denomination in which the observations are registered, as exhibited in Cacciatore's late valuable publication The situation of the superior microscope is a little under the upper stage, where a frame of parallel bars connects the upper ends of the two front pillars, as seen in the plate, and affords the means of making the proper adjustments for distinct vision, for the value of the screw, and for bisecting the circle, as the position regards the lower microscope. The situation of the latter microscope is regulated by a similar frame, screwed to the lower parts of the same pillars, above the inferior stage, and has the same adjustments and value of its micrometrical screw, as the other microscopes, each of which will read separately to the accuracy of a single second of a great circle.

7. When the zeroes of the two microscopes are adjusted to the zero points of the circle, separating the semicircles by an imaginary vertical line, the circle may be turned half round, or the telescope inverted, and if the same zeroes again coincide, then the microscopes are properly opposed to each other, and also the circle is properly divided, in the direction of that diameter, into two equal semicircles. When there is a difference indicated between the two semicircles, after the inversion of any given position has taken place, one half of the difference, shown by one of the microscopes in the second position, will be the error belonging to each semicircle, which error will have the sign + in one semicircle, and − in the other, but one fourth of the said difference will be the error belonging, with its proper sign, to each quadrant; on which account the whole error thus observed is called by Troughton the *quadruple* error, and if the maximum of this error is small, when the circle has been examined by opposite microscopes in all diametrical directions, the circle may be said to have but little excentricity, and also equal divisions, which is the most desirable property a graduated circle can have.

8. Besides the two reading microscopes just described, which are employed solely for reading the subdivisions of the circle, a pair of smaller or secondary ones are placed on the same frames respectively, for viewing a plumb-line, suspended from a small adjustable cock, placed above the upper frame, which plumb-line descends down a square pipe of wood, attached to the right hand pillar, and carries a weight immersed in a water-vessel, standing on a small stage that may be raised or lowered at pleasure, by a vertical screw, for regulating the depth of the immersed weight, and as this small stage is fixed upon the larger one, that revolves with the vertical axis, it is evident that this plumb line may be used in any azimuthal position that the telescope can take, and therefore that the axis may be adjusted by it into a position that will be perfectly vertical in all azimuthal directions, and, what is very important, will watch this adjustment at all times, by preserving its own vertical position, and exhibiting any deviation that may take place in the cock of suspension carried by the vertical axis, and consequently in the perpendicular direction of the axis itself. The *Ghost* apparatus, composing the microscopes for viewing the plumb-line, has been described already [§ XLIX. 2.], to which description our readers may turn for information, with respect to its construction, adjustments, and mode of application When any the least inclination of the vertical axis towards a given point in the horizon has been detected by the plumb-line, it must be re adjusted by the screws

acting on the circular plates, supporting the lower end of the axis when this inclination is towards a point lying in the middle between the two adjusting screws, they must both be turned an equal quantity in the same direction, but if it be directly towards one of the screws, that screw only will require to be turned in general that particular screw must be most turned in making the adjustment, towards or from which the inclination or reclination is greatest.

9. But there is a second useful purpose to which the plumb line is applied, the horizontality of the vertical circle's axis is thereby insured, as often as any inclination is detected in it. The method of accomplishing this object has already been described, and fully explained in our fifty-ninth section, in which the Moscow Transit Instrument, by the late W. Cary, one of Ramsden's pupils, is described. If we suppose the forked measuring bar there used with the microscope, forming the ghost apparatus, to be applied to the plane of the vertical circle, so as to measure its distance from the plumb-line at two points, successively taken in a vertical diameter, the equality of the measures will show that the circle's plane is parallel to the plumb-line, in the same way that the telescope of the transit-instrument was shown to be parallel to its plumb-line when properly adjusted, and as the circle was formed in the lathe upon the pivots of its own axis, its plane stands by construction at right angles to the line passing through the axis, that joins the centres of the pivots; and therefore when the plane of the circle is adjusted to become vertical, its axis necessarily becomes horizontal. It is affirmed that in this way an error of a single second of inclination in the axis may be detected; but as the transits taken by this circle, are intended to be only approximate, to identify the body observed, and as its principal use is to measure correct zenith distances, a very nice adjustment of the horizontal axis is not so material, as that of the vertical axis, on which the accuracy of the observation depends. Instead of measuring from the plane of the circle itself to the plumb-line, Ramsden however fixed a small bridge to the object end of the telescope, in which was inserted a pin, from which the measurement by the forked rod was taken alternately above and below, after the circle had been turned half round, which mode of measuring, he thought, insured the motion of the telescope to be in a vertical circle in the heavens, without considering the question of its parallelism to the plane of the vertical circle; and this was precisely the plan adopted by Cary. Whenever Piazzi rectified the superior and inferior microscopes of his vertical circle, and of his plumb-line, he was accustomed to use the zero points of his circle, as the points that bisected it most perfectly into two equal semicircles; and, as a reason for such preference, he affirms that these points did not deviate more than a quarter of a second from their true places The clamp of the vertical circle and the tangent-screw of slow motion are made fast to the left-hand short pillar, carrying one of the pairs of rollers, and the handle seen depending near its lower extremity, and parallel to it, communicates the slow motion, by taking hold of the screw's arbor with the hollow squared end of its universal joint.

10. *Adjustments.*—When the telescope has been brought to distinct vision of a terrestrial object, the first adjustment will be that which makes the vertical axis perpendicular in all directions, which may be performed by means of the plumb-line, by halving the error, partly by the screws of the subjacent circular plates, and partly by the adjusting screws of the cock of suspension, or by turning round the excentric disc of mother-of-pearl forming the object of the compound microscope, or ghost apparatus, which, for small quantities, is a more conve-

ment operation. When the vertical axis is adjusted, the microscope, reading the azimuthal circle which is now perfectly horizontal, must be so placed and rectified, that it may view the dividing strokes of the limb, and the wires in the common focal point, distinctly at the same time, and also make just six revolutions in measuring one space the directions for effecting which requisites, are given in the section above referred to. The relative positions of the balustrade, that carries this reading microscope, and of the zero of the azimuthal circle, must also be so situated, that when the telescope is brought truly into the meridian, the zero of the microscope is capable of being made coincident with that of the circle, which may be effected by turning the ring carrying the balustrade a little round the pivot of the vertical axis The collimation in azimuth must be rectified by turning the vertical axis half round till the microscope reads the opposite zero, and by observing a distant mark in both positions, before and after the telescope has been reversed, when the error will appear, which must be done away by continual halving, partly by the proper screws in the eye-piece that move the vertical wires, and partly by turning the axis a little, and altering the reading on the scale of the microscope, till the distant object is bisected in both of the reversed positions. after which the telescope may be finally placed in the meridian, and the zeroes of the microscope and of the circle's limb be made again to coincide. The error of collimation in zenith distance may lastly be adjusted, either by a distant meridian mark, or by the pole star, at its meridian passage; for if the reversed positions of the circle and inversion of the telescope give the same zenith distance, no error exists, but when there is an appreciable difference it may be made to disappear by repeated halving, partly by the contrary screws in the eye-piece, intended for this purpose, and partly by displacing the scale of one of the vertical circle's reading microscopes, till the observation is the same in the inverted positions of the telescope, and reversed positions of the circle; after which the second microscope must be put to correspond to the position, exactly opposed to that of the first; and the instrument will be fit for use, provided that, in making these adjustments, the plumb-line still shows that the axis of the vertical circle is horizontal, for adjusting which it has the usual vertical screw at the *Y* bearing the front pivot.

11. With respect to the mode of making the observations, suppose of zenith distances; if the instrument had no error in collimation, the two readings of the microscopes would at once give a mean, when the star is seen crossing the middle of the field of view upon the horizontal wire; but where there are so many pillars and arches of metal exposed to different strata of air, not precisely of the same temperature, it is found from experience, that the adjustments will not be permanent, and that good results may be obtained, particularly with this instrument, rather from the application of known corrections, than from a dependence on the continuance of perfect adjustments for any considerable time. When the vertical axis is truly perpendicular, and the error of collimation in zenith distance known, this error applied to a mean of the readings will be sufficiently correct, but the change in the temperature of the internal air, that is continually taking place from the opening and shutting of doors and shutters, nothwithstanding every precaution, will always render such single observations, made in one position, doubtful, and therefore corresponding observations of the same body, made in reversed positions, should always be preferred; and when several of these are taken, in which there exists but slight discrepances, a mean of the whole is most to be depended on If the reversed observations, with the face of the vertical circle alternately placed to the east and

west, be made on the same evening, the circumstances affecting the adjustments are most likely to be similar, and the error in collimation will disappear, by having contrary signs in the different positions, though in this case the body observed cannot be on the meridian at both instants of making the observations; but the tables of reduction to the meridian, given in our first volume, will, with a little trouble, remedy this inconvenience. Yet the circle's vertical axis must have its position correct at both instants, or there will exist an error arising from the inclination towards either the north or south, that will affect both observations alike, for which there is no correction, but what must be estimated from the situation of the plumb-line, that has no scale for indication. If the star is observed on the meridian on two successive nights, or after an interval of some days, a slight change will have taken place in the zenith distance itself from precession, aberration, and nutation, and also the instrument may not be in the same state of adjustment, as to collimation, and perpendicularity of the axis, that it was at the first period: hence difficulties present themselves in either case which minute attention and delicate management alone can overcome. The instrument could not have been in better hands than those of Piazzi, and of his highly gifted assistant Cacciatore, who, fortunately for astronomy, succeeds him in the Observatory at Palermo.

12. The late proprietor of the instrument we have here described, has enumerated eight advantages which it possesses over its quadrantal predecessors, which are as follow;—first, the graduated circles are not encumbered with verniers, so as to have their divisions defaced, or steadiness molested: secondly, the subdivisions are read by microscopes that magnify nine times, so that the least quantity may be appreciated: thirdly, the vertical circle has its plane made by revolving on its own axis, and also its circular lines struck therefrom, consequently a deviation of the plane, and an eccentricity of the divided circles are both avoided: fourthly, the compound circle preserves its figure much better than it would have done, if it had been cast in one solid piece: fifthly, the observations may be reversed with respect to both zenith distances and azimuths; therefore a mean of two reversed observations of a zenith distance will correct the errors of the limb arising from eccentricity, and also the error of collimation in zenith distance, which will be plus in one position, and minus in the other: sixthly, the instrument may be clamped to the balustrade, and used as a transit instrument: seventhly, it gives zenith distances and azimuths at the same time, and therefore is particularly useful in single observations of a comet, or other temporary phenomenon: lastly, the refraction of the atmosphere, corresponding to a given temperature, may be experimentally determined by comparing an observed zenith distance of a known star, with its computed zenith distance, in a known latitude, when the azimuth has also been observed. Indeed it was in this way that this zealous and persevering astronomer determined the mean refractions contained in his table at page 16 of our first volume.

13. Among the various astronomical works published by Piazzi, may be mentioned one deserving particular attention, printed in folio at Palermo in 1806, entitled *Del Reale Osservatorio di Palermo*, in six books, containing a variety of useful practical information, particularly respecting his comparisons of the stars *Altair* and *Procyon* with the sun, at five successive equinoxes, in the years 1803, 1804, and 1805. We may congratulate the cultivators of astronomy on the continuation of this useful work by Niccolo Cacciatore, the present superintendant of the observatory at Palermo, whose first volume, having the same title, was published in 1826,

and contains in three books, numbered VII. VIII. and IX., various lists of reduced observations of the planets, sun, stars, and comets made by the circle now under our notice, as well as by the transit-instrument. The author has not given the observations precisely as they were journalized, with notices of the errors of the clock and of the instrument, as is done in most other observatories, but has himself applied the corrections, and given the observations reduced to some epoch, and has moreover, given such explanations as leave the reader little more to wish for. The volume may be considered as a ledger, in which separate current accounts are opened, and continued from year to year, with each separate planet, and with the other bodies, that have been successively observed We cannot convey to our readers a better idea of this mode of publishing the results of observations, cleared of all errors, and arranged under distinct heads, instead of giving the observations themselves, than by copying a few specimens from Cacciatore's first volume, after premising that the capitals in the seventh column of the first specimen, viz. I. S. and D. S. denote the two positions of the circle.

14 PLACES OF THE PLANETS OBSERVED BY THE CIRCLE

1815	English Barom	Fahrenheit's Thermom			Names of the Planets and Stars	Posit	Passage by Cumming's Clock	Zenith Distance observed	Remarks
		Attach	Intern	Extern,					
Aug 1	20 940	78.0		67 7	P 330 XX......	D S	20h 43m 30s 3	62° 33′ 31″ 0	
					Vesta		20 48 23 0	61 40 39 0	
2	20 966	77.5		66.7	P 339 XX .	I S	20 43 31 2	62 32 18 0	
					Vesta		20 47 31 8	61 52 43 5	
3	20.910	77 3		67.0	P 330 XX .. .	D. S	20 43 33 2	62 33 25 5	
					Vesta		20 46 31 0	62 0 53 0	
5	20.884	77 6		68.8	Vesta	I S	20 44 35 0	62 13 35 0	
					Saturn lower limb		20 46 48	56 58 19 0	
					Saturn upper limb			57 42 0	
7	20 886	70 3		75.0	P 339 XX	D S.	20 43 33 3	62 33 32 0	
					Vesta... . .		20 42 43 0	62 28 5 0	
					Saturn lower limb			57 1 56 0	
					Saturn upper limb			1 12 0	
8	29.714	77 0		63.7	P 339 XX.. .	I S	20 43 33 3	62 32 19 0	
					Vesta... ...		20 41 46.3	62 33 17 5	
					Saturn lower limb.		20 45 55 4	57 2 1 0	
					Saturn upper limb			1 22.5	

In the preceding specimen of the selected observations, the places of the two planets, Vesta and Saturn, are compared with a star of the sixth magnitude, which, in Piazzi's Catalogue of 1814, is No. 339 under the hour XX, where its place is given for the beginning of the nineteenth century, viz Æ = 310° 18′ 58″ with an annual precession in arc = 53″.02, and dec. = 21° 34′ 17″.5 *S.* with the annual precession = − 12″.98. If we compare the zenith distance of the star given by the instrument on August 1 with that of August 2, we shall find the correction in collimation was ± 36″.5; from a comparison of the second day's observation with that of the third, it will be ± 33″.75, and from the observations of August 7 and 8, it will again be found to be ± 36″.5, then if this quantity were applied with a positive sign in the position I. S. and with a negative one in the position D S. the zenith distance of the star might have been obtained with equal accuracy on any of the two first and two last evenings of observation, without reversing the position of the axis, though not with the same degree of confidence, since the intermediate observations indicate a change in the error of collimation, or of the position of the axis, or otherwise in the observer's use of the telescope.

15 After we have been presented with several series of comparative observations of all the planets with their neighbouring stars, we then find the reduced places of each of them classed in succession under the individual planet, out of which we may examine the places resulting from the particular observations above quoted, which stand thus; viz.

Vesta.

1815.	Mean time of passage.	Right Ascension observed.	Declination observed	Longitude observed	Latitude observed
Aug. 1	12^{h} 8^{m} 20^{s} 27	20^{h} 47^{m} 3^{s} 88	23° 41′ 0″.4 S.	10S. 7° 47′ 46″.0	5° 32′ 15″.9 S.
3	11 58 33 .84	20 45 8 .98	23 55 19 .1	10 7 18 28 .9	5 39 7 .6
5	11 48 47 .79	20 43 14 .43	24 9 9 .2	10 6 49 28 .6	5 45 34 .1
&c.	&c.	&c.	&c.	&c.	&c.
Saturn.					
7	11 42 33 .76	20 44 52 .49	18 55 37 .1	10 8 33 48 .0	0 48 56 .1
8	11 38 19 .11	20 44 33 .70	18 56 59 .1	10 8 29 4 .5	0 49 14 .9
&c.	&c.	&c.	&c.	&c.	&c.

This method of giving the places, resulting from the observations, affords much employment for assistants, who may be occupied in making the computations, and the convenience thus afforded of readily comparing the observed with the computed places, as given in the different ephemerides, is calculated to detect errors in the planetary tables, and may lead to a reform of the elements, particularly of the small bodies recently discovered. The author has generally given the formulæ and co-efficients from which the reductions were derived; for instance, in computing the longitudes and latitudes from the right ascensions and declinations, he has taken the mean obliquity of the ecliptic for 1800 = 23° 27′ 56″, with an annual diminution of 0″.42, and the lunar equation of the obliquity = − 9″.63 cos long. ☾'s ☊. He has not observed the planets Venus and Ceres.

16. After having arranged the observed places of eight out of the ten planets, the author favours us with a list of the apparent oppositions of six of them, as observed in different years from 1794 to 1821 inclusive, when opportunity occurred: and as these observations and places

computed from them are not numerous, we will give them entire, as another specimen of this astronomer's felicitous disposition of the materials furnished by the circle, of which we have given an account in this section.

APPARENT OPPOSITIONS OF THE PLANETS, OBSERVED AT PALERMO.

Year	Month.	Mean times				Observed Longitude				Observed Latitude.			
MARS													
1794	April	23^{d}	18^{h}	35^{m}	$49^{s}.2$	7^{s}	4°	13′	44″.2	1°	12′	30″.3	N
1796	June	14	15	5	40.5	8	24	34	47.5	3	38	18.3	S
1798	August	31	12	51	35.3	11	8	42	44.4	6	22	48.7	S
1800	November	8	13	41	34.5	1	17	30	32.3	0	19	15.4	S
1807	March	4	0	50	57.0	5	13	2	31.5	4	9	15.7	N
PALLAS													
1803	June	30	0	44	18.5	9	7	39	38.0	46	27	5.0	N.
VESTA.													
1815	July	31	18	12	22.3	10	7	31	47.5	5	30	30.6	N
JUPITER.													
1794	June	19	13	41	50.4	8	28	44	48.0	0	16	15.5	N.
1796	August	30	15	51	16.2	11	8	0	14.0	1	25	18.0	S
1803	March	22	1	47	23.0	6	0	58	49.5	1	36	10.0	N
1806	June	25	6	17	15.3	9	3	22	29.0	0	8	15.5	N.
1807	July	31	2	7	52.4	10	7	13	46.5	0	47	29.0	S
1808	September	5	9	55	22.2	11	13	2	26.4	1	26	25.2	S
1816	April	25	4	23	24.0	7	5	15	38.0	1	27	2.3	N
1818	June	30	22	14	51.5	9	7	58	10.5	0	0	55.0	N
SATURN.													
1808	May	9	15	10	50.3	7	18	55	55.0	2	28	41.5	N
1809	May	21	21	45	28.3	8	0	32	13.0	2	10	36.0	N
1811	June	14	23	48	42.5	8	23	22	21.3	1	20	2.0	N
1812	June	26	21	32	56.0	9	4	38	7.3	0	49	50.0	N
1815	August	1	18	10	42.0	10	8	59	8.2	0	48	28.0	N
1816	August	13	5	7	4.0	10	20	42	45.0	1	20	5.0	S
1821	October	15	17	51	59.5	0	23	19	6.0	2	46	2.0	S
URANUS													
1794	February	14	8	36	18.0	4	26	25	49.4	0	47	1.0	N.
1796	February	24	7	37	28.5	5	5	57	35.7	0	48	52.6	N
1797	February	28	7	36	55.1	5	10	44	8.0	0	49	1.8	N
1811	May	7	22	36	47.3	7	16	51	10.5	0	21	39.5	N
1814	May	21	18	46	4.5	8	0	26	49.3	0	10	46.0	N.

17. When this circle is partially exposed to the solar rays a deviation from the true perpendicular position, of 4″ or 5″, usually takes place, and a difference in the two semicircles of 10″ or 12″ is also frequently produced, though the greatest error in simple graduations does not exceed 3″ in the vertical circle. In the azimuthal circle the error in each of two quadrants is + 6″, and in the other two − 6″. But notwithstanding this influence of the sun's heat on the expansion of the vertical circle, and perpendicularity of the vertical axis, so many solstitial observations of the sun were made in the years 1817, &c. to 1825 inclusive, as give the obliquity of the ecliptic in the most satisfactory manner, viz. for the year 1809 = 23° 27′ 51″.64 with an annual variation of − 0″.45 very nearly. This determination accords very well with a mean of all the determinations made at the principal observatories, but yet, as has been the case at many of them, there is here a difference of upwards of 5″ between the determination arising from the solstitial observations of summer and those of winter, the obliquity appearing to be at Palermo always the greatest in summer, as at Greenwich, though Bessel's observations show no such constant excess in summer, but sometimes the contrary. [Vol. I. p. 436, Tab. 4.]

18. A second azimuth and altitude circular instrument of much larger dimensions, was nearly constructed by Ramsden, and afterwards finished and divided by Berge, his successor, for the observatory of Trinity College, Dublin The diameter of the vertical circle, of this equally celebrated instrument, was ordered to be ten feet, and the materials were actually formed and put roughly together to suit such dimension; but the structure was considered too bulky and heavy, and was therefore reduced to nine feet, in which state, we understand, it was actually divided, but, on further consideration, it was reduced a second time, to eight feet, in which state it was not completely finished when Ramsden died but fortunately his assistant and successor was competent to do ample justice to the completion of what work remained imperfect. This instrument, in the hands of one of the first theoretic and practical astronomers of the age, has put itself in competition with the more recent circles erected at Greenwich by Troughton and T. Jones, and up to the present moment disputes with them the claim to accuracy, with respect to that *single second* which constitutes the apparently existing difference between them, as it regards the subject of parallax of certain stars in north polar distance, *sub judice lis est.* Dr. Robinson, professor of astronomy at Armagh, affirms, notwithstanding the size and weight of the Dublin instrument, that* "there does not seem to exist any excentricity in it " and on this account the relative positions of the three reading microscopes, when the fourth is neglected, are not so objectionable as they would have been, if excentricity had existed in a considerable quantity. Four equidistant microscopes were supplied to the circle, and were at first all read after each observation, but as the fourth was placed too far from the ground to be read with convenience, and as the three remaining ones gave a mean equally correct, the trouble of reading the fourth was dispensed with. This disposition of the three microscopes, two of which bisect the circle horizontally, while the third bisects only the lower semicircle, has been objected to theoretically, but when it is considered that the readings of the first and third microscopes will always show whether or not they continue to bisect the circle; and that the two semicircles change places at each reversion of the instrument's position in azimuth, the second microscope may be considered as correcting for the effects of temperature on

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* Paper "On Correcting Errors of the Astronomical Circle."

the different semicircles, when a star is observed in both positions in immediate succession, at or before, and after the meridian passage this appears to have been Dr Brinkley's practice; though his observations themselves have not been published, that furnished the data for his many elaborate computations and valuable deductions, respectively contained in the transactions of the Royal Society of London, and of the Royal Irish Academy. The desirable property, that this construction of a circular instrument possesses, of obviating the effect of its own errors, including that of collimation in zenith distance, by a reversion of its position, has been duly appreciated and insisted on by the learned and dignified professor, who has experienced not only the utility but the convenience of such property. Each double observation is thus independent and conclusive.

19. The Dublin instrument has the advantage of pillars of masonry, instead of metallic pillars, for supporting the superior end of the vertical axis; which improvement must prevent the frequent derangement of the perpendicular position of its vertical axis, as well as insure steadiness in the azimuthal motion, in the act of reversing the instrument, which is essential to the accuracy of the observations The inferior end of this axis, rests on a block of stone so placed as not to touch the pillars or floor itself, while the upper pivot is adjusted by a system of screws carried by the frame placed on the stone pillars, and as the whole building stands on an extensive solid rock of limestone, no changes of temperature affect the stability of the building. The plan of this observatory was designed by Dr. Ussher, the first practical astronomer at Trinity College, Dublin, to whose superintendence the arrangement and execution of such parts were committed, as demanded particular nicety and attention; and to whose choice the ordering of the first instruments was confided. His "Account of the Observatory belonging to Trinity College, Dublin", was read before the Royal Irish Academy, on June 13, 1785, and forms the first article of the Transactions of that Society The reader who has access to this account, and to the papers published in the second volume of the same scientific work, by this author, will be satisfied that the wants of a new observatory could not have been better supplied. For not only did he employ the first artist of the day to construct all the instruments, but, what perhaps can be said of no other observatory, the solid pillars of Portland stone were selected from the quarry as they lay, side by side, so that their corresponding parts, at the same height from the ground, might be homogeneous, and affected with similar expansions and contractions by changes of temperature. The room which contains the circular and transit-instruments, together with the observing clock, is spacious, and the openings are six feet wide, besides which are semicircular apertures for the admission of external air, so that the temperature of the room can be brought to assimilate with that of the external air, before the observations commence.

20 As we have given some of the tabulated results derived from the use of the Palermo instrument, we will conclude this section by giving some of the equally interesting and important conclusions drawn from the observations made by the Dublin instrument, which has still greater claims to confidence. The well known contest on the subject of annual parallax, that has existed for the last few years between Mr. Pond and Dr Brinkley, has called forth extraordinary exertion of talent and assiduity, both in making and reducing the respective observations; and it still remains a matter undetermined whether the trifling difference, arising from a comparison of the respective conclusions, is occasioned by the instruments themselves,

or by the tabular corrections which have been applied as reductions. None but instruments of the highest order can compete, in determining the limit of accuracy that is attainable in observations of such extraordinary delicacy, that all the sources of error may be reduced within the compass of a *single second*. In Dr. Brinkley's third communication to the Royal Society of London, published in 1821, the following process is adopted in reducing the observations, which will be best described in his own words.—" The observed zenith distances of a given star were reduced to Jan. 1, 1819, by the common equations, taking the constant of aberration = 20″.25. The mean of these was taken. The correct mean zenith distance was supposed equal to this mean — *e*, the constant of aberration = 20″.25 + *x*, and the semi-parallax = *p*. The equations of condition, resulting from the respective observations, thus contained three unknown quantities. These equations were reduced to three by the method of making the sum of the squares of the errors a minimum*. The solutions of these three equations give the values of *e*, *x*, and *p*, and thence the values of the mean polar distance, constant of aberration, and semi-parallax " The respective quantities accruing from this method, when applied to the observations of thirteen stars, are contained in the subjoined Table, which explains its own columns.

21. A TABLE OF THE CONSTANTS OF ABERRATION AND SEMI-PARALLAXES.

Stars	No of days of observation	No of observations 1818–1821	N P D. Jan 1, 1819 Co-lat. 36° 36′ 46″ 5	Const. of Aber	Semi-paral or *p*
Polaris . . .	77	343	1° 39′ 24″.95	20″.18	— 0″.03
Polaris S. P. .	80	337	1 39 25 .16	20 .12	+ 0 .12
β Ursæ Majoris .	75	75	32 38 59 .61	20 .16	+ 0 .02
γ Ursæ Majoris .	105	105	35 17 55 .15	20 .48	+ 0 .39
ε	109	109	33 8 19 .54	20 .29	+ 0 .33
ζ	94	94	34 7 34 65	20 .23	+ 0 .28
η Ursæ Majoris .	99	99	39 46 47 .18	20 .76	+ 0 .13
Arcturus . .	94	259	69 52 13 .66	20 .04	+ 0 .61
β Ursæ Minoris .	53	131	15 6 17 .74	20 .49	— 0 .13
α Ophiuchi . .	97	228	77 17 58 .23	20 .39	+ 1 .57
γ Draconis . .	152	152	38 29 7 .51	19 .86	— 0 .08
α Lyræ . . .	157	227	51 22 42 .84	20 .36	+ 1 .21
α Aquilæ . . .	135	320	81 36 5 11	21 .32	+ 1 .57
α Cygni . . .	94	154	45 21 42 .30	20 .52	+ 0 .33

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* Vide " Theoria Motûs Corporum Cœlestium ", by GAUSS, " Theorie des Probabilités ", by LA PLACE, the Appendix to " The Elements of the Theory of Plane Astronomy ", by W MADDY, M A. Cambridge, 1826, and " Connaissance des Tems, pour l'an 1827 ", p 273, et seq. by M POISSON

In the preceding list the stars near the zenith were observed on the meridian, the stars on the south side of the zenith were observed once in each position of the circle on each day; those which are 30° or more from the zenith, were twice before, and twice after, the reversion of the instrument, but the others were observed several times before and after their meridian passages. Whenever the semi-parallax has a negative sign, the error may be considered as arising from the observation. The whole number of observations is 2633, and the mean of all the constants of aberration = 20″.37. One remarkable presumption that arises from an examination of these different constants of aberration is, that the apparently largest stars have not their constants the greatest, and consequently cannot be placed the nearest to the earth, and this presumption is confirmed by the circumstance that the brightest stars have not generally the greatest proper motions. Another inference, tending to lessen the difficulties of the question at issue between the two observatories, is "that in a certain part of the heavens of considerable extent, many of the stars exhibit a sensible parallax "

22 The method of arranging and reducing the observed zenith distances at Dublin Observatory, when extracted from the journal, will appear without explanation from the following specimen.

β Aquilæ.

	Face of Circle	Mean of three Microscopes	Refraction.	Mean Z D Jan. 1, 1819	x	p.		Face of Circle.	Mean of three Microscopes.	Refraction	Mean Z D Jan. 1, 1819	x	p.
1818							1818.						
July 17	E	47° 25′ 20″ 83	1′ 1″ 14	47° 25′ 26″.04	+ .15	+ .46	Oct 16	W	47° 23′ 37″.93	1′ 1″.94	47° 25′ 26″ 21	+ 46	− .14
24	W	23 37 53	1 0 48	25 31	21	.44	17	E	25 13 17	1 2 22	25 29	46	.15
25	E	25 24 77	1 1 33	23 09	.21	43	21	W	23 30 73	1 2 25	25 21	45	.18
Aug 6	E	25 22 13	1 1 95	22 83	29	39	Nov 1	E	25 16 73	1 2 17	28 20	.41	26
9	W	23 32 53	1 2 33	24 48	.31	37	2	W	23 38 53	1 1 .48	25 70	40	.26
10	E	25 23 90	1 2 23	25 41	32	36	3	E	25 14 57	1 1 35	25 11	40	.27
11	W	23 32 73	1 2 43	25 04	33	36	7	E	25 15 17	1 2 43	26 50	38	30
12	E	25 22 40	1 2 30	21 24	33	35	8	W	23 30 97	1 2 78	28 02	38	31
13	W	23 32 07	1 2 36	25 47	34	35	14	E	25 16 63	1 1 36	26 35	34	35
14	E	25 24 .43	1 2 29	26 49	35	34	20	W	23 48 57	1 1 95	27 88	29	39
16	E	25 24 63	1 2 22	26 84	36	33	23	E	25 11 .53	1 1 30	27 30	27	40
							24	W	23 47 .40	1 2 36	26 71	27	41

From these series of observations it appears that, agreeably to theory, as the quantity x, expressing the error of aberration in N P. D. increases, the quantity p, expressing the semi-parallax, decreases, and vice versâ, one becoming a maximum when the other is a minimum, and the contrary. In reducing the observed to the mean north polar distances the precessions, corrected for proper motion, were used as given in the Nautical Almanac.

23. The correction used for lunar nutation was $-\ 8''.28 \sin(\text{Æ} - ☊) - 1''.22 \sin(\text{Æ} + ☊)$ but by a comparison of the observations made from 1809 to 1814 with those made in 1818—1820, the author finds that this correction ought to be $-\ 8''06 \sin(\text{Æ} - ☊) - 1''.19 \sin(\text{Æ} + ☊)$; but the adoption of the latter will make no sensible difference in the determination of

the constant of aberration and of parallax. The solar nutation used was $-0''.48 \sin(\text{Æ} - 2\odot)$, disregarding the smaller term; but with the improved lunar nutation, the solar nutation will become $-0''.52 \sin(\text{Æ} - 2\odot) - 0''.02 \sin(\text{Æ} + 2\odot)$; not materially different from what was used. In strictness, the stars used for determining the greatest co-efficient of lunar nutation of the obliquity of the ecliptic, should have their zenith distances observed through the whole period of the revolution of the nodes, in order to obtain the annual variation of zenith distance for each star; but as that had not been done, such stars were selected, for the two periods above specified, as had their nutation respectively a maximum, and with contrary signs; so that no uncertainty might arise from the omission of the other portions of the period. According to the lunar nutation used in the correction above stated, the greatest term will be $9''.50 \cos \text{☊}$, and supposing the true co-efficient to be $= 9''.50\,(1 + y)$, the illustrious author found from his two series of observations the equations, and mean of all the co-efficients of nutation, agreeably to the contents of the following Table.

24. A TABLE OF THE GREATEST CO-EFFICIENT OF LUNAR NUTATION OF THE ECLIPTIC'S OBLIQUITY.

	No of Observ 1808–1814	No of Observ 1818–1820	Equations deduced	Greatest Co-efficient of Nutation of Obl. Ecl
Capella . .	30	96	$54''.20 + 8.49\,y = 53''.50 - 7.79\,y$	9''.09
β Tauri .	18	84	$21.73 + 8.66\,y = 21.65 - 7.63\,y$	9.45
α Orionis .	18	148	$9.24 + 8.81\,y = 7.98 - 7.09\,y$	8.75
Castor . .	10	66	$30.23 + 8.92\,y = 30.42 - 5.18\,y$	9.62
Procyon. .	16	136	$8.41 + 8.91\,y = 7.30 - 4.88\,y$	8.74
Pollux .	10	65	$44.98 + 8.79\,y = 44.29 - 4.81\,y$	9.01
γ Draconis .	27	132	$7.54 - 8.65\,y = 7.90 + 5.80\,y$	9.26
α Lyræ. .	126	155	$42.69 - 9.14\,y = 42.94 + 7.89\,y$	9.36
α Aquilæ .	76	238	$4.94 - 8.74\,y = 5.10 + 7.42\,y$	9.40
α Cygni .	47	120	$42.15 - 7.48\,y = 42.77 + 4.97\,y$	9.03
	378	1240		9.25

Since the number of observations taken of some of the stars at the first period was comparatively small, a mean was obtained by giving each result a weight proportional to the number of observations of each star at that period, which produces a mean result of $9''.25$. Taking this determination as correct, and omitting the small terms depending on 2 long ☊, we shall have

The nutation in N. P. D. $= -8''.06 \sin(\text{Æ} - \text{☊}) - 1''.19 \sin(\text{Æ} + \text{☊})$
The nutation in Æ $= (-8''.06 \cos(\text{Æ} - \text{☊}) - 1''.19 \cos(\text{Æ} + \text{☊}) \cot \text{N. P. D.}$
Equation of equinoxes in Æ $= -15''.86 \sin \text{☊}$

Equation of equinoxes in long = $-17''.29 \sin \Omega$
Equation of the obliquity of ecliptic = $9''.25 \cos \Omega$
The mass of the moon = $\frac{1}{80}$, that of the earth being unity
Force of the moon on the sea = $2\frac{1}{5}$, that of the sun being unity.

These are most important results, as they regard the future reductions of astronomical observations, and such as none but an astronomer of the first eminence, possessing superior means, could have accomplished in so satisfactory a manner.

25. With respect to the performance of the circle's indications in the opposite extremes of temperature, the author has given a small Table containing the observations made at various times on nine principal stars, from which a comparison is made of the north polar distances deduced in the opposite seasons, and it appears that the mean of the differences, of 343 summer observations, and of 414 winter observations, is only $\frac{8}{100}$ths of a second. The observations of the pole-star also, as taken in the four seasons of the year, accord so remarkably, in giving the co-latitude of Dublin Observatory, that a transcript of the deductions given in a small Table, will satisfy the most scrupulous inquiry.

OBSERVATIONS OF THE POLE-STAR.

	No of Observations.	Zen Dist of Pole-Star	Co-Latitude of Observatory
Autumn.	72 76 S. P.	34° 57′ 21″.24 38 16 11.84	36° 36′ 46″.53
Winter.	72 64 S. P	34 57 21.51 38 16 11.89	36 36 46.70
Spring.	64 71 S. P.	34 57 21.26 38 16 11.71	36 36 46.49
Summer	72 60 S. P.	34 57 21.87 38 16 12.13	36 36 47.00

26. We have only to observe further on the subject of the Dublin observations, what is important to be tried at other observatories, that the true correction of the refraction seems to be that which depends entirely on the *internal* thermometer; the results depending on it being more accordant, than when the external thermometer is applied either singly, or jointly with the internal one. and lastly, with respect to the errors of observation, of α Lyræ for instance, out of 157, in two only the error exceeded 3″, in ten it exceeded 2″, in fifty-one it was more than 1″, and in 105 it was less than a single second, the observations in any one day being considered as a single observation.

27. The smaller instruments made on a similar plan, by the late W. Cary, but so as to be portable, may be considered as of the same school, and one of the larger ones, used by Bessel, has been held in high estimation.

§ LXVII. THE WESTBURY ALTITUDE AND AZIMUTH CIRCLE BY TROUGHTON [Plate XXIV.]

1. After Mr Troughton had constructed several small altitude and azimuth circles, he made one with a vertical circle of two feet in diameter for Count Bruhl, so long ago as in the year 1792, that probably exceeded in accuracy every instrument that preceded it, but as the observations made with it were few, we propose to describe in this section an instrument by the same maker, which, in the hands of Mr. Pond, detected the errors of the Greenwich quadrant, about the beginning of the present century; and brought him into notice as a skilful and accurate observer. Fig. 2 of Plate XXIV gives a perspective view of this instrument, such as exhibits the principal parts so clearly to view, that the description may be understood without letters of reference. As it was originally intended to form an equatorial instrument, the circles were made light, and the supporting parts proportionally slender; but it was at length determined to finish it so, that it might be used only in a vertical position. Troughton seems always to have entertained an objection to a vertical axis supported, at its upper end, by metallic bearing pieces, that must be occasionally exposed to the solar rays, or that may be subject to variable expansions in the different states of the atmosphere: he therefore contrived to conceal the vertical axis, on which the azimuthal circle revolves, in a stone pedestal, into which it descends to a considerable depth.

2. This axis, which is not seen in the figure, is firmly attached by screws to the cross horizontal bar, or thick plate, which bears the long vertical pillars of brass, that support the vertical circle, and is also made fast to the centre of the azimuthal or horizontal circle, the stone pedestal is hollow, and holds a brass conical socket firmly fastened to it, that reaches down almost to the ground. This socket receives the vertical axis, and consequently bears the weight of all the moveable part of the instrument, which turns on an obtuse point at the lower extremity, while the upper end of the axis is kept in its place, by a right angled hole, having two springs opposite the points of contact, which press it against its bearings, while it turns in contact with only four points with a steady, easy, and pleasant motion. The bar in which the vertical axis is thus centered, is acted on by two adjusting screws, that are independent of each other, and stand at right angles the one to the other, by means of which the axis is adjusted to its true vertical direction, while the blunt point continues in its subjacent cup. The frame, to which this apparatus is attached, is composed of a central portion and six strong conical tubes of brass, screwed to as many bearing pieces, standing on and made fast to the six corners of the hexagonal cap stone of the pedestal, as exhibited in the engraving.

3. The azimuthal circle is composed of smaller conical radial tubes and a circular limb, of two feet in diameter, divided into spaces of 5' of a degree all round, and, being made fast to the vertical axis, turns round between the radiated frame, just described, and the upper end of the pedestal; its divisions are read by two opposite micrometrical microscopes, of the construction we have described [§ XLVIII.], and the slow motion is regulated by a screw with two milled heads, seen near the right-hand microscope in the figure, which acts with the exterior end of a compound bar, that, terminating with a large ring cut through at the remote

side, clamps the top of the axis above the frame, by means of a strong screw entering the ears of the semi-rings, and closing them or opening them, accordingly as the motion is required to be slow or quick; that is, accordingly as the clamp is required to be attached or detached. The contiguous compound frame, carrying the two reading microscopes at its opposite ends, surrounds the central part of the main frame of radial cones, that carries the adjusting screws, and is fixed over the opposite zeroes of the horizontal circle, when the instrument is placed truly in the meridian

4 The vertical circle is thirty inches in diameter, and is divided into spaces of 5′ of a degree, figured into two semi-circles. its horizontal axis is supported by the two strong vertical pillars, that turn with the concealed vertical axis, and is composed of three strong tubes, the middle one cylindrical and the two end ones conical, admitting of the transmission of light, like the axis of a transit-instrument, but is only about twelve inches long, its steel pivots rest in adjustable Ys, similar to those of a transit instrument. The circle is composed of two limbs having their separate conical radial tubes connecting them with the cylindrical portion of the axis, and being themselves united by a number of bars interposed between and made fast to both, at right angles to their faces. The divided face is read by a pair of opposite reading microscopes of the micrometrical kind, that at first were fixed in a stationary position, but were afterwards, at the suggestion and desire of Mr. Pond, made to revolve through an arc of 60° from their horizontal position, for the purpose of taking observations from a new and distant zero, which, when there are only two microscopes, is a desirable property, checking the divisions of the circle. A revolving level remains constantly suspended, parallel to the horizontal axis, as is usual with the axis of a transit-instrument, to watch its position in all elevations of the telescope, which is achromatic, having three feet and a half focal distance and two inches and three quarters aperture, with different magnifying powers; but a principal feature in the construction of this instrument is the *manner*, in which a plumb-line is applied to insure the perpendicular position of the vertical axis during any alterations that may take place in the indication of the telescope's level by changes of temperature.*

5. In zenith sectors, and astronomical quadrants, points had usually been made on the faces of the graduated limbs, which were bisected by the plumb-line at two distant places in a vertical line, in reading which there was always some parallax; and besides, the line could not be referred to at the moment of making an observation with such instruments; in that case the plumb-line had reference to the situation of the fixed points, to which it could be applied only occasionally, and therefore was no check on the permanence of the instrument's position at the times of making the observations; but here the plumb-line has reference only to the vertical position of the axis, which it not only insures at first, but watches at all times. A long tube of smaller calibre than the pillars, is attached to one of them in a vertical position, and contains the plumb-line within it, which is suspended from the apparatus placed at its upper end, that has a system of adjusting screws and an angular point of suspension; at its lower end, near the water vessel, are placed across it, at right angles to each other, a pair of microscopes of the ghost kind, with each an adjustable excentric disc, to be bisected by the plumb line

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* On this subject see the papers of Carlini and Bianchi in the "Effemeridi Astronomiche di Milano," 1827, pp 77—97, Appendice.

when in perfect adjustment. The plumb-line will place the vertical axis truly perpendicular, when the discs of the microscopes continue to be bisected during a whole revolution of the azimuthal circle, and when this is not the case, one half of the error must be adjusted by the screws of the suspension apparatus, at the top of the long tube, and the other half by the screw adjustment at the upper end of the axis, by repeating the operation till the bisections are right in every position of the lower circle. When this adjustment is completed, the telescope must be placed horizontal, by means of the level carried by it, which it will be, when the level will reverse in position, and then the microscopes must be put to their respective zeroes, by means of their revolving double bar, and the usual adjustments made for distinct vision, and value of the screw, as already explained. When these adjustments are perfect, the instrument is in a state to be employed in measuring zenith distances in the following manner.

6. Having released the clamp of the vertical axis, turn the axis round till the telescope points due north or south, as near as can be ascertained by ordinary means, and put the microscopes of the horizontal circle to their zeroes; proceed with the adjustments of the telescope for position in the meridian, horizontality of its axis, and collimation in azimuth, in the same manner as if it were simply a transit-instrument; the reversion of the axis being made by turning the circle half round in azimuth; then re-adjust the lower microscopes to their true places on the horizontal circle: in this state of preparation the telescope may be pointed to any particular star, that is passing the meridian, which may be made to pass along the horizontal wire in the field, supposed to be duly placed; this is easily effected by clamping the circle as soon as the star is in the field, and by using the tangent screw for placing it on the wire. When this is done examine the plumb-line, and see that the disc, affecting the altitude in the present position, is properly bisected, if it is, the microscopes must be now read, and their readings noted in the journal; this constitutes one half of an observation only, because at this stage, the error of collimation in zenith distance is not known; on the following night, or as soon as opportunity will permit, turn the horizontal circle half round by its microscopes, and examine the adjustments; then, when the same star passes again in the new meridian position, let it be brought to the horizontal wire again, while the plumb-line cuts the proper disc, and the observation will be complete; the microscopes of the vertical circle must now be read, and the readings noted as before; then if the sum of the two means, with the addition of 90°, make exactly 180°, there is no error in the collimation; and one part of the observation will give the true apparent zenith distance, and the other the altitude, when 90° are subtracted; but if the amount is more or less than 180°, one half of the excess or defect will be the error, and must be applied, as a correction, with a contrary sign, to each portion of the entire observation, to give the exact zenith distance or altitude, as the observation may require; and after this a single observation may be made in one position of the instrument, as long as the adjustments and collimation error remain the same.

7. It will however be safer to consider the error variable; to make every observation in the reversed positions; and to take half the sum of the two zenith distances, as the true zenith distance observed, provided the interval be short, and as observations, made before and after the meridian, can be reduced to the meridian with but little trouble, the interval may be confined to five or ten minutes, during which time the star may be observed more than once in each position, and consequently while the adjustments remain unaltered. Mr. Pond how-

ever took the reversed observations on successive days, as soon as the weather permitted, and, having obtained the zenith distance corrected for refraction, he afterwards reduced it into mean polar distance by the tabular corrections in the usual way. The following example of his method will render all further directions unnecessary.

α Pegasi.

1800		Observations corrected for refraction			Barom.	Therm	Refraction	Zen distances corrected for refraction			Mean Polar Dist for Jan 1800, co-latitude = 38° 45′ 43′ 0.		
Nov.	17	142°	54′	25″.9	29 .7	46	43″ 5	37°	5′	53″.0	75	51	57″.2
	18	37	6	11 .9	30 .1	44	44 .4						
	19	37	6	10 .0	30 .2	40	45 .0	37	5	54 .1	75	51	58 .0
	20	142	54	21 .8	30 .3	40	45 .2						
	26	37	5	49 .8	29 .5	44	43 .5	37	5	54 .8	75	51	58 .7
	27	142	54	00 .1	29 .6	42	43 .9						
Dec.	24	142	53	30 .0	29 .3	47	43 .0	37	5	56 .8	75	51	58 .7
	25	37	5	23 .7	29 .4	47	43 .0						
	26	37	5	25 .0	29 4	43	43 .4	37	5	56 .0	75	51	57 .2
	31	37	5	24 .0	29 .8	38	44 .4						
Jan.	1	37	5	22 .8	29 .7	45	43 .6						
	3	142	53	32 .1	29 .7	47	43 4						
										Mean =	75	51	58 .0

8. The principle of this method of observing may now be thus explained, when the telescope has been pointed to a star in the southern quadrant of the circle, at an elevation of say 40°, it is then turned half round in azimuth, and in this second position will retain its elevation of 40°; then in turning it over from the north to the south aspect again, to catch the same star, the limb of the circle, connected with the telescope, will pass over the zenith distance twice over, viz. 50°+50°, and as much more or less as is equal to twice the error in collimation; for turning the telescope back, from north to south again, inverts the eye-piece, and turns the horizontal wire the lower side upwards, and produces a positive error in one position, and a negative one in the other, supposing the adjustment incomplete, but as one part of the double observation is given in altitude and the other in zenith distance, the sign of the correction must remain the same in both positions; because a *plus* correction in altitude is a *minus* correction in zenith distance, and the contrary; consequently leaving the signs alike, when applied to observations of the two different denominations, is the same thing as changing the signs, when the denominations of the observations are similar. Now as the two halves of the arc passed over, in turning the telescope back, lie at each side of the line pointing to the zenith, it is evident that half the arc passed over must be the zenith distance, when the error is applied; or half the sum of the whole arc will be the zenith distance without reference to the error, which in this case disappears by an opposition of signs. All therefore that the plumb line has to do, is to keep the line pointing to the zenith in its true place, as being the line that bisects the total arc moved over. This beautiful and accurate application of the

plumb-line is peculiar to Troughton's circular instruments; and is attended with only this inconvenience, that if the line should break by accident before an observation, or series of observations, is complete, it cannot be replaced with certainty by a new wire, and consequently the first part of the observations is lost: but this will seldom happen with ordinary care, except in frosty weather, when the water in the vessel becomes congealed.

9 When this instrument came out of the maker's hands, Mr. Pond has affirmed, that, on examining the divisions by the opposite readings, the greatest error of the vertical circle was only 1″.25, and the mean error 0″.7; but in the carriage into the country, some trifling alteration took place; and a constant difference was detected in the collimation error, as determined by γ Draconis and by an object in the horizon, of 3″; and this quantity was very uniformly distributed through the intermediate arc; which circumstance seems to indicate a slight bending of the telescope, that projected at both ends beyond the circumference of the circle nearly six inches. After about a quarter of a century's exposure to the air, and for some portion of the time to the destructive air of London, the circular limbs and conical tubes of this instrument actually perished, so as to require being replaced with new metal, requiring new divisions; which work was recently performed by Mr. Simms, without making any other alteration in the construction, than shortening the telescope about six inches, to allow it to pass the brace, that binds the two pillars together under the circle, and to prevent a tendency to flexure. The instrument thus repaired is in the possession of Dr. Scott of Bedford Square, London.

10. We will conclude this section by giving a copy of Mr. Pond's comparison of the observations made at Greenwich by the quadrant, at Armagh by an equatorial instrument by Troughton, at Palermo by Ramsden's circular instrument, and at Westbury by the instrument above described, with a catalogue deduced from a mean of all these, together with the respective columns of differences from the mean catalogue.

II COMPARATIVE MEAN POLAR DISTANCES OF 29 STARS

January 1800.	Greenwich	Diff	Armagh	Diff	Palermo	Diff	Westbury	Diff	Mean of all
γ Draconis	38° 28′ 53″ 0	−0″ 8	38° 28′ 52″ 8	−1″ 3	38° 28′ 53″ 0	−0″ 8	38° 28′ 53″ 8	+0″ 0	38° 28′ 53″ 8
Capella	44 13 21 5	+1 5	44 13 20 0	0 0	44 13 18 5	−1 5	44 13 18 5	−1 5	44 13 20 0
α Cygni ...	45 25 41 4	+2 6	45 25 38 0	−0 8	45 25 39 6	−0 8	45 25 37 0	−1 8	45 25 38 8
α Lyræ ...	51 23 41 1	+3 4	51 23 35 8	−1 9	51 23 37 8	+0 1	51 23 36 0	−1 7	51 23 37,7
Castor	57 41 14 0	0 0	*57 41 8 0	...	57 41 14 0	0 0	57 41 14 0	0 0	57 41 14 0
Pollux	61 30 9 8	−2 2	*61 30 3 8	..	61 30 12 2	+0 2	61 30 13 7	+1 7	61 30 12 0
β Tauri	61 34 30 9	−1 6	61 34 31 0	−1 5	61 34 34 8	+2 3	61 34 33 7	+1 2	61 34 32 5
α Andromedæ	62 0 45 8	−1 2	62 0 43 7	−3 3	62 0 48 5	+1 5	62 0 50 0	+3 0	62 0 47 0
α Coron Bor	62 36 10 5	+0 3	*62 36 6 0	...	62 36 10 8	+0 6	62 36 13 0	+2 8	62 36 10 2
α Arietis	67 29 20 1	−1 3	67 29 22 0	+0 6	67 29 22 7	+1 3	67 29 20 6	−0 8	67 29 21 4
Arcturus	69 46 7 8	−1 2	69 46 9 7	+0 7	69 46 11 0	+2 0	69 46 7 5	−1 5	69 46 9 0
Aldebaran ...	73 54 16 6	−0 2	73 54 17 0	+0 2	73 54 18 0	+1 2	73 54 15 5	−1 3	73 54 16 8
β Leonis	74 18 34 5	+0 8	74 18 32 7	−1 0	74 18 35 0	+1 3	74 18 32 5	−1 2	74 18 33,7
α Pegasi	75 51 57 0	−2 0	75 51 59 8	+0 8	75 52 1 0	+2 0	75 51 58 0	−1 0	75 51 59 0
γ Pegasi	75 55 36 3	−0 7	75 55 36 2	−0 8	75 55 38 5	+1 5	75 55 37 0	0 0	75 55 37 0
Regulus	77 3 35 1	+1 1	77 3 30 7	−3 3	77 3 37 5	+3 5	77 3 34 0	0 0	77 3 34 0
α Ophiuchi .	77 16 54 0	+0 6	77 16 52 1	−1 3	77 16 54 0	+0 6	77 16 53 5	+0 1	77 16 53 4
α Aquilæ	81 38 52 0	+1 5	81 38 49 3	−1 1	81 38 55 0	+4 5	81 38 51 5	+1 0	81 38 50 5
α Orionis	82 38 30 8	−0 7	82 38 30 5	−1 2	82 38 33 0	+2 3	82 38 31 5	0 0	82 38 31 5
α Serpentis .	82 56 1,2	−0 8	82 55 58 5	−3 5	82 56 5 8	+3 8	82 56 2 2	+0 2	82 56 2 0
Procyon	84 16 17 4	−3 1	84 16 18 0	−2 5	84 16 22 0	+1 0	84 16 21,5	+1 0	84 16 20 6
α Ceti	86 42 6,1	−1,5	86 42 6 0	−2 2	86 42 10 8	+0 6	86 42 10 2	+2 0	86 42 8 2
α Aquarii .. .	91 16 59 8	−3 2	91 17 3 3	+0,3	91 17 3 7	+0 7	91 17 4 6	+1 6	91 17 3 0
α Hydræ	97 47 49 1	−1 5	97 47 47,5	−3 1	97 47 54 0	+3,4	97 47 53,0	+2 4	97 47 50 6
Rigel	98 26 28 8	−5 3	98 26 31,7	−2 4	98 26 35 5	+1 4	98 26 36 5	+2 5	98 26 34,1
Spica Virg .	100 6 37 0	−3 0	100 6 37 5	−2 5	100 6 42 8	+2,8	100 6 43,0	+3 0	100 6 40,0
α Capric	103 9 3 2	−4 9	103 9 12 0	+3,9	103 9 9 2	+1 1	103 9 8 0	−0 1	103 9 8 1
Sirius	106 26 56 3	−5 5	106 27 3,8	+2 0	106 27 5 0	+3 2	106 27 2 0	+0 2	106 27 1 8
Polaris	1 45 34 5	...	1 45 34 5	..	1 45 36 2	...	1 45 36 0	.	1 45 36 0

The observations marked * are omitted in the calculation

§ LXVIII THE SOUTH KILWORTH ALTITUDE AND AZIMUTH CIRCLE, BY TROUGHTON

[PLATE XIX]

1. WHEN Troughton undertook to make an altitude and azimuth circle for the Royal Academy of St. Petersburgh, of larger dimensions than he had previously chosen, he formed the axis of vertical motion in a manner, that admits of easy packing for carriage, as well as of great convenience in giving the requisite position on any single pedestal previously constructed; and at the same time the axis itself is guarded from the influence of the solar rays and variable temperature of the internal air, as well as that of the Westbury circle Instead of

allowing the axis to descend into the cavity of the pillar, he made a strong conical axis of hard metal to ascend in an inverted position, and to remain vertically fixed, while a hollow cone of brass, forming the axis of the azimuthal circle, is made to fit it in the nicest manner, and to revolve round it without the least play or deviation. When the instrument had been nearly constructed it was countermanded, under an apprehension that Saint Petersburgh might eventually undergo the same fate as Moscow had previously done, during the attack of the French army under Buonaparte, but on condition that the deposit money might be returned. This proposal was immediately complied with, and the instrument became the property of an English purchaser, whose time, since he received it, has been principally occupied in composing and printing the present work, so that the use which might otherwise have been made of it has hitherto been necessarily suspended. The circles, both horizontal and vertical, were divided by Thomas Jones, at a time when Troughton was so seriously indisposed, as to despair of ever being able to perform so nice an operation again; and the work was performed in so accurate and delicate a manner, as drew from the maker himself an expression not simply of satisfaction, but of admiration, after he had examined the divisions by the opposite microscopes. *A Report on the properties and powers* of this instrument was read before the Astronomical Society of London on May 13, 1825, and was afterwards published in the second volume of their Memoirs, but as no plate or description of the constituent parts accompanied the report, we will here supply those omissions.

2 Plate XIX. exhibits a perspective view of this altitude and azimuth circular instrument, on so large a scale, and from such a point of view, that all the essential parts of the structure are presented to the eye, and may be explained without the tediousness attending a reference to letters or figures of indication. The instrument is exhibited standing in its proper position, on the cap stone of a strong pedestal, that is erected on a solid foundation of gravel, at the depth of five feet within the ground, and that ascends to a proper height for the convenience of the eye of an observer. The stone is represented as being circular, but is square and rounded at the four corners. Three bearing cups of solid brass are let into the stone and fixed with plaster of Paris, in such relative situations, as to form an equilateral triangle, the angular points of which were previously determined to be the places, where the three microscopes, used for reading the azimuthal circle, were required to stand, when the plane of the vertical circle is placed in the meridian, in order to be adjustable to zero of the horizontal circle. This desideratum was easily accomplished, by making the incisions in the stone too large for the brazen supporters, so as to admit of adjustment to the meridian, with the instrument standing on them, before the plaster was poured in. In describing the construction of the instrument, it will be convenient to consider it as composed of three principal parts, having each its own appendages, which parts we propose to describe separately in succession, beginning at the basis, and ascending as we proceed

3. The solid metallic cone, round which the horizontal circle revolves, rises nearly eighteen inches from an hexagonal solid of brass, to which it is well secured; each face of which solid is a square surface of upwards of three inches in each direction, three strong conical radii, tapering from three inches in diameter to one and three quarters, and each fourteen and a half long, are inserted into three of the faces of the hexagonal solid as firmly as possible, so as to make angles with each other of 120°, and a solid piece of brass is fastened into

the remote end of each radial cone, of a substance sufficient to admit of a foot-screw being tapped into it, these three strong screws have each a milled head above their lower extremities, which, being rounded in the lathe, fall into the cups formed on the upper ends of the bearing pieces finally made fast to the stone. By these three feet screws the whole weight of the instrument is supported, as well as the vertical position of the fixed conical axis regulated, therefore the screws have strong threads cut in steel, which act with a female screw through a space of two inches, where thirty-six threads are engaged at the same time, thereby ensuring steadiness in the position, which is a most important requisite. To the outer end of each of the three conical radii a frame, including an edge-bar, is screwed fast in a vertical position, of such a height as to hold each a reading microscope of the construction already described (Plate XI. figs. 9, 10, and 11.) at their due relative distances from each other, when adjusted, which microscopes read all in the same direction A fourth similar radial cone is made fast between two of the said cones, at 60° from each, the sole use of which is to hold the clamp of the construction exhibited in figures 13, 14, and 15, of Plate IV and explained in the third paragraph of § XLVI. The manner in which this clamp is fixed, and held by the fourth cone, is clearly seen in the engraved figure. Thus the first portion of the instrument consists of the hexagonal solid and its vertical cone, or fixed axis, both concealed from the view; the four radial cones, the feet-screws, the frames holding the three microscopes; and the clamp with its tangent screw of slow motion, which is so situated, that a rod, having a squared hole with an universal joint, can reach it with convenience, when the observer has the telescope placed in or near the direction of the meridian.

4. The second portion consists of the horizontal circle, the hollow cone at its centre, the two vertical pillars that stand on it at each side of the centre, and the appendages borne by them. The diameter of the circle is such as to admit of a radius of curvature of eighteen inches to the extreme curve of graduation, the whole circle is divided into four quadrants, terminating respectively with the letters S, E, N, and W; and at every ten degrees the figures of indication are large, viz 10, 20, 30, &c., and the intermediate single degrees are denoted by the smaller figures 1, 2, 3, &c., up to 10, the degree is divided into four larger spaces of 15', 30', &c, and these again sub-divided each into three 5' spaces by shorter lines, so that one of the subdivisions of 5' is that which the microscope has to measure by five exact revolutions, when properly adjusted for distance from the divided limb, while the eye piece has distinct vision, both of the limb, and of the crossed lines in the field of the microscope, as directed in § XIX. The three microscopes have a peculiar advantage in measuring azimuths, or other horizontal angles, in the reversed positions of the telescope, inasmuch as they read at six equidistant points of the circle in every double observation. The cone of brass made fast to the centre of the horizontal circle, turns very steadily and with great freedom round the corresponding fixed cone of steel, by bearing chiefly near the extreme ends of the respective cones against each other. The diameter at the base of the outer cone is six inches. The two pillars standing upright at each side of this circle's centre, are hollow but very strong, they are each three feet two inches long, tapering upwards from four and a half to three inches in diameter, and are made fast below to a thick plate, into which the twelve smaller radial cones are screwed fast at their inner ends, before their outer ends are fastened by screws to the lower face of the divided limb. These pillars are united by a strong horizontal bar, through which the upper

end of the large brass cone passes, which brace keeps them steady in their parallel position. A strong plate of brass covers each upper end of these pillars, and extends outwardly about six inches to allow the pivots of the vertical circle's axis to be supported by their extreme ends, which ends therefore are formed into gibbets by the reclining supporting bars, attached at their lower extremities to their respective pillars · two of these gibbet-bars appear dark in the figure, one at each side of the included pillar. A long bar of solid brass lies over one of the preceding covering plates, at right angles to it, and therefore parallel to the divided limb of the vertical circle; its length is thirty-seven inches, its breadth two and a quarter, and its thickness a full quarter of an inch. As this long and strong bar is intended to bear the two opposite microscopes, that bisect the vertical circle in a horizontal line, it was necessary that it should keep its position unchanged by any accidental force that might be applied to it, and therefore, the edge-bars that support the extreme ends are curved downwards till they meet with one of the upright pillars, to which they are screwed fast in the manner of gibbet-pieces, that completely answer their purpose of securing the station of the long horizontal bar. The manner in which the two opposite microscopes are supported at the ends of this bar, may be easily apprehended from the representations in the figure. If the instrument had been forwarded to Russia agreeably to the original order, the two microscopes here spoken of were all that would have been applied; but at our request two additional microscopes, capable of being moved into any relative positions and kept there, were without hesitation supplied by the maker, and the large ring, of about two feet two inches diameter, was placed parallel to and concentric with the plane of the vertical circle, by means of strong screws, fixing it in three places to one of the long pillars and to the long bar made fast to its superior end, as may be better understood by examining the parts of the figure The two additional microscopes stand opposite each other on the strong ring, that holds them by means of the thumb-screws of pressure that clamp them, which happened to be the situation in which they were placed, when our draftsman made his drawing, but the situations, in which they have been advantageously used, are those which form an equilateral triangle with the right hand horizontal microscope, that has got the index for pointing out the sub-divisions of the degree on the adjoining limb, by means of a single stroke carried by an adjustable pointer. In these situations one of the additional microscopes stands just as far above the horizontal diameter of the circle, as the other does below, and it has been found from experience in variable temperatures, that, when the readings of one of these two microscopes are in excess, as compared with the first or horizontal right-hand microscope, denominated A, the other will generally be found in defect by a similar quantity, or so nearly so, that no doubt can arise as to the dependence that may be placed on a mean of the three. The upper microscope is denominated B, and the lower one C, D being the opposite horizontal microscope, which is not read for the purpose of giving the measurement of any angle of elevation or depression of a star, as it has reference to the horizon or zenith, but as it serves to show that the circle is or is not bisected at any time by the horizontal line joining the zeroes of the opposite micrometers. If the two additional and moveable microscopes were placed so as to bisect the circle vertically, then it would be necessary to read all the four microscopes at each observation; and each pair of microscopes would separately correct for excentricity, or in other words, they would do the same service twice over. But the microscope C cannot be placed exactly in the vertical line by reason of the union of the ring with the pillar at the very place where the clamp ought to fix it An equilateral triangle may however be formed with the microscope D occasionally for a new series of measures, and the last microscope may be read in conjunction with B and C, and then the index of A alone may be consulted for the degree and subdivision, to which the microscopic readings may be respectively added, for the minutes shown by the notched scales of the micrometers, and for the seconds indicated on the divided heads of the different screws. We have not yet noticed that each pillar contains a strong spiral spring within it, that, by pressing upwards, supports a large proportion of the vertical circle's weight, which we shall have occasion to notice presently; and on this account the *Y*s carried by the gibbet-pieces, in which the pivots of the vertical circle rest, are covered by a pair of small cross bars, pressed down into contact with the upper parts of the pivots, to prevent the upward pressure of the spiral springs from lifting the circle's axis, these bars also serve the purpose of creating some friction on a third point of the pivot, thereby preventing the cylinder becoming longer in one diameter of its section than in the other. These small bars which are omitted in the plate, have each a longitudinal opening to admit of the *Y*s of the riding level of the axis being applied to the pivots The plumb-line and the spirit-level are also appendages connected with this second portion of the instrument, they will separately adjust and also watch the position of the vertical axis, so that when they agree, there can be no doubt as to the verification, and when one of them receives injury, or suffers derangement, the other will be competent to superintend the position of the axis, during the remainder of an unfinished series of observations. The plumb-line is suspended from an adjustable suspension apparatus made fast to the upper end of the long tube of small bore, seen to the right of the pillars, to one of which it is made fast at two places, distant from each other two feet and four inches; the tube itself being four feet six inches long, consequently the line, that descends into the attached and subjacent water vessel, is about five feet in length. The weight of the perforated small vessel, containing the shot, is as much as the silver wire will bear out of water without breaking, and therefore the wire is stretched as much as is consistent with its safety the upper end of the wire lies in the angular point of an adjustable bearing piece of metal, that is moved by two pairs of contrary screws, acting at right angles to each other. A pair of microscopes of the ghost-kind, crossing the lower end of the long tube at right angles, look at the line, and also at the image of an excentric luminous disc of thin mother-of-pearl, in each of two directions, one showing the state of the adjustment with respect to east and west, and the other with respect to north and south, agreeably to the account we have given of the plumb-line, and of its application, in a former section (§ XLIX.). The hanging level is suspended by two similar cylindrical pins of steel fixed in a pair of cocks, screwed to the plane of the long horizontal bar, above described, and may be seen lying horizontally under the projecting end of the vertical circle's axis. This level has an ivory scale, with a zero at each end of the bubble, when the temperature is 60° of Fahrenheit's thermometer, there are thirteen divisions in the inch, which were intended to indicate single seconds, but on trial in actual measurement, by the circle, of a small distant object, we found that the value of each division is 1″.5. Our description of the level and of its modes of application, which the reader may refer to, (§ L.) will render a further account of it here unnecessary Besides the two levels, which have been here mentioned, a third is seen in the figure, suspended from pins borne by two cocks, affixed to the telescope's tube, for the purpose of levelling it, but as no use is made of such level, in an instrument that reverses in position, it may be considered superfluous.

5. The third portion of this instrument consists of the vertical circle and its telescope, which together are supported by the two pillars and their included spiral springs. This circle has two limbs, one graduated, and the other without divisions, enclosing the telescope between them in close contact near both ends, thereby preventing any tendency to flexure of its tube. These two circular limbs are of the same dimensions as the limb of the horizontal circle, and have in like manner each a dozen light conical radii uniting them firmly with the middle portion of the horizontal axis, which is cylindrical. The distance between the limbs is 3.75 inches, and the union is effected by a species of lozenge-work of light bars of brass, that cross one another before they are made fast at both edges to the limbs, which they thus unite. The tube of the telescope is just thick enough to enter two lozenges thus made, one at each side of the double circle, and is kept firmly in its position in all directions at both ends The graduation of the vertical circle is similar to that of the horizontal one in all respects, and all the microscopes are precisely alike The ends of the axis are composed of strong inverted cones of brass, firmly attached to the central cylinder, and terminating with bell metal pivots; the length of the axis is eighteen inches, exclusive of the pivots, which rest in *Y*s not requiring an adjustment for collimation in azimuth, and but only for levelling· each pair of radial cones, that lie parallel to one another, are united by cross pieces at one third of their length from the circumference, which complete their union. The clamping apparatus is fixed to the interior side of the remote pillar, in such a situation, that it takes hold of the limb having no graduations, and though the construction of the clamp is of the ordinary kind, its tangent screw regulates the altitude of the telescope with sufficient precision, the clamping screw is turned by an indented wheel instead of a milled head, which is easily turned in either direction by the application of a single finger to its periphery. The object glass of the telescope is three inches and a quarter in diameter, and has a solar focal length of 44.4 inches: it has been esteemed one of Tulley's best object-glasses, and was therefore exchanged for the original glass of smaller dimensions. At the eye-end there is a micrometer with three fixed spider's lines and one moveable, for measuring small angles, or differences of the altitudes of any two bodies, seen at the same time in the field of view, besides which there is a line having a circular motion by means of a thumb-screw, by which angles of position may be measured on the meridian without interfering with the position of the micrometer or adjustments of the instrument. The value of one revolution of the screw is 44″.7, from which a table of values for the exclusive use of this micrometer is easily constructed by continual addition. The field of view is limited to 54′, and the common eye-pieces magnify from 50 to 80 times; but with the diagonal eye-pieces used for high altitudes, and having three lenses, the powers are much greater; with these an observation may be made in the zenith with as much ease as in any other degree of altitude. The diaphragm, containing the five vertical and three horizontal fixed lines, is moveable in two directions, at right angles to each other, by means of two pair of adjusting screws, that assist in exterminating the errors of collimation in azimuth and altitude respectively. The two extreme horizontal lines nearly include the sun's disc.

6. When the three portions of this instrument had been unpacked, and put together on the pedestal, it was found to be a good test of its stability of position, to observe that, when

the telescope was pointed to a fine mark on a distant object, turning the instrument round in azimuth, either backwards or forwards, or both, did not make the middle horizontal line depart from the said mark, at its final return, but with respect to azimuth the stability was not found so perfect, on reference to the indication of the horizontal circle, there frequently appeared a deviation in azimuth when the telescope was only elevated to the zenith and brought down again to the horizon, even while the horizontal circle was clamped; and particularly when the direction of the pressure that produced elevation was partly lateral, which cannot always be avoided in taking altitudes. This defect produced a temporary feeling of disappointment, and as it was apprehended that the first or lowest portion of the instrument allowed of this deviation, from a yielding of some of its parts, the first and second portions being united only by one slender clamp, having no connexion with the stone on which the instrument stands, it was proposed to fix a second clamp of strong materials to the stone itself, which might clamp another part of the limb at a considerable distance from the former, which was afterwards done in the most satisfactory manner, in the form of a triangle, as seen at the left hand side of the horizontal circle: this application has cured the defect, and rendered the instrument sufficiently steady to be used occasionally as a transit-instrument, and to give observations of right ascension and of zenith distance at the same time, when the circle's position is in the meridian, and the collimation in both respects known. It may be remarked further, that the bar which holds the additional microscopes B and C, being of the same material as the graduated limb, and being formed also into a circle standing parallel to its plane, and concentric with it, not only admits of a change of position of these microscopes, but will keep their places relatively to the divisions on the limb, for the changes produced by variable temperature will only increase or diminish the radius of the circular bar, and slightly vary the distance of the micrometers from the centre; which change will not sensibly affect their measures.

7. *Adjustments* After the incidental observations we made in our last section, respecting the adjustments of the Westbury circle, which is used in the same manner as the instrument now under our notice, it will not be necessary to dwell long on the measures to be successively attended to in effecting the requisite adjustments. When the telescope has its glasses so arranged, that a star of the first magnitude will appear round, small, and well defined, at the same time that the spider's lines are distinctly seen, the adjustments may be made in the following order: in the first place, when the feet screws are raised or lowered, till the limb of the horizontal circle will pass freely through the released clamp, attached to the stone, while the circle is turned round, the vertical axis will not be far from a perpendicular direction; then if the telescope be turned towards either the north or south point approximately, it will lie in a direction over the foot-screw nearest the said clamp, by which the deviation of the axis towards the north or south will require to be regulated, and the level, suspended on the long diametrical bar, will point north and south also; secondly, while in this situation adjust the level till it will reverse in position, end for end, partly by the foot-screw facing the south, and partly by the screws of one of the suspension pins, and after repeating the operation of halving the error in the two positions, till the bubble will remain at the same part of the scale in both, the level will be adjusted; and also the upright conical axis will be vertical in the direction of north and south; in this position the plumb-line may be conveniently applied to its suspension apparatus, and if the bubble still retains its place, this line may be adjusted by the

proper screws, at the top of the long tube, till it intersects the disc of the micrometer, standing at right angles to the telescope, in which situation it will be partially adjusted, provided the excentric disc be truly placed, thirdly, the telescope must now be turned through a quadrantal arc, into the situation pointing east and west, when it will stand parallel to a line joining the second and third feet screws, which will also be the position of the level, and if, in this situation, the level has the bubble as before in the middle of its cylinder, the axis will be vertical in the direction of east and west also, and consequently in every other azimuthal direction: in this case the plumb-line may be re-adjusted, one half by the screws at the suspension, and the other half by turning round the luminous disc of the ghost-micrometer: but it will rarely happen that the adjustment for north and south, will be good for east and west also, and any alteration of the second or third foot-screw will derange the previous adjustment by the first or southern foot screw. In the management of an adjustment to a truly vertical position depending on the feet-screws of a perfect *tripod*, the readiest and surest way of succeeding is, when the axis is nearly vertical, but not quite so, to bring the level parallel to two of the feet, namely the second and third, which in our case is in the line of east and west, then, if the axis is made truly vertical by one of these screws and by the screws of the level in the usual way of halving, and reversing the telescope's position at each mediation, till the adjustment is complete in both positions, moving the first foot-screw *alone* will not derange the said adjustment, as being in a point at right angles to the line joining the other two screws, and the level being previously adjusted, will now show when the axis is vertical also in this rectangular direction. This mode will answer equally well without any reference to the tripod's position, as it regards the cardinal points. When the vertical axis is thus perfectly adjusted by the level, the bubble will remain stationary while the horizontal circle gradually revolves, and the plumb-line may then be finally adjusted with but little trouble, by attending both to the luminous disc and point of suspension alternately, till the former continues bisected during a slow revolution of the horizontal circle. As the whole accuracy of a careful observation made with this instrument, as well as with its predecessor, depends on the accurate position of the vertical axis, we could not be too particular in our directions respecting its adjustment. Fourthly, when the axis is known to be perfectly vertical, which may be so preserved by an occasional application of the southern or first foot-screw in the principal direction of north and south, accordingly as the level or plumb-line may indicate, the collimation both in altitude and azimuth may be adjusted successively, either by the help of a distant mark or by the pole star, according to the method of halving the errors which we described in our section LVI., when treating of the transit-instrument; except that here the tangent screw of the azimuth circle must be substituted for the horizontal screw that there moves the *Y*: the methods of levelling the horizontal axis, and also of placing the instrument in the meridian, are the same as there described. When these adjustments have been carefully made, and the values of the equatorial spaces, contained between the vertical lines of the field of view, ascertained, the instrument may be considered as prepared for use, provided the microscopes have been previously adjusted in all respects, and put to their proper zeroes.

8. The manner of using this instrument is the same as that which we described in our last section, except that here we have two arbiters of the vertical position of the upright axis, which operate as checks on each other; and also more microscopes. The quadrantal arcs of

this vertical circle are figured so as to read altitudes in one of the reversed positions, and zenith distances in the other, thereby affording the ready means of ascertaining the error of collimation, by comparing a mean of the sums with 90°, and taking half the excess or defect as the correction to be applied, with the *same sign*, to either the zenith distance or altitude, as we before explained when describing the Westbury circle. The delicacy of the level is such, that it indicates the slightest changes of temperature affecting one side of the pillars more than the other, which changes may be inadvertently produced by opening the door of the observatory, or by the observer's touching or even breathing on one side of the pillars in taking a zenith observation, in the former case the bubble recedes from, and in the latter advances towards the observer; and when such changes are noticed, it is proper to wait for the return of the original temperature, before the contact is finally made, and the observation registered. The plumb line, however, is by no means so sensible, and as the level has a scale of known value, it is usually referred to in each observation. The re-adjustment of the vertical axis, after reversion, is likewise made by the level in preference to the plumb-line, which requires more time, in coming to a state of quiescence, than is convenient on many occasions, when the instrument is required to follow the moving body in azimuth. As the face of the vertical circle has two positions, and the telescope may be directed to either north or south in each of these, there will be four varieties in the readings of the microscopes, two of altitudes and two of zenith distances, and it is of importance to be certain what *sign* properly applies to the indication of the level, when the vertical axis is not truly perpendicular, in each of these four varieties. The rule is this, which may be easily remembered, viz whenever the bubble approaches the *object end* of the telescope, the correction given by the level must have the sign − when *zenith distance* is read, and + when *altitude* is indicated by the circle, and, on the contrary, when the bubble runs towards the *ocular end* of the telescope, the signs must be the reverse, i. e. the correction of zenith distance will be +, and of altitude −. In order to know whether zenith distances or altitudes are the arcs measured, it is only necessary to examine whether the numerals engraved on the limb count 1, 2, &c. from the zenith, or from the horizon, which will vary accordingly as the face of the circle is turned to the east or west, and as the telescope is pointing towards the northern or southern quadrant, in taking the observation. The method of determining the value in seconds of a single division on the scale of any level, and the formula by which the existing error, as indicated by the bubble, is appreciated, have already been explained in our Section I., and need not be repeated When the bubble retains its place after reversion of the circle in azimuth, it is a proof that the vertical axis and the level are both well adjusted, but this will not often be the case in changeable states of the atmosphere, and whenever the level has a new indication in the reversed position, its value must be applied with the proper sign to the observation, or otherwise the bubble must be brought to its original place by the foot screw, and then the errors of the reversed positions will counteract one another by what is called a *collimated* observation. In all instances, when the bubble runs towards the north, the plumb-line will incline towards the south, and the contrary.

EXAMPLE 1.—We will take as our first example of the method of using this circular instrument those successive observations of certain stars, from which the latitude of South Kilworth Rectory was determined, as reported in the second part of Vol. I. of the Memoirs of the Astronomical Society of London (p. 268), and the tabular arrangement that we have made, will render particular explanation unnecessary, when it is understood that the apparent zenith distance is represented by Z, and the apparent declination by D, for then $Z+D$ will be equal to the latitude, when the star is seen in the quadrantal arc remote from the upper pole; but when the star lies between the zenith and pole, the latitude will be represented by $D-Z$, and when the star is taken below the pole, $Z-P$ will give the co-latitude, where P is the apparent polar distance, or complement of the declination.

Date 1824	Bar	Int Ther	Stars	A	B	C	Mean of 3	Level	Coll	Brad refrac	Z	D	N latitude
Jan 27	29 33	45	α Orionis	45° 3′ 47″	37″	39″	41″ 00	. .	+13′ 7	+57″ 14	45° 3′ 51″ 81	7° 21′ 50″ 5	52° 25′ 51″ 34
28	29 20	41		41 56 58	45	55	52 66		+13 7	−57 53	45 3 51 17	7 21 50 5	52 25 50 67
July 8	29 58	64	γ Draconis	0 54 53	56	55	54 66		+1 67	+0 88	0 55 0 21	51 30 53 0	52 25 53 21
9	29 50	63	. . .	89 4 56	55	57	56 00		+1 67	−0 88	0 55 0 13	51 30 53 3	52 25 52 42
16	29 70	68	.	0 51 42	44	47	44 33	.	+11 33	+0 80	0 54 56 40	51 30 55 4	52 25 51 86
17	29 75	61	. . .	89 4 55	54	53	54 00		+11 33	−0 81	0 54 55 48	51 30 55 7	52 25 51 18
21	29 80	65	β Draconis	0 0 21	26	17	21 33	−3 .75	+0 04	0 00	−0 0 26 02	52 26 15 82	52 25 49 20
21	29 80	65		89 59 25	22	20	21 33	.	+0 01	0 00	−0 0 28 03	52 26 16 70	52 25 50 07
Aug 9	29 25	65	α Lyræ	13 47 31	41	28	33 33	. . .	+16 25	+13 32	13 48 3 00	38 37 45 6	52 25 49 50
15	29 10	62	.	76 11 46	69	40	51 .66	+1 5	+16 25	−13 35	13 48 3 01	38 37 46 5	52 25 50 41
27	29 80	61	α Aquilæ	43 59 23	24	23	23 33	+6 0	+20 35	+53 72	41 0 52 40	8 21 57 6	52 25 50 00
28	29 50	65	.	45 59 32	33	34	33 00	.	+29 35	−52 83	41 0 50 18	8 21 57 7	52 25 48 18
Sep 13	29 60	60	α Cygni	82 11 9	10	11	10 00	−6 0	+2 58	−7 50	7 46 1 00	41 39 48 .7	52 25 49 .70
14	29 50	61	.	7 45 42	45	43	43 33	+7 5	+2 58	+7 48	7 46 0 80	41 39 49 0	52 25 49 80
13	29 60	60	α Cephei	9 21 53	51	58	54 00	.	+4 50	+9 12	−0 25 7 62	61 50 59 29	52 25 51 .07
15	29 70	61		80 31 51	54	54	51	+3 00	+4 50	−9 06	−0 25 7 56	61 50 59 99	52 25 52 43
												Mean latitude	52 25 50 74

9. In selecting well known stars for determining the latitude of a place, it is desirable to take such as have considerable altitude; under which restriction it will not much signify what tables of refraction are used. In the preceding reductions Bradley's tables were preferred for no other reason but because the Greenwich apparent places, as given in the Nautical Almanac of 1824 have been adopted, to save the trouble of computing the corrections, except in the instances of β Draconis and of α Cephei, not there included, the apparent north-polar distances of which were computed from Mr. Pond's Catalogue of 60 Stars for 1823, and from our Table of Corrections of 48 principal stars, in the preceding volume. The stars contained in the list were all observed on the meridian, alternately in the reversed positions of the vertical circle, to avoid reductions to the meridian, consequently the error of collimation is, in every instance, that which is due to both evenings; but as the latitude is deduced from each observation, and as the collimation does not vary but with sensible changes of temperature, no erroneous conclusion is likely to arise from short intervals, according to the method of observing above exemplified. If several observations had been taken of each star, for so many means, the computed latitude would have been more correct; but our object here is to show the powers of the instrument as exhibited in single observations, and the mode of using it with advantage. The stars might easily have been observed in both positions on the same evening, but as the times given by the clock must have been noted to correspond, as data for the reductions to the

meridian, and as some derangement of the vertical position of the upright axis might be apprehended, it is more convenient, in favourable weather, to reverse and re-adjust on the alternate evenings The reading microscopes were re-adjusted on the 16th of August, to equalize the readings of all the three, but if their relative discordances had continued unaltered, the accuracy of any pair of observations, taken in the reversed positions, would not have been affected thereby while no change was produced in the collimation The level was also re-adjusted about the beginning of September, when the thermometer stood at 60°, as it regarded the zero of the microscopes, which occasioned a diminution of the error of collimation without affecting the correctness of the measures

EXAMPLE 2.—When the pole star, or δ Ursæ Minoris, is observed for the purpose of determining the latitude, the observation may be conveniently repeated after the position of the instrument has been changed and corrected, since the slow apparent motion does not very sensibly change the altitude for several minutes, and the reductions to the meridian will enable the observer to arrive at a good conclusion at one passage, since the apparent declinations of these stars are given, in one or other of the ephemerides, for every day of each year. And when the successive passages above and below the pole are thus observed, the height of the polar point, or latitude, may be known without an ephemeris or other tables, by merely taking the mean of the two observed altitudes cleared of the effect of the corresponding refractions.

Date 1824	Bar	Ther	Star	Mean of 3 readings	Level	Reduc to merid	Coll.	Refrac	D	L	N Latitude
Aug 27	29 80	67	Polaris	39° 10′ 35″ 60			+27″ 01	+44 96	88° 22′ 20″ 1	39° 11′ 47″ 63	52° 25′ 52″ 27
			below	50 48 48 33	+15 0	−33″ 01	+27 01	−44 96	88 22 20 1	39 11 47 62	52 25 52 28
Sep 1	29 55	77	below	39 10 37 33			+25 53	+45 02	88 22 21 72	39 11 47 88	52 25 51 40
				50 49 5 33		−33 72	+25 53	−45 02	88 22 21 72	39 11 47 88	52 25 51 40
Nov 12	29 ,70	45	above	35 56 26			− 8 03	+41 84	88 22 47 9	−35 56 59 81	52 25 48 00*
				54 3 47 ,60		+ 2 40	− 8 03	−41 84	88 22 47 9	−35 56 59 81	52 25 48 00
19	29 20	47	above	54 3 56 0			−12 86	−40 93	88 22 48 3	−35 56 57 79	52 25 50 51
				35 56 28 66		+ 0 97	−12 86	+40 93	88 22 48 3	−35 56 57 70	52 25 50 00
26	29 20	38	below	39 10 32	+ 4 5		− 7 00	+47 17	88 22 52 2	39 11 16 07	52 25 51 13
				50 49 30 00	+ 7 5	0 00	− 7 00	−47 17	88 22 52 2	39 11 16 07	52 25 51 13
Dec 1	28 95	38	above	54 3 12 60			+22 72	−41 31	88 22 53 5	−35 57 5 93	52 25 49 00
				35 56 0 66	− 3 00	+ 1 27	+22 72	+41 31	88 22 53 5	−35 57 5 96	52 25 49 11
2	29 15	39	below	39 10 4 60		+ 0 42	+21 74	+46 79	88 22 53 7	39 11 13 61	52 25 52 74
				50 49 16 56	− 3 00	− 2 12	+21 74	−46 79	88 22 53 7	39 11 13 61	52 20 52 74
13	29 ,85	45	below	50 49 19			+15 79	−47 36	88 22 56 1	39 11 12 57	52 25 51 ,33
				39 10 9		+ 0 12	+15 79	+47 36	88 22 56 1	39 11 12 57	52 25 51 33
										Mean latitude =	52 25 50 82

If we take the observed zenith distances of the pole star on the 1st and 2d of December, viz. 35° 57′ 5″.96 and 39° 11′ 13″.61, when reduced, the mean 37° 34′ 9″.78 will be the co latitude, and its complement 52° 25′ 50″.22, the latitude, without any previous knowledge of its declination, which result is independent of other observatories.

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* The latitude arising from this observation, as given in the Memoirs of the Astronomical Society of London, is erroneously stated, Schumacher's apparent declination having been extracted for December 12 instead of November 12

10. With respect to the use of the azimuth circle in astronomical observations, we have stated in our first volume (p 301), that Piazzi determined a scale of celestial refractions by measuring the altitudes and corresponding azimuths of certain well known stars, in various situations, both above and below the pole, where the differences between the observed and computed apparent zenith distances, supposing no errors to exist in the observations or computation of the apparent places of the stars, were taken as the simple effect of refraction. Observations of this kind are practicable only in the open air, or under a rotative dome. When the latitude of the station, the apparent polar distance of the star, and the azimuth are correctly known, the apparent zenith distance may be computed by the following formula, viz.

$$\text{Cos}\,(z-\varphi)=\frac{\cos\Delta\cos\varphi}{\cos\lambda} \qquad \text{(No. 24. in § LXX.)}$$

where tang $\varphi=\cos\alpha$ tang λ, α and λ respectively denoting the azimuth of the star, and co-latitude of the place. As an example of the application of this formula, let it be required to compute the apparent zenith distance of α Serpentis on the evening of April 30, 1828, when its azimuth from the north point was 130°, and observed zenith distance 55° 49′ 26″.2; the apparent polar distance (Δ) given in the Nautical Almanac being at that time 83° 1′ 40″, the latitude of the place 52° 25′ 51″ north, the barometer being at 29 .6, and thermometer 50°.

First let us find φ thus.	Cos α . . .	130°	0′	0″	. . . . .	−9 .8070675
	Tang λ . .	37	34	9	. . . . .	9 .8860657
	Tang φ . .	−26	15	30	. . .	−9 .6931332
Then we have	Cos Δ . . .	83	1	40	. . . . .	9 .0841762
	Cos φ . . .	26	15	30	. . . .	9 .9526997
					Sum . .	9 .0368759
	Cos λ . . .	37	34	9	Sub. . .	9 .8990639
	Cos $(z-\varphi)$	82	6	20	. . . . .	9 .1378120
	Add φ . .	−26	15	30		
Computed zenith distance		=55	50	50	without refraction.	
Observed zenith distance		55	49	26 .2	with refraction.	
Refraction at a mean temp.			=1	23 .8		

As it is important, in computations of this nature, that the signs of the sines, cosines, tangents, &c. should be properly applied in the different quadrants, it may not be irrelevant to remark here, that whenever a sine, cosine, &c. passes through 0, or infinity, the sign changes, hence may be derived an useful rule for the mutations, which will be conveniently referred to, when tabulated in the following manner.

TABLE OF SIGNS OF THE SINES, COSINES, TANGENTS, &c

		Sin	Cos	Tang	Cotang	Sec	Cosec
1st . . . 5th	Quadrants	+	+	+	+	+	+
2nd . . . 6th		+	—	—	—	—	+
3d . . 7th		—	—	+	+	—	—
4th . . 8th, &c.		—	+	—	—	+	—

11. Another use that may be made of the azimuth circle, is the determination of the true meridian of a place, either by circum-meridian observations of equal altitudes, with their corresponding azimuths, or by measuring the horizontal angle of a circum-polar star, at the moment of its greatest elongation, when its apparent polar distance is known but as we shall have occasion to consider these operations hereafter, we merely advert to them here, as being within the reach of our present instrument, which may be also said of zenith observations Indeed there is scarcely any species of observation, not requiring an equatorial motion, to which this instrument is not competent, whether it regards declination, right ascension, or time, in or out of the meridian.

§ LXIX THE COLLIMATOR [Plate XXI]

1. It is a well-known dioptric fact, that if a luminous body be situated exactly in the solar focal point of either a single or double convex lens, the rays of light issuing from it, and passing through the lens, will, at their exit, become parallel; and any telescope viewing the said luminous point in a backward direction, at right angles to the surface of the lens, will have good vision of it at any distance, with the same ocular adjustment, as is suitable for a heavenly body. Opticians and practical astronomers have availed themselves of this property of a lens, which is still better for being achromatic, of rendering a near mark visible, when a distant one cannot be obtained, or when, being obtained, it cannot be seen by night. For many years past Mr. Troughton has been accustomed to supply a lens of very long focal distance, to be put occasionally over the object-glass of his 30-inch transit instrument, which will render any meridian mark distinctly visible, that is placed in the focal point of this lens, and when it is nicely centered in its cell, which may be known to be right by turning it round while viewing the mark, it is found very convenient for this purpose, by reason of its requiring no adjustment for focal distance, or direction of the optical axis, when once properly fitted.

2. Rittenhouse has given an account, in the second volume of the Transactions of the American Philosophical Society, of his employing the object-glass of a telescope 36 feet long, for the purpose of rendering a metallic plate, containing several concentric circles, visible as a meridian mark, and Professor Gauss has recently proposed a short telescope to be used for a meridian mark, by illuminating the cross wires in the common focus of the object and ocular lenses, and fixing it so, that the telescope of another instrument directed towards it may view the illuminated lines through its object glass, the intersection of which lines becomes the re-

quired mark, when adjusted to the meridian. This method of fixing a small telescope, instead of a simple lens, affords the ready means of giving a proper direction to the optical axis, as it regards the instrument that is destined to view the cross lines as a fixed mark. Soon after this proposal was notified, Professor Bessel applied the same contrivance to a new purpose, which is described in the 61st No. of Schumacher's *Astronomische Nachrichten*, he there shows how the flexure of a long telescope's tube may be ascertained, when attached to the horizontal axis of a graduated circle, having a level, provided it be first adjusted for collimation in altitude, for when the horizontal position of the telescope is reversed, by turning the circle half round in a vertical direction, the excess above 180°, as measured by the circle, when the telescope views telescopic horizontal marks to the north and south alternately, will be double the horizontal flexure. He also proposes to find the zenith point on a divided vertical circle, without turning it in azimuth, by the help of a small vertical telescope, having a level attached very nearly at right angles to its optical axis, and being suspended over the circle's zenith in such way, that it may turn half round in azimuth with the attached level, to watch its vertical position, for when the cross lines of this small telescope have been viewed in the first position, and the point of intersection has been covered by the horizontal wire, and read on the circle, it may be turned half round, and the intersection again observed and referred to the circle, then the mean of the two readings will give the zenith point on the divided limb of the circle. It does not, however, appear certain that this plan was carried into execution.

3. Captain Kater, whose ingenuity in the application of mechanical contrivances to practical operations is generally known, substituted mercury for the level, and, improving on the suggestion of Bessel, constructed an instrument for the express purpose of determining and verifying at pleasure the place of the horizontal or of the zenith point on a vertical circle, without the assistance of plumb-line, level, or reflecting surface, and that at any time almost instantaneously. The details of various experiments, tending to prove the practicability and efficiency of the plan, are given by the author in the first part of the volume of the Philosophical Transactions of the Royal Society of London, for the year 1825 (p. 147 and seq.), which will be read with interest by every practical astronomer. The author avails himself of the telescopic mark constituted by two crossed slender springs, in the focal point of a small achromatic object-glass, carried by a cast-iron plate floating on mercury, and, finding that the angle of inclination of the optical axis, as compared with the surface of the mercury, is *constant*, or sufficiently so for his purpose, he substitutes the floating small telescope, or rather the lenses and cross of flat springs, without a tube, for a fixed mark, and relies on the permanency of the inclination of the optical axis, for such time as he has occasion to refer to it, which is seldom more than a few minutes. His first object was to find the error of collimation in altitude by means of a small telescope floating horizontally, or nearly so, and when he had succeeded in doing this, which gave him some trouble, he proceeded to determine the same error by such an alteration in the mechanism as allows the small telescope to float nearly in a vertical position. These two constructions of the horizontal and vertical collimators were executed, under the contriver's directions, by Robinson of Devonshire Street, Mary-le-bone, who has supplied us with a specimen of each kind, which we shall now describe.

4. *Horizontal Collimator.* A mahogany box 15 inches long, 6½ broad, and 5 deep, is represented in figure 1 of Plate XXI, as having only one side and two ends, that the enclosed

float, and parts carried by it, may be exposed to view, for the sake of easier description; *A B* is the float of cast-iron partly immersed in the mercury covering the bottom of the box, *C* and *D* are two iron bearing pieces screwed to the bottom of the box by short iron screws, each piece has two vertical plates turned up, the inner one of which has a longitudinal slit in it, into which slits the iron pivots, screwed into the sides of the float, are demitted, the use of these parts is, to keep the sides of the float parallel to the sides of the box, and at an inch or more from contact with any part of the box, that the mercury may assume a flat surface at *E* and *F* two holding pieces of metal, cast along with the float, stand up and are perforated, to receive each a socket, that at *E* holds the cross before the eye-piece, and the opposite one at *F* receives the object-glass, which is adjustable by a screw for its focal distance, while the lens held by the end of the box at *G* constitutes the eye-piece It will therefore depend upon the relative heights of these two sockets from the float, what the inclination of the optical axis, or of the line joining the centre of the object-glass and intersection of the cross, shall be, as compared with the surface of the fluid this inclination has no mechanical adjustment, but may be modified by the addition of perforated circular pieces of iron, surrounding the vertical pin at *I*, as weights, to depress or balance the remote end *A* of the float, which, however, should be sparingly used, lest the float should sink sensibly deeper into the mercury at one end than at the other The mercury must be as pure as can be obtained, and particles of dust must be constantly excluded by a lid, that embraces the top of the box. At the end *H* is a circular hole, also occasionally closed, through which the large telescope of the circular instrument is directed, when viewing the cross at a short distance, and a lamp or lantern is placed behind the eye-lens *G*, to illuminate the cross Several precautions are necessary in fixing and using this collimator, which we will describe in the author's own words, as best conveying his own ideas. "The instrument being placed on the north or south side of the observatory, with its telescope pointed to the centre of the circle, and nearly in its plane, it is to be directed so, that the wires of the telescope of the circle may be seen through it, when reciprocally the cross wires of the collimator will be visible through the telescope of the circle, and the collimator is to be so placed, that the cross wires may appear in the centre of the field of view. The place of the box should then be carefully marked, to ensure its being at once restored, as nearly as possible, to the same situation The collimator is then to be removed to the opposite side of the observatory, and the same process repeated, the situation of the box being here also carefully marked. In observing, the star having been taken, and the readings of the microscopes registered, the telescope is to be depressed to the collimator, and the angle formed by the cross wires carefully bisected. The collimator is then to be taken to the opposite side of the observatory, and the cross wires again bisected; the mean of the readings at the bisections will give the inclination of the collimator to the horizon, and the difference between this and the apparent inclination, at either position of the collimator, will be the correction to be applied to the mean of the readings registered at the bisection of the star For example, let the mean of the readings of the bisection of the cross wires, when the collimator is to the south of the instrument, be 7′ 30″ of altitude, and when it is to the north 8′ 40″, the mean of these readings, 8′ 5″, is the true inclination of the collimator to the horizon, and the difference between this and 7′ 30″ (0′ 35″) must be added to all altitudes taken to the south, or subtracted from those to the north of the zenith."

5. This transportation of the vessel, containing the mercury, from one side of the room to the other at every observation, is very troublesome, and attended with risk, we have therefore adopted a plan, in all respects preferable, of using the horizontal collimator in one place only, where it remains permanently fixed. As the only use of carrying the vessel and float across the room, is to have reversed positions of the ocular end of the telescope, as the horizontal spider's line has reference to a section of the optical axis, that the error of collimation may be + in one position, and − in the other, the same effect is produced by turning the circle half round in azimuth, and then the telescope half round in elevation and depression, till it looks again into the hole at the end of the collimator, when the position of the eye-piece and of the horizontal spider's line will be equally reversed, as by the other method of looking first to the north and then to the south alternately. Captain Kater's method supposes no level or plumb-line to be required, in gaining the error of collimation in altitude, but presumes on the permanency of position of the vertical axis, on which the amount of the error depends; but by our method either the level or plumb-line is used to watch this position, which a small variation of temperature, partially applied on either side, will sensibly affect, but the changes may be corrected by the foot screw; and as observations are made with most confidence when the vertical axis is known to be truly perpendicular, the most satisfactory way of using the horizontal collimator is to use it instead of a distant mark in the usual way, with face east and face west alternately, in which case the sum of the reversed readings, being nearly 90°, will show by half the excess or defect, on a comparison with a quadrant, what is the amount of the existing error; which is not simply the error of collimation, but combined with the error of the level and index error of the microscopes; a change in any one of which will affect the compound error in question in very slight variations of temperature. When the collimator is thus used, as a substitute for a distant horizontal mark, it becomes peculiarly useful in making observations by night, because, when it is illuminated, the error in the measure of any observation may be known as soon as the observation is made, before any change has taken place in the circumstances under which it was made. We have used our circular instrument, by Troughton, in this way with entire satisfaction, and therefore hesitate not to recommend it to others, who have instruments that will reverse in position. This collimator, we understand, has been used with success at Dublin, Armagh, and Bedford, and probably in the way we have described, at Greenwich also it has been tried, but not considered so correct in its indications, as to supersede the method now adopted of observing by two circles at the same time, one of which measures by direct vision, and the other by reflection from mercury; which method will be described and exemplified in one of our subsequent sections Our method of substituting the cross lines, in the focal point of the floating telescope, for a distant meridian mark, will be readily comprehended from an inspection of the following example.

EXAMPLE. The sun's meridian altitude had been observed with our circular instrument on the 20th of December, 1826, after which the following determination of the error was made immediately after the observations were finished; viz

Before reversion, the apparent altitude of the cross was found A 0° 1′ 12″ After reversion its zen. dist. was 89° 58′ 14″
B 0 48 15
C 1 7 2

Mean of three readings 0 1 2 33 . . . . 89 58 10.33
Add 1 2 33

Sum 89 59 12.66

Complement to 90° 47 33

Correction of the error of collimation, &c. $\frac{47''.33}{2}=$ + 23.66

Altitude of the cross corrected . . = 1′ 2″ 33 + 23″ 66 = 1 26.00
Zenith distance of the cross . . 89° 58′ 10″.33 + 23″.66 = 89 58 34 00

Sum 90 0 0.00

6. Our horizontal collimator was fixed on a strong board extending across the southern opening in the wall of our observatory, and was laden with a small weight to diminish its inclination, on the 26th of June, 1826; in which situation it remained unmolested till April 19, 1828, during which period its inclination evidently varied, as well as the error of collimation of the vertical circle the latter may be accounted for as depending on changes effected in the circle, in its level, and adjustments of the microscopes, by variable temperature; but the former was not so easily referable to natural causes, since any change of place occasioned by shrinking and swelling of the board, would not affect the site of the focal representation of the cross in the large telescope, seeing that the incident rays coming from the cross, by falling on *any part* of the object-glass, would form an image in the same focal point, which is a great advantage possessed by this telescopic mark. On removing and examining the float, however, at the latter period above mentioned, it was found that an oxide was formed on the inferior surface of the float, in contact with the mercury, to which several amalgamated particles adhered, and when this was smeared with lard and scraped clean, and the mercury strained through chamois leather, the inclination increased from 50″ to 1′ 4″.33, though the error in the collimation, &c remained unchanged, as then determined, and compared with that of the preceding day. The subjoined Table will exhibit the changes that took place in the inclination, and in the error corresponding to it, as copied from our register on the days there specified, which we leave without comment for the inspection of our readers.

7 OCCASIONAL NOTICES OF AN HORIZONTAL COLLIMATOR

Date 1826		Barom	Ther	App alt of ×	App zen dist of ×	Corr of col	True alt of ×	Remarks
June	27	29 60	83°	0° 1′ 59″ 00	89° 59′ 28″ 33	+16″ 33	2′ 15″ 33	
	28	29 65	83	0 1 50 33	89 57 13 00	+28 33	2 18 66	Microscopes adjusted
Dec	11	29 30	42	0 1 3 00	89 58 11 00	+23 00	1 26 00	
	20	29 25	40	0 1 2 33	89 58 10 33	+23 66	1 26 00	
	21	29 28	40	0 1 1 20	89 58 8 66	+25 07	1 26 27	
	22		32	0 0 49 00	89 58 8 66	+31 16	1 20 16	Frosty night
	22		32	0 1 6 00	89 58 27 33	+13 33	1 19 33	The microscopes put 15″ forwards
	22		32	0 1 19 66	89 58 42 33	− 1 00	1 18 66	Do put forwards 13″ 33 more
	27	30 05	41	0 1 21 00	89 58 31 33	+ 3 83	1 21 83	
1827								
Jan	4	29 20	28	0 1 15 00	89 58 48 75	− 2 87	1 12 13	Snow
	23	29 05	33	0 1 10 33	89 58 32 00	+ 8 83	1 19 16	
	25	29 20	31	0 0 58 66	89 58 22 66	+19 33	1 18 00	Level 10″ out, adjusted
	27	29 75	33	0 0 58 00	89 58 22 66	+19 66	1 17 66	
Feb	12	29 50	37	0 0 52 00	89 58 33 33	+17 33	1 9 33	Level out again 11″
	19	29 40	31	0 0 36 66	89 58 10 66	+21 33	0 58 00	Corr by mural mark +21″ 83
	21	29 60	40	0 0 33 33	89 59 1 33	+12 66	0 46 00	Adjusted the circle's horiz axis
March	1	28 85	49	0 0 50 66	89 58 46 33	+11 50	1 2 16	No rain to day
	4	28 30	45	0 0 38 66	89 59 8 33	+ 6 50	0 43 66	
	24			0 0 34 33	89 59 18 66	+ 3 50	0 37 83	
April	3	29 60	52 5	0 0 32 33	89 59 15 33	+ 6 16	0 38 49	
	6	29 45	60	0 0 32 33	89 59 15 33	+ 6 16	0 38 49	
	15	29 60	55	0 0 22 33	89 59 20 33	+ 4 16	0 26 49	Current of air from door
	28	29 55	57	0 0 30 00	89 59 14 66	+ 7 66	0 37 66	
June	20	29 35	66	0 0 25 00	89 59 21 33	+ 5 33	0 30 33	After a solar observation
	21			0 0 26 33	89 59 26 00	+ 3 83	0 30 16	
	21			0 0 35 00	89 59 35 66	− 5 66	0 29 34	The level was first adjusted
	22	29 55	64	0 0 31 33	89 59 39 00	− 3 66	0 27 66	After the solar observations
	23	29 65	60	0 0 30 66	89 59 38 00	− 4 33	0 26 33	Level two divisions to right
	24	29 65	63	0 0 33 66	89 59 38 00	− 5 83	0 27 83	After solar observations
	29	29 20	68	0 0 38 00	89 59 31 66	− 4 83	0 33 16	Ditto
July	16	29 60	71	0 0 19 33	89 59 35 00	+ 2 83	0 22 16	
Aug	12			0 0 32 66	89 59 46 33	+ 4,50	0 37 13	
Sept	26			0 0 46 60	89 59 30 00	+11,30	0 57 90	Level deranged
	26			0 0 22 33	89 59 26 33	+ 5 66	0 28 00	Micros B and C put to A
Nov	17	29 15	45	0 0 31 66	89 59 12 66	+ 7 83	0 39 49	Corr by northern mark +5″ 0
	17			0 0 30 66	89 59 14 66	+ 7 33	0 38,00	Sun shining
	17			0 1 0 00	89 58 36 00	+ 9 50	1 9 50	Level purposely put wrong
	17			0 1 5 00	89 59 51 66	−32 50	0 37 83	Level further deranged
	17			0 0 26 33	89 59 7 00	+13 33	0 39 66	Level adjusted now
	17			0 0 25 66	89 59 4 00	+15 16	0 40 82	Very foggy now
Dec	6	29 45	45	0 0 29 00	89 58 46 33	+22 33	0 51 33	Before solar observations
	6			0 0 25 33	89 58 42 33	+26 16	0 51 49	After solar observations
	8	29,10	47	0 0 23 66	89 58 48 00	+24 33	0 48 00	Ditto
	18			0 0 29 66	89 58 42 66	+23 83	0 53 49	
	20			0 0 36 66	89 59 42 33	+21 00	0 57 66	
	21	29 10	41	0 0 33 33	89 58 44 00	+21 33	0 54 66	After solar observations.
	27	29 85	47	0 0 33 66	89 58 39 66	+23 33	0 57 00	Ditto
	30	29 75	35	0 0 33 00	89 58 44 40	+21 30	0 54 30	Ditto.
1828								
Feb	15		32	0 0 42 00	89 58 41 00	+18 50	1 0 50	Snow, level wrong.
March	1		47	0 0 29 33	89 58 32 66	+29 00	0 58 33	By the lamp
	5			0 0 19 33	89 58 46 33	+27 16	0 46 49	Snowing
	16			0 0 9 66	89 58 52 00	+29 66	0 39 33	
April	12	29 12	58	0 0 19 00	89 58 35 33	+32 83	0 51 83	
	13	29 04	53	0 0 18 00	89 58 16 33	+27 83	0 45 83	In the day
	13			0 0 18 00	89 58 50 33	+25 83	0 43 83	Midnight
	18	28 98	55	0 0 19 00	89 58 52 00	+21 50	0 43 50	
	18			0 0 36 33	89 58 31 66	+20 00	0 58 00	Float cleared of rust and mercury strained
	19	29 25	51	0 0 28 66	89 58 20 00	+35 66	1 4 33	Micros C and D put to A
	20	29 37	47	0 0 34 00	89 58 21 00	+32 50	1 6 50	An accident deranged the adjustments

8. *Vertical Collimator.*—The second construction of the collimator is represented by fig. 2. of Plate XXI., in which *A B* is the side of a mahogany frame, moving on four friction rollers on any horizontal plane, and carrying the superincumbent metallic parts along with it. At the centre of this frame, which is composed of a solid board and two parallel subjacent bars, is a large circular hole, into which a ring or circular edge bar of iron is driven fast, round which the collimator itself will revolve in azimuth. The large rim *C D* forms the circumference of a cast iron vessel that contains the mercury, and *E F* is a smaller rim forming the interior boundary of the vessel in which the fluid assumes the form of a broad ring between the said two rims of the containing vessel, cast in one piece with the bottom, on which the mercury lies. This circular vessel has therefore a round hole within the interior rim, which embraces the concealed circular edge-bar, made fast to the mahogany frame, and fits it so as just to turn smoothly round it when necessary. Another ring of cast iron, *G H*, lies with its flat side on the mercury, and floats upon it by virtue of its smaller specific gravity; this piece is called the *float*, and into its upper surface are screwed fast the extreme ends of two large cocks of iron, *I* and *K*, which hold the small telescope *L* between them in a vertical position, so that this telescope may also float, together with the cocks. The telescope's object end drops into the cavity at the centre of the rim *E F*, and may be seen by an eye directed upwards from below the frame that slides along a rail-way formed on the sides of two joists, lying above the superior end of the telescope that views the crossed slender springs in the focal point of the small telescope's object-glass; hence either a circular instrument or a simple vertical telescope having a micrometer, may stand under this collimator when it is in a situation to be used. The object-glass of the floating telescope, which in this construction has a tube, is made fast to a socket, that screws round the lower end of this telescope's tube, for the sake of adjustment to distinct vision of the cross, which adjustment is not easily accomplished but by frequent trials by candle or lamp light, since the strong light of a clear sky makes the cross invisible, if not partially excluded by a dark shade. After this adjustment is perfect the vision will be good, when the cross is well defined at any part of the field of view; which is the best criterion of good arrangement. A long rod of steel, *M*, is screwed fast into a plate attached to the wooden frame, and rises vertically high enough to hold another rod in its sliding clamp, that carries an adjustable reflector, turning on pivots to any inclination over the eye-piece of the vertical telescope, for the purpose of illuminating the cross, when a lamp is placed on the opposite end of the frame, facing the reflector. When this instrument is placed over the upper end of a vertical telescope, the illuminated cross will appear as a dark body, because the opaque side is turned towards the ground, and the horizontal wire of the observing telescope may be made to bisect at the same time both the external and internal angles formed by the cross; but if not, the tube of the floating telescope must be gently turned in azimuth till this will take place, while the said horizontal wire lies in the direction of east and west; then on turning the mercurial vessel and float together 180° in azimuth, it will be seen whether or not the cross is again visible; if not, the angle of inclination of the floating telescope's optical axis is too great, and will require adjustment, till the cross will appear near the middle of the field in both of the reversed positions. This adjustment may be made approximately by unscrewing one of the cocks, *G* or *K*, and pushing under it a thin slip of metal, such as will bring the small telescope nearer into a vertical position; or the object-glass itself may be in fault by its excentric attach-

ment, and may be turned half round, not by its screw which adjusts for distinct vision, but by turning the whole tube half round in azimuth when the vertical position is nearly adjusted, one or more of the small metallic weights may be slided by their central holes upon one of the upright cylindrical pins on the cocks *G K*, to balance the float more exactly; but it is not necessary that the cross should be found in precisely the same place in the field of the large telescope before and after reversion, for it will be seen presently, that a little inclination of the floating telescope's optical axis is serviceable in pointing out the exact zenith point on the limb, by circulating round it as the float is turned round. To prevent the float from remaining stationary while the vessel of mercury turns round, a couple of cylindrical iron pins are screwed into the edges of the float as pivots at opposite sides, which, entering a pair of slits made in two fixed pieces screwed fast to the bottom of the vessel of mercury, keep the float always in its relative situation, and concentric with the vessel, so that one portion cannot turn without the other, and yet the float is at liberty to vibrate on these pivots whenever it receives motion from the agitated mercury. Just above these pivots a second pair of cylindrical upright pins ascend from the float on which other circular weights may be put, to balance the float in a contrary direction, and to bring the centre of the cross into the middle of the field of view. The diameter of the outer rim of the mercurial vessel, *C, D,* is 11.75 inches, and that of the inner one only 4.25, so that the breadth of the circular surface covered by the mercury is 7.5 inches, allowing a ring of mercury of about an inch in breadth to remain uncovered at each side of the float. A japanned cover of tin encloses the outer rim, and has a chimney rising up nearly to the reflector, in the centre of which a circular piece of glass ground parallel, lies directly over the upper end of the floating telescope, which has a focal distance of eight inches, and allows the light to pass downwards in such quantity only, as is proper to render the cross visible by skylight, at the same time shading the superfluous light, and guarding the mercury from particles of dust, which might affect the position of the float, by operating as partial wedges. The round pin, projecting from the exterior rim, acts as a stop to its motion at the end of the semicircle, by sliding along the inclined slender spring *N*, till it falls into an angular notch that limits the excursion, and a similar spring lies at the opposite side, not seen, both which springs are adjustable to their diametrical positions, by screws passing through elongated holes.

9. The method of using the vertical collimator with a simple telescope, of either the refracting or Newtonian construction, furnished with a wire micrometer, is very easy, and may be thus explained let the telescope stand erect on its stand, or be fastened to a firm wall in an erect position, and be supplied with adjusting screws for position in two directions at right angles to each other, so that the tube may be made vertical as it regards both the east and west, and also the north and south points of the horizon; the feet screws of a tripod stand answer this purpose very well, when the centre of the telescope's motion is so placed, as to allow the tube to take a perpendicular direction, which construction we shall have occasion to show hereafter: then the collimator must stand on a perforated shelf, or slide between two joists placed horizontally, till it will take a position exactly over the centre of the object-glass, as nearly as the eye can judge; and when the light is reflected down the collimator, the place of the cross being adjusted as we have already described, it is more than probable, indeed it is almost certain, that the optical axis of the small floating telescope will have an inclination with respect to a true perpendicular line, this will be discovered by turning the mercurial vessel round an exact semicircle in azimuth, by

means of its stops, when it has been found that the centre of the cross has been displaced by turning, the middle point between the two positions will be the true zenith. This appearance will be better explained by reference to fig 3 of the same plate, in which a and a stand at opposite sides of the small circle described by the centre of the cross as the mercurial vessel revolves, the optical axis of the floating telescope having an inclination represented by $a\ b$ in the first position, and by $a'\ b'$ in the second; and the point at the centre of this small circle being the true zenith point, while the subjacent telescope retains its position, whether exactly vertical or not, the micrometer will determine the place of this point on its scale and divided head, by reading the quantities corresponding to the two bisections of the point of intersection, of the inner edges of the lines forming the cross, at the reversed positions respectively, for their difference will be the diameter of the small circle, and if we denote the first quantity by a and the second by a', we shall have $\frac{a+a'}{2}$ for the reading due to the zenith point or centre of the circle, and $\frac{a-a'}{2}$ for the inclination of the optical axis of the floating telescope, describing this circle or otherwise, if the micrometer's head be put to zero in the first position, by laying the moveable line over the fixed one, and if this double line be brought by the proper foot screw to bisect the centre of the cross, the moveable line of the micrometer will at once give in the second position the diameter of the small circle in seconds, one half of which will be the inclination and when the screw of the micrometer is turned back to read this half, the moveable wire will then bisect the true zenith point For instance, on the 23d of April we placed a zenith telescope with three equidistant foot screws on a solid stand, insulated from the floor of our observatory, under one of the openings above the vertical collimator, which we had previously adjusted, and having bisected the cross with the moveable line at zero by means of a foot screw, the reading in the reversed position was one revolution and eight hundredths of the micrometer's head, and as the value of one revolution with the telescope in question, viz No. 2. (§ XIX. 6.) is 46".12, the measure of the diameter was found 49" 8, and consequently the inclination or distance from the zenith point 24".9, then if a zenith star were to be observed passing the telescope in the position now given it, the micrometer would give the apparent zenith distance of it correctly, when the error ± 24".9, as the case might be, has been applied to the star's place, as read by the micrometer This quantity has been improperly called the error or correction of the *collimation* of the observing telescope, for it is evidently the inclination of the floating telescope's optical axis, and what is properly called the collimation in altitude of the large telescope is altogether out of the question, when thus used with a zenith micrometer.

10. When the vertical collimator is applied over the telescope of a vertical circle having reading microscopes, it will determine not only the inclination of the floating optical axis, as has been explained, but will show the *compound* error comprehending all the errors of collimation in altitude, of the level, of a deviation in the circle's vertical axis, and of the index or zero of the microscopes. We have made several experiments with our vertical collimator acting in conjunction with our altitude and azimuth circle by Troughton, described in the last section, and are now satisfied that what is usually denominated the error of collimation in altitude may be readily determined as correctly by it, without turning the circle half round in azimuth, as by

the horizontal collimator when the faces of the circle are reversed, or the collimator removed to the opposite horizon Fig. 3 will assist in rendering the explanation intelligible to any amateur astronomer. If, as before, we consider the small circle *a a'* to be that which is described by the floating optical axis, passing through the centre of the cross, the dotted line *c c'* passing through the vertical observing telescope to be the line of collimation, and 0, 50, *a*, *a'*, to be a portion of the graduated vertical circle, the point 90° of the arc not being coincident with the zenith point, indicated by the centre of the circle, it is evident that the true horizontal point lies below 0, the zero of the circle, and that the point 90° falls in the next quadrant, as well as the line of collimation, which we suppose adjusted to 90°. If then, a star *d* be observed by the telescope, and its altitude 50°, or zenith distance 40°, be read on the circle so circumstanced, the former will be indicated too much and the latter too little by the small arc or error, that is intercepted between the central point of the small circle (the true zenith) and the ostensible zenith (90°) shown by the circle, this error, from whatever inaccuracy in the adjustments it may arise, will, we affirm, be determined by the collimator, for, supposing altitudes to be read, half the sum of the readings *a* and *a'* will give the zenith point on the circle, and the difference between this mean and 90° will be the *error* required, and being applied as a correction, must have the sign + when the mean of the two readings is less than 90°, and the contrary but if zenith distances be read, the signs must be the reverse under the same circumstances. The simplest mode, however, of using the circle is, to read the zenith distance of the star first, and then determine the zenith point on the circle by the collimator, for in all cases the difference of the two will be the apparent zenith distance of the star, without reference to the adjustments. On examining the measure of the small circle's diameter, $a + a'$, on the 24th of April by the circle we found it 42″, on the 25th it was 34″, and on the 26th only 28″, the vertical axis of the circle being always kept adjusted by the level, that the experiments might be comparative; from which we conclude, that the angle of inclination in the floating telescope's axis, in this collimator, experiences variations depending probably on the state of the atmosphere; for, during the above stated interval of two days, the barometer gradually ascended nearly four degrees and the thermometer two This conclusion was confirmed by experiments made on the same days, with the simple telescope of a similar focal length with that of the circle, when adjusted to its vertical position under the same collimator; for though there was always a difference of about 4″ between the measures $a + a'$ of this telescope gained by its micrometer, and $a + a'$ given by the circle, from some cause not ascertained, yet the daily differences in the gradual diminution of the floating circle's inclination were alike in the two instruments. The error of collimation in the circle, determined under these variations of the inclination, continued notwithstanding the same during the two days, as nearly as could be ascertained by both collimators.

11. The application of the vertical collimator to a circle without reference to a level, a plumb line, or reflecting surface when a star is observed, and of the horizontal collimator substituted for a distant mark, may be illustrated by the following example

Example 1. On the night of the 28th of April, 1828, when the barometer indicated 29.75 and the thermometer 59°, the altitude of Polaris was observed at South Kilworth by the circle described in the last section in due adjustment, at 44 minutes before the star passed the meridian at the lower culmination, and was found 50° 51′ 53″; immediately after which observation the vertical collimator gave the zenith point on the circle 89° 59′ 40″.66 before reversion, and 90° 0′ 3″ after, from the mean of the three microscopes, and when the horizontal collimator was afterwards observed in the reversed positions of the circle, the mean readings were 89° 58′ 47″ and 0° 0′ 58″ let it be required to determine the latitude from each of the two collimators separately?

OPERATION BY THE VERTICAL COLLIMATOR.

Collimator's first position .	89	59	40 .66
2d ditto	90	0	3 .00
Zenith on the circle	89	59	51 .83
Observed altitude of * . . .	50	51	53 00
Observed zenith distance . .	39	7	58 .83
French refraction (Vol. I). .			+46 22
True zenith distance	39	8	45 05
Vol. I. Tab. IV. p. 271 . . .			− 4 .23
Cos 11° (=44^{m})=9 .9919466			
Polar dist. 96′.5=1 .9845278			
94′.727 . . . . 1 .9764739 +	1	34	43 .62
Polar distance subtract	1	34	38 .39
Co-latitude	37	34	6 66
Latitude	52	25	53 .33

BY THE HORIZONTAL COLLIMATOR.

Circle's 1st position . . .	89	58	47
2d ditto	0	0	58
Sum	89	59	45
Complement to 90°	0	0	15
One half of the difference	0	0 +	7 .50
Observed altitude	50	51	53 .00
Apparent altitude	50	52	0 .50
Refraction subtract			−46 .22
True altitude	50	51	14 .28
Corresponding zenith dist. .	39	8	45 .72
Computed polar distance . −	1	34	38 .39
Co-latitude	37	34	7 .33
Latitude	52	25	52 .66

In this example the horary angle exceeded 35^{m}, the extent of the table for the reduction to the meridian given at pages 99—101 of our first volume, and therefore the Greenwich table for determining the polar distance at the time, viz Table IV. at pages 271—273, was used instead. The inclination of the floating telescope's vertical axis $\left(\frac{\alpha-\alpha'}{2}\right)$ was 11″.165, and the correction of the observation $\left(90^{\circ}-\frac{\alpha+\alpha'}{2}\right)$ = 8″.17, which, applied to the observed altitude, would have given the same result.

12. EXAMPLE 2.—The pole star was again observed by the same circle in the reversed position reading zenith distances, on the night of May 1, 1828, when the barometer and thermometer indicated 29.82 and 54°, and to save the trouble of making reductions, the observation was taken on the meridian, the observed zenith distance being 39° 9′ 43″ 33, and the readings of the vertical collimator 89° 59′ 22″ 66 and 90° 0′ 15″, without reference to the position of the circle's axis (which happened to be about 3″.7 inclined towards the south), or to the error of collimation: let the latitude be determined again from this observation?

First position of the vertical collimator gives .	89°	59′	22″.66
Second ditto	90	0	15.00
The place of the zenith point	89	59	48.83
Observed zenith distance subtract	39	9	43.33
Apparent altitude of ⁎	50	50	5.50
French refraction (Vol. I.)			−46.85
True altitude	50	49	18.65
Apparent polar distance by the Nautical Almanac	1	36	33.54
Latitude by this observation	52	25	52.19

In this example the inclination of the small telescope's optical axis was 26″.16, and the error at the zenith of the circle + 11″.17, which, if added to the observed zenith distance, would have given the complement of the star's apparent altitude, and the latitude the same as it stands in the operation.

§ LXX. ON THE USES OF A PORTABLE ALTITUDE AND AZIMUTH CIRCULAR INSTRUMENT.

1. THE portable altitude and azimuth instrument, as now constructed by the best makers, differs very little from our instrument described in § LXVIII., except in size, and in having verniers instead of reading microscopes; it will not therefore be necessary to give a new figure for its description, nor any account of its adjustments, which are similar to those we have already explained. The diameter of each of its two graduated circles is usually about a foot, and a pair of opposite verniers indicate the two readings; but, when the diameter of the vertical circle is extended to fifteen or eighteen inches, three or four verniers may be used with it, according to the wish of the purchaser, or reading microscopes may be substituted for verniers, when the packing-box is well contrived for securing the delicate parts in carriage. The size must always depend on the accuracy that is expected from the use of the instrument, and as circles of eighteen inches diameter, and under, are usually divided by an engine, the expense of reading microscopes is generally avoided. When there are three or four verniers, each reading to the accuracy of 5″, the mean of the whole may be expected to give the truth within two or three seconds, or even less when the observation is repeated, so as to give an average: but as four verniers only perform the same thing twice over in the reversed positions, Mr. Troughton has lately preferred three, placed at 120° from each other, which require less time to read, and are considered competent to give as good a result. When the workmanship of this instrument is good, and the circles well divided, its powers may be advantageously applied to determine a variety of data, from which both time and the places of the heavenly bodies may be computed by direct logarithmic processes, which have generally been explained

by verbal rules, too numerous and sometimes too prolix to be well remembered. To particularize all these rules, and to illustrate them by appropriate examples, would engross too many of our pages, large as they are, particularly if we comprehend all that have been applied in nautical astronomy, as well as in fixed and temporary observatories, including geodesic operations. We propose, therefore, to arrange such a collection of formulæ, as will direct the logarithmic computer (which every practical astronomer must be supposed to be) to solve the different problems, that are most likely to occur in the use of this comprehensive instrument, beyond what belong exclusively to the transit-instrument already explained, for which it is the best substitute. When the portable altitude and azimuth circles are used out of the meridian, in a state of proper rectification, they are peculiarly adapted for giving, at the same instant, such measures as constitute the ground-work of various important computations, and on this account, as well as by reason of the transits that may be taken in the meridian, these instruments, considered as portable instruments, are peculiarly useful in voyages and travels of discovery, where a solid rock will supply the place of a pedestal or firm tripod, on any shore or place that may be accessible. When the daily rate of a clock or chronometer is required to be determined, without reference to the *absolute* time, it will not be necessary to regard either the exact altitude or azimuth of the star to be observed, for Mr. Riddle has shown that, if the exact times be noted when the star has the *same altitude* on successive days, either in the eastern or western hemisphere, by a clock or chronometer, going equably, the differences of the observed times, separately compared with $3^m\ 55^s.91$, the star's gain in a sidereal day, will give the gain or loss of such solar clock or chronometer, but if a sidereal clock be observed, the differences themselves will be the daily errors, and their mean may be taken as the proper rate. But as the long intervals that must elapse between such equal altitudes, taken on one side of the meridian, by an instrument remaining in a fixed position, will prevent opportunities of taking observations, that may be employed for gaining the *real* time, and other desiderata; a sextant and artificial horizon may be employed for gaining the rate; and then the circle will be at liberty to be used for other purposes. On whatever part of the globe the traveller may have occasion to place his instrument, there are always three points of reference in the heavens with which his observations are connected, viz. the upper pole of the equator, P, the zenith of his place, Z, and the point S apparently occupied by the star or other body observed. The meridian line in the heavens connects the pole and zenith, a circle of declination the pole and star, and a vertical the star and zenith; hence a triangle, which may be denominated the *astronomical triangle*, is always formed by the three lines PZ, PS, and ZS, which represent respectively the co-latitude of the place, λ, the polar distance of the body observed, Δ; and the zenith distance, z. If we call the horary angle formed at the pole, h; the azimuthal angle formed at Z, α, and the angle of variation formed at S, v, then when any three, out of the six parts of the spherical triangle, are given by the instrument and chronometer, or clock, the other three may be computed from the following formulæ, which afford several varieties of interesting deductions, where ϕ is introduced, in some of the more complex cases, as an auxiliary arc or angle, to simplify the formula, and to adapt it for logarithmic computation. In the following Table as many expressions are included, for directing the operations, as could be contained in an ordinary page.

No	Given	Required	Auxiliary Angles	Formulæ
1	z, Δ, λ	h	....	$\tan \frac{1}{2} h = \sqrt{\frac{\sin \frac{1}{2} (z+\Delta-\lambda). \sin \frac{1}{2} (z+\lambda-\Delta)}{\sin \frac{1}{2} (z+\Delta+\lambda) \sin \frac{1}{2} (\Delta+\lambda-z)}}$
2.	Δ, λ, α	h	$\tan \phi = \frac{\cot \alpha}{\cos \lambda}$	$\cos (h \sim \phi) = \frac{\cot \Delta \cos. \phi}{\cot \lambda}$
3.	z, λ, α	h	$\cot \phi = \frac{\cot z}{\cos \alpha}$	$\cot h = \frac{\cot \alpha. \sin \lambda \sim \phi}{\sin \phi}$
4	z, Δ, α	h	....	$\sin h = \frac{\sin z, \sin \alpha}{\sin \Delta}$
5	$z, \Delta, \lambda.$	v	....	$\tan \frac{1}{2} v = \sqrt{\frac{\sin \frac{1}{2} (\lambda+z-\Delta). \sin \frac{1}{2} (\lambda+\Delta-z)}{\sin \frac{1}{2} (\lambda+z+\Delta). \sin \frac{1}{2} (\Delta+z-\lambda)}}$
6.	$\Delta, \lambda, \alpha.$	v	....	$\sin v = \frac{\sin \lambda. \sin \alpha}{\sin \Delta}$
7	$\Delta, z, \alpha.$	v	$\tan \phi = \frac{\cot \alpha}{\cos z}$	$\cos v \sim \phi = \frac{\cot \Delta. \cos \phi}{\cot z}$
8.	$z, \lambda, \alpha.$	v	$\cot \phi = \frac{\cot \lambda}{\cos \alpha}$	$\cot v = \frac{\cot \alpha \sin (z \sim \phi)}{\sin \phi}$
9.	Δ, λ, h	v	$\cot \phi = \frac{\cot \lambda}{\cos h}$	$\cot v = \frac{\cot h \sin (\Delta \sim \phi)}{\sin \phi}$
10.	Δ, z, h	v	$\tan \phi = \frac{\cot h}{\cos \Delta}$	$\cos (v \sim \phi) = \frac{\cot z \cos \phi}{\cot \Delta}$
11.	z, λ, h	v	....	$\sin v = \frac{\sin \lambda. \sin h}{\sin z}$
12.	z, Δ, λ	α	....	$\tan^2 \frac{1}{2} \alpha = \sqrt{\frac{\sin \frac{1}{2} (\Delta+\lambda-z). \sin \frac{1}{2} (\Delta+z-\lambda)}{\sin \frac{1}{2} (\Delta+\lambda+z). \sin \frac{1}{2} (z+\lambda-\Delta)}}$
13.	Δ, λ, h	α	$\cot \phi = \frac{\cot \Delta}{\cos h}$	$\cot \alpha = \frac{\cot h \sin (\lambda \sim \phi)}{\sin \phi}$
14.	z, λ, h	α	$\tan \phi = \frac{\cot h}{\cos \lambda}$	$\cos (\alpha \sim \phi) = \frac{\cot z. \cos \phi}{\cot \lambda}$
15	$z, \Delta, h.$	α	....	$\sin \alpha = \frac{\sin \Delta. \sin h}{\sin z}$
16.	z, Δ, α	λ	$\tan \phi = \cos \alpha. \tan z$	$\cos (\lambda \sim \phi) = \frac{\cos \Delta. \cos \phi}{\cos z}$
17	$z, \Delta. h$	λ	$\tan \phi = \cos h \tan \Delta$	$\cos (\lambda \sim \phi) = \frac{\cos z. \cos \phi}{\cos \Delta}$
18	$z, \alpha, h.$	λ	$\cot \phi = \frac{\cot z}{\cos \alpha}$	$\sin (\lambda \sim \phi) = \frac{\cot h. \sin \phi}{\cot \alpha}$
19	$\Delta, \alpha, h.$	λ	$\cot \phi = \frac{\cot \Delta}{\cos h}$	$\sin \lambda \sim \phi = \frac{\cot \alpha \sin \phi}{\cot h}$
20	$z, \lambda, \alpha.$	Δ	$\tan \phi = \cos \alpha. \tan z$	$\cos \Delta = \frac{\cos z \cos (\lambda \sim \phi)}{\cos \phi}$
21.	z, λ, h	Δ	$\tan \phi = \cos h. \tan \lambda$	$\cos (\Delta \sim \phi) = \frac{\cos z \cos \phi}{\cos \lambda}$
22.	$z, \alpha, h.$	Δ	....	$\sin \Delta = \frac{\sin \alpha \sin z}{\sin h}$
23	$\lambda, \alpha, h.$	Δ	$\tan \phi = \frac{\cot \alpha}{\cos \lambda}$	$\cot \Delta = \frac{\cot \lambda. \cos (h \sim \phi)}{\cos \phi}$
24.	$\Delta, \lambda, \alpha.$	z	$\tan \phi = \cos \alpha \tan \lambda$	$\cos (z \sim \phi) = \frac{\cos \Delta. \cos \phi}{\cos \lambda}$
25	Δ, λ, h	z	$\tan \phi = \cos h. \tan \lambda$	$\cos z = \frac{\cos \lambda \cos \Delta \sim \phi}{\cos \phi}$
26	$\Delta, \alpha, h.$	z	....	$\sin z = \frac{\sin h. \sin \Delta}{\sin \alpha}$
27	λ, α, h	z	$\tan \phi = \frac{\cot h}{\cos \lambda}$	$\cot z = \frac{\cot \lambda \cos (\alpha \sim \phi)}{\cos \phi}$

2. Before the azimuth circle can be usefully employed in observations, its zero must be brought into the meridian, or otherwise its index error ascertained, by means of equal altitudes taken successively on the eastern and western sides of the meridian. In doing this it will be convenient to measure the altitude of the body observed approximately, in the first place, with the instrument in proper adjustment in all respects, and allowing for refraction at that altitude, the verniers must afterwards be set to some degree and convenient minute, that, with the addition of the refraction, exceed the observed altitude, then if the observer wait, till the star approaches the intersection of the vertical and horizontal wires, in the telescope's field of view, while moving the tangent screw of the azimuth circle to keep the star in view, the time must be noticed, and the subsequent beats counted till the contact takes place, the value of which, added to the time noticed, will give the exact time to be registered for the first observation, which should be at not less than three hours before the meridian passage. Provided the star be known, this requisite can be previously determined, by comparing the star's right ascension with the sidereal time at the moment. When the focal distance of the telescope is from twenty to thirty inches, its highest power will be competent to discriminate the time to a single second, when the star is observed near the prime vertical, or even in other situations when it is properly chosen for the purpose, with respect to its distance from the meridian, which distance is best when the star's motion in altitude is nearly a maximum. After the time has been noted, the azimuth circle, which we suppose at first erroneously placed, must be read, and the degrees, minutes, and seconds shown by the verniers put down. When the star has passed its greatest altitude, and is declining towards the west, its altitude must be watched by the instrument remaining in adjustment, or rendered again vertical by the level and foot-screw, if any inclination or reclination has taken place during the interval, but without touching the telescope, or disturbing its elevation. The star, which, on the eastern side of the meridian, appeared to descend to the horizontal wire, will now appear to ascend, and must be watched, and followed by means of the azimuthal screw of slow motion, till the contact again takes place at the central intersection; when the time must be noted as before, the azimuthal circle read, and the measure put down. From these two corresponding observations, carefully made, two essential particulars will be determined, the meridian point on the circle of azimuths; and the error of the chronometer or clock, supposing the time to have been indicated uniformly; for as equal azimuths correspond to equal altitudes, at each side of the meridian, it is obvious, that the greatest altitude, or altitude on the meridian, will take place at the middle of the observed horary interval, and also that the middle of the azimuthal arc passed over will be the point indicated, when that greatest altitude took place. Hence, when the verniers are turned back, to read the middle point of the azimuthal arc passed over, and the telescope has been cautiously depressed to the horizon, while its axis remains level, a fine mark, placed exactly in a situation to be bisected by the vertical wire, will be the meridian mark, as nearly as can be obtained by one pair of observations; and when the middle time is converted into sidereal time (Vol. I. pp. 334, 335 *), if not already in that denomination, the difference between this time and the star's apparent right ascension, on the given day, will be the error of the chronometer or clock and if one of the 60 stars given in the Nautical Almanac be chosen

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* See also the *Supplement to the Nautical Almanac*

for this operation, the trouble of computing the corrections, for obtaining the apparent from the mean right ascension, will be avoided. When the telescope has several horizontal wires placed parallel to one another, above and below the central one, the contacts may be taken at each, with the corresponding times, and azimuthal readings, and then as many corresponding ones taken beyond the meridian will give, in pairs, so many separate determinations, of which a mean may generally be taken as the most correct, when but little disparity exists between the corresponding pairs.

3. Should the sun be chosen as the object, in order to have the interval in the day, in preference to the night, his daily change of declination, which is considerable near the equinoxes, and but little near the solstices, will render a correction necessary, from our solar Tables 21 and 22 by Delambre, or from 23 and 24 by Zach, for obtaining which examples have been given in our first volume (pp 361—364). When the instrument is thus placed in the meridian, it will be near enough for determining the meridian altitude of any body that may be passing within the hours limited for observations, and comparative transits taken on successive days, when the same star is observed, will give the rate of the chronometer or clock, but to determine absolute right ascensions with accuracy, the position should be examined, and if necessary corrected, either by high and low stars, or by circumpolar stars, as the transit instrument has been directed to be managed in some preceding sections.

4. After the portable altitude and azimuth circle has been fixed in the meridian, and a permanent mark put up at a proper distance for distinct vision, observations may be taken *out of the meridian*, and at any horary angle removed from it, that necessity or convenience may require, and it is with a view to the reduction of these observations, that the formulæ, for determining all the values of the sides and angles of the astronomical triangle, have been introduced in this section, as a tablet of directions to the computer. A few computations, depending on some of the formulæ, will render the application of all the rest sufficiently easy, without verbal directions.

EXAMPLES

Given, the zenith distance of α Aquilæ corrected for refraction $= 65° 18' 45''$, and its corresponding azimuth from the north point $= 109° 27' 48''$, on the 20th of May, 1828, to determine the hour angle, and latitude of the place from the proper formulæ?

In the Nautical Almanac for 1828 the apparent north polar distance (Δ) of α Aquilæ is given $= 81° 34' 39''$, and its right ascension $19^h\ 42^m\ 26^s$, neglecting the fractional parts; so that we have the three parts z, α, and Δ as data, for determining the postulata by the formulæ 4 and 16 in the following manner.

Formula 4. $\sin h = \frac{\sin z . \sin \alpha}{\sin \Delta}$.

Sin z	65°	18′	45″	. log	9 9583724
Sin α	109	27	43	. .	9 9744486
or	70	32	17		
Sum . .				. . .	9 .9328210
Sin Δ	81	34	39	. . sub	9 9952906
Sin h	=60°	0′	0″	. .	9 .9375304
or	= 4^h	0^m	0^s	sidereal time.	
*'s R. A.	19	42	26		
Diff	. 15	42	26	=h, if in the east.	
Sum . . .	23	42	26	=h, if in the west.	

Formula 16. $\cos (\lambda \sim \phi) = \frac{\cos \Delta . \cos \phi}{\cos z}$,
and aux. angle, $\tan \phi = \cos \alpha . \tan z$.

Cos α . . .	109°	27′	43″	. . .	9 .5226798 −
Tang z .	65	18	45	. .	0 .3375408
Tan ϕ . .	−35	56	5	.	9 .8602206 −
Then cos Δ	81	34	39	. . .	9 .1657531
Cos ϕ. . . .	35	56	5	. . .	9 .9083167
				Sum	9 .0740698
Cos z. . . .	65	18	45	. . sub.	9 6208321
Cos ($\lambda \sim \phi$)	73	30	15	. . .	9 .4532377
Add ϕ . .	−35	56	5		
λ.	=37	34	10,	or latitude = 52° 25′ 50″	

If we suppose the apparent polar distance of the star to be unknown, and instead thereof observe the true time by the sidereal clock to be $15^h\ 42^m\ 26^s$, corresponding with the same corrected zenith distance and azimuth, observed by the instrument, the polar distance of the star and latitude of the place may be thus determined from α, z, and h.

Formula 22. $\sin \Delta = \frac{\sin \alpha . \sin z}{\sin h}$

Sin α	109°	27′	43″	log.	9 .9744486
Sin z	65	18	45	. . .	9 .9583724
				Sum	9 .9328210
Sin h	60	0	0	sub	9 .9375306
Sin Δ	81	34	39	. .	9 .9952904

As the zenith distance (z) was observed, the polar distance (Δ) is *apparent*, and will require the precession, aberration, and nutations to be applied to get *mean* Δ.

Formula 18 $\sin (\lambda \sim \phi) = \frac{\cot h . \sin \phi}{\cot \alpha}$.

Aux angle $\cot \phi = \frac{\cot z}{\cos \alpha}$

Cot z	65°	18′	45″	. .	9 .6624597
Cos α . .	109	27	43	sub.	9 .5226798
Cot ϕ . . .	35	56	5	. .	0 .1397799
Then cot . .	60	0	0	. . . . .	9 .7614394
Sin ϕ . .	35	56	5	.	9 7685368
				Sum	9 .5299762
Cot α . . .	109	27	43	. . sub.	9 .5482312
Sin ($\lambda \sim \phi$)	73	30	14	. . .	9 .9817450
ϕ sub.	35	56	5		
λ.	37	34	9,	or latitude 52° 25′ 51″.	

When neither the time nor azimuth of the star is known, at the place and moment of an observation of the zenith distance of a known star, taken out of the meridian; provided the latitude be previously known, or subsequently determined by means of the greatest altitude of another known star, or by the highest and lowest altitude of a circumpolar star, so as to complete the three sides Δ, z, and λ of the triangle, as it regards the star out of the meridian, which in our case is α Aquilæ, the horary angle, and consequently the longitude of the place, by the aid of a chronometer, as well as the azimuth and position of the meridian, may be determined by the formulæ 1 and 12 respectively, by the following processes

$$\text{Formula 1}\quad \operatorname{Tan}\tfrac{1}{2}h=\sqrt{\frac{\sin\frac{1}{2}(z+\Delta-\lambda).\ \operatorname{Sin}\frac{1}{2}(z+\lambda-\Delta).}{\sin\frac{1}{2}(z+\Delta+\lambda).\ \operatorname{Sin}\frac{1}{2}(\Delta+\lambda-z).}}$$

z . 65° 18′ 45″
Δ . . 81 34 39
146 53 24
λ . . −37 34 9
2) 109 19 15
Sin . . 54 39 37 .5 = log. 9 .9115507

Again z 65 18 45
λ . . 37 34 9
102 52 54
Δ −81 34 39
2) 21 18 15
Sin . . 10 39 7 5 log. 9 .2668072

Sum of numerators . . . 9 .1783579

z . . . . 65° 18′ 45″
Δ 81 34 39
λ 37 34 9
2) 184 27 33
92 13 46 .5
Sin of sup. 87 46 13 .5 log. 9 .9996711

Again Δ 81 34 39
λ . . 37 34 9
119 8 48
z . . −65 18 45
2) 53 50 3
Sin . . 26 55 1 .5 log. 9 .6558110

Sum of denominators . . . 9 .6554821

Subtract . . . . 9 .6554821

Extract the root . 2) 19 .5228758, radius being added.

Tan. 30° 0′ 0″ . . 9 .7614379 = tang $\frac{1}{2}h$
Multiply by 2

60 0 0 = h = 4ʰ 0ᵐ 0ˢ sid. time from the meridian, as before.
R.A. of ∗ . . 19 42 26 taken in the eastern hemisphere.

The diff. . 15 42 26 = sidereal time at the place on May 20, 1828.

The corresponding solar time was 11ʰ 48ᵐ 2ˢ 8, the difference between which and the Greenwich time, shown by a chronometer, will give the longitude of the place in true solar time nearly; and when the allowance in sidereal time is previously made, from the *Supplement to the Nautical Almanac*, for the difference of longitude, the result will be correct.

$$\text{Formula 12. } \operatorname{Tan} \tfrac{1}{2}\alpha = \sqrt{\frac{\sin \frac{1}{2}(\Delta+\lambda-z).\ \ \operatorname{Sin} \frac{1}{2}(\Delta+z-\lambda).}{\sin \frac{1}{2}(\Delta+\lambda+z).\ \ \operatorname{Sin} \frac{1}{2}(z+\lambda-\Delta).}}$$

$\operatorname{Sin} \frac{1}{2}(\Delta+\lambda-z)$ 26° 55′ 1″.5 log.	9 .6558111	$\operatorname{Sin} \frac{1}{2}(\Delta+\lambda+z)$ 92° 13′ 46″.3 }	log. 9 .9996711
$\operatorname{Sin} \frac{1}{2}(\Delta+z-\lambda)$ 54 39 37 .3	−9 .9115507	The suppl. 87 46 13 5 }	
Sum of logs. of numerators . . .	9 .5673618	$\operatorname{Sin} \frac{1}{2}(z+\lambda-\Delta)$ 10 39 7 .5	9 .2668072
Subtract	9 .2664783	Sum of logs of denominators .	9 .2664783
Extract the root. . . . . 2)	0 .3008835		

$\sqrt{}$ = 0 .1504417 = tan 35° 16′ 8″2, or tan ½ α.

Hence α = 70 32 16 .4 from the south point.

Therefore whatever may have been the accidental indication of the verniers on the azimuth circle, moving them carefully towards the south, over 70° 32′ 16″.4 will bring the telescope into the true meridian In like manner the other formulæ may be logarithmically applied to similar purposes, as occasion may demand. We have not used the angle *v*, as one of the data, which would have extended the number of the formulæ; because it is not obtained from direct observation it is notwithstanding very useful in a variety of computations connected with occultations, and is in this example, by any of the formulæ, = 35° 31′ 53″.

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§ LXXI A NEW PORTABLE ALTITUDE, AZIMUTH, AND ZENITH INSTRUMENT, MADE BY FAYRER [Plate XXIII Fig 2]

1. The figure numbered 2, in Plate XXIII., is a perspective representation of an instrument contrived by the Author of the present work, and made for him by Fayrer of White Lion Street, Pentonville, who has been long employed by Mr. Troughton in manufacturing sextants and small circles of different descriptions. The portable instruments that existed, when the plan before us suggested itself, had their powers limited by the smallness of their telescopes; and as repeating instruments could not give greater accuracy than the limited powers of vision would allow, after all the time and labour bestowed on taking and reducing observations made by them, a good telescope was chosen as the basis of the construction, and a new method of applying the verniers all round the divided circle, without the trouble of repeating the observation, otherwise than once in the reversed position of the circle and telescope, was adopted, as being more simple than the repeating principle, and yet sufficiently accurate for even the nicest purposes, that a small circle is capable of being applied to. for it will be seen that by this method three equidistant verniers, reading to the accuracy of 5″ each, will give twelve readings at two observations taken successively, and in a very short interval, at the two reversed positions. In determining the construction, it was necessary to guard against the bending of a telescope of 43 4 inches focal length, which was the one provided for the purpose; and also that it should be so mounted, as to be capable of reversion both in altitude and

azimuth, while it preserved the plane of the attached circle perfectly vertical, both before and after reversion. It was also requisite, that the axis of the circle's motion in altitude should be adjusted and preserved horizontal, at the same time that the axis of the azimuthal motion remained perfectly vertical; that the position of the verniers should be regulated by a good level, under the command of a clamp and tangent screw, and that all the parts of the instrument should be strong enough to bear the telescope and circle steadily, so that the adjustments might not be deranged by subsequent motions in elevation or in azimuth, and yet that the whole would admit of being readily dismounted, and the principal parts separated, for the purpose of being packed as a portable instrument. How far these preliminary conditions have been actually accomplished, will be seen in our following description of the instrument itself, for as it is at present unique, and the production of our own contrivance, we cannot with propriety speak of it in terms of encomium, but simply state the uses to which it may be advantageously applied.

2 The base on which this instrument stands is a tripod, composed of three strong arms of brass, standing as edge-bars, each ten inches long, and terminating with enlarged round ends, to admit the feet screws, and a slit, made horizontally across the female screw, produces an elasticity that makes the two parts open from each other a little, and keep the threads in close action with the feet screws these bars were cast in one having a strong circular piece at the centre, which is perforated, and also a circular edge-bar, twelve inches in diameter, concentric with it. A circle of twelve inches in diameter, having its limb strengthened by a circular edge-bar beneath, is fixed upon the circle of these arms, and the ends of the three feet are rounded in the lathe, to fit the three cups, formed at the upper ends of the solid bearing pieces, that may be placed on a pillar or other firm support. In the centre of the tripod a very strong tapering tube, or pillar of brass, is made fast to a thick flanch, which is then fixed to the tripod by three thumb screws, ascending from the lower face of the tripod, this fixed tube forms the perpendicular axis for the azimuthal motion, its length being upwards of two feet. Round this fixed tube an external one of larger dimensions tapering from three and a half to two and a quarter inches in diameter, revolves in nice contact with rings of bell-metal at the two extreme ends: this exterior tube carries the two opposite verniers at its lower extremity, and at its upper a bracket, or outrigger, on the two ends of which the horizontal axis of the vertical circle rests. The outer end of this bracket is supported by a strong brass bar, screwed to it, and also to the outer tube at about two thirds from the upper end, so that when the long tube is turned round, the circle and parts connected with it turn also into any required azimuthal position, indicated by the lower verniers, which read to 10″ each. The horizontal plate of the bracket has a couple of cocks ascending from it, one at each end, which receive the strong pivots of the circular and perforated bearing pieces, instead of *Ys*, the outer one of which has screws of adjustment for the horizontality of the vertical circle's axis. this axis, composed of bell-metal at the places of contact, has its remote end so confined in the bearing piece just mentioned, that no counterpoise is required to balance the circle, and its appendages, attached to the other end of it; for the plane of the circle lies only two inches and a half beyond the centre of the included pillar, and the horizontal axis is a solid body of eleven inches in length hence the position of this part of the instrument greatly resembles that of the Greenwich mural circle. The cylindrical bearing piece, near to the circle, is cut into two half cylinders, which may be pressed together

by the two thumb-screws, that appear in the figure, so as to be capable of fixing the circle in any given position, independently of the ordinary clamp of the circle, which has its tangent screw and clamping parts made fast to the revolving tube, below the edge of the circle, where the two milled heads of the fixing and tangent screws are clearly seen. The vertical circle is sixteen inches and a half in diameter, having its limb supported by eight conical radii, and is divided into 5′ spaces by the engine; the inner end of the horizontal axis is left with a shoulder thicker than the cylinder that appears over and parallel to the bracket, for the sake of strength where the weight falls, and the central piece of the three armed vernier circle revolves on a long bearing round this thick part of the axis. The vernier circle has an edge-bar surrounding its external face, which not only strengthens it, but affords the means of clamping it to the revolving vertical axis, in order that the vertical circle may move either with or without the vernier circle. This clamp is of a peculiar construction, having both vertical and horizontal motions, by means of two suitable joints; and being made fast to nearly the upper end of the revolving vertical axis, will either turn back from or take hold of the vernier circle, as occasion may require, the second clamp, having a tangent screw, is, as usual, attached to one of the verniers.

3. The instrument has two hanging levels and also a plumb-line, the last of which may be used or not, as the levels alone are competent to perform all that is required, in effecting and watching the adjustments; one of these levels may be seen hanging by its angular hooks round the bell-metal pivots of the vertical circle's axis, and the other is suspended by pins, fixed in a pair of cocks screwed to the face of the vernier circle, and carrying the requisite adjusting screws. The plumb-line, which is about two feet and a half long, is suspended from the centre of an adjustable plate, that covers the top of the long vertical tube, and, descending down the fixed pillar, is viewed below by a pair of microscopes, of the ghost kind, fixed at right angles to each other, so as to detect any inclination towards either of the two directions. The suspension plate might have been borne by the fixed pillar, but it was placed, as an experiment, on the top of the bracket plate, attached to the revolving tube, where the squared arbors of the adjusting screws may be more easily approached by a long key, but as the plumb-line suspended here must necessarily descend in the central line of the pillar and of the revolving tube, and is viewed by stationary microscopes, the delicacy, in completing the adjustment of this line to the exact central point, requires some time, and is rather a troublesome operation, but when adjusted the indication is very sensible. The reading microscope, with a reflector of plaster of Paris, will apply to any one of the pins carried by the three arms of the verniers, but a microscope constructed on the principle of a short telescope, such as is seen in figure 3, enables the observer to read with more convenience, as the heat of an illuminating taper or lamp regards his face, and this microscope clamps to the supporting bar of the bracket, and views from thence the divisions on the limb, and also the verniers brought to it in succession, after the observation is made, for the field of view is large enough, to take in all that is wanted in one position.

4. The telescope consists of three parts, the eye-end, the object-end, and a short middle piece into which the other two are separately screwed; the middle piece, which is a short frustum of a cylinder, has a flat square plate of considerable thickness made fast to one side of it, and four thumb-screws, one at each corner, fix it firmly to a similar plate, that forms the strong flanch of the vertical circle's axis, at the posterior face; and in this way, the optical axis of

the telescope being fixed parallel to the graduated face of this circle, the two are strongly united, and move or rest together. Then, to prevent the injurious bending of the telescope, a pair of brass cocks are attached to the telescope, at 16.5 inches distance from each other, just opposite the edges of the circle that are diametrically opposed to each other, and a pair of thumb-screws, passing through the projecting ends of the cocks, enter the solid ends of two conical radii, and thus fix the telescope's tube to the circle at two additional and distant points opposite to each other, and near the very parts of the tube that are most liable to flexure. Besides this protection against curvature, strong rings of brass forming diaphragms, are forced into the tube at different places, that not only brace the tube, but afford the means of balancing the opposite ends of the telescope when the object-glass and eye-piece are attached. Mr. Troughton has had occasion to notice, that when a telescope is bent alike at both ends, the errors produced in altitude will balance each other. The reading microscope, seen attached to the vertical axis, above its lower extremity, is intended to watch the perpendicular position of the telescope, when directed to the zenith, which it does by its spider's line bisecting a dot on a piece of silver carried by a cock, made fast to the telescope's tube near the eye-piece, which dot is adjustable by a pair of opposing screws. The tubular nozzle of a small lantern, inserted into the cylindrical opening in the middle piece, opposite the telescope's horizontal axis, affords light in the usual way, for illuminating the wires before the positive eye-pieces, that have various powers adapted to the telescope.

5. It is easy to perceive from our description, that this instrument, having tangent-screws for slow motion, both in altitude and azimuth, may be applied to almost all the purposes of practical astronomy, besides the ordinary purposes of a good three feet and a half achromatic telescope, with the various micrometers successively applied to it, as far as a clear aperture of 2.65 inches supplies sufficient light. But the chief uses for which the construction is best adapted, are those by which *time* and the *latitude* of the place may be accurately determined. As an equal-altitude instrument, and also as a zenith instrument, it is inferior to none of a portable size, and is competent to perform all the work that can be required from any instrument of the same dimensions; indeed the length of the vertical axis, as well as of the telescope, brings it into competition with many of the larger instruments on the score of accuracy, and yet when the five principal portions are dismounted, by means of thumb-screws with milled heads, and packed for carriage, they are contained in a box of 31.5 inches long, 16.5 broad, and seven deep, which will go under the seat of a travelling carriage. As a zenith instrument, used with a spider's line micrometer, the easy reversion in position, by the help of stops applied to the edge of the azimuthal circle, gives a facility to the operation of determining the latitude by a known zenith star, that yields not to that of the vertical collimator, when the levels, plumb-line, and reading microscope, co-operate in watching the position of the vertical axis, and confidence in the result is strengthened by these mutual checks, beyond what can be implicitly placed on any single resource hitherto employed. If hereafter the vertical collimator should gain a preference, in point of accuracy, over the level and plumb-line, the construction of this instrument is peculiarly adapted for availing itself also of this advantage.

6. The utility of the instrument for measuring equal altitudes also is quite unobjectionable; for, if we admit, that the telescope may be liable to flexure to an extent, that may render it unfit for taking absolute altitudes with great accuracy near the horizon, yet the double

clampings, of the circle, and of its horizontal axis, will detain the telescope any length of time, that can be required, in the same state of elevation, so long as the position of the vertical axis remains unaltered; for all that is required in such observations is, that the altitudes before and after the meridian should be precisely *the same*, without regard to their absolute quantities. The five horizontal lines afford the observer the opportunity of taking as many corresponding instants of time at each position, before the azimuth circle is released; and if a meridian mark be required, for gaining absolute azimuths, the middle point of the total arc, passed over during the interval, will give it very nearly, when a star has been twice observed at the intersection of the two middle lines, so that in the course of about eight hours, the travelling observer may obtain his time, his latitude, and the situation of his meridian, as directed in our last section, with but little trouble; after which he will be prepared to take altitudes and azimuths in the reversed positions of the instrument, with such other observations, either stellar, solar, lunar, or planetary, as may best suit his purpose. When the adjustments are all perfect, and the instrument brought into the meridian, by several pairs of stars, it will even become a tolerable substitute for a transit instrument, when delicately handled in giving elevation, not by the telescope, which should never be touched, but by the radii of the circle; or by the gibbet when turned quickly in azimuth, after which the tangent-screws will complete the observation.

7 The adjustments of the instrument before us are neither numerous nor complicated, being of the same nature as those of other altitude and azimuth circles, already described, and therefore need not be again particularized We must bear in mind that the vertical position of the upright axis of motion is the main object of attention, during a zenith observation with this instrument, when used in conjunction with a wire micrometer, and also when equal altitudes and equal azimuths are to be determined. but when collimated altitudes or zenith distances are obtained, the level of the revolving vernier circle is that on which the readings ultimately depend, and when the telescope is doubly clamped in altitude, its elevation will be steadily preserved, while the position of the level and of the attached verniers is reversed, and readjusted by the tangent screw, which is very convenient for this purpose After this reversion of the level and verniers, a second set of readings is procured, at each position of the telescope; and when the level is a good one, the twelve readings, by the three verniers taking four different positions, with only two sights of the telescope, approximate to the accuracy of the repeating principle, which we shall have occasion to explain in a future section. In general, however, six collimated readings will be obtained in the two reversed positions, without the revolving property of the verniers and level; and in many cases these may be deemed sufficient, when the time will not allow of six readings being taken at the first position, but after the observation has been made in the reversed positions of the circle and telescope, in the usual way, the other six readings may also be obtained, the first three while the telescope retains its second position, being securely clamped, and the second three when replaced in its previous position, by means of the verniers and level, which are competent to reinstate the telescope's original elevation. Since our readers have been instructed in the management of a tripod, and of a level or plumb line, by the method of mediating the errors repeatedly explained, the ordinary adjustments for vision, levelling, collimation, and the index will be as obvious, as they must now be familiar. We proceed therefore to give an example of the use of our instrument, applied to determine the latitude by the zenith observation of a star.

8. *Example* 1.—On the evening of July 28, 1824, the star β Draconis was observed, in passing the meridian of South Kilworth rectory, by the telescope of Fayrer's circle in due adjustment, with a spider's line micrometer by Troughton applied to it, that gives 45″.85 in a revolution; when, with face west, the measure of the star from the fixed line was 1.38 revolutions by one of the micrometer's screws, in a forward direction; and with face east, only 0.07 by the other screw, moved in a retrograde direction, the bubble remaining unaltered by reversion: let it be required to determine the latitude from this observation?

Operation.

Face west . . .	+1.38
Face east	−0.07
Mean towards S.	0.655 = 29″.95

	s			
Refraction				0″.00
Aber. with arg	7	12°	25′ .	−13.23
☾ nut. with arg.	0	14	9 .	+ 2.34
⊙ nut. with arg.	11	18	54 .	− 0.08
Sum of the corrections				−10.97

β Draconis, Jan 1, 1820	37	33	37.60
Four years' ann. var. add			+11.88
July 28, add more (TAB. XV)			+ 1.65
App. N.P.D.	37	33	40.16
Observed zenith distance towards the south			+ 29.95
Co latitude	37	34	10.11
Latitude	52	25	49.89

In this observation the star was so near the zenith, and the fixed line of the micrometer so far from the optical centre of the telescope, that the error of collimation exceeded one of the measures, which therefore became negative.

9. *Example* 2.—On the 28th of June, 1828, the sun's upper limb was in contact with the middle horizontal wire, at 5^m 50^s before true noon by a sidereal clock, with the plane of the circle turned towards the east, when the mean of the three verniers gave the apparent zenith distance 28° 54′ 13″.3; and when the face was turned towards the west, the zenith distance of the lower limb, on taking a mean of the same verniers, was found 30° 35′ 48″.3; there being at the time a considerable index error of the verniers, as they regarded the level carried by the revolving circle: then, the observation being concluded, the verniers were carried round through 180°, while the telescope retained its second position, and the bubble was adjusted by its tangent screw to the middle of the scale, as before, when the mean of the three readings was found 30° 37′ 33″.3; lastly, the position of the telescope was changed to its original situation, by turning the face of the circle back again, replacing the verniers at 28° 54′ 13″.3, as at first read, and adjusting the elevation by the clamped level and telescope's screw of slow motion; it was then doubly clamped, till the verniers had been turned through 180° by their level, when the mean of the fourth set of readings was 28° 52′ 40″· in the mean time the

barometer indicated 29.65 inches, and the thermometer 78° let it be required to determine the index error common to the verniers, and the latitude of the place of observation from the preceding data?

Operation.

Face east . . . 28° 51′ 13″.3 Face west . 30° 35′ 48″.3
28 52 40.0 30 37 33.3

Mean . . . . . 28 53 26.6 . . . . . 30 36 40.8 Sum 59° 30′ 7″.4 diff. 1° 43′ 14″.2

Then ½ sum = 29 45 3.7 is the collimated zen. dist. of the sun's centre, as observed; and ½ the diff. lessened by the sun's diameter (31′ 31″) gives the index error = 35′ 51″.6.

Hence we have . . . . 29° 45′ 3″.7 = app. zen. dist
+ 0 31.2 = refraction } Vol. I.
− 0 4.2 = parallax in alt. } Vol. I.
− 1 16.8 = reduction to the merid. } Vol. I.
− 35 51.6 = index error with contrary sign.

True zenith dist of the sun's centre . . . 29 8 22.5 reduced to the meridian.
Sun's declination from Naut. Alm. . . 23 17 30.0

Latitude of the place . . . . 52 25 52.5 disregarding the sun's latitude.

10. When equal altitudes and equal azimuths of the sun's centre are observed, it becomes necessary to notice both limbs, as they regard both the vertical and horizontal wires, and when there are five vertical, as well as five horizontal, several pairs of observations may be made both in azimuth and altitude. The ingenious author of the *Fasciculus Astronomicus* first recommended this mode of using the azimuth, altitude, and transit circle, and as he may be considered the father of such construction of an instrument, we will give his directions for using it advantageously in nearly his own words. "In the morning, two, three, or more hours before noon, let him" (the observer), says Mr. Wollaston, "point the telescope toward the sun, and a little above it, and, clamping the vertical circle, let him follow the sun till its upper limb touches the first horizontal wire Then, noting down the exact second of time, as shown by his chronometer, when that happened, let him follow the sun again till its upper limb just arrives at the second horizontal wire. After setting that down, as before, let him prepare for the third or central wire, by now clamping the instrument in azimuth likewise, and holding its adjusting screw between his finger and thumb, let him bring the preceding limb of the sun just to touch the third or central perpendicular wire, at the same instant that the upper limb just touches the third or central horizontal one. Noting that instant, and setting it down, let him now read off the azimuth marked on the azimuth circle, and set it down under the other, and then prepare for making the preceding limb to touch the fourth perpendicular wire, at the same instant that the upper limb arrives at the fourth horizontal one, setting that time down again, and reading off the azimuth again, and setting it down, let him do the same by the fifth wire each way, and record them as before. He will now find the lower limb of the sun, and

its second or following limb ready for observing in the same way at the first, second, and third wires let him make each perpendicular wire a tangent to the sun's last limb, at the instant that its lower limb just leaves the correspondent horizontal wire, setting down the time, and after reading off the azimuth, setting that down too under the other. After these operations, the instrument may be released in azimuth, and the lower limb alone be observed, as it quits the fourth and fifth horizontal wires respectively. As soon as the sun has thus passed all the wires the observer should read off, at both the microscopes (or verniers), the zenith distance or altitude at which he had clamped his circle, and also the indications of his barometer and thermometer for though he may have no occasion to regard the precise altitude at which he made these observations, yet, if any thing should deprive him of the correspondent ones, he may wish to have it in his power to deduce his time or his azimuth from these and reading the microscopes, after all is over, is attended with very little trouble. These things at first will appear hurrying, and till a person becomes a little accustomed to such observations they will certainly be so, but after a little practice there will be found time enough to go through the whole with ease, for the vertical circle remains clamped the whole time, and all the six azimuths lie much within the limits of their adjusting screw."

11. Leaving the instrument clamped in altitude, and well screened from the sun's rays, the observer must wait till this body is at the same distance from noon in the evening, before he can resume his task. In the interval the state of the adjustment for vertical position must be examined, and brought right if necessary, that the evening observations may be made under the same circumstances as the morning ones were. When the time has arrived, the same method of observing must be pursued that was observed in the morning, those wires being considered as the first which the sun's limb first approaches, the times of the different appulses of both limbs to the respective horizontal wires must be noted, and the azimuths read corresponding to the contacts with the perpendicular wires, as in the morning series. When all the corresponding altitudes have been observed in this manner, there will be ten pairs registered, and six pairs of azimuths, which must be properly classed, by taking the last of the morning in conjunction with the first in the evening, and so on till the contacts are all paired in their respective denominations. The method of deducing the time from the horizontal wires, by Delambre's or Zach's Tables of correction of noon gained by equal altitudes, has been explained in our first volume, and referred to in the last section, but the error in azimuth, arising from the sun's change of declination, during the interval between the morning and evening observations, will require a correction to be applied to the middle point of the horizontal arc, passed over in that interval. Delambre has given a formula for this purpose in the first volume of his *Astronomie*, p. 567, which is of easy application, thus·

$$\text{Cor.} = \frac{\frac{1}{2}(D'-D)}{\cos\text{lat} \times \sin\frac{15^\circ}{2}.(T'-T)}, \text{ or } = \tfrac{1}{2}(D'-D)\,.\,\sec\text{lat}\,.\,\operatorname{cosec}\frac{15^\circ}{2}(T'-T)$$

in which $(D'-D)$ is the change of declination, and $(T'-T)$ the interval in time. When the sun is advancing towards the north pole, this correction will carry the middle point towards the west of the approximate south point, but when he is approaching the south pole, it will carry the same towards the east, and must be applied accordingly.

12. If we suppose an observation to have been made on July 10, 1828, when the sun's daily change of declination was 7′ 50″, receding from the north, or tending towards the south, and $(T'-T)$ the interval $= 8^h\,0^m\,0^s$, then $\frac{15^\circ}{2}\,(T'-T)$ will be $= 60^\circ\;0'\;0''$, and $\frac{1}{2}\,(D'-D) = 78''\,33$, the change due to 4^h, or half the interval; hence, if we take the latitude at 51° 28′, we shall have

Log 78″.33	1 .8939281
Cosec 60°	10 .0624694
Sec lat 51° 28′	10 .2055330
Correction 145″ 19	2 .1619305

Now as the sun's daily change of declination is, at this time, increasing his distance from the north pole, or shortening the diurnal arc, this correction of 2′ 25″ 2 must carry the middle point of the observed arc towards the eastern side of the said point, to gain the true southern point for a meridian mark

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§ LXXII THE GREENWICH MURAL CIRCLE [PLATE XX]

1. A MODEL of this grand instrument was offered by Mr. Troughton to the President and Council of the Royal Society in the year 1806, and recommended to them as exhibiting the most proper construction that could be devised for the use of the royal observatory, but the instrument was not finished until six years afterwards. The whole fabric being very different from any thing that had preceded it, even the learned body above mentioned did not readily comprehend its nature, and much warm discussion took place before it was adopted The artist who proposed the new circle, had, long before the period above mentioned, conceived the idea of measuring polar distances from the pole itself, as a zero, to the utter exclusion of plumb-line or spirit-level. Before entering upon a description of the mural circle, it may be proper to give an account of the new principles which Mr. Troughton had in view at the time he first proposed it; when, having fairly before him every former construction of the circle, as well of those that succeeded as of those that had failed, he determined to adopt this peculiar construction, and gave to astronomy a new instrument

2 The artist contemplated with what simplicity a horizontal angle between two terrestrial objects is measured with the theodolite, where, without regarding any zero to reckon from, without reversion of the instrument, or reference to any secondary artifice, the thing wanted is comprehended between two points upon the limb, and at once obtained by the difference of two readings it was therefore easy to see, that a circle steadily fixed in the plane of the meridian would, without the assistance of plumb-line or level, give angular distances of the heavenly bodies, and from time to time show their relative positions respecting each other. For the purposes of geodesy, where the objects are at rest, the horizontal angle is generally all that can be wanted, but the celestial bodies being subject to various motions, to ascertain and

measure which is the intent of the instrument, an immoveable point to reckon from becomes indispensably necessary: without such a point an instrument might indeed show any change of place among the heavenly bodies respecting each other, but would be found defective, because altogether incapable of assigning locality to any individual body. The idea of a circle fixed to a wall, in the plane of the meridian, did not escape the artist, the steadiness, simplicity, and elegance of such a position had fixed it in his mind. All the different applications of the plumb-line, for proving its position or for supplying a zero, had been examined and rejected, and it was at no inconsiderable expense of time and thought, that the want of a better substitute was supplied. At length it occurred, that such an instrument was no other than the meridian circle of an equatorial, the polar axis of which is the axis of the earth produced to the starry regions. In the polar point nature has provided for the astronomer an unerring zero, and has given him an unexceptionable criterion for the position of his instrument, a point permanent, free from the errors of manipulation, and equally independent of the celestial bodies and of the instrument. Invisible indeed is this point, but precisely determinable, and easily referred to the instrument, by observations of the stars in its vicinity, which apparently revolve round it.

3. It must be mentioned that the polar zero is somewhat defective, inasmuch as it cannot show zenith distances, and of course cannot *alone* give the latitude of the observatory: to remedy this defect, a ten feet zenith micrometer was erected at the back of the pier, by which the point zenith could be found, and referred to the divisions of the circle. The zenith is no otherwise essential to astronomy than that its distance from the pole is the co-latitude, and also that it is the point of no refraction. The zenith micrometer, however, has never been used: there was something defective in the illumination, besides, it had been thought that, as it reversed by stops very quickly, there would have been time for making two corresponding observations of a star at the same passage; but this being found impracticable, the zenith sector of the observatory (one of the most perfect of the old instruments) was substituted. The zenith micrometer, however, made no part of Mr. Troughton's original plan; it was annexed to the mural circle to reconcile contending parties, many individuals of whom were shy of the polar zero, and insisted on a plumb-line being employed in some way or other. But there is another and better way of coming at the same thing; and which was strongly recommended by the artist before the instrument was made. We here allude to the method of observing by reflection from a plane of naked mercury. The same laws of nature which point the plumb-line, form also the surface of the fluid at right angles to it. But the plumb-line is the work of man, and the reflecting surface the appointment of nature as much as the polar zero. The zenith sector ranges about 6½° on each side of the zenith point, the micrometer scarcely one half of a degree, whereas the reflecting surface serves equally well from 15° above the horizon to 12° below the zenith, a range of 63° on each side of the zenith. Observations by reflection were not practised at Greenwich for twelve years after the mural circle had been put up, but since that time have been employed constantly. It was thought that such a mode of observation would be very troublesome, and that the air would seldom be in so calm a state as not to disturb the surface of the mercury: the former objection was obviated by a convenient gallery, which Mr. Pond contrived for the observer to stand in, and the latter never occurs but in rough weather. The polar zero is used to most advantage in high latitudes,

where the refraction at the pole is not great, and the small arc of the meridian, terminated by a circumpolar star, may be corrected by the Tables and instruments that give the density of the air, within a sensible quantity. A six-feet mural circle has been sent to the Cape of Good Hope, where the polar zero will fail. The south pole has not within several degrees of it a star that can be seen in the day time; and at the Cape, where the pole is not elevated more than 36°, if there were a star of the proper magnitude and distance, the uncertainty of refraction would make it of very little use; therefore at that place the horizontal zero must of necessity be employed. The Rev. Fearon Fallows, the present astronomer at the Cape, will not suffer much from being deprived of the use of the polar zero; for at Greenwich, where both zeroes are independently used, the same results are obtained, and thus the instrument demonstrates its own correctness. There are 126° of the meridian that may be seen by reflection, and any star observed in this way, and also by direct vision, gives the horizontal point, by simply bisecting the angle comprehended between the readings of the two observations. thus the horizontal zero is found by two observations of any star, and that without regarding the refraction, for it is obvious, that a star elevated by direct vision will be just as much depressed by reflection. But as both observations cannot be made by one instrument on the same day, the thermometer and barometer may have altered, and of course the difference of refraction must be applied in finding the horizontal zero· hence, for this purpose, high stars are best, because the refractions are smaller, as well as less liable to alteration, which is not the case with the lower ones. The horizontal zero has many beauties, one of which, and not the least of them, is, that as the uncertainty of the lower refractions vanishes when a great number of observations have accumulated, the method of observing by reflection becomes a criterion for ascertaining if there be any flexure in the instrument, and when there is, of ascertaining its quantity, and furnishing data for correcting it.

4 In making observations with the mural circle, the telescope and circle move round together, and while this is the case, no greater approach to accuracy can be made than that which the divisions command, for as far as the fixed stars are concerned, each of them for ages will be read off upon the same lines, and after the astronomer has multiplied his observations to a certain extent, he will, as far as those stars are concerned, have gotten out of his instrument all that is in it this is what might be said of every other instrument, excepting the repeating circle. But the mural has within itself a contrivance for approaching the truth much more nearly, than can be attained by the most accurate graduation. This is brought about by giving the telescope a motion concentric with that of the circle, and firmly clamping the former to any part of the limb of the latter. the number of changes, when six microscopes are read, is 720, and when the telescope is attached to any of these points, the effect, as far as graduation goes, is like that of a new instrument. For some years after the mural circle was erected, Mr. Pond made the change frequently; the stations at which he fixed the telescope made it point to the pole at 0°, 10°, 20°, and 30° on the limb, as shown by the zero microscope. He has, however, for many years used the instrument without this change, finding from experience that, when six microscopes are read, the errors of division become almost insensible.

5. In constructing this instrument the maker arranged all the parts, as he conceived, best suited to the polar zero, every thing was made subservient to that end; the horizontal zero with all its advantages indeed fully possessed his mind, but, like every other person at that

time, he deemed the operation by reflection a troublesome mode of observing; and great praise is due to Mr Pond for contriving a reflecting apparatus, by which the difficulty of that mode of observation is nearly obviated and it is worthy of remark, that whatever is favourable to one of the natural zeroes must be equally favourable to the other. All astronomical circles, whether they reverse on a vertical or horizontal axis, may be used to advantage by employing the two natural zeroes, although their construction is not well adapted thereto, the artificial zeroes of the plumb-line, and of the spirit-level, are what belong to them. In old observatories of the most complete kind, such as Greenwich was, and Oxford is, there is one quadrant for the south meridian, and another for the north; these become united into one instrument by the intervention of a zenith sector, and may use the polar zero, but certainly under obvious disadvantage, because of the great number of readings required to effect the purpose. The horizontal zero cannot be employed. There is in the King's Observatory at Kew a quadrant, or rather mural arc, of six feet radius, which seems peculiarly adapted to the use of the polar zero. The arc is continued beyond the zenith, towards the south, by some degrees more than the co-latitude of the place; and therefore Polaris may be observed by it both above and below the pole. But such is the slow progress of both art and science, that neither the maker of the instrument (Sisson, Jun.), nor the astronomers who have used it, seem to have had any idea of the polar zero; for, as we understand, the position of the arc has always been ascertained by the plumb-line, and the collimation of the telescope found by the zenith sector.

6. There has been at the Royal Observatory, for several years, another mural circle of the same dimensions as the former one, and nearly a copy of it; this was constructed by Mr. Thomas Jones. The two circles are placed in the observatory with their axes in a right line, where, with their faces opposed to each other, at about seven feet distance, they seem to regard each other as antagonists; yet is there the same cordiality between them, as there has subsisted between their respective makers for many years It was said above, that the mural circle of Troughton demonstrated its own correctness, by giving the same results whichever of the two natural zeroes was had recourse to: as much may be said in favour of Jones's, for they both give the same results from one horizon to the other; that is, within a reasonable allowance for inequality of division, and the unavoidable effect of partial temperature. The two circles, TROUGHTON and JONES, so called in the columns of the Greenwich observations, may be used either separately, or as one instrument; for, in the latter case, an observation may be made with T by reflection, and another with J. at the same instant, by direct vision, on the same star The two telescopes are thus alternately pointed to the direct and reflected object, and bring out the true horizontal zero, independently of refraction, even by the lowest stars: the circle J. may be considered as T. reversed on either a vertical or horizontal axis, and an index error applicable to all the readings of each circle is derived from the observations, as will be explained hereafter.

7. The mural circle and its inventor have been severely censured because it did not perform well as a transit instrument. The author of this work has in his possession a copy of Mr. Troughton's three communications to the committee of the Royal Society, in which there is not a word said about the circle's use for this purpose. in fact, the instrument was in great forwardness before such a requisite was thought of. Dr. Maskelyne first suggested the idea, he being then grown feeble from age, so as seldom to go into the observatory, and Mr. Troughton, con-

sidering how desirable it would be, could both the N. P D. and Æ be observed by the same instrument (there being at the royal observatory at that time but one assistant), at the astronomer-royal's request promised to use his best endeavours to make the circle perform as a transit instrument, and Æ wires and other contrivances were added, but if, in this respect, the instrument had ever a fair trial, they were added in vain

8. *Description*—The principal figure of Plate XX represents, on a scale of about one eighteenth of the real dimensions, the front or eastern face of the wall, with the circle and the greatest part of the apparatus attached to it The wall is in breadth from north to south seven feet, four feet in thickness from west to east, and ten feet high it is formed of four stones laid one upon another The third stone has in its under side a semi-cylindrical groove cut in its middle from west to east of six inches radius. The upper side of the second stone, being worked smooth and level forms the diameter of this semicircular *arch-hole*, and supports the axis-work of the circle, at about five feet above the floor the real centre of the instrument, however, is about five inches higher. The nucleus of the Greenwich circle is an octagon of eight inches diameter at the corners, its depth is three inches, and a circular perforation of six inches and a half is made through its whole depth. The outward faces of the octagon, which are each three inches square, support eight of the circle's conical radii, to which they are screwed and steady-pinned. The other eight radii are fitted in closely, each one between two of the former, by filing them on opposite sides, till their lower ends come down and fit upon the corners of the octagon. The eight first-mentioned cones are solid, for half an inch at the faces of the octagon, and strengthened within as far as where they separate; the other eight are solid above the point of separation, and strengthened three inches upwards, each of the former is fastened to the octagon by two screws, the heads of which are concealed within it, and each of the latter by one very strong screw, long enough to reach the solid part of the cone, so as to bring it in close contact with the strengthened sides of the adjoining cones, and also with the contiguous corner of the octagon. In former constructions of the circle, the nucleus was a polygon of as many sides as there were radii in this there are sixteen; and, looking with a mechanical eye at the figure, perhaps no one will think that there are too many, but if so, it becomes a matter of some difficulty, how to support the centre-work between the axis and a polygon requiring to be sixteen inches in diameter. To support it, however, may not be impossible, though perhaps hitherto not done, but at any rate, it would exhibit an awkwardness disagreeable to the eye.

9 The limb of this circle consists of two rings, the interior one having its plane parallel, and the exterior one perpendicular to the plane of the circle, so that when united their section will be represented by the letter T. The interior, or flat ring, has the appearance of passing through clefts in the middle of the upper ends of the sixteen radii the cones are made solid, for about four inches at their upper ends, a part of which is cut away in each for receiving the flat ring, to give the latter a central position, and the solid part of the former, which was cut away, being reduced to the substance of the part remaining, these parts are then united by screws which fasten the circle between them. The perpendicular ring is fitted close upon the exterior edge of the other, being also in firm contact with the ends of the conical radii, to which it is screwed, as well as to the other ring. There is a circle of bracing bars, which being interposed bind the cones together, at half the distance from the double ring to the centre.

The circular aperture of the octagon is shut up by plates both before and behind, which plates are fastened to the octagon by strong steel screws; the posterior plate has a large circular hole, and the anterior a smaller one; these circular openings are worked with the greatest care, for into them the axis of motion is fitted, and, by means of screws, the latter is made one with the octagon and circle. The axis is a cone of brass, nearly seven inches in diameter in front, but behind only about half as much, and nearly four feet long. This axis works in a socket, which at each end receives it, and in which it fits with the greatest possible exactness: the two parts which fit the axis, are soldered into a strong brass tube, of greater dimensions than the tube of the axis, but nearly of the same shape. On the tube of the sockets, in front, is soldered a very strong perforated plate, or upright bearing piece, at right angles to the axis; which nearly fills the semi-cylindrical aperture in the wall, and at the remote end is soldered a short cylinder, the use of which will be shown below: it is there that the adjustments for placing the circle in the meridian, and for levelling the axis, are performed. Upon the horizontal surface of the second stone, which forms the diameter of the perforation through the wall, are fastened two strong horizontal plates, one before, and the other behind; the bearing piece of the socket in front only rests upon its plate, but behind the bearing cock and plate are screwed to each other: in front the plate and bearing piece are connected by a conical piece of hardened steel, which is fixed under the middle of this piece, and fits nicely into a hole in the plate, but so as to revolve. At this end of the axis these parts do not come quite in contact, for there are fixed under the bearing piece at each extremity, about ten inches apart, two short props, like buttons, of hardened steel, the spherical surfaces of which rest upon planes of hardened steel fixed in the plate. The central conical piece prevents the circle from sliding sideways from its place on the wall, when angular motion is given round this conical piece to bring the instrument into the plane of the meridian. The parts last described, being made of hardened steel, were intended to reduce the friction, in order that they might the more promptly produce the action of the adjustment. It was said above, that a short cylinder was soldered on the remote end of the cone of the sockets: this passes into a perforation in the cock behind, which perforation is of greater diameter than that of the cylinder. Two strong fine-threaded screws, at right angles to each other, work in the cock; one vertically, to elevate or depress the axis in levelling, the other horizontally for meridional adjustment: the two screws only press with their points against the sides of the short cylinder, but opposite to them are the ends of two small cylinders, standing in the same line, which, being urged forwards by strong spiral springs, force the short cylinders into contact with the screws; thus, when either of the screws advances, the opposite cylinder retires, and when the former retires the latter advances.

10. The telescope is shown in fig 1, elevated above the south horizon about 34°: its focal distance is six feet two inches, which is exactly equal to the exterior diameter of the circle; the aperture is four inches, and its common magnifying power about 150. The telescope is attached to the circle at the centre by a steel axis, which passes through the proper axis of motion from end to end, and was indeed the arbor on which the axis was turned: the motion of the telescope round its own axis is, therefore, perfectly concentric with that of the circle. The weight of the telescope is supported upon its own axis, and for the purpose of fixing it in any position respecting the circle, there are two clamps, one at each end, which bite the exterior

border of the circle, and unite them during a series of observations, as before mentioned. The clamps are seen in both figures.

11 The graduation of this instrument is made on the broad surface of the exterior ring, which we have said is a plane at right angles to that of the circle, and therefore the reading microscopes have their direction parallel to the latter plane From the plane surface of the second stone, upon which the instrument rests, to the central point of the axis is a space of about six inches of brass this, from its alteration with change of temperature, will cause the circle to rise and fall as it regards the microscopes, which are fixed to the wall, but to prevent the errors occasioned by this expansion, the microscopes are so fixed on plates, as to compensate it the two upper plates are wrongly represented in fig. 1, the draftsman (perhaps for the sake of uniformity) having placed them to act in the contrary direction. The divisions are made upon a narrow ring of white metal, composed of four parts of gold, to one of palladium, and the figures which count the degrees are engraved upon a similar ring of platina neither of these metals tarnish in the least degree, which property renders frequent cleaning of the surface (which in time wears out fine divisions) quite unnecessary. These divisions are by lines, not dots, and suited to acutely-crossing wires in the reading microscopes the degrees are cut into twelve parts, or 5′ spaces, and are numbered from the pole southward to the same pole again, viz. from 0° to 360°. The 5′ spaces are sub-divided by the microscopes to single seconds, and a division representing this quantity on the micrometer-head, may easily be estimated to the tenth of a second. There are six reading microscopes, which, when they are all read, cause much labour and loss of time, on which account the two horizontal ones only are often used; but three are much better than two; and in this case but one of the horizontal ones should be used. If in employing combinations of three, in making a series of observations, the other three were alternately used on the same star, half the trouble of reading would be saved, the same index error obtained, and the errors of division, and of partial expansion, reduced very nearly as much, as if the six microscopes had been constantly read

12 In order that the circle should move round easily on its axis, there is an apparatus for counterpoising it, or for lifting the whole weight, without which the load would press altogether on the lower side of the front socket. This is brought about by means of two large rollers, shown below the axis in fig. 1 the rollers, fixed in a double frame, act upon the edge of the centre flanch, nearly in contact with the radial cones. Two perpendicular bars of steel, at about the height of the centre, are connected by hook and eye with the frame of the rollers, and these bars, in a similar manner, are suspended by two beams, each resembling a common balance, at the top of the wall. The part which appears in front is well enough represented in fig. 1, and one of the beams, its fulcrum, and counterpoising weight are shown in dotted outline near the top of the wall in fig. 2. Now these parts of this apparatus, being all pendent and pliant, produce a simple lift without the least tendency to influence the due motion of the circle's axis in any direction.

13. There is another flat circular ring, somewhat larger in diameter than the graduated one, which has its position nearly in contact with the wall, to which it is fastened at several places on this ring the clamp and screw for slow motion slide, and may be clamped to it in any part of the whole ring, at the other end of the moving screw, is a clamp which bites the

circle, and both clamps being set the screw gives motion to the circle, the clamp of the fixed ring acting as a fulcrum. By this contrivance, the observer has it in his power, to place the slowly moving apparatus either above or below the telescope, and at any distance from the eye-piece that he may find convenient The ring at the back of the circle is divided roughly, so as to show polar distances, and an index to this graduation was fixed to the telescope, for the purpose of pointing the latter to a star, whatever its position might be respecting the proper circle: but as all the changes of the telescope have been through arcs of 10° nearly, it was easy to apply this, without computation, as an index error to the divisions of the circle, the graduation of the fixed ring has, therefore, never been used.

14. The plumb-line of the mural circle is for the purpose of adjusting the axis truly horizontal: it is shown in fig. 2, but would have been much better exhibited, had the telescope been represented in a perpendicular position. The apparatus, from which the plummet is suspended, applies by dovetail fittings occasionally to the wall near the top, directly over the centre, as seen in fig. 1, between the suspending rods, and the apparatus itself is seen in fig. 2. In the former figure, at a small distance from each end of the telescope, fixed microscopes are represented, attached to its tube, for viewing the plumb line; these are nearly similar to others that have been before described, as used for similar purposes. The microscopes have their axes parallel to the plane of the circle, and in the middle of their length have their tubes cut away beyond the centre, to allow the plumb-line to pass through the axes of vision without touching any thing: the apparatus has a pinion motion at the centre of suspension, by which the plumb-line is brought forwards, so that it may not be injured while the circle and telescope are reversed for adjustment. The plummet has its situation several inches below the circle, where it swings in a water-vessel, that stands on a tripod, placed upon the floor.

15. The wires of the telescope are illuminated by a diagonal reflecting plate, fixed in the middle of the tube, which receives the light through a circular aperture seen in fig. 1, exactly in a line with the centre of the circle. A lantern at four or five feet distance, placed in the line of the axis, throws light on the field of view, equal to the brightness of day-light, which brightness may be regulated and modified by coloured glasses, that apply to the aperture of the tube; perhaps, those of a green or blue colour produce the most pleasing ground, that a star and the wires can be seen upon.

16. It has already been said that the zenith micrometer, applied to the back of the wall, has never been used at Greenwich; but as it was a new instrument, at the time it was made, and as it forms a prominent feature in fig. 2, it seems to deserve a description. The tube is ten feet in length, and five inches in diameter, and is a reflecting telescope of the Newtonian kind: it has, from end to end, half its diameter let into the wall, as shown in the figure. The tube of the telescope itself forms the vertical axis, on which the reversion is made, for measuring zenith distances with the micrometer, both to the east and west: at the lower end it turns on a hollow pivot, which works in a socket fastened to the wall; at the upper end, the tube is embraced by a collar, worked circular with the greatest care; which collar bears against an angle made in a strong plate, connected with the wall, and is pressed by two springs, into close contact with the angle. There are three spring rollers affixed to the tube, which roll on a horizontal circular plate at the top, these relieve the pivot below from a great part of the weight, and assure an easy and steady motion. The large speculum at the lower end is fixed in the tube so

as to form a small angle with the axis, in the manner that Sir W. Herschel constructed his larger telescopes: this mode was adopted in order to place the small mirror about half way between the axis of the tube and the wires of the micrometer; the plumb-line occupies the axis of the tube, and when every thing is adjusted, the instrument may be reversed, or turned half round, without giving sensible motion to the plumb-line; which expedient prevents the loss of time that an excentric plumb-line requires in coming to rest. The large mirror is perforated through its centre, for the plumb-line to pass through; for the plummet and water-pot have their station much below. The lower end of the plumb line is viewed as bisecting the luminous discs of two microscopes, placed at right angles to each other; at the upper end it is suspended by an apparatus, in which there are adjustments for making it central, and which do not intercept many of the incident rays. The parallel rays from a star, falling upon the large mirror, are returned in a converging state upon the small one, by which they are turned into a horizontal direction, and come to a focus in the field of view of a parallel line micrometer, where the image is seen by an eye-glass magnifying about 100 times. It is evident that the range of observation in this instrument is confined within the field of view, which is not more than 15' on each side of the zenith; yet this is amply sufficient for the purpose it was intended for.

17. *Adjustments.*—To render the axis horizontal, place the telescope perpendicular by estimation, the anterior plumb-line being in its place; by the pinion motion at the top bring the plumb-line upon the luminous disc of the upper microscope, and by turning the circle round a little, procure distinct vision of the line and disc, and then bisect the latter correctly by the former. If the disc of the lower microscope is not now bisected by the plumb-line, it must be made so, by turning round the cell, in which it is excentrically set, within the tube. The circle must next be turned half round, which is best done by its own divisions and by one of its microscopes, when the opposite end of the telescope will be uppermost: now, by the pinion, bisect the disc as before, when, if the lower disc is also bisected, the axis will be level; but if not, one third of the apparent error is the real error belonging to the position of the axis, the other two thirds being divided between the microscopes and circle on reversion: this third part of the apparent quantity must therefore be corrected by estimation, by the proper screw at the remote end of the axis: when the quantity of error is found great, this process should be repeated. The adjustment for collimation, or for making the axis of vision parallel to the plane of the circle, is effected as usual by two opposite screws, that act on the wire plate. Where there is a good transit instrument, as at Greenwich, the easiest way of adjusting the axis is, to make a zenith star pass the meridian at the same instant that the star transits the middle wire. In like manner, the circle may be adjusted to the meridian by comparison with a transit: in this case a star near the horizon, either to the north or south, must be chosen; and its passage over the wire made to coincide, in time, with that over the adjusted transit-instrument, by means of the screw at the remote end of the axis, which gives to the latter horizontal motion. This instrument is however complete in itself: and the astronomer, without the assistance of a transit, will find little difficulty in performing the two last-mentioned adjustments. From observations of three stars of known right ascension, but differing in declination, he will have sufficient data for separating and correcting the errors that may belong to each, as we have before explained (§ LVIII. 4.). The collimation of the instrument, as it respects polar distance, is performed by setting the reading microscopes to show the apparent place of a known star (Po-

lairs is the best, because of its slow motion) and then by moving the telescope independently, of the circle, by means of the parts already described, until the middle horizontal wire bisects the star. In this manner the adjustment may be made correct to within a few seconds, which is near enough, for it will ever be necessary to employ some *index error*. There is an adjustment in each of the reading microscopes [§ XLVIII.] by which the remaining error might be brought to zero, but which has been exclusively used for setting those microscopes at exactly 60° distance from each other Neither is the latter adjustment required to be perfect, for, any small discordance in the places of the microscopes, will merge in the mean of the readings, and, commingled with the former, will produce a simple index error The microscope placed at the north end of the horizontal diameter is called *A*, and the one opposite to it *B*; the two superior ones are denominated *C* and *D*, and the two inferior ones *E* and *F*, so that if three only were read, they must be those that form an equilateral triangle, viz. *A C E*, or *B D F*, and when these are read alternately, in any series of observations, the index error will be obtained as well as if all the six were read at each observation. It is convenient to make out a list of stars that are intended to be observed in any series of observations, and to tabulate the degrees and minutes that will be read by microscope *A* on the circle for each star, when it passes the meridian, accordingly as it is intended to be observed by direct vision or by reflection, to facilitate the setting of the telescope to the proper altitude or depression, for finding the star, or its image. In other respects the observation of a star is made during its bisection by the middle horizontal wires, as in other circles.

18. In this account of the mechanical construction and adjustments of the Greenwich mural circle, we have entered into detail more minutely than we should have done, partly because this magnificent instrument has never been before described, (though an engraved representation has been given in some of the volumes of the Greenwich observations,) and partly because we have been favoured by the inventor himself with the particulars, that constitute the preceding part of this section, which we have given chiefly in his own language. For the first twelve years, from 1812 to 1824, the polar point was regularly determined by a circumpolar star, as the zero of angular measurement, and the differences of the measured distances of the various stars from such point, constituted so many intercepted arcs, which were at first measured on different parts of the circle, and compared with each other; from which comparisons it appeared that the circle was in all respects as complete as art could make it. Since February 1825, when the second circle by Thomas Jones had been finished and adjusted, the reciprocal method of observing by a combination of the two circles has been chiefly practised, where one instrument has been used by direct vision, and the other by means of reflection, and as this method is new, we propose to give an explanation of it supplied by the present Astronomer-royal. "A mural circle," says Mr Pond, "is merely a *differential instrument*, it can measure nothing but the angular distance between two given points: by an extension of this principle the angular distance between an object and its reflected image may be obtained. But to apply this principle to astronomical purposes, we must suppose the instrument to remain perfectly in the same state, wit respect to its surrounding microscopes, for at least twenty-four hours, and in most cases for much longer period. Moreover, the frequent occurrence of windy or cloudy weather so limit the employment of this method, that for many important investigations a sufficient number observations can seldom be obtained within the year; both these objections cannot fail to b

greatly diminished by the introduction of the new circle, and perhaps the advantage in the latter case will be found to be even more important than in the first. When a star has been observed directly with one instrument, and its reflected image with the other, the relative state of each circle with respect to its microscopes being quite unknown, it is evident that sufficient data for deducing the altitude are not supplied by this double observation. It becomes absolutely necessary at the same time to investigate the relative position of one circle with respect to the other now this can only be done by observing a number of stars on various points of the meridian with both circles in a similar position, and in proportion as their relative differences are well determined, so will the required altitudes be correctly deduced. The altitude of a star thus found gives the horizontal point in each circle, and thus an index error may be obtained for each of them by a process purely mechanical, without the aid of any astronomical consideration whatever. By means of a number of horizontal points determined in this manner by stars at various altitudes, the index error of each circle may be ascertained to any required degree of accuracy. To derive, therefore, the greatest possible advantage from this combination of the two circles, it is necessary to institute two distinct series of observations, each equally essential for the correction of the other. The stars, which are observed in the same manner with each instrument, give differences, from which the altitudes of those stars are deduced, which have been observed reciprocally, and the stars, which have been observed reciprocally, give horizontal points, by means of which the apparent places of those stars may be determined, which have been observed for the purpose of obtaining differences."

19. The astronomer-royal has given an example, in the end of that number of the GREENWICH ASTRONOMICAL OBSERVATIONS, which was published for the months of April, May, and June of the year 1825, by which the index errors and horizontal points of the two mural circles are deduced from one day's observations, and the north polar distances computed; but as the sheet, containing the work, is too large to be comprised conveniently in our volume, even as a folding sheet, and as it explains only two of the methods of coming at the results, we propose to introduce shorter examples of three different methods, presuming that all practical astronomers have access to the sheet referred to, in which the numerous columns are spread over a wide surface. We will give, for a specimen of a day's work, the mode of registering the observations made by each separate instrument on July 3, 1826, as given in the Greenwich observations, except that we shall place one series under the other in the same page, instead of side by side on opposite pages, as in the original, and shall transpose the two final columns, in the page JONES, to the position denoted by the second and third columns, headed "Difference of Jones from Troughton", that the principal columns may range under their respective titles. The figures 2 and 6, standing over these two columns, imply the mean of two, and of six microscopes, from which the differences were computed, it being unnecessary to repeat the state of the barometer and thermometers already given in the page TROUGHTON, for the same observations.

OBSERVATIONS OF NORTH POLAR DISTANCE (A DAY'S WORK)

IN THE YEAR MDCCCXXVI

TROUGHTON

Day of the month	Barom	Thermom In	Thermom Out	Names of Stars	Degrees and Min	Microscopes A	B	C	D	E	F	Mean of two	Mean of six	
☾ July 3	30 12	73	77	Polaris s p R	258° 30	12″ 0	16″ 7	7″ 5	8″ 5	10″ 5	8″ 6	14″ 3	10″ 6	
	30 11		76½	ζ Ursæ Maj	34 9	18 1	50 2	15 7	17 2	17 .1	45 0	19 1	17 2	
		72	71½	η ——— . .	39 48	46 7	19 3	17 5	16 3	51 3	44 2	18 0	17 0	
			73	α Draconis .	24 47	30 0	31 5	30 1	28 7	31 6	26 2	32 2	30 2	
			72½	Arcturus R	187 8	45 0	17 .7	12 4	12 2	17 1	41 1	16 3	14 3	
		71	70	ι Bootis R	191 51	10 8	53 2	50 1	50 0	53 0	17 9	51 0	50 8	
			69	β Ursæ Min R	211 54	25 3	32 1	25 .2	23 0	26 5	22 3	28 .7	25 7	
		69	67	α Cor Bor R	191 21	29 6	35 0	27 2	27 8	29 9	27 2	32 3	29 5	
			66	α Serpentis R	174 2	19 9	26 0	19 0	21 5	21 5	21 1	23 0	22 0	
				δ Scorpii . .	112 4	3 8	9 2	58 2	0 .0	6 9	3 7	7 5	3 6	
			65½	β¹ ———	109 16	41 2	16 8	36 2	37 3	44 3	36 2	41 0	10 3	
	30 10	66	64	τ Scorpii	117 45	50 5	5 5	55 0	51 0	59 4	57 2	2 5	58 .4	T T
				ι ———	123 48	3 3	10 .1	2 8	58 6	10 1	3 0	6 7	1 7	
	30 09		63	α Herculis R	181 30	12 6	10 5	9 2	9 9	11 9	11 9	14 5	12 0	
				α Ophiuchi, R	170 45	11 .5	15 0	7 5	6 5	11 6	10 3	13 3	10 1	
		65	62	γ Draconis ...	38 20	3 8	8 .6	4 1	0 5	3 0	2 0	6 1	3 7	
				α Lyræ	51 22	0 6	6 .5	50 .1	55 0	50 6	0 9	3 5	0 .3	
				β ———	56 49	32 6	37 7	33 8	28 1	31 9	34 7	35 1	33 1	
	30 08	61		ζ Aquilæ ...	76 22	22 .5	26 4	10 4	14 6	21 2	22 .5	24 5	21 1	
				δ Draconis ...	22 38	18 5	48 7	15 3	46 3	44 7	45 4	48 6	46 .5	
			61	δ Aquilæ	87 12	12 .4	15 8	9 6	5 6	11 6	12 2	14 1	11 .2	
	30 .07	61½	60½	γ ———	79 47	12 0	16 0	9 2	4 7	13 5	10 2	14 0	10 .9	R
				α ———	81 33	55 6	2 1	51 2	50 1	57 5	55 2	50 0	55 8	
				β ———	84 0	3 5	8 9	2 0	58 0	7 .2	1 3	6 2	4 0	
			60	α¹ Capricorni .	103 0	3 9	9 0	0 0	55 0	6 8	3 5	6 .4	3 .0	
		63½		α Cygni ... .. R	211 42	41 5	16 2	41 8	37 5	18 6	41 9	13 9	13 1	
	30 02	64	63	Polaris	1 37	56 8	57 0	52 .7	51 5	59 2	53 9	56 9	55 2	
		66	69	α Arietis .	67 21	17 0	19 0	11 7	7 4	16 2	14 8	18 0	14 4	
		70	72	β Ursæ Min s p	314 53	23 0	25 0	19 4	19 6	26 2	20 .8	24 0	22 3	
			73	α Persei	40 46	1 5	1 6	56 3	54 6	2 6	56 3	3 0	59 3	

JONES.

	Diff. of Jones from Troughton 2	6												
☾ July 3		. . .	Polaris s p	358 23	11 .0	48 .2	38 .3	42 .0	37 0	39 1	12 .1	40 1	
	+ 9′ 2	+ 9′ 7	ζ Ursæ Maj. ..	34 0	52 0	4 7	58 1	57 0	55 3	54 2	58 .3	56 .9	
	+ 11 1	+ 10 1	η ——— . ..	39 48	53 2	5 0	50 1	57 2	58 .0	54 0	59 1	57 7	
	+ 10 2	+ 10 2	α Draconis ...	24 47	30 0	48 9	10 3	10 .2	38 9	37 8	12 5	40 1	
	. .	. .	Arcturus	69 54	5 0	11 1	8 4	7 2	5 .8	6 0	8 1	6 4	
		. .	ι Bootis .	62 10	58 8	2 .7	57 1	1 4	57 1	57 2	0 8	59 1	
	..	.	β Ursæ Min. ...	15 8	22 9	28 3	23 1	25 2	21 9	20 8	25 6	23 7	
	...	.	α Cor Bor	62 41	20 8	23 1	18 1	21 3	19 3	19 6	22 0	20 4	
		.	α Serpentis ...	83 0	27 3	33 0	27 9	29 8	25 2	27 8	30 2	28 .5	
	+ 6 1	+ 8 7	δ Scorpii .. .	112 4	9 2	18 0	13 3	13 .0	10 .0	10 2	13 6	12 8	
	+ 6 4	+ 9 1	β¹ ——— ———	109 16	47 0	53 7	40 .2	51 6	48 8	46 3	50 4	49 4	
	+ 6 4	+ 8 9	τ Scorpii ..	117 46	3 4	14 .4	8 7	8 2	4 7	1 6	8 9	7 .3	W R
	+ 7 3	+ 8 8	ι ——— . .	123 48	7 6	20 4	14 6	13 4	11 1	13 8	14 0	13 5	
	. ..	.	α Herculis . ..	75 23	32 .4	39 8	33 9	34 5	32 .6	33 0	36 1	34 4	
	...	.. .	α Ophiuchi	77 17	37 .4	41 6	36 8	30 6	35 6	35 3	39 .5	37 .7	
	+ 9 7	+ 10 4	γ Draconis	38 20	10 .6	21 0	14 8	14 6	11 .3	9 .4	15 .8	14 1	
	+ 6 3	+ 8 .0	α Lyræ	51 22	5 .4	14 2	7 6	8 .0	6 .0	7 .2	9 8	8 3	
	+ 7 2	+ 7 4	β ——— . .	56 49	38 .2	46 4	38 2	41 0	41 0	38 .2	42 .3	40 5	
	+ 7 8	+ 10 .1	ζ Aquilæ .	76 22	29 .4	35 2	30 .8	31 .4	29 8	30 4	32 3	31 2	
	+ 9 8	+ 10 5	δ Draconis .	22 38	54 0	2 7	58 0	57 0	57 4	52 7	58 .4	57 0	
	.....		δ Aquilæ .. R	109 50	36 .2	44 .0	36 2	36 .0	36 6	36 .6	40 1	37 7	
	+ 6 8	+ 8 5	γ ———	79 47	17 8	23 .8	17 3	23 0	17 6	17 1	20 .8	19 .4	E
	......	.. .	α ——— . .. R	175 28	52 2	0 .5	51 0	51 2	53 8	49 5	56 4	53 1	
	+ 7 2	+ 8 0	β ———	84 0	11 6	16 .1	11 0	13 .4	11 0	8 9	13 4	12 .0	
	+ 8 0	+ 9 5	α¹ Capricorni ..	103 0	11 .0	17 .8	10 5	13 .0	11 1	11 .1	14 .4	12 5	
	. ..	.	α Cygni	45 20	5 1	13 .8	6 7	7 .0	8 4	7 .0	9 5	8 0	
	..	.	Polaris .. ...R	255 21	53 4	58 2	50 0	52 8	56 0	54 .0	55 8	54 1	
	+ 6 .8	+ 8 4	α Arietis	67 21	22 4	27 1	10 0	23 .4	21 .6	22 0	24 8	22 8	
	+ 8 6	+ 9 6	β Ursæ Min s p	344 53	30 .9	31 .2	28 1	31 .8	32 6	31 7	32 6	31 .9	
	+ 6 6	+ 9 1	α Persei . .	40 40	5 0	14 .2	7 0	8 8	8 2	6 9	9 6	8 4	

20. *An illustration of the different methods of obtaining the Index Error.*—The first method of ascertaining the index error which we shall explain, is by means of stars observed above and below the pole, which, theoretically speaking, is the most obvious that presents itself to us, when we consider that the mural circle is constructed to measure angular distances in the plane of the meridian only, that the degrees of the circle are numbered from 0° to 360°, and that, when the telescope which is clamped to the circle, and consequently the circle also, are directed to the pole, one of the microscopes, designated by *A*, is adjusted to read off 0° or 360°, plus or minus the index error, which for convenience is now generally very small. The observations for this purpose are separated by an interval of twelve hours, and not unfrequently of several days, in consequence of unfavourable weather. We will confine our examples, in the first place, to determine the index error of Troughton's circle, from observations taken on the 3rd and 4th of July, 1826, some of which will be found in the day's work recorded in our last page, and the rest are extracted from the Greenwich observations of the following day, after which we will show how the same may be obtained for either of two circles used in conjunction, as now practised at the Royal Observatory, and also how a star's altitude may be determined, by means of the horizontal points ascertained by the respective instruments

CIRCUMPOLAR METHOD

1826	Names of the Stars	Observed North Polar Distance	Refraction	Sum of Prec Aber and Nut	Sum of the five Equations	North Polar Distance corrected by the five Equations
July 3	Polaris	1° 37′ 55″2	−41″88	−5″38	−47″26	1° 37′ 7″94
4	S P	358 23 29 ,2	−45 61	+5 36	−40 25	358 22 48 95
3	β Ursæ Min S. P	344 53 22 ,3	−74 56	−8 27	−82 83	} 314 51 59 74
4	....	314 53 22 4	−73 99	−8 40	−82 39	
4	β Ursæ Min.	15 8 13 3	−23 59	+8 33	−15 26	15 7 58 01

Sum .. .. = 719 59 54 07
½ Sum . . = 350 59 57 33
½ Sum − 360° = twice the index error . = − 2 67
And the index error . . . = − 1 33

With respect to the preceding example, it is almost unnecessary to add any further explanation, than what is given at the top of each column, only, that the refractions are computed from Dr. Bradley's tables, given in the first volume of this work, making use of the barometer, and interior thermometer, annexed to the observations. Also, the corrections in the fifth column are computed from the tables given for that purpose in the same volume, the signs of the equations being changed, in order to reduce the apparent place of the star to its mean place at the beginning of the year. The sign prefixed to the resulting index error, shows the error of the instrument; but when used for correcting the observations, it must be changed.

21. The second method of determining the index error, which we shall notice, is that by means of a catalogue, and for this purpose we shall make use of the observations of those stars of Mr. Pond's Standard Catalogue, which were made on the two days chosen in our first example.

METHOD BY A GOOD CATALOGUE

1826	Names of the Stars	Observed North Polar distance	Refraction	Sum of Prec Aber and Nut	Sum of the preceding Equations	North Polar Distance corrected by the preceding Equations	Seconds of N P D by the Catalogue	Index Error
July 3	ζ Ursæ Maj	34° 9 47"2	− 4"19	+ 3'43	− 0'76	34° 9' 46'44	48"2	−1"76
	η . .	39 48 47 6	+ 1 21	+ 3 26	+ 4 50	39 48 52 10	54.6	−2.50
	α Draconis ...	24 47 30 2	−13 41	+ 6 16	− 7 25	24 47 22 95	21.7	−1 75
	γ .	38 29 3 7	− 0 03	+ 8 93	+ 8.90	38 29 12 60	13 7	−1 10
	α Lyræ .	51 22 0 3	+12 68	+ 9 29	+21 97	51 22 22 27	22 5	−0 23
	β .. .	56 49 33 1	+18 41	+ 9 41	+27 85	56 50 0 95	1 0	−0 05
	ζ Aquilæ .	73 22 21 1	+43 46	+10 21	+53 67	73 23 14 77	16 6	−1 83
	δ Draconis .	22 38 46 5	−15 88	+ 8 14	− 7 74	22 38 38 76	40 0	−1 24
	δ Aquilæ .	87 12 11 2	+63 52	+11 72	+75 24	87 13 26.44	27 8	−1 36
	α . . .	81 33 55 8	+52 11	+12 74	+64 88	81 35 0 68	2 8	−2 12
	Polaris	1 37 55 2	−41 88	− 5 38	−47 26	1 37 7 94	9 4	−1 46
	α Arietis	67 21 11 4	+30 60	+ 5 98	+36 58	67 21 50 98	52 5	−1.52
	α Persei . ..	40 45 59 3	+ 2 10	− 3 80	− 1 70	40 45 57 54	58 7	−1 16
4	Polaris S P	358 23 29 2	−45 61	+ 5 36	−40 25	358 22 48 95	50 6	−1 65
	η Ursæ Maj	39 48 48 5	+ 1 23	+ 3 33	+ 4 56	39 48 53 06	54.6	−1 54
	α Draconis	24 47 29 4	−13 26	+ 6 23	− 7 03	24 47 22 37	21.7	−2 33
	Arcturus .	69 53 56 7	+33 12	− 4 27	+28 85	69 54 25 55	26.3	−0 75
	ι Boötis . ..	62 10 49 4	+23 88	+ 0 21	+24 09	62 11 13 49	14 1	−0.61
	β Ursæ Min	15 8 13 3	−23 59	+ 8 33	−15 26	15 7 58 04	59.7	−1 66
	α Cor Bor .	62 41 10 2	+21 51	+ 2.34	+23 85	62 41 37 05	38 6	−1 55
	α Serpentis .	83 0 20 6	+53 60	− 2 03	+51.57	83 1 12 17	12 3	−0 13
	α Herculis .	75 23 25 9	+41 17	+ 4 94	+46 11	75 24 12 01	14.1	−2 09
	α Ophiuchi . .	77 17 29 6	+44 07	+ 5 80	+49 87	77 18 19 47	20 4	−0 93
	γ Draconis ...	38 29 2 6	− 0 03	+ 9 23	+ 9 20	38 29 11 80	13 7	−1 90
	α Lyræ	51 21 58 5	+12 53	+ 9 60	+22 13	51 22 20 63	22.5	−1 87
	β	56 49 32 0	+18 21	+ 9 71	+27 92	56 49 59 92	1.0	−1 08
	ζ Aquilæ .	73 22 21 5	+42 83	+10 43	+53 26	73 23 14 76	16 6	−1 84
	δ Draconis ..	22 38 45 8	−15 65	+ 8 49	− 7 16	22 38 38 64	40 0	−1 36
	α Cygni	45 19 58 3	+ 6 59	+ 9.06	+15 65	45 20 13 95	11 8	−0 85
	α Arietis	67 21 14 6	+30 22	+ 6 13	+36 35	67 21 50 95	52 5	−1 55
	α Persei . .	40 46 0 2	+ 2 09	− 3.88	− 1 79	40 45 58 41	58 7	−0 29
							Mean index error =	−1.36

This method of deducing the index error is extremely convenient as well as correct; especially as the Greenwich catalogue is now determined by successive approximations to a great degree of accuracy it affords the most powerful means of determining the index error of an instrument, by tending to exterminate every source of error arising either from partial expansion, or from defects of division, as well as those of bisection and reading, which property is very essential in determining the absolute polar distance of a celestial object, and more particularly when the stars, chosen for this purpose, have not very low altitudes, so as to be affected by the variability of refraction. The seconds in the eighth column, employed for obtaining the index error, exhibited in the last, are taken from the catalogue in page 4, at the end of the first quarter of the Greenwich observations for 1826.

22. The third, and last method, which we shall exhibit for finding the index error of a mural circle, is by means of observations made by direct vision of a star in the usual way, and also of its reflected image, from a trough of mercury, or oil, supposing the latitude of the place of observation known but before we give an example of this method, we shall point out the

necessary rules for directing the instrument in observing the reflected image of a star, supposing its polar distance known nearly, and also for determining the altitude of the star when it has been observed in both positions with the instrument, and here it may be observed that, supposing there is but one instrument, twenty-four hours must elapse, and sometimes several days, owing to very windy or cloudy weather, before we can obtain sufficient data for our purpose, except in the case of the pole star, which certainly may be observed both ways, whilst passing the field of the telescope of the instrument, the proper reduction being made when observed off the meridian. If we put Δ for the polar distance of a star, Δ' that shown by the instrument, A its altitude measured from the south horizon in all cases, L the latitude of the place, H its supplement, and H' the supplement shown by the instrument, or what at Greenwich is called the *horizontal point*, the difference between H and H' will be the index error. By the principles of catoptrics, the image of the star is as much depressed below the horizon of the reflecting surface, as the star is elevated above It is evident then, that, in order to observe by reflection, the instrument must be moved through an angle $= 2A + \Delta$, call this R. Now $A+\Delta+L=180°$, and $2A+2\Delta+2L = 360°$, therefore $2A+\Delta$, or $R=360°-2L-\Delta$; which gives a general rule for setting the circle to observe by reflection. But for a star passing below the pole, the rule will be $R=360°-2L-(360°-\Delta)$, or $R=360°-2L+\Delta$. For Greenwich, taking $L=51° 28' 39''$, we have $R=360°-102° 57' 18''-\Delta=257° 2' 42''-\Delta$ when the star passes above the pole, and $R=257° 2' 42''+\Delta$ when the star passes below. Now to determine H' and A, the observations must furnish us with Δ', and $R'=2A+\Delta'$. It is evident that $H'=A+\Delta'$. Now $R'+\Delta' = 2A+2\Delta' = 2H'$; therefore $H'=\frac{R'+\Delta'}{2}$, and $A = H'-\Delta'$, consequently the altitude is obtained independently of the index error. The following example will sufficiently illustrate these rules. The eighth column is here copied from the seventh, in the example by the second method.

METHOD BY REFLECTION AND DIRECT VISION

1826	Names of the Stars	Observed Polar Distance of the Reflected Image R'	Refraction	Sum of the Prec Aber and Nut	Sum of the preceding Equations	R' Corrected by the preceding Equations	Polar Distance corrected from last Example Δ'	$R' + \Delta'$	$\frac{R'+\Delta'}{2}$ or the Horiz Point H'	$H'-\Delta'$, or the True Altitude for the beginning of 1826 A
July 3	Polaris S P	258° 30' 10".6	+46".24	− 5".10	+40".01	258° 30' 51".41	358° 22' 48".05	257° 2' 40".36	128° 31' 20".18	130° 8' 31".23
	Arcturus	187 8 44 3	−38 49	+ 4 37	−29 ,12	187 8 15 18	69 51 25 55	40 73	20 37	58 36 54 ,82
	ε Bootis	194 51 50 8	−24 15	− 0 08	−24 23	194 51 26 57	62 11 13 49	40 06	20 03	66 20 0 54
	β Ursæ Min	241 54 25 7	+ 23 84	− 8 21	+ 15 63	241 54 41 33	15 7 58 01	39 37	19 68	113 23 21 64
	α Cor Bor	194 21 29 5	−24 ,85	− 2 17	−27 02	194 21 2 48	62 41 37 05	39 53	19 77	65 49 42 ,72
	α Serpentis	174 2 22 0	−54 30	+ 2 14	−52 16	174 1 29 84	83 1 12 17	42 01	21 ,01	45 30 8 84
	α Herculis	181 39 12 0	−41 74	− 4 ,75	−46 49	181 38 25 51	75 24 12 01	37 52	18 ,76	53 7 0 75
	α Ophiuchi	179 45 10 4	−44 70	− 5 ,61	−50 31	179 44 20 09	77 18 19 47	39 56	19 78	51 13 0 31
	α Cygni	211 42 43 4	− 6 69	− 8 74	−15 43	211 42 27 97	45 20 13 05	41 02	20 06	83 13 7 01
4	δ Aquilæ	169 50 28 2	−62 ,73	−11 88	−74 61	169 49 13 59	87 13 26 44	40 03	20 02	41 17 53 58
	Polaris	255 24 46 4	+41 26	+ 5 33	+ 46 59	255 25 32 99	1 37 7 94	40 93	20 47	126 54 12 53

The mean of 11 united observations give the supplement of latitude, or $H' = 128\ 31\ 20\ 09$

The assumed supplement of latitude of Greenwich ... $H = 128\ 31\ 21\ 00$

Difference, or $H'-H$, the index error ... $= -\ 0\ 91$

This is the most independent method of determining the altitudes of stars by means of one mural circle, when the weather continues favourable for some days together, but as this is very seldom the case, in this climate at least, the astronomer is often disappointed after he has been employed in making a set of observations in one position of the instrument, by the intervention of unfavourable weather, preventing him from completing his series. Since a second mural circle has been erected at the Royal Observatory at Greenwich, the above objections are obviated, by observing the reflected image of a star with one instrument, and at the same time the star itself with the other. Now if the two instruments were identical with respect to their index errors, the operation of determining the horizontal points of the instruments, and the altitudes of the stars, would be precisely the same as given in the last example; but as this cannot be the case, it will be necessary to determine the difference between the two instruments, which is done by observing a number of stars in the same position with each instrument, in the manner practised at Greenwich, when this is done, it will be easy to determine the horizontal points of each instrument, and the altitudes of all stars observed by reflection with one instrument, and by direct vision with the other. For if Δ represent the polar distance of a star shown by one instrument, Δ' that by the other, and d the difference between Δ and Δ', Δ' being $=\Delta+d$, also H and H' the horizontal points of each instrument, and R and R' the reflected polar distances shown by each instrument, R' being $=R+d$, we have from the principles already given,

$$H=\frac{R+\Delta}{2}=\frac{R+\Delta'}{2}-\tfrac{1}{2}\,d, \text{ and substituting for } \Delta \text{ its value } \Delta'-d,$$

$$=\frac{R'+\Delta}{2}-\tfrac{1}{2}\,d, \text{ since } R=R'-d,$$

By a similar process
$$H'=\frac{R'+\Delta'}{2}=\frac{R'+\Delta}{2}+\tfrac{1}{2}\,d$$

$$=\frac{R+\Delta'}{2}+\tfrac{1}{2}\,d$$

And
$$A=H-\Delta,$$
$$=H'-\Delta'$$

The following example will fully illustrate this method. It may be well however to observe, that, to determine the apparent altitude of the stars, and the horizontal points of each instrument, it is unnecessary to correct the observations for refraction, precession, &c.; since the observations as read off from the instrument may be used instead of the reduced observations, which will greatly abridge the process of computation, and will produce apparent results with perfect accuracy. A plate in outline is given, in the number containing the observations of January, February, and March, 1826, which exhibits the relative positions of the two circles, and situation of their troughs containing the mercury, which sketch conveys a good idea of the manner in which they are alternately used for reflected vision.

UNION OF THE TWO INSTRUMENTS

1826	Names of the Stars	Observed R and R'	Observed Δ and Δ'	$R' + \Delta$ and $R + \Delta'$	$\frac{R + \Delta'}{2}$ and $\frac{R' + \Delta}{2}$	½ Mean Diff d	H by Troughton's Circle	H' by Jones's Circle	App. Altitude A
July 3	Polaris . S P	258° 39′ 10 6	J 358° 23′ 40″ 1	257° 2′ 50 7	128° 31′ 25″ 35	4″ 58	128° 31′ 20″ 77	128° 31′ 29″.93	130 7 49 03
	Arcturus .	187 8 44 3	J 69 51 6 4	50 7	25 35		20 77	29 93	58 37 23 53
	ε Boötis ..	194 51 50 8	J 62 10 59 1	49 9	24 95	..	20 37	29 .53	66 20 30 43
	β Ursæ Min.	241 51 25 7	J 15 8 23 7	49 4	24 70	.	20 12	29 28	113 23 5 58
	α Cor. Bor. ..	194 21 29 5	J 62 41 20 4	49 9	24 95	.	20 37	29 53	65 50 0 13
	α Serpentis ...	174 2 22 0	J 83 0 28 5	50 5	25 25		20 67	29 83	45 31 1 33
	α Herculis .	181 39 12 0	J 75 23 34 4	46 4	23 20	...	18 62	27 78	63 7 53 38
	α Ophiuchi .	179 45 10 4	J 77 17 37 7	48 1	24 05	.	19 47	28 63	51 13 50 93
	δ Aquilæ ..	169 50 37 7	T 87 12 11 2	48 9	24 45	..	19 87	29 03	41 19 0 70
	α	175 28 53 1	T 81 33 55 8	48 9	24 46	.	19 87	29 03	16 57 21 07
	α Cygni . .	211 41 43 4	J 45 20 8 0	51 4	25 70	...	21 12	30 28	83 11 22 28
	Polaris	255 24 51 1	T 1 37 55 2	40 3	24 65		20 07	29 23	120 53 21 87
4	... ... S P	258 39 19 4	T 358 23 29 2	48 6	24 30	4 78	19 52	29 08	130 7 50 32
	Arcturus	187 8 52 3	T 69 53 56 7	49 0	24 50	.	19 72	29 28	58 37 23 02
	ε Boötis .	194 52 0 6	T 62 10 49 4	50 0	25 00		20 22	29 78	66 20 30 82
	β Ursæ Min	241 54 30 6	T 15 8 13 3	49 9	24,95	.	20 17	29 73	113 23 0 87
	α Cor. Bor. ..	194 21 38 8	T 62 41 10 2	49 0	24 50	.	19 72	29 28	65 50 9 62
	α Serpentis ..	174 2 31 6	T 83 0 20 6	52 2	26 10		21 32	30 88	45 31 0 72
	α Herculis ...	181 39 22 6	T 75 23 25 9	48 5	24 25	..	19 47	29 03	63 7 53 57
	α Ophiuchi	179 45 20 1	T 77 17 29 0	49 7	24 85		20 07	29 63	51 13 50 17
	δ Aquilæ .	169 50 28 2	J. 87 12 21 8	50 0	25 00	.	20 22	29 78	41 19 7 98
	α Cygni . .	211 42 51 6	T 45 19 58 3	49 9	24 95	..	20 17	29 73	83 11 21 07
	Polaris .	255 24 46 4	J. 1 38 5 0	51 4	25 70		20 92	30 48	120 53 25 .48

The mean of 23 observations each way give H and H' .... =	128 31 20 16	128 31 20 51
The supplement of latitude of Greenwich	128 31 21 00	128 31 21 00
And the index error by Troughton and Jones's circles are	—0 84	and + 8 51

It will be seen by a reference to the Greenwich observations, from which the data for these examples have been taken, that the direct observations in the third column having T. or J. affixed to them, are intended to show by which instrument these observations were made. This distinction is necessary to assist the computer in deducing the altitudes in the last column.

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§ LXXIII CIRCULAR REPEATING INSTRUMENT BY REICHENBACH [PLATE XXII.]

1. SIG. CARLO BRIOSCHI, in his *Commentaries on the Royal Observatory at Naples*, (1824—1826) has given an account of the different instruments made use of by him, with references to various engravings, which enable us to gain a competent knowledge of their construction Among the collection of instruments, constituting the apparatus of that observatory, are two similar circular instruments by the late eminent artist Reichenbach*, one of which stands in the eastern and the other in the western part of the building. Figure 1, of our Plate XXII., exhibits the plan of one of these instruments, having two vertical circles, that revolve one within the other, one

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* Reichenbach is succeeded by Ertel.

forming the graduated limb, and the other being a circular quadruple vernier, sunk into the same plane; it also exhibits the vertical axis, telescope, and other appendages. Figure 2 gives a section of the various parts, that could not otherwise be so well comprehended, by a reference to the first figure only.

2. The vertical axis is composed of a strong tube of brass sufficiently long to allow the axes of the circle, verniers, and telescope to pass across it, at a height from the floor suitable for the convenience of an observer's eye, a little above its centre; its upper pivot is cylindrical, entering a box having a triangular hole, that is furnished with two pair of antagonist screws, and that is made fast to a metallic architrave *A*, broken at both ends in figure 1, but which is supported at the extremities by a pair of vertical pillars, not introduced in our plate; the lower pivot is also cylindrical, and being blunted rests in a metallic cup inserted into a marble slab, supported by a pillar of masonry under the floor. This slab *B* is also represented as broken at the ends in both figures. A sensible level is suspended at right angles across the axis behind the part exhibited to view, and may occasionally be suspended by a long cylindrical rod extending parallel to the plane of the vernier circle. A section of the level is seen behind the axis in fig. 2. The diameter of the graduated circle is upwards of thirty-nine inches, viz. equal to the French metre, and has eight radial tapering tubes braced together, connecting it with a strong concentric cylinder, to which its tubular horizontal axis is made fast; which is also the construction of the inner circle, having four verniers marked at equal distances on its four quadrantal arcs, each reading to 2″. The graduated circle, the vernier circle, and the telescope have their respective axes of motion, passing through one another in as close contact, at the two extremities of each, as good fitting can effect; the axis of the telescope being solid, and both the others tubular: they are all kept close home by collets and tapped nuts, so that the two circles may revolve in the same plane, and also the telescope, which is screwed fast to the vernier circle at the eye-end, in a plane parallel to the same. The telescope and vernier circle may therefore be considered as having only one common axis. The adaptation of the different parts here described may be understood from an inspection of the second figure, where the axes are represented by different shades in the section. The fixed strong tube, through which the three concentric axes pass, is adjusted at right angles to the long vertical axis by four screws, two above and two below, that pass through four strong cocks attached to the opposite sides of the said axis, as seen in this second figure. The divisions on the circle are subdivided into spaces of 5′, and read in connexion with the four verniers taken successively, by the aid of a pair of eye-glasses carried on a long double lever, that revolves round the telescope's axis, directly over the graduations.

3. The telescope *C* extends beyond the circle's periphery, with a smaller tube inserted, which is connected with the screw of the clamp having balls peculiar to this maker's construction [§ XLVI. 4.]. The smallest tube, or drawer, that terminates the telescope, holds a box containing the adjustable wires and eye-piece, and is kept in its place by a pair of fixing screws entering the exterior end of the smaller fixed tube. At the upper edge of the circle a second similar clamp is attached to the front of the vertical axis, which in its turn will clamp the graduated circle, whenever the first clamp is released, as will be explained presently, when we come to speak of the repeating principle. As the telescope is made fast to the vernier circle only at one end, its tendency to bend towards the object-end required some contrivance for

preventing flexure of the tube; which is accomplished by a long lever, turning on a fulcrum, and entering a hole made in a cock fixed to the telescope, nearly at one third of the distance from the axis, the counterpoise near the remote end of the lever being adjusted by trial, during the observation of a distant horizontal mark.

4. The horizontal circle for measuring azimuths, which is seen in section in both of these figures, has a smaller diameter than the vertical circle in the proportion of 75 100, and reads by two opposite verniers only, each to the accuracy of 4″. The positions of the reading lenses and of the verniers are sufficiently seen, and require no particular explanation; the subdivisions being here also spaces of 5′.

5. From the description which has been given of the concentric position of three axes, passing one through another, and being loaded respectively with the telescope, verniers, and graduated circle, at their projecting ends, our readers may have anticipated, that a cause of error may arise from such concentric position of these essential parts of the instrument, which no ordinary caution can guard against, since, supposing the fittings to be perfect, the elasticity of the metallic portions will produce deviations from the vertical plane, and eccentricities, that may vitiate an observation. The mechanist was aware of such objection, and therefore contrived an apparatus to obviate it, by means of counterpoises. Towards the upper end of the vertical axis a couple of rings surround its surface, and are kept from sliding downwards by a pair of strong steel pins with milled heads, screwed into the large tube, as two pair of fulcra, round which the two double levers, *D* and *E*, if overloaded would turn, that are seen in a horizontal position, sustaining each an adjustable load, nearly at their extreme ends, which are made fast by screws when equipoised. The interior ends of the said levers carry each a descending bar terminating with a ring having a pair of rollers, which embrace, as a pair of stirrups, the short cylinders attached to the external ends of the telescope's and circle's axes respectively, and lift the principal portions of their weights, without impeding their circular motions; thereby preserving the vertical direction of the planes in which they move and it is for this reason, that the object-end of the telescope could not be made fast to the vernier circle. The vertical rod of suspension, carried by the superior lever *D*, has a ring formed in it, opposite the readings of the verniers, and contains a lens proper for rendering the strokes distinctly visible. The fulcra of both these levers are better seen in fig. 1, and also the front descending bar that terminates with a ring, or round stirrup, embracing the grooved cylinder; but the bar behind the circle is not there so well seen. Fig. 3. is a representation of the second level which is applied to the telescope's axis occasionally, when turned into a certain position; one of the suspending rods of which is hooked at the upper end, by the formation of a rectangular notch, as seen in figure 4.

6. *Adjustments.*—The first adjustment of this instrument is that by which the long vertical axis is rendered perpendicular in all directions, which is managed by means of the level that applies, at right angles, to the back of the vertical tube forming the axis. In doing this our readers will now understand that one-half of the error in a given direction, is corrected by the proper pair of screws at the superior pivot, and the other half by the adjustment of the level itself, till after some trials it is found, that the instrument will turn quite round in azimuth, without displacing the air bubble of the level The second adjustment is performed by means of the second level, which is seen in figures 3 and 4, and which applies to the axis of the tele-

scope through an opening in the surrounding tubular axis of the vernier circle, in that particular position that directs the telescope to a horizontal mark; for then one of the hooks, seen in fig 4, embraces the left hand end of the axis, through the said opening, and the other hook hangs on the end of the axis, that projects towards the right hand, as now seen in the figure; for the level may be introduced to its place, by passing it between the conical radii of the two concentric circles: if the level will not reverse in position, now that the vertical axis is truly vertical, the adjusting screws opposed to each other in the four cocks, above and below the horizontal axes, will adjust the surrounding tube, against which they press, till the bubble of the level assumes its proper station at the middle of its tube; and if the level will not then reverse, its Ys will require adjustment, till, by the operation of halving the remaining error, it will show that the axes stand exactly at right angles to the vertical axis, in which case the telescope and circles will move as nearly in a true vertical plane, as the imperfection of the construction will admit of. Thirdly, the telescope must be adjusted for distinct vision by means of the drawer; and for the position of the ocular wires, as it regards both the vertical direction of the parallel wires, and the collimation in altitude, the modes of doing which we need not repeat, after our former directions for these adjustments of the altitude and azimuth circles of other makers. In the instrument before us, however, the vertical position of the parallel wires is regulated by turning the drawer bearing the small box round, in the small fixed tube, when the two opposite screws are released, and then fixing it in its proper position by their means; after which the thumb-screw above the box will adjust the place of the horizontal wire for collimation in altitude. Lastly, the collimation in azimuth is necessarily effected by a very distant mark in the horizon, on account of the telescope being placed at one side of the vertical axis; for in the reversed position the eccentricity of this position is doubled; and accordingly we find, that the mark necessary for the adjustment of the collimation in azimuth is placed at the distance of *twenty Italian miles!* A circumpolar star, however, would be better, at the moment of its greatest elongation from the polar point. When this adjustment had been finished partly by the side screws in the box of the eye-piece, and partly by moving the horizontal circle in azimuth, the verniers were put to read opposite points on it, and as the zero of this circle was afterwards found to be a little out of the meridian, the deviation is always allowed for in observing azimuths.

7. One of the first objects of remark that occurs to an English astronomer, on examining the instrument now under our consideration, is, that the readings of the circles are by means of *verniers*, now that the use of micrometrical microscopes is become general in England, whenever instruments are made on a scale large enough to admit of accurate subdivisions to spaces of 5'. An idea, however, prevails on the continent, that verniers marked on a concentric entire circle are less likely to be displaced by expansion and contraction, as they regard the contiguous graduated circle, than microscopes attached to bars, cocks, or pillars; and a prejudice of this nature is very likely to be perpetuated by the cheapness of verniers, as compared with reading microscopes; but in our opinion whoever has experienced the accuracy and convenience of the latter, will seldom wish to return to the former. The Neapolitan astronomer confesses, what might be anticipated, that when a quick motion is suddenly given to the telescope, and to its attached vernier circle, the graduated circle yields a little from its original position, and nothing but the most firm clamping of its limb will remedy the evil, and that pro-

bably not completely; on which account an excess of elevation is first given in observing, and then a retrograde motion is supposed to restore the due position, that the two circles in contact ought relatively to maintain. For these reasons it is found to be of the utmost importance, that the *balls* of the clamps be made to bind their corresponding screws as closely as the action will allow. The evil is considered to be cured, when one of the levels suspended by pins on the face of the graduated circle, has its bubble remaining steady, while the telescope is alternately elevated and depressed, in connexion with the contiguous vernier circle. It appears, however, from the published account that *lard* is used to lubricate the parts moving in contact with one another, in preference to *oil*, which is applied only to pivots that are accessible.

8 *Flexure*.—Though the permanency of the horizontal position of the common axis of the telescope and verniers, and of the circle's tubular axis, was attempted to be secured by the counterpoises *D* and *E*, yet when the two circles came to be tried against each other in actual observations, there was generally found a difference of 2″, or nearly so, in the zenith distances, and also between the observations of the summer and of the winter solstices, in using both instruments. Such discrepancies induced an opinion that *flexure* in some way or other might be the cause of this want of agreement, and experiments were instituted to ascertain in what way such effect would be produced. From observations of high and low stars it was found that an error existed nearly proportional to the zenith distance, and it was inferred, that a flexure of the telescope took place towards the object-end, where there is no brace to attach it to the vernier circle; for with the eastern circle the error at the horizon was found to amount to 5″.06, and with the western to 5″.61; the counterpoise lying parallel to the tube, which has been already spoken of, was therefore applied, but after adjustment the error was found to be only partially corrected. It was then suspected that the radii of the vernier circle might be liable to partial flexure also, and a plan for determining this point was determined upon. The whole weight of the telescope was detached from the vernier circle by cords suspending it from both ends, and when the level was hung on the vernier circle, the ocular micrometer was used to measure the zenith distance of an object, first with the weight of telescope included, and secondly, with the weight detached, when it was found that the radial flexure of the eastern instrument was 3″ 03, and of the western one 4″.75. Since the principal tube is made fast towards the eye-end to the plane of the vernier circle, it cannot bend at that end, but the drawer, or ocular small tube may, and the effect produced by such bending would in some measure counteract the flexures of the object-end, and of the radial cones, by having a contrary sign. Braces were therefore applied to keep the eye-piece in the same line with the main tube, in both instruments, and when the counterpoises were applied to suspend the telescopes, as in the trial of the radial cones, the level being now suspended from the graduated circle, the mean quantity of ocular flexure in the eastern instrument was found without the brace − 2″.68 and with the brace − 1″.07. The western instrument had nearly the same quantities. These various experiments were registered in seven tables which were completed for every degree of altitude, but which would be of no use for other instruments, and therefore are here omitted.

9. The corrections resulting from these experiments, when the telescope is not braced at the eye-end, but counterpoised at the object end, may be stated in the following manner, viz.

	EASTERN.	WESTERN
Flexure at the object-end . .	+0".42	+1".03
Radial flexure	+3 .03	+4 .75
Ocular flexure	−2 .77	−2 .58
Sum of the errors	+0 .68	+3 .20

Hence the difference of the errors comes out 2".52, not much different from what simple observations had produced, and this quantity, modified for the position of the telescope, diminishes the observed zenith distance in every observation

10. But these were not the only experiments made with the two similar instruments, to detect the constant difference of their measures taken at the horizon, or nearly so: recourse was lastly had to reflection. A wooden vessel was prepared for holding *mercury*, and another of tin painted black for *water*, which, when properly filled and conveniently placed, were successively used with both instruments, when their object-ends were counterpoised, and then eye-tubes unbraced After a mean had been taken from several observations of an object, at the distance of 2500 metres, with diminished diaphragms of black paper, with the eastern instrument, the measure by reflection from water exceeded the direct measure by 1".38, and by reflection from mercury by 1".21, but when the eye-end was braced the excess from water was 3".13 making a mean = 2".25 With the western instrument used in the same way with mercury and with water, the mean of the excess was 4" 23, so that 4".23−2".25 = 1".98 was the difference from this experiment, between the horizontal measures of the two instruments.

11. *Repeating principle.*—Though the vertical circle of Reichenbach's instrument may be used in the same manner, as the English instruments before described are employed, when the upper clamp *a* is fast, and the lower one *b* at the telescope is released, in which situation the graduated circle is fixed and the vernier circle moveable in conjunction with the telescope, yet in all nice observations made with small instruments, where extreme accuracy is aimed at, the observations are more correctly taken on a number of successive arcs, by repeating the observation on every new arc till the whole circle has been gone over by the verniers, after which it is presumed that the effects of eccentricity in the circles, and of inequality among the divisions will be annihilated When the adjustments of the long vertical axis, and of the telescope's axis are properly made, and the zero of the circle, which we suppose to be graduated all round to 360°, (or to 400° according to the French new system) is adjusted to the zenith point, by means of a zenith star, with face east and face west alternately; the level being suspended on the vernier circle with its bubble at right angles to the telescope's line of collimation, if the telescope be brought down to a given star on the meridian, together with its attached vernier circle, while the graduated circle remains fixed near the upper end of the pillar, by the clamp *a*, it is obvious, that it will have moved over an arc of the clamped circle equal to the star's zenith distance, and the four verniers, if now read, will give a mean of such distance approximately, or indeed truly if all the adjustments, including the collimation, were complete, and the circle's divisions perfect, but as such presumption would leave a doubt on the mind of the observer, he is not satisfied with this first, or single measure, though taken at the same time on four separate arcs of the limb, he therefore disregards these first readings of the four verniers, and now fixes the telescope and verniers to the graduated circle by the clamp

b, before clamp *a* is unscrewed, then, clamp *a* being released, the two circles and telescope, being now all united in one, are brought back till the telescope is again pointing to the zenith nearly, in which situation they are made fast by the clamp *a*, then by means of the vernier's level and the tangent screw of clamp *a*, the vertical position of the telescope is made correct: in this situation clamp *b* is released, and the telescope is again at liberty to descend to the same star, when a small motion has been given in azimuth, equal to the star's change in that direction, the graduated circle in the mean time remaining clamped: the arc now passed over by the verniers is a multiple of the first arc, and if the readings were now registered, the mean would be not a fourth but an eighth part of the sum, and in all probability the resulting zenith distance would be more correct than the arc giving one fourth of the first sum of the four readings; but still it is not necessary to read at this stage of the operation. The exact times however at which the contacts in the observations are made must be marked down in connexion with the corresponding readings, because the zenith distance of the star is constantly varying out of the meridian: as clamp *b* was made fast to complete the observation by the aid of its tangent screw, clamp *a* is now released, and the telescope again brought into the vertical position as above described, to measure a similar arc the third time, and when a repetition of measures, along the limb of the graduated circle, has carried the verniers round an entire revolution, in the way here described, by screwing and unscrewing the clamps *a* and *b* alternately, at each repetition, the sum of the final readings divided by four will give the whole arc passed over, and this again divided by the number of repeated operations will give the single arc, which would be the measure of the star's zenith distance, if it had no apparent motion during the whole interval. In this explanation of the principle of observing with a repeating circle, we have assumed, for the sake of illustration, that the first measure shall commence at *zero*, but this is not necessary to the accuracy of the result, nor yet always convenient, for if the mean of the incipient readings be subtracted from a mean of the final readings, the remainder, adding an entire circle if necessary, will be the total arc passed over, the same as if the measure had begun exactly at zero. This measure however will require to be corrected by certain quantities depending on the star's declination, and distance from the meridian at the instants of observation.

12. *Repetition by double arcs*.—In the preceding paragraph the measures were described as taken by repeating the operation along *single arcs*, taken in succession; which mode of observing is simple, but requires, in small zenith distances particularly, many repetitions, and as many corresponding intervals of time; and consequently allows the star to vary its zenith distance very considerably; but the operation may be accelerated, by reversing the face of the vertical circle after each alternate observation, and by gaining thereby *double arcs* at each motion of the telescope. Let us suppose the star to be to the south of the zenith in any given latitude, then the observation will be made thus; clamp the telescope and vernier circle to the limb of the graduated circle at any convenient point near the zero, and reading all the four verniers register them in their proper columns; carry the telescope and circle so united to the star, and fix the circle by the clamp *a*, in order that its tangent screw may complete the contact of the star with the horizontal wire; note the instant by the clock, or time-piece, when such contact took place, to the fraction of a second, and enter it in a column in the same line with the readings already taken, the mean of which may be called the point of departure on

the circle; the vertical axis is here supposed to be adjusted truly vertical by its attached level, and to remain so during the subsequent operations, turn now the circle and telescope together half round in azimuth, and the telescope will preserve its zenith distance, but will point towards the north instead of the south, release the clamp *b*, and carry the telescope with the attached verniers over the zenith, and back again to the star, where the contact must now be made, by fixing this clamp again, and using its tangent screw, while the circle remains fixed as before; in this situation the telescope and verniers have passed over a *double arc* from the north side of the zenith to the south, and this arc is the measure of *twice* the zenith distance of the star, corrected too for collimation, because the telescope's position has been reversed, as it regards the horizontal wire in the eye-piece; the time of this second contact must be noted as before, but the readings are disregarded for the present, the circle and telescope being now united are turned another semi-circle in azimuth, when the telescope again points to the north as before, and if it were now carried back separately as at first, it would come to the star at the original point of departure, and the same arc would be used that has been already employed for a double measure, in which case the advantage of repetition would be lost the telescope therefore must now be carried over to the star in *connexion with* the graduated circle, in order to gain a new point of departure, and the observation must be finished by the tangent screw of clamp *a*, as at first, the original point of departure being thus sent forwards towards the south. The time having been noted, and also the position of the level's bubble, the same operation is resumed, of turning the circle and telescope together a semi-circle in azimuth, first for the telescope to be brought singly, and then, after another semi-revolution, for the circle and telescope to move in connexion, when a second double arc will be measured, and a third point of departure brought to the star, in doing which the tangent screws of the two clamps are alternately used for completing the contacts. After four, six, or eight repetitions of this operation have carried the original point of departure round the circle, or nearly so, the time of the last contact made by the telescope, carried over without any connexion with the circle, must be noted, and the final readings put down to correspond, as well as the state of the level; at which period the series of observations will be complete.

13 Suppose now that eight pairs of observations have been made of a star's zenith distance, with the telescope alternately placed at the east and west face of the circle, each pair of such observations would separately give a double arc, differing from a mean of the whole number by the quantity that the star has ascended before the meridian, or descended after, at the moment of making the last observation, after the telescope has moved separately over the said arc, hence a reduction of the star to the meridian altitude is necessary for each instant of such second observation of the pair; and when the sum of all the reductions divided by their number is applied, with its proper sign, to the mean double arc, the true double measure is obtained; one half of which will be the apparent zenith distance, which, by the proper corrections, may then be converted into the true zenith distance. The Tables of Delambre, or that by Dr Young, given at pages 99—103 of our first volume, and explained at pages 329—330, may be conveniently used for effecting the reductions in question; and the examples we have given, at the latter place of reference, are sufficiently applicable to our present purpose. The mode of registering the observations, and method of reducing them adopted by Brioschi will be seen, without further enlargement of our section, from an inspection of the two annexed specimens of solar observations taken near the solstices, at the Royal Observatory erected at Naples, in latitude $40^\circ\ 51'\ 46''.63$, and longitude $0^h\ 47^m\ 44^s.3$ east of Paris.

CIRCUM-MERIDIAN ZENITH DISTANCES OF THE SUN,

BY REICHENBACH'S REPEATING CIRCLE

IN THE YEAR 1820

SUN

Day		18 June		19		20		21		22	
Repeating Circle		Eastern.		Western		Eastern		Western.		Eastern	
Column 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col 2.
Time by the Clock.	Level	45^m 34^s	398	44^m 8^s	392	51^m 56^s	380	51^m 30^s	371	0^m 32^s	378
		46 22	360	45 9	374	53 40	414	52 31	382	1 26	413
		47 14	419	45 58	392	54 54	429	53 30	375	2 43	118
		48 12	383	46 53	377	56 28	382	54 36	385	3 33	375
		49 8	426	47 48	291	57 35	412	55 30	377	5 14	388
		50 0	379	48 40	278	58 20	401	56 31	387	5 53	408
		50 56	428	49 35	291	59 14	416	57 42	378	6 55	398
		51 47	388	50 29	279	1 8	401	58 44	387	7 40	415
										8 48	390
										9 30	411
Noon.		5^h 48^m 10^{s}5		5^h 47^m 59^{s}7		5^h 56^m 32^{s}1		5^h 56^m 32^{s}2		6^h 4^m 54^{s}4	
REMARKS		Rather trembling and confused.				Behind clouds The axis altered after the second, and after the fourth observation		Quiet behind clouds		The axis moved after the second, and after the fourth observation	
Barometer		27^i * 7^l.9		27^i 9^l 7		27^i 10^l 3		27^i 9^l.7		27^i 8^l 8	
Thermometer within		18° 0		18° 0		17° 5		17° 3		17°.7	
without		18 0		18 .5		17 .7		18 5		19 0	
Num of observations		8		8		8		8		10	
Beging. of the arc I		209° 38′ 0″		131° 20′ 14′		116° 2′ 28″		90° 0′ 10″		213° 2′ 10′	
II		6		16		28		13		10	
III		37 58		8		32		10		6	
IV		38 2		6		26		6		10	
End I		310 6 16		273 46 2		254 18 32		220 12 52		67 6 20	
II		12		45 58		33		54		23	
III		14		46 0 *		28		52		25	
IV		18		0		30		52		20	
Arc measured		139° 28′ 13″.5		139° 19′ 40″.0		139° 16′ 2″ 2		139° 12′ 42″.8		174° 4′ 14″ 5	
Reduction to zenith		− 30 .4		− 26 .1		− 7 2		+ 18 .0		+ 8 .2	
Reduction to meridian		− 2 46 .1		− 2 50 .8		− 4 51 .7		− 4 9 8		− 6 44 .1	
Measured arc reduced		139 24 57 .0		139 16 32 .1		139 11 3 3		139 8 51 .0		173 57 38 .6	
App. mer zen. dist		17 25 37 .1		17 24 31 0		17 23 52 7		17 23 36 .4		17 23 45 .9	
Flexure		+ 0 2		+ 0 0		+ 0 .2		+ 0 9		+ 0 2	
Refraction		+ 17 .1		+ 17 .1		+ 17 .2		+ 17 2		+ 17 .0	
Parallax		− 2 .7		− 2 7		− 2 7		− 2 .7		− 2 7	
True zen. dist.		17 25 51 7		17 24 49 .3		17 24 7 .4		17 23 51 7		17 24 0 4	

* i = inch. l = line French.

CIRCUM-MERIDIAN ZENITH DISTANCES OF THE SUN,

BY REICHENBACH'S REPEATING CIRCLE

In the Year 1820

SUN

Day		20 December		21		22		23		27	
Repeating Circle		Eastern		Eastern.		Western		Eastern		Western.	
Column 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col 2	Col 1	Col. 2
Time by the Clock	Level	58^m 35^s	398	59^m 21^s	411	54^m 30^s	402	12^m 47^s	415	19^m 25^s	401
		59 32	387	0 20	379	55 28	390	13 43	397	20 30	388
		1 20	401	1 55	392	56 33	403	14 36	416	21 56	396
		2 28	398	2 51	417	57 38	390	15 50	395	22 53	390
		3 30	404	4 1	406	58 57	404	17 9	413	23 47	397
		4 26	390	5 0	403	0 0	397	18 16	388	24 52	396
				5 57	403	1 6	401			25 48	398
				6 58	410	2 10	393			26 57	396
				7 57	402	3 23	401				
				9 5	407	4 5	394				
Noon		17^h 59^m $45^s.9$		18^h 4^m $15^s.1$		18^h 8^m $20^s.4$		18^h 13^m $10^s.5$		18^h 22^m $28^s.2$	
Remarks		Distinct although behind clouds.		Limb trembling The axis moved after the second and fourth observations		Limb confused and trembling The axis moved after the fourth observation		Behind clouds, well defined		Behind clouds, not very distinct The axis moved after the second observation Very moist air	
Barometer ...		27^l $11^l.7$		27^l $10^l.3$		27^l $8^l.0$		27^l $6^l.9$		27^l $7^l.8$	
Thermometer, within		$9^0.7$		$7^0.2$		$7^0.0$		$6^0.5$		...	
without		10.6		6.5		6.3		6.0		$8^0.5$	
Num of observations		6		10		10		6		8	
Begins of the arc I		6° 5′ 7″		119° 55′ 0″		81° 0′ 26″		67° 27′ 47″		175° 53′ 42″	
II		8		54 56		33		48		44	
III		4		55 0		32		52		38	
IV		6		0		30		54		36	
End I		31 50 4		42 54 20		6 59 56		93 12 10		220 18 18	
II		8		24		57		7		25	
III		3		18		55		10		22	
IV		5		24		53		10		23	
Arc measured		385° 44′ 58″.8		642° 59′ 22″.5		642° 59′ 25″.0		385° 44′ 19″.0		513° 24′ 42″.0	
Reduction to zenith .		− 5.6		+ 0.4		− 21.1		− 12.8		− 5.9	
Reduction to meridian		− 1 11.7		− 2 21.4		− 2 44.4		− 1 17.9		− 1 14.1	
Measured arc reduced		385 43 41.5		642 57 1.5		642 56 19.5		385 42 48.3		513 23 22.0	
App. mer zen. dist ...		64 17 16.9		64 17 42.1		64 17 37.9		64 17 8.0		64 10 25.3	
Flexure		+ 2.1		+ 2.1		+ 2.8		+ 2.1		+ 2.8	
Refraction		+ 1 57.8		+ 1 59.7		+ 1 59.0		+ 1 58.7		+ 1 57.3	
Parallax.		− 8.0		− 8.0		− 8.0		− 8.0		− 8.0	
True zen dist.		64 19 8.8		64 19 35.9		64 19 31.7		64 19 0.8		64 12 17.4	

§ LXXIV. REPEATING CIRCLE BY TROUGHTON [Plate XXIII Fig 1]

1. The principle on which the construction of a repeating circle is founded, was first suggested by Professor Mayer, in a paper presented by him in 1758 to the Royal Society of Gottingen, in which he described an instrument for repeating horizontal angles, which he called a *Goniometer*, and which he had contrived eight years before. It had no circular limb, but consisted of a pair of bars, of a foot each in length, that opened at a joint at one end, like a pair of dividers, and were held by a stand in such way, that they would turn round it either jointly or separately, so that when the lower was pointed to a left-hand object, and the upper one bearing a telescope to a right-hand one, the included angle was measured simply from a chord line by a pair of compasses, extending from a fixed point in the first rod to a similar point in the second, after making an allowance for the superposition of the rods, and as the upper rod could be brought several times to the first object, and opened again to give a view of the second without interfering with the position of the under rod, every successive opening gave an additional arc of measure, and when the lower rod had been thus gradually carried backwards, at every repetition, nearly through an entire circle, the supplemental arc measured by the chord line, when subtracted from 360°, gave the large arc passed over, and consequently a mean of the whole number of arcs, separately passed over, when the large arc was divided by this number.

2. The first person however who applied this principle to measure round the limb of a divided instrument was Borda, who about the year 1789 caused a repeating circle to be constructed, that would measure with equal facility horizontal and vertical angles, and with much greater accuracy than the imperfect state of dividing would at that time admit of in the common way. While astronomers on the continent have bestowed unlimited praise on the utility of the repeating circle, both before and since great improvements have been made in the nice art of graduating instruments, the English astronomers have preferred those of larger radii, which admit of more correct subdivisions by the improved methods of graduating, thereby avoiding much trouble in making and reducing their observations, and also saving much time spent in repetitions of the same observation. When, however, a circular instrument is made so small as to be portable, and yet is destined to perform accurate work, the trouble and time become secondary considerations, and the English artists object not to construct it on any principle that may promise to secure the proposed object. Accordingly Troughton, who was never deemed remarkable for copying other men's inventions, improved the construction of Borda's original instrument by the introduction of different contrivances that ensure, at the same time, its superior accuracy and convenience in use. we propose therefore to describe in this section the astronomical repeating circle as constructed by this eminent maker.

3. Figure 1 of our Plate XXIII. gives such a perspective view of Troughton's repeating circle, as will enable us to convey a general idea of its constituent parts by the aid of letters of reference, which in this complicated single figure become essential but after the descriptions we have given of other larger instruments, it will not be necessary to be very circumstantial, as to the dimensions of the various parts, which may be judged of from the proportions exhibited in the representation. As we have not given a drawing and corresponding description

of Borda's instrument without reflection, we shall occasionally point out how it differs from the instrument we propose to describe. The base of this repeating circle, like the French instruments made after Borda's pattern, is a tripod of brass bearing a clamp and tangent screw for giving slow motion in azimuth, but has three equidistant verniers that read each to the accuracy of 10″ on a twelve-inch horizontal circle, the original one having but one vernier, and one of the feet screws, *a*, has a contrivance for preventing the feet from slipping on a stand, and also for giving very slow motion to this foot, which in use is placed towards either the north or south, this additional detached piece stands on two sharp points besides the end of a screw, which together form an isosceles triangle, having a gutter in which the screw of the tripod rests, and the slowness of the adjustment depends on the distance of this foot from the two sharp pins, which constitute a fulcrum for the lever forming the triangle. A strong conical spindle of steel fifteen inches long, and tapering upwards from an inch to a little more than half an inch at the top, is fixed perpendicularly in the centre of this tripod, as an axis for the horizontal motion. A strong brass pillar, made hollow, fits this spindle at both ends, so as to turn smoothly on it without perceptible play, and, resting on the central part of the tripod, supports the whole weight of the body of the instrument, and also carries the graduated horizontal circle above mentioned so close to the arms of the tripod, that it may just revolve without touching. Two of the three feet screws, *b* and *c*, rest on brass cups made fast to the wooden stand, on which the instrument usually stands, and a simple microscope, *d*, forming a positive eye-piece, is attached by a lever to a ring that embraces the pillar, and allows it to view any of the verniers that may require to be read, by means of its revolving motion. In Borda's instrument this circle is only four inches in diameter, and reads with a single vernier, which indicates 3′, and sometimes only 5′, its edge is indented, and the motion is given by a pinion, as in the case of a common theodolite.

4. Across the upper end of the pillar is fixed a strong bar of brass, *e*, which consequently partakes of its revolving motion, and upon this bar is fixed a similar one, face to face, by a pair of thumb screws, one of which appears at the end opposite *e*; to this second horizontal bar a pair of upright supporting bars are firmly attached, which together form a strong frame, with two sides and a bottom, and when the fixing thumb-screws are withdrawn, the body of the instrument may be detached from the tripod and pillar, which separation affords great convenience in packing the two parts separately for travelling this is also the construction of Borda's circle. The ends of an horizontal axis are centered into pivot holes made near the upper ends of the supporting bars of the frame, round which the whole of the superincumbent portion of the instrument may turn, and by means of this axis the vertical circle and its appendages may take either a vertical, horizontal, or inclined position, at the pleasure of the observer. One end of this horizontal axis, which lies parallel to the plane of the circle, may be seen at *f*, above the back telescope. To this end of the axis a semicircle is made fast, having its plane parallel and contiguous to the inner face of the left-hand supporting bar, and is held by a clamp fast to this bar, in any given position. This semicircle cannot be seen in the figure. The graduated vertical circle, which is eighteen inches in diameter, has eight conical radii, the inner ends of which are fastened to a strong octagonal centre piece, and the outer ends are screwed to the circular limb, at equal distances from each other; while the limb itself is strengthened by a circular edge-bar, large enough to be held by a clamp. this edge-bar, being

on the back of the circle is consequently not seen. A thick socket or cylinder of brass passes at right angles across the middle of the horizontal axis, which there swells to a large diameter, and is made perfectly fast to it, so as to ascend or descend as the axis turns round; at the remote end of this cylinder a solid piece of metal is fixed, sufficiently heavy to counterpoise all the assemblage of metallic parts, that are placed on the near side of the said horizontal axis, namely the circle, vernier bars, telescopes, and level. This fixed cylinder allows the long socket to pass through it, that constitutes the circle's axis, when made fast at the outer end to the back face of the octagonal centre piece, and, having its remote end passing beyond the extremity of the counterpoise, which is the frustum of a cylinder, the bearing of this tubular axis within the fixed cylinder is long enough to allow of good fitting, and of steady motion in the circle's altitude. Another axis of solid steel, which fits the bore of the circle's tubular axis, passes through the whole length of it, and carries the front telescope in connexion with the four armed verniers, that move as one piece along with this solid interior axis, and though the verniers are in contact with the divided limb, their motion in its plane is not guided, as in Borda's circle, by the face of the limb, but by the perpendicular direction of their long axis, passing about eight inches through the circle's concentric hollow axis. One of the four verniers, standing at *g*, has a clamp and tangent-screw for giving the slow motion, which is purposely omitted in the drawing; and by which the contact of a star with the horizontal wire of the front telescope is completed. This telescope is placed at an angle of 45° with each of the four vernier bars, either above or below, and preserves this distance constantly. Another bar, like one of the vernier bars, but not graduated, revolves on a socket round the circle's hollow axis, beyond the posterior face, and has a clamp, by which it may be made fast to the circular edge-bar, that strengthens the limb, at any part of it; the back telescope is made fast to this single clamping bar at the distance also of 45°, and as near to the axis of the circle as its thickness will permit; hence it may be perceived, that either of the two telescopes may be attached to, or detached from the circle, by means of their respective clamps; and that they may be made to move, all as one piece, or any of them separately but neither of these clamps will fix the circle in a stationary position, or regulate its motion. In Borda's construction a racked circular plate is fixed on the remote end of the circle's tubular axis, beyond the counterpoise, and an endless screw, fast to the counterpoise, gives motion to the circle at the opposite end of the axis, which is too far removed from the screw to be immediately obedient to its action. In our instrument this motion is better regulated; a circular jointed clamp surrounds the circle's axis near the cross horizontal axis, and carries an arm ascending towards the upper limb of the semicircle for clamping, as above described, while another arm is made fast to the fixed cylinder carrying the counterpoise, that ascends in the same direction, then a long screw passing through the remote ends of these bars, or levers, will make them approach or recede, according to the direction in which the screw is moved, the head of which is visible at *h*; and as the moveable lever is connected with the circle's axis, by means of the clamp, and the fixed one fast to the cylinder of the counterpoise, it is easy to conceive, that the action of the screw will move the circle, when the clamp is fast; but that the circle will be free to take a quick motion when the clamp is released The back telescope is used only when horizontal angles are measured in terrestrial operations, and instead of it a good level is fixed parallel to it, at the opposite side of the circle's axis, in which situation it forms a counterpoise to this telescope, by being made unusually heavy for this purpose it has the same advantage of quick and slow motion that this telescope derives from the tangent-screw of the clamp, carried by the single bar.

5. The telescopes are achromatic, and are similar to each other, each having a focal length of twenty five inches, magnifying from thirty to sixty times, according to the eye-pieces used, and having apertures of an inch and three quarters each, with illuminating reflectors. The telescope, placed behind the circle, has only two lines in its focal point crossing each other at right angles, but the front telescope has six lines crossing one another, three in each direction, the plates that hold this system of spider's lines being adjustable by two pairs of screws, one pair acting in the horizontal, and the other in the vertical direction. In the former telescope the adjusting screws are designed to place the line of collimation parallel to the plane of the circle, and also to the bubble of the level, but in the latter the second adjustment is not wanted. These telescopes, being intended to view objects at different distances, have each a drawer for distinct vision adjustable by rackwork, as in an ordinary telescope, carried by a common tripod. The level *i*, lying parallel to the back telescope, is ground to a radius, that gives a scale generally of an inch and a half to the minute, marked on two slips of ivory sliding on the glass tube, and adjustable to zero. Another level is fixed at one end to the cross axis, and at the other to the surface of the counterpoising frustum of a cylinder, the use of which is, to place the circle's plane in a vertical position, which it will always do by bringing the bubble to the middle of the glass tube, when once properly adjusted by a detached plumb-line, suspended parallel to the said plane. The upper or remote end of the back clamping bar, the tangent screw of which is seen at *k*, is formed into a semicircle denoted by *l*, of about half the radius of the circle itself, and on its face are roughly divided two quadrants of zenith distance; and a blank of about half an inch separates their zeroes from each other Two small sliding stops are made to fit the limb of this semicircle, in such way, that their fiducial edges may be set and fixed to any given division. Near the object end of the telescope an index *m* is made fast to the tube, and reaches down to the semicircle's limb, where its breadth just covers the space left between the two zeroes of the graduated quadrants; and when the stops are set to the zenith distance of a star to be observed by repetition, the index, meeting with the stops alternately, will limit the arc of the telescope's motion, at the very points where the star will become visible in the centre of the field of view, thereby ensuring the finding of the proper star without loss of time. The motion in azimuth is also limited to a semi-revolution by stops, but differently applied; a small cylinder of pointed steel is lodged in the hole of a piece of brass, attached to one of the tripod's arms under the vernier, and urged forwards by a spiral spring against the edge of the horizontal circle, when in use, but at other times put out of action. When the leading vernier arrives at the zero 360°, or at 180°, the pointed end enters a shallow hole made purposely at the edge of the circle, but so slightly as just to be felt, on turning the instrument round in azimuth; for the point will retire of itself when an additional pressure is applied, and then, this point being displaced, the motion in azimuth will become free. By the help of these stops applied to the two circles, a bright object may be repeatedly observed by day light, or a faint one by night, without fear of losing it, or danger of mistaking another for it, and even without regarding the graduated circles, during the interval of repeating the observations. When the repetition is completed the verniers of the vertical circle

are read by means of the two simple microscopes, placed at the opposite ends of the double bent lever, which revolves round the central portion of the quadruple vernier bars.

6. *Adjustments.*—The adjustments of this instrument are neither numerous, nor difficult to be executed. As in other cases where a vertical axis is supported by a tripod, this axis must be first made perpendicular by the foot screws and levels in the way that has been already explained . secondly, the plane of the upper circle must be placed truly vertical, in the first instance by a detached plumb line held parallel and close to its face, while a slight motion round the horizontal axis will complete the parallelism; but this need not be repeated, when the second level, applied to the axis of the circle, has once been adjusted by the plumb line, for its bubble will perform this adjustment afterwards with sufficient accuracy, though the best criterion of the circle's vertical position is, when an elevated object can be brought to coincide with its image, seen by reflection thirdly, the line of collimation of the telescope must be brought parallel to the plane of the circle, as both telescopes are in eccentric situations, as they regard the vertical axis, a very distant object must be chosen as a mark, and as nearly as can be estimated in the horizon; when that mark is bisected by the middle wire, the verniers of the azimuth circle must be read, then, turning the vertical axis half round till the circle's face stands at the contrary hand, read the azimuth circle again, when the same mark is a second time bisected, now move the azimuth circle through half of the difference of these readings, abating 180°, and the plane of the vertical circle will pass through the distant object chosen as a mark, in this position move the vertical wires of both telescopes to bisect the said mark, which may be done by the proper ocular screws, and this adjustment will be finished. the fourth adjustment is that by which the horizontal wire of the back telescope is set parallel to the level placed above it, but this is only wanted when the operation of levelling is the object of the observation; for this purpose a very distant object is again necessary, because the back telescope is situated lower than the front one with the former adjustments remaining unaltered, an object nearly in the horizon must be bisected by the middle wire of the front telescope, and the four verniers read, in the next place reverse the position both in azimuth and altitude, and read the verniers again, then half the difference of these readings will be the error in collimation in altitude, and half the sum the altitude or depression of the object, set the verniers to the horizontal point on the circle thus pointed out, and turning the instrument slowly round in azimuth till a new object is seen in the horizon, after having adjusted the half difference for collimation, bisect that object, or again turn to one that, being exactly in the horizon, will be a proper mark for bisection, then lastly, turn the back clamping arm by its tangent screw till the bubble of the parallel level stands at zero, and the adjustments will all be complete.

7. The frame constituting the support of the body of this instrument is made deep and wide enough to admit of the counterpoise passing through it, as the horizontal axis revolves, and the vertical circle may readily be so placed, as to have its face perfectly horizontal for measuring angles between terrestrial objects, or even in any oblique direction, that may require to be reduced to the horizon, but as our business is confined to practical astronomy, and as various formulæ, or tables, for making the necessary reductions would be required for explaining the use of the instrument in geodætical measurements on a large scale, we will refer those readers who wish to make such use of this instrument, to the ample work published by the French Institute, called *L'Arc du Meridien*, to Puissant's *Geodosie*; and to Delambre's

Methode Analytique pour la Determination d'un Arc du Meridien, where the subject is treated with all requisite precision. As, however, the Astronomer-royal of Greenwich has lately proposed to refer the points of greatest elongation of circumpolar stars to marks in the horizon, by perpendicular lines demitted by means of an altitude and azimuth circle, and then to measure by a repeating circle the angle subtended at the horizon, by a line joining such horizontal marks, we will explain here how such angle may be determined by the repeating process, that this instrument is capable of performing; and in a subsequent section we will show, how such horizontal circle may be converted into the diameter of the circumpolar circle, which will then be twice the polar distance of a star, apparently moving in such diurnal circle, without the influence of atmospheric refraction.

8. *Horizontal angles.*—When the line subtending the angle to be measured is quite horizontal or nearly so, the preparatory operation is, to place the plane of the circle, that repeats, in a horizontal position, by turning the whole body of the instrument, including the telescopes, round the horizontal axis, till the intersection of the wires of the moving telescope will pass over both the marks that bound the subtense; but as it will seldom happen that both marks are accurately in the horizon, this operation is not easily performed; since the plane of the circle requires to be adapted to the given obliquity of the subtending line. Mr. Troughton has given the following practical rule, which will greatly facilitate this preparatory operation:—"Set one foot of the tripod, as nearly as you can guess, in a line with that object of the two, which you judge to have the least elevation or depression, and with the plane of the circle vertical, and the back telescope horizontal (both to the exactness of two or three minutes), bring back the telescope to the object, partly by turning in azimuth, and partly by turning or propping the foot-screw; next turn the circle round on the cross axis, until it seems by the eye to occupy the proper position; then a second time bring the back telescope to the object by the foot-screw, and also by turning in azimuth; lastly, complete the operation by bringing the upper telescope to the other object by its own proper motion, in conjunction with that of turning round the cross axis." The principle of the rule is this; the cross axis and the lower telescope being made parallel to each other, and pointed to the same object, the circle may be turned round that axis, without changing the angular position of this telescope. When the plane of the circle is thus made to pass through both objects, the operation of measuring the angle by repetition may be conducted in the following manner. Let the verniers connected with the fore telescope be set to zero, or be clamped at any convenient part of the limb; and let the readings be noted down; in the next place by means of the general motion place the intersection of the ocular wires exactly on the middle of the object towards the left. then by its own proper motion bring the back telescope to cover the object to the right in like manner; and while in this situation examine both objects by their respective telescopes, and be satisfied that the angle included between them is correctly contained between the two telescopes; now by the general motion move the back telescope until its wires coincide with the object on the left. The front telescope is by this motion carried the space of a single measured arc to the left of the left hand mark, unclamp it after fixing the circle, and carry it round to the object to the right, and its verniers will have passed over an arc equal to double the angular subtense connecting the objects; the whole operation is now once performed. To read this double arc would be rather

injurious than useful, by prolonging the time, and is only done to gratify curiosity, in keeping an account of each separate step in the gradual progress of the work, therefore, with the front telescope clamped in this part of the limb, the same operation that has been described must be gone through a second time, when the readings would give the sum of four single arcs, if noted, but a third, fourth, fifth, &c. course of the same operation is usually gone through, till a sufficient number of repetitions are accomplished, to ensure accuracy, that is, to do away the inequality of the circle's divisions. Then when the final readings are obtained in degrees, minutes, and seconds, on an average of the four verniers, the total arc, divided by *double* the number of repetitions of the whole operation, will give the single angle required The readings at the commencement and termination of the series of operations are all that are required in this method of observing, for it is of no importance whether the whole arc be well divided or not, provided the first and last readings fall in a graduated portion, that nearly begins and terminates the operation. If the result of each separate repetition should be taken singly, the errors of division would be included in each measure, but a mean of the whole would not in principle differ from the one final result, obtained by an uninterrupted continuation of the repetitions. A mean of all the double arcs taken separately without the repeating principle would not be so accurate. In a series of operations, for obtaining the horizontal angle, the levels are of no use. Each telescope views both the marks alternately, and as they contain permanently between them the single arc, this arc is doubled, as it regards the right hand mark and the left hand or fore telescope, when carried away beyond the left hand mark by the general motion, in placing the second telescope to the left hand mark, for then, the circle being clamped, the final motion of the left hand telescope to the right hand mark, in which it crosses the other telescope, measures the double arc at once. The principal advantage of the instrument is, that, however unequal the divisions of the intermediate limb of the circle may be, they are of no importance, as not being read, and the incipient and final readings, being all that are taken, are charged with but few errors, and those such as will merge in the number of arcs passed over. If, however, there is any error in the resistance of the centre-work, it will probably be constant, and the observer has no means of detecting it

9. *Example.*—We will give an example of a terrestrial angle measured by a repeating circle by Troughton, in which a series of ten repetitions, or twenty single arcs, carried the verniers more than three times round the graduated limb of the circle, before the final readings gave at once the measure of the angle; but as all the readings after each successive operation are registered, it will afford us the means of computing the angle due to any number of operations in the series, and also to each double arc, taken singly without the repeating principle, from which a *mean* may be deduced this mode of treating the example will exhibit a comparison of all the results. The first column contains the number of single arcs measured, the second gives the mean of the four readings of the verniers at the corresponding points of the limb, with the entire revolutions to be added, the third shows the angles gained by repetition at the end of each pair of single arcs, and the fourth exhibits the same as they are deduced from each double arc separately. The first operation begins at zero, and the ninth is not registered

No	Double observations	∠ by repetition	∠ by successive pairs.
2.	127° 7′ 45″ + 0°	63° 33′ 52″.5	63° 33′ 52″.5
4.	254 15 24 + 0	51 .0	49 .5
6.	21 23 00 + 360	50 .0	48 .0
8.	148 30 30 + 360	48 .7	45 .0
10	275 38 10 + 360	49 .0	50 .0
12.	42 45 50 + 720	49 .1	50 .0
14.	169 53 20 + 720	48 .6	45 .0
16	297 0 50 + 720	48 .1	45 .0
20.	191 16 17 + 1080	63 33 48 .8	51 .8
		Mean of all the nine	63 33 48 .5

In this example, it may be remarked, that the measure of the horizontal angle derived from the last readings, after ten repetitions, viz 63° 33′ 48″.8, is almost the same as arises from the average of all the results contained in the last column, where it must be taken for granted, that an opposition of contrary errors has produced a sufficient correction. But the angle thus obtained requires to be corrected for excentricity of the lower telescope; and the quantity of this correction depends on the distance of the objects from the observer, and upon the distance of the axis of vision of the lower telescope from the centre of the instrument In Troughton's circles the excentricity of the lower telescope is usually one inch and four-tenths, and if we calculate by the formula given by Delambre for this purpose, we shall find that the correction, due to 1000 French fathoms of distance, is 2″, and also that the amount of this correction varies inversely as the distance, viz. that at 2000 it will be only 1″, and at 3000 0″.67, &c. But when the line connecting the two objects is not horizontal, the principal correction will be to reduce the observed oblique angle to the horizontal angle, which is most readily done by the French Tables, it will however be sufficient for our purpose, to give the formula by which the computation may be effected, when the obliquity is not considerable, viz. if we put

$A =$ the angle of position observed,
$H =$ the altitude of the mark a,
$h =$ the altitude of the mark b,
and $n = \sin^2 \frac{1}{2}(H + h) \,.\, \text{tang}\, \frac{1}{2} A - \sin^2 \frac{1}{2}(H - h) \,.\, \text{cotang}\, \frac{1}{2} A.$
Then the correction $x = n.\ \sec H\ \sec h.$

But if the zenith distances differ more than 2° or 3° from 90°, the following formula may be substituted, viz

$$\operatorname{Sin} \tfrac{1}{2} Z = \sqrt{\frac{\sin\left(\frac{C+\delta+\delta'}{2} - \delta\right) \sin\left(\frac{C+\delta+\delta'}{2} - \delta'\right)}{\sin \delta - \sin \delta'}}$$

where Z is the angle reduced to the horizon, C the angle at the centre, δ and δ' the respective zenith distances of the objects but in general when the three sides of the triangle are measured by the instrument, the angle at the zenith, being subtended by the horizontal line, may be had by spherical trigonometry. When the altitudes of the objects above the horizon are small, we may conveniently use the subjoined formula, viz.

$$x = \left\{ \left(90^\circ - \frac{\delta+\delta'}{2}\right) . \operatorname{tang} \tfrac{1}{2} C - \left(\frac{\delta-\delta'}{2}\right)^2 . \operatorname{cotang} \tfrac{1}{2} C . \sin 1'' \right.$$

which formula may be thus exemplified,

$\delta = 89^\circ\ 41'\ 54''.6$
$\delta' = 88\ \ 49\ \ 15.6$

$\delta+\delta' = 178\ \ 31\ \ 10\ 2$ $\qquad$ $\delta-\delta' = 0^\circ\ 52'\ 39''$
$90^\circ - \frac{\delta+\delta'}{2} = 0\ \ 44\ \ 24.9$ $\qquad$ $\frac{\delta-\delta'}{2} = 0\ \ 26\ \ 19.5$

Put $2664''.9 = p$, and $2579''.5 = q$,

Then we have sin 1″ . . . 4.68557 . . . . . . 4.68557
Log. p^2 . . . . 6.85136 . . . log. q^2 6.39704
Tang ½ C . . . . 9 46228 . cotan ½ C = 0.58764

+0.99921 = 9″ 98 . . −1.62025 = 41″.71
−41.71

Reduction required . . . . −31.73
Observed angle (say) 32 20 15.70

Angle reduced to the horizon = 32 19 43.97

10. *Vertical angles.*—After having comprehended the method of observing an horizontal angle by repetition, our readers will perceive that the same adjustments and operation would apply to the measurement of a vertical angle, such as the altitude of a star from the horizon, provided an horizontal mark could be seen by night, and also provided the star had no apparent motion here there are two difficulties to overcome; the former is easily obviated by substituting a level for the horizontal mark, and the latter by tabular reductions of the star to the meridian, at each moment of observing it. It may be of use to the young observer to bear in mind, that the circle must be alternately clamped to the telescope, and to the level, and must always be connected with one or other of them in the front observation, which brings the point of departure to the star, the telescope must form one piece with the circle, so as to move together, and the clamp that connects them must on no account have its tangent screw or fixing screw touched, till the star is bisected, on the contrary, in the second part of the observation, in the reversed position, the level must be made fast to the circle, and equal care taken not to disunite them, till the star has been again bisected. For want of this caution a series of observations has frequently been vitiated, after several good repetitions have been gone through. Turning a wrong screw is fatal to the accuracy of the observations. When the four verniers have been clamped and read, a mean of them must be registered as the point of departure, which may be at zero or otherwise, in the next place turn the circle by the azimuthal and vertical motions till the telescope, now connected with it, is pointed to the star, and brought into contact by the tangent screw *h*, that gives slow motion to the circle, then adjust the level to the horizontal position, by its tangent screw *k*, taking care that this is the case

exactly at the moment when the star is bisected, which moment indicated by a chronometer or good clock must be registered, for on the coincidence of these three requisites depends the accuracy of the observation, and therefore an assistant for noting the time and level should be always ready, and well practised the circle, telescope, and level are now all connected, and in the first position, the vertical axis being supposed previously adjusted to its perpendicular attitude, turn now the whole instrument half round in azimuth, without deranging the connexion of its three material portions, and if the level is not found truly horizontal, make it so by the foot-screw under the edge of the circle, and the telescope will now point to the same zenith distance that the star gave it, but on the contrary side of the zenith point, release only the telescope, and, leaving the circle and level clamped, turn it over to the star again by its own proper motion, and bisect a second time by the tangent screw *g*, of the first vernier bar, when clamped again to the circle, but by no means touch any other screw, the telescope by passing over the zenith to the star a second time, passed over a double arc, and provided the bubble of the level retains its place, the verniers, if read, would show the distance of this double arc from the point of departure, when the reading due to that point is subtracted in the form of an index error; but the amount of this arc is disregarded, and could only be useful as an approximate double arc in case the series should by any accident be vitiated, or the number of repetitions forgotten. These two bisections of the star, in the first and second positions of the circle, which turns alternately to face east and face west, constitute the first *pair* in the series; and considering the present readings as a new point of departure, a repetition of the same operation will give the second pair of the series, which may now be described in a few words, first by making the bisection with the telescope in connexion with the circle, and then after reversion with the level in connexion, and in both situations perfectly adjusted. Then when a third, fourth, &c repetition have been carefully gone through, the difference between the index error and final reading when 360°, 720°, &c have been added to it, after being divided by the number of bisections of the star, or by double the number of repeated operations, will give the simple arc representing the apparent zenith distance of the star, and free from the existing error of collimation, which in each reversed position changes its sign. A similar process may be obtained with equal ease, by measuring double arcs of altitude instead of those of zenith distance, when the level is suspended so as to turn round on pivots, and the circle is furnished with a *stop* that limits its plane to a vertical position, when turned over its horizontal axis, to the contrary side of the pillar, for then the telescope is depressed below the horizon after such turning over, as much as it was elevated above in the first position in other respects the operation of repeating is the same as by the method of double zenith distances, and the figuring of the divisions may be engraved accordingly, or otherwise the *complements* of the readings may be substituted for altitudes. When the sun, moon, or a planet is observed, the upper and lower limbs must be taken alternately an equal number of times, and then the mean will give the zenith distance of the centre.

11. *Example.*—We will take as an example for the method of obtaining vertical angles by repetition, the observations made at Dunkirk of the pole star before and after its upper meridian passage on December 19, 1808, in which series thirteen repetitions or twenty-six single arcs were gone through to obtain the true latitude of the place, which had been previously determined to be 51° 2′ 5″. the apparent declination of the star was then known to be

88° 17′ 41″ 41, and consequently the zenith distance $Z\,(=D-L)$ was 37° 15′ 36″.41; the French barometer was at 0.75425, and the centesimal thermometer at −4°. When the verniers had travelled twice round the circle, the remaining arc was 248° 38′ 45″.46, and 968° 38′ 45″.46 divided by twenty-six single arcs give 37° 15′ 20″.21 $= z$, or measured zenith distance. The times at each bisection were registered, as in the subjoined type, as they differ from $0^h\ 24^m\ 44^s$, the time of the star's meridian passage.

True times of observation			Horary angle		Part I of reduction	Part II of reduction
H.	M.	S.	M.	S.		+
23	57	2	27	42	1504″.7	5″.49
	58	18	26	26	1370.4	4.55
	59	6	25	38	1288.8	4.03
	59	47	24	57	1221.0	3.62
	0	27	24	17	1156.8	3.25
	1	5	23	39	1097.2	2.92
	1	46	22	58	1034.8	6.60
	9	26	15	18	459.5	1.06
	10	10	14	34	416.6	0.42
	18	57	5	47	65.7	0.01
	22	23	2	21	10.8	
	22	59	1	45	6.0	
	25	43	1	1	2.0	
	29	19	4	35	41.2	0.01
	41	59	17	15	583.9	0.83
	45	54	21	10	879.0	1.87
	46	36	21	52	938.1	2.13
	47	12	22	28	990.3	2.38
	47	52	23	8	1049.8	2.68
	48	31	23	47	1109.6	2.99
	49	3	24	19	1159.9	3.26
	50	21	25	37	1287.1	4.01
	51	4	26	20	1360.1	4.48
	52	47	28	3	1542.9	5.77
	54	40	29	56	1756.8	7.47
1	0	18	35	34	2478.8	14.89
Vol. I. pp. 99—101.			Part I.		24811.8	80.72
			Part II.		+80.7	
			Sum . .		24892.5	

For the Reduction.

(Vol I. p. 329)

Log cos L	9.7985466
Log cos D	8.4735794
Log sin Z	0.2189328
Const. log . . .	8.4910588
Log 24892.5	4.3960685
Sum	2.8871273
Log 26 sub. . . .	1.4149733
Reduc. = 29″.66 . . .	1.4721540

For the Latitude.

Zen dist. obs. (Z)		37°	15′	20″.21
Cor. for level + 0″.68 Refraction +46.41 Reduction −29.66	}			+17.43
Merid. Z		37	15	37.64
Polar dist. of ∗ . .		1	42	18.59
Co-latitude		38	57	56.23
Latitude		51	2	3.77

12. Though it is agreed on all hands that greater accuracy may be obtained by a repeating circle, than by any other having the same radius, where time and trouble are of little comparative importance, yet there are many objections to its use, which do not apply to the altitude and azimuth circle, and which have been enumerated by an artist who has improved both constructions, and has had frequent opportunities of making the requisite comparisons. We

will therefore conclude our section with a list of the objections alluded to, as given in the beginning of the first volume of the MEMOIRS of the Astronomical Society of London.

"1 The origin of the repeating circle is due to *bad dividing*, which ought not to be tolerated in any instrument in the present state of the art

"2. There are three sources of fixed error which cannot be exterminated, as they depend more on the materials, than on the workmanship, first, the zero of the level changes with variations of temperature, secondly, the resistance of the centrework to the action of the tangent screws, and thirdly, the imperfection of the screws in producing motion, and in securing permanent positions

"3. This instrument is applied with most advantage to slowly moving or circumpolar stars, but in low latitudes these stars are seen near the horizon, where refraction interferes

"4. Much time and labour are expended, first in making the observations, and again in reducing them

"5. When any one step in a series of observations is bad, the whole time and labour are absolutely lost

"6. When the instrument has a telescope of small power, the observations are charged with errors of vision, which the repeating principle will not cure:

"7. This instrument cannot be used as a transit instrument, nor for finding the exact meridian of a place

"8. The structure of the instrument is unsightly, topheavy, and unsteady."

Some of these objections however, it must be allowed, apply to other small instruments, though to few of them in the same degree.

13. Before concluding our remarks on the repeating circle, it is but justice to state, that it is capable of being used with advantage in determining the zenith distance, or polar distance of a star, without the assistance of the repeating principle; for when the back telescope and circle are well clamped, with the zero of the limb directed to the zenith or pole, a series of observations may be taken with face east and face west alternately; by keeping the bubble of the level in the same place at all times, in the same manner as the altitude and azimuth circle is employed, either on alternate nights, or on the same night, by reducing to the meridian: and as the circle can be turned round to a new position after every series, considerable accuracy may be obtained by the aid of four verniers, by reversing in azimuth and changing the circle's position occasionally, notwithstanding each vernier does not separately read to greater accuracy than 10″.

§ LXXV NEW REPEATING CIRCLE BY GEORGE DOLLOND. [PLATE XXIV FIG 2]

1. ABOUT the time when the Astronomical Society of London was first formed, in 1819, Mr George Dollond had contrived and constructed an instrument of a portable size, for measuring both horizontal and vertical angles, the construction of which is an union between the altitude and azimuth, and the repeating circles, and partakes of the properties of both. A short description of this instrument is given in the first volume of the said Society's MEMOIRS, ac-

companied by an engraving by Turrell, which represents in perspective all the parts that can well be shown at one view, and which is the same that is contained in fig. 1. of our plate XXIV. We will describe the lower part of this instrument first, partly from an examination of the representation before us, and partly from our recollection of its structure when exhibited to the Society, and will then proceed to the upper part, in the order just the reverse of that adopted by the maker.

2 The tripod that forms the base of this instrument is a triangular frame, having three pairs of equidistant arms separated at the extreme ends by as many upright sockets, and about the middle of each by strong cylindrical rods of brass, while the central parts are united by a strong perforated cylinder, that constitutes the centre of motion of the superincumbent parts of the instrument. The three sockets, uniting the extreme angular points of the frame, admit the feet screws to ascend into them, and to act with their inferior ends in making the principal adjustment of the instrument's position. Two of these feet screws are well seen at *a* and *b*, and the third is partially visible at *c*. The horizontal circle, which is twelve inches in diameter has a socket for its central axis, as long as the sockets of the feet screws, the external face of which fits the larger fixed socket, uniting the central portions of the tripod's frame, and the internal bore admits the solid axis of the strong oblong plate *d*, which covers this part of the frame. The flanch, or circular plate, to which the circle and its tubular axis are both made fast, is so formed at the circumference as to admit of a strong clamp taking hold of it to fix it to the upper part of the frame when necessary, the thumb screw of this stationary clamp may be seen near the foot screw *c*, within the radii of the circle. To the lower face of the strong plate *d* three verniers are screwed fast when adjusted to equal distances and to the face of the circle, which verniers therefore revolve with the said plate. The principal vernier, next to the foot *b*, has an attached clamp and tangent-screw for giving slow motion in azimuth, when the circle is made fast by the inferior clamp embracing the flanch Each vernier has its own attached microscope for viewing the divisions. Besides these essential parts of the lower half of the instrument, there is a new contrivance for watching the stability of the horizontal circle while the verniers are moving over it, in its fixed state, this consists of a delicate level, *e*, clamped to the limb of the circle and connected with a supporting frame *f*, by means of its pivots in such way that the smallest motion of the circle, that carries the level, will displace the bubble, by lifting one end of the tube sufficiently to produce this effect. How far this contrivance is competent to the purpose of giving delicate indications we cannot affirm from any experience of our own The weight *g* acts as a counterpoise to this mechanical contrivance.

3 On the oblong plate *d* are fixed perpendicularly four pillars of brass, in pairs parallel to each other, one of which we will call the front and the other the back pair, as they regard the circle in its present position. These pairs of pillars are respectively united together by strong short bars at their upper ends which bear the *Y*s holding the pivots of the telescope's horizontal axis, one of them having an adjustment by contrary screws for horizontal position, as in the transit-instrument The telescope passes through and is made fast to the thick part of the transverse axis, which is hollow and resembles that of the transit-instrument; and a graduated circle of six inches in diameter seen at the front side of the telescope, is fixed on the same axis at a little distance from the front pivot, and is read by a pair of verniers, made fast to the cross bar connecting the pillars, in order that when the telescope is used in the meridian as a simple

transit-instrument, the altitude or zenith distance of a star may be regulated by this circle and its tangent-screw when clamped. When this use is made of the instrument, no attention is paid to the principal vertical circle and its appendages, which we have yet to describe. We may here state that the focal distance of the telescope is seventeen, and its clear aperture two inches; it is supplied with eye-pieces that magnify twenty, thirty, fifty, and one hundred times, as the observation may require, and has power and light enough to show the pole star by daylight, under favourable circumstances. The eye-end has five vertical and three horizontal spider's lines, the latter of which are adjustable for collimation in altitude, and the former we must suppose placed so by construction, that the optical axis passing through the middle one is parallel to the plane of the vertical circle, which is essential even in the operation of repeating. A lantern with a bent chimney, and having a pair of notched plates already described (§ XXII 5.), for limiting the transmitted light, is applied before the remote pivot, and completes the transit portion of the instrument, since the tangent-screw of the horizontal circle will adjust the optical axis of the telescope into its meridian position, provided, as we have supposed, the middle vertical line is already in the axis of vision, but if not, the side screws at the eye-end of the telescope will be required for effecting a moiety of the said adjustment, on reversing the ends of the transverse axis.

4. The vertical circle is fifteen inches in diameter and is read by three verniers, which, like those of the lower, read each to 10″ these verniers are made fast to the axis of the telescope, so as to move always with it, and one of them, seen between the front pillars, clamps the circle to the telescope when necessary, and has a tangent-screw of slow motion. The axis of the circle is a socket surrounding and nicely fitting the telescope's axis, which socket carries a second small circle, of six inches in diameter, at its remote end, near the back pivot of the transverse axis, the sole use of which circle is, to hold a clamp and tangent screw made fast to the cross piece, that connects the tops of the back pillars, by means of which slow motion may be given to the vertical circle, and its position fixed. A second but shorter socket, exterior to the one just described, revolves round it, and carries a delicate level, *h*, between the large and small circles, which is furnished with a clamp and adjusting screw of slow motion attaching it to the circle, as occasion may require. A counterpoise, *i*, is screwed to the flanch of the verniers, to balance the clamp attached to the vernier opposed to it; and the reading microscope *k*, carried by the front pillars, reads all the three verniers in succession, before and after a series of observations have been concluded, the circle being gradually moved round for this purpose. The level of the transverse axis is of the hanging kind, and is suspended between the pillars over the surface of the oblong plate *d*, by means of long descending bars, suspended by inverted *Y*s on the ends of the pivots of this transverse axis. The level in this situation may remain during an observation or be displaced at pleasure, for the hinges of the suspending bars, as at *l*, allow the *Y*s to be withdrawn, by their backward motion.

5. We have not yet mentioned the finders applied to limit the excursions of the telescope, in its horizontal and vertical motions, while it is directed towards the object of observation, these contrivances do not appear in our figure but may be understood from verbal description. A small portion of the horizontal circle's socket, or axis, projects downward below the frame composing the tripod, and receives a double lever so tightly pushed on by friction that it will keep any position given it, then when the telescope is carried to the object in measuring angles,

one arm of the lever is brought into contact with a small pin projecting from the lower face of the frame, and when the instrument is turned half round in azimuth, the other arm meets with the same pin, and stops the motion at the proper place, where the telescope must be reversed to find the star again the other finder regards the telescope's elevation, and consists of two small levels that are attached to its main tube in such way, that, when the object is in the field at the first observation, moving one level, till the bubble stands in the middle of its tube, sends back the other level through an equal but contrary angle, so that when the instrument has been moved half round in azimuth by the lower stops, and the telescope turned back over the zenith, the second level is ready to indicate by its bubble, at what elevation the telescope must stop to find the star again these contrary motions of the two levels round their centres are produced by two pinions, one fixed under the centre of each level at such distance from each other as to include a contrate wheel, that, acting with both pinions at the same time, turns one by a direct and the other by a retrograde motion, and inclines the tubes of the levels alike, but in opposite directions. the wheel is moved by a milled-head inserted on its own arbor, which by its friction will retain any situation that it is turned into This finder must be seen in order to be useful, but would be more serviceable if it could be felt

6 With respect to the adjustments of this instrument, it is obvious that the back level and feet screws will place the bearing pillars vertical, and that the hanging level will place the transverse axis of the telescope horizontal, when the plane of the vertical circle must by construction be vertical and that the system of lines in the eye end may be rectified as they are in a transit-instrument, particularly if the ocular side screws were added. When the telescope is brought into the meridian by the proper means explained in our sections LVI and LVII., and both the circles clamped, it may be used to gain the time of the clock, the right ascension of a star, with its azimuth and zenith distance, as an altitude and azimuth circle, in the manner already directed; but as the clamping circles are small in comparison with the other parts of the instrument, the clamps must be very good to keep the circles steady, and the motion of the telescope in a vertical plane both in ascending and descending. The operation of repeating vertically is the same with this instrument as with the one immediately preceding it, in the last section, but as this circle never departs from its vertical position, we have yet to show, how the measure of an horizontal arc is obtained by repetition. As there is only one telescope single arcs only can be here repeated, which may be done very simply thus, set the verniers to zero, or other point of the horizontal circle, by turning the circle only round, while the telescope is directed to the first object, to the proper place for the verniers to read, then clamp this circle and turn the telescope to the second object, which will carry the verniers over the single arc once, clamp the verniers to the circle, and release the lower clamp of the flanch, and go back with the telescope to the first object again, here clamp the circle again and release the verniers, and come again to the second object, when the second single arc will have been passed over by the verniers; and when the operation of clamping first the verniers and then the circle has thus been repeated several times, while the telescope vibrates from one object to the other alternately, the whole arc, as finally read and corrected for the index error, divided by the number of operations, will give the correct single arc taken in a horizontal line, even though one of the objects be more elevated than the other, because the vertical lines of the ocular diaphragm are, or ought to be, by adjustment truly vertical. We understand that this

unique instrument was purchased by Dr. Scott, of Bedford Square, London, but as we are not aware that any observations taken with it have been committed to paper, it will not be expected of us, to give any examples of the work that may have been performed by it

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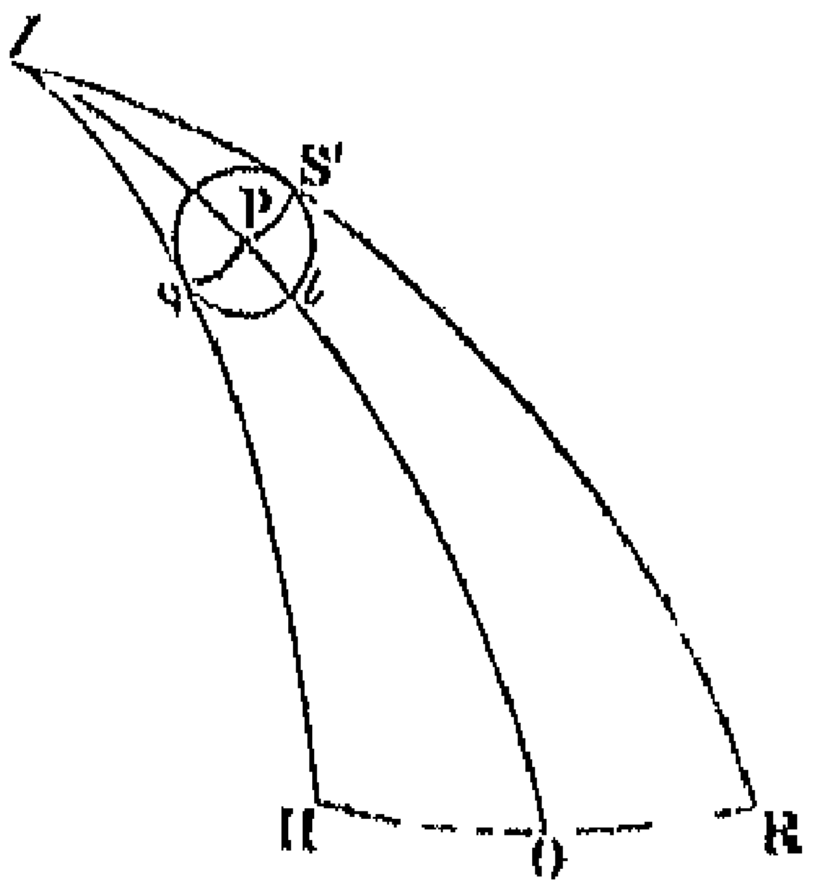
§ LXXVI REPEATING TRIPOD [Plate XXIX]

1. It is sometimes convenient to make use of a small instrument, such as a portable theodolite, for measuring azimuths, or horizontal terrestrial angles in a survey, by converting it into a repeating instrument, which may be done by the addition of a tripod having the means of supplying the repeating principle to an horizontal circle not originally formed for repetition A tripod of this description, made by G. Dollond, at the suggestion, as we understand, of the Astronomer Royal, is represented by fig. 7, of Plate XXIX., and answers the purpose most satisfactorily. The body of this tripod is formed of mahogany, and braced by crossbars in the ordinary way at the three sides, as seen in the representation. It stands on three foot-screws upon a three-legged stool or common tripod, for the sake of steadiness, and carries a strong covering board at its upper end, rounded, with a central perforation, and projecting outwards for bearing a tangent screw of slow motion, of the spiral spring construction [§ XLVI. 5] A conical axis, fixed to the centre of a surmounted triangular plate, descends to the middle of the three-armed brace that connects the feet screws, by means of which and a level, either attached or detached, it may be rendered exactly vertical by the usual adjustment. A circular clamp made fast to the tripod surrounds the cylindrical or upper portion of the vertical axis, and by means of its horizontal screw, passing through the ear where it is nearly slit into two, will set it fast or release it as occasion may require the tail-piece of this clamp, which is of the construction described in § XLVI. 7, extends to the tangent screw of slow motion, above mentioned, and is acted on by it in gaining the contact of a given object, seen through the telescope of the theodolite, or other instrument having an horizontal graduated circle. Each projecting corner of the surmounted triangular plate has a chamfered groove extending from the extreme end towards the centre, upon which the feet-screws of the theodolite are demitted, and as they are usually placed at equal distances from one another, the only position that the super-posed theodolite will take on the tripod will be concentric, or as nearly so as good workmanship can make it, and when this takes place, the two instruments become connected, and form a repeating instrument of the simplest construction; or such as will repeat the measurement of single arcs, that form separate measures of the angle to be ascertained by repetition.

2. When the theodolite is placed on the tripod, its level will serve to place first the axis of the tripod vertical, and then its own axis in the same situation; and when the usual adjustments of the telescope and horizontal circle are completed, the verniers may be put to zero, and the contact made with the first object, by releasing the tripod's clamping screw and turning it round on its vertical axis, in which motion the theodolite will participate, and finishing with the tangent screw, which gives a beautiful slow motion in either direction, without loss of motion in the screw, then the tripod's axis being again clamped during the contact, the

telescope is turned to the second object, and the theodolite clamped nearly in contact with it; then this contact must be completed by the theodolite's tangent screw, in which situation a single arc, if read, would be indicated by the verniers, but the clamp of the tripod being again released, the contact must be made a second time with the first object, and this clamp being again made fast, and the theodolite's clamp released, a second contact must next be made with the second object, when a double angle will be indicated and in like manner as often as the two clamps are alternately released and the respective contacts made, the final arc will be triple, quadruple, &c. according to the number of repetitions

3. Our Astronomer Royal lately proposed to determine the polar distances of circumpolar stars, free from the effects of refraction, by means of an altitude and azimuth circle, that, being well adjusted, will first demit the points of greatest elongation of such star to the horizon, where marks may be made, and then measure the horizontal angle included between those marks, the middle point in which arc will be the meridian mark, if the elongations be observed after a short interval Application was made for a portable instrument to answer this purpose to the visitors of the Greenwich instruments in June 1827, at the annual visitation, and it is understood that the horizontal angles are intended to be measured with extreme accuracy by the aid of the repeating stool, in the way that has been described As this is a new method of determining polar distances, we will explain how it may be carried into effect, by illustrating the formula by which the observations may be reduced to polar distances.



4. In the annexed figure let Z be the zenith point of any given latitude, $H\,O\,R$ a portion of the horizon, P the pole of the equator, and the points S and S', of the circumpolar circle S to S', the points where a star has the greatest elongations, then $Z\,P\,O$ will be a secondary to the horizon passing through the pole from the zenith, and $Z\,S\,H$ and $Z\,S'\,R$, the two other secondaries, passing through the points S and S', will be tangents to the small circumpolar circle. Now, as refraction takes place only in a vertical circle, the lines $P\,S$ and $P\,S'$, representing the star's elongations, being parallel to the horizon, will not be affected by refraction, and when the points S and S' are demitted perpendicularly to the horizon, by a telescope having motions in altitude and azimuth, they will fall at H and R respectively, and the arc $H\,R$ will be the measure of the whole angle $H\,Z\,R$, being the arc of a great circle 90° from the zenith, and meeting $Z\,H$ and $Z\,R$ at right angles. Then to find the polar distance of the star $S\,P$, or $S'\,P$, we have in the right-angled spherical triangle $Z\,S\,P$, or $Z\,S'\,P$, $Z\,R$ for the co-latitude, which we will call λ, and the angle $P\,Z\,S$, or $P\,Z\,S'$ equal to half the angle $H\,Z\,R$ which we may call $\frac{\varepsilon}{2}$, ε denoting the arc $H\,O\,R$, which measures double the azimuth, also we may designate either of the perpendiculars, $P\,S$ and $P\,S'$, by the character Δ; then by Napier's rules for the circular parts we shall obtain the formula,

$$\text{Sin}\,\Delta = \sin\lambda \sin\frac{\varepsilon}{2} \quad . \quad . \quad . \quad . \quad (a)$$

5. The application of this formula will be easily understood from the work of the two subjoined examples

Example 1.—If we take the angle ε, as derived from the points of greatest elongation of the pole star, supposed to have been observed at Greenwich, $= 5° 8' 49''.2$, which is the angle due to the apparent polar distance $1° 36' 9''$, given in the Nautical Almanac January 0, 1828, and if we assume the co-latitude of Greenwich $= 38° 31' 21''$, then we shall obtain Δ by the following simple process, viz

$$\begin{array}{llllll} \text{Log sin } \lambda - 38° & 31' & 21'' & \ldots & & 9.7943639 \\ \text{Log sin } \tfrac{1}{2}\varepsilon - \;\; 2 & 34 & 24.6 & \ldots & & 8.6522549 \\ \hline \text{Sin } \Delta \;\; = \;\; 1 & 36 & 9.0 & \ldots & & 8.4466188 \end{array}$$

Example 2.—The apparent polar distance of γ Ursæ Minoris will be $3° 24' 47''.44$ on January 0, 1829, and the angle ε corresponding thereto at Greenwich will be $10° 58' 14''.28$, which we will take as the angle observed free from refraction, then we shall have

$$\begin{array}{llllll} \text{Log sin } \lambda \;\; 38° & 31' & 21'' & \ldots & & 9.7943639 \\ \text{Log sin } \frac{\varepsilon}{2} - \;\; 5 & 29 & 7.14 & \ldots & & 8.9804155 \\ \hline \text{Log sin } \Delta \;\; 3 & 24 & 47.44 & \ldots & & 8.7747794 \end{array}$$

In like manner if we take $\frac{\varepsilon}{2} = 24° 47' 44''.56$, for β Ursæ Minoris, we shall have Δ $15° 8' 28''.95$.

6. This method of gaining the apparent polar distance of a star, free from refraction has a further advantage over the common method of measuring the vertical angle in this respect, that an error in the co-latitude of the place (λ) arising from refraction will not affect the polar distance (Δ), derived from it, in so great a degree. This property may be thus explained; if we take ε to be a constant quantity, since it is charged with no error except that of observation, or of the divisions of the instrument, and consider Δ and λ variable, by differentiating the equation (*a*) we shall have

$$\operatorname{Cos} \Delta \,.\, d\Delta = \cos \lambda \,.\, d\lambda \,.\, \sin \frac{\varepsilon}{2}$$

$$\text{and } d\Delta = \frac{\cos \lambda \,.\, \sin \frac{\varepsilon}{2} \,.\, d\lambda}{\cos \Delta} \quad . \quad . \quad (b)$$

By the aid of this latter formula (*b*) we can now assume the co-latitude $1''$ too much, and see what effect such assumption will have on the value of Δ, thus,

For Polaris . . .	Cos λ 38° 31′ 21″ .	9.8934087
	Sin $\frac{\varepsilon}{2}$ 2 34 24.41 . . .	8.6522549
	$d\lambda = 1''$	0.0000000
	Sum	8.5456636
	Cos Δ 1° 36′ 9″ . . . . . sub.	9.9998301
	$d\Delta = 0''.0352$	8.5458335
For γ Ursæ Minoris .	Cos λ as before	9.8934087
	Sin $\frac{\varepsilon}{2}$ 5° 29′ 7″.14	8.9804155
	$d\lambda = 1''$	0.0000000
		8.8738242
	Cos Δ 3° 21′ 27″.44 . sub.	9.9992290
	$d\Delta = 0.0749$	8.8745952

For β Ursæ Minoris $d\Delta$, by a similar process, will come out $= 0''.3399$, so that this differential seems to be nearly proportional to the polar distance of any circumpolar star.

7. The principal difficulty attending the practical application of this method of determining polar distances from the horizontal angle ε, is the obtaining of the exact places of the marks proper for stars observed in the night, which can only be done by leaving the instrument in the position given it by the observation, till it can be referred to the horizon by daylight, and on this account it will not be convenient to observe more than one star by the same instrument during any single evening, unless it be large enough to be seen while the mark is visible. Such observations taken of well known stars will however be found very useful in ascertaining the true place for a meridian mark, when the horizontal circle indicates single seconds, or when the repeating principle can be applied to a portable instrument that is too small to admit of single readings of so minute a quantity. Hence observations of the pole-star's greatest elongations have proved peculiarly serviceable in trigonometrical surveys, with regard to the bearings of the principal lines; for by a transposition of our formula (a) we have

$$\text{Sin}\,\frac{\varepsilon}{2} = \frac{\sin\Delta}{\sin\lambda} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (c)$$

which is of easy application, when the latitude of the station, and polar distance of the star at the given time, are previously known.

8 In the practical application of these formulæ, it is necessary that the polar distance of the star chosen be less than the co-latitude of the place of observation, that it may pass between the zenith and the pole; and then, since $Z\,S\,P$ is equal 90°, the horary angle of the star may be found by the following formula,

$$\text{Cos}\,h = \text{cotan}\,\delta\,.\,\text{tang}\,L \quad . \; . \; . \; . \; . \; . \quad (d)$$

where h represents the horary angle, δ the declination of the star, and L, as usual, the latitude of the place.

Example of the Pole-Star

Cotang δ . . . 88° 23′ 51″ 8.4467887
Tang L . . . 51 28 39 . . . . 0.0990448
Cos h = . . 87 59 9.83 or $5^h\ 51^m\ 56^s.65$ 8.5458335

Then app. meridian passage Jan. 0, 1828 . = $0^h\ 59^m\ 29^s.47$ taken from the Naut. Alm.
± 5 51 56 65 = h
Sidereal time of greatest eastern elongation . . 6 51 26.12
Ditto of western ditto . . . 19 7 32.82

Example of δ Ursæ Minoris.

Cotang δ . . . 86° 34′ 44″.16 . . . . 8.7765600
Tang L . . 51 28 39 . . . . 0.0990448
Cos h = . 85 41 36 or $5^h\ 42^m\ 46^s.4$. 8.8756048
Then app. merid passage of δ Ursæ Minoris . 18 28 55.1 for Jan. 0, 1828.

Sidereal time of greatest eastern elongation . 0 11 41.5
Ditto of western ditto . . . 12 46 8.7

§ LXXVII EQUATORIAL INSTRUMENT [Plate XXV]

1. When we described the equatorial or parallactic stands, for directing the motion of a telescope along a parallel of declination, in § X., we stated that "the most perfect stand that a telescope can have, for effecting this purpose, is a large equatorial instrument, furnished with graduated circles, and having the nicer adjustments." The basis of all equatorial instruments is a revolving axis placed parallel to the axis of the earth, by which an attached telescope is made to follow a star, or other celestial body, in its arc of revolution, without the trouble of repeated adjustments for changes of elevation, that circles with vertical and horizontal axes require. Such an instrument is not only convenient, but essential in examining and measuring the relative positions of two contiguous bodies, or in determining the diameters of the planets, when the spider's-line micrometer is used. So long ago as in the year 1544 John Muller, sometimes called Regiomontanus, contrived his *torquet*, which is described to be a species of portable equatorial, and Tycho Brahé called some of his instruments *equatoriæ*, but until telescopes were invented, these contrivances might be considered rather as machines of explanation than as instruments of observation. Christopher Scheiner was probably the first astronomer who, in the year 1620, soon after Galileo had invented the dioptric telescope, made use of a polar axis, but without any appendage of graduated circles. It was not until the year 1741 that the

means of determining the apparent place of a moving body were added to the polar axis by Henry Hindley, an ingenious clockmaker of York, who, according to Smeaton, affixed an equatorial plate, a quadrant of latitude, and declination semicircle to his axis, and racked the edges to suit endless screws, which at the same time gave motion to a refracting telescope, and measured, by a species of micrometer, the elevation and angular motion of any body, without the assistance of graduated circles. This mechanism was sent to London for sale in 1748, but was not disposed of till the year 1761, when Mr. Constable, of Barton Constable in Yorkshire, purchased it for the purpose of observing with it the transit of Venus over the sun's disc.

2 Mr Short however profited by his inspection of Hindley's instrument, and gave to the Royal Society of London his "Description and Uses of an Equatorial Telescope", which was read on the 7th of December, 1749, and published in their Transactions. This telescope was of the reflecting kind, and was mounted over a combination of circles and semicircles, that were strong enough to support a speculum of the Gregorian construction of eighteen inches focal length. About the year 1770, public attention was called to the convenience of the equatorial mounting of an achromatic refracting telescope, and various modifications of the instrument were executed by different instrument-makers, but particularly by Nairne, Ramsden, and the Dollonds Nairne, it is said, was assisted by Ludlam of Cambridge in the formation of his plan, and Ramsden, always depending on his own copious resources, took out a patent for his construction in 1775, and, having made a convenient arrangement of the circular portions, added a refraction apparatus to the eye-piece, which, by means of a small quadrant, level, and micrometrical screw, enabled him to move the horizontal wire in the field as much as the refraction at any given altitude displaced the object observed. These instruments were made on a scale to render them portable, and therefore did not admit of powerful telescopes being supported and steadily guided by them, though they were necessarily expensive, from the complexity of their construction The large instrument made by Ramsden, for Sir George Shuckburgh, may be considered as an exception, but the frame-work was found too slender for the length of its polar axis, and it remains at Greenwich, a proof of its former proprietor's munificence, rather than of its maker's success in the strength and stability of its essential parts.

3. About the year 1795, Captain Huddart, of Highbury, undertook the construction of a large equatorial instrument, in the formation of which his object was to unite strength with lightness, by making its body entirely of tin plates, braced and soldered together in different directions, and, calling in the assistance of Troughton, to finish the brass work, which the tinman could not accomplish, he succeeded in producing a construction that combines all the requisites of a first-rate instrument, and having been improved by the addition of new parts, since it came into the possession of Mr. South, this valuable piece of mechanism, by means of its superior telescope, has performed a great service to astronomy, in giving correct measures of the relative positions and distances between double stars, by the aid of a delicate spider's-line micrometer In performing this work, the equatorial and declination circles were found very useful as *finders* of the stars to be observed, and also as giving the approximate places of such as had not been previously included in the catalogues A description of the last named instrument has been given by its present possessor, with a reference to plates of illustration, in Part III. of the volume of the Philosophical Transactions of London for 1824,

which must be in the recollection of our readers, and which we need not repeat. With respect to the portable constructions, and Sir George Shuckburgh's, we have had occasion to give detailed accounts of them in another publication (Cyclopædia by Dr A Rees), where our readers will find their comparative merits contrasted with each other. Ramsden's equatorial has also been minutely described by the late Professor Vince, in his *Treatise on Practical Astronomy*.

4. Mr. Troughton was prevailed on to make a new equatorial instrument for Mr J. H. Magellan, in 1788, of larger dimensions than had been previously constructed, with the exception of Ramsden's for Sir George Shuckburgh, which preceded it about three years this instrument was sent to Coimbra, where it has been little if at all used, and where consequently its merits have been withheld from the public. On this account, and because Mr. Troughton considers this as the best model of an instrument for such a purpose, we have selected it for the subject of our present section, in preference to his other equatorial now used at the Armagh Observatory. We may however just mention that this latter is formed of two cones, uniting at the intermediate circle that constitutes the horary circle, moving in the plane of the equator, on pivots united by conical tubes meeting at the poles, and containing within it the circle of declination and telescope, with the horizontal axis of motion resting on the rim of the said horary circle. These two equatorials are both described in the work to which we have just referred, the plates of which are engraved by that late eminent artist, Wilson Lowry. As we were assisted in the composition of the article in question by Mr. Troughton himself, we shall have no hesitation in availing ourselves of it in the remainder of this section.

5. *The Coimbra Equatorial.*—This instrument is of that kind which is called *universal*, being adapted for all latitudes, and may be considered as *moveable*, in opposition to *fixed*, but can hardly be said to be *portable*, as it stands seven feet high, and is too bulky to be carried by one person. Plate XXV exhibits a perspective figure of this excellent instrument in its equatorial position, the different parts of which we propose to describe in the order of their ascent from the ground. The support is a tripod of mahogany, well braced by cross bars at the three sides, and carries a circular table at its upper end. The frame of the tripod is further united by three vertical pillars round the centre, one of which is removed in the drawing, to expose to view a more essential part. Immediately over the table, and parallel to it, is a very strong azimuth circle of brass, twenty-four inches in diameter, this circle rests on its conical axis, passing through a collar of brass, attached to the centre of the table, the point of which axis is supported by a stud made fast to the centre of the tripod This length of axis keeps the azimuth circle steady under all circumstances of motion or rest, which is of the greatest importance. The tripod stands upon three equidistant feet-screws, which serve to adjust the circle into a truly horizontal position, and three handles, having each an universal joint, take hold of the said screws, and enable the observer to make the adjustment in an erect position The apparatus for quick and slow motion is attached to the table, and clamps the adjoining circle. As three verniers had not been thought of when this plan was adopted, two only were applied opposite to each other, by being made fast to the table, but in such way that they are capable of adjustment for position whenever the table and frame alter by shrinking, or otherwise. Two vertical supporters of brass frame-work are erected on the plane of the azimuth circle, one at each side of the centre, which are too clearly seen to require a parti-

cular reference by letters of indication, these supporters have therefore a motion in azimuth in common with the circle the distance between them is nineteen inches, and an horizontal axis of the same length holds them together at their upper extremities The posterior supporter has an apparatus for rendering this axis perfectly horizontal, which is not seen in the figure. At the middle of this axis, which may be called the axis of latitude, is a solid cube of brass, to which the long brass socket is made fast, which is inclined, and through which the steel polar axis passes by good fitting, exactly at right angles to the axis by construction. To the lower end of this polar axis the equatorial or horary circle is made fast, also at right angles, and on its superior end, above the axis of latitude, a small platform is fixed, with parallel sides, to bear the superstructure. The equatorial circle is divided so as to show a single second of time, by each of its two opposite verniers, carried by the lower end of the socket in which this circle's polar axis turns. To each pivot of the axis of latitude is made fast a quadrant, one graduated into 90° and its parts, and the other containing 96 spaces and its subdivisions, by particular desire, as a check on the former, the verniers being placed, in an adjustable manner, on the upper ends of the respective supporters The use of these quadrants is to place the polar axis exactly parallel to that of the earth, which is an important adjustment. Upon the plane of the azimuth circle, and between the supporters, are fixed two good spirit levels, at right angles to each other, which, by the assistance of the feet-screws, enable the observer to place the circle's plane exactly horizontal. The mechanism, for adjusting the inclination of the polar axis and its socket to any given quantity, is seen above the plane of the azimuth circle, near its centre; and the two rows of perforated holes across its diameter serve to fix it at any required distance, the upper end of the cylinder, projecting at right angles from the inclined axis, is attached to a broad ring embracing the socket, which will slide up and down according to the distance of the pair of holes that it may be screwed to on the plate, then an universal joint above the holes allows the sliding motion to take place down the socket of the axis, till the inclination is nearly right, when the milled head of the screw that enters the cylinder being turned, this slow adjustment will complete the inclination, provided the mechanism be screwed fast to the plate by their proper holes. It is easy to perceive that, when the supporting cylinder stands at right angles to the socket of the polar axis, which can have no lateral motion by reason of its long axis, the position of all the parts must be firm and steady, and the equatorial circle will always move in the plane to which it has thus been adjusted. The nearer to the bottom of the socket the ring holds it, the firmer will be the position Again, upon the platform above the polar axis are fixed a second pair of smaller supporters, at the distance of fifteen inches from each other, united, at their upper ends, by the axis of the declination circle, these supporters being connected with the equatorial circle, by the intervention of its polar axis, partake of its circular motion at all times, and bear the circle of declination, or meridian circle, and also the telescope surmounted over all. At the upper end of one of these smaller supporters an apparatus is attached for adjusting the axis of the declination circle to a right angle with the polar axis, which is another important adjustment Under the declination circle may be seen the two milled heads of a tangent screw of the clamp, that produces slow motion of this circle when fast, and quick when released. Both faces of the declination circle are divided, one as usual into degrees and parts, which reads with its own vernier made fast to the platform, and the other, by desire, into four times ninety-six divisions, with their subdivisions, indicated by another vernier, attached also to the platform. Each vernier reads to the accuracy of 10″ only, but might be furnished with a reading microscope The unincumbered situation of the telescope, which has a focal length of three feet and a half, is clearly intelligible without further description, the supporting bars being seen, some attached to the declination axis, and some to the circle itself. As the declination circle is complete, the telescope will take observations in any altitude, and follow a star without impediment for any length of time A very sensible spirit level with adjusting screws is made fast to the upper edge of the declination circle, in the direction of a tangent that is parallel to the line of the telescope's collimation. Upon the upper surface of the telescope's main tube are mounted two small microscopes, that read the plumb-line suspended alternately at opposite ends of this tube, as it regards a fixed point made on the tube, which is used for levelling the declination axis. Indeed many of the adjustments may be verified both by the levels and plumb-line. The telescope has, as usual, various eye-pieces, direct and reflecting, for viewing the stars at all altitudes, and under all circumstances. Lastly the refraction apparatus, invented by Ramsden, was made an appendage to this instrument, that the moveable wire in the focus might give the true instead of the apparent place of a moving body, so far as refraction is concerned, to save the trouble of computation. As the polar axis of this instrument may readily be fixed in a vertical position, in which case the equatorial circle becomes parallel to the azimuth circle, it may be used for all the purposes to which an altitude and azimuth circle can be applied, which we have already explained. The instrument we have here described is the only one of the same construction, that has been made on so large a scale, but several small ones have been constructed nearly on the same plan by Fayrer, which are well calculated for a lecture room.

6. *Adjustments.*—The more complex any instrument is in its parts, the greater the difficulty, as well as necessity, of having its adjustments well made, and preserved during use There is a certain order of succession according to which this delicate business will be best performed, that one adjustment may not interfere with another, during the progress of the operations. When Professor Vince received, from Mr. Troughton, instruction how to proceed with the adjustments of Ramsden's equatorial, he followed the directions only in part, and so far puzzled himself with his own methods, that he has not failed to puzzle also those amateurs in practical astronomy, who have been taught by him to undo one thing by doing another. Each instrument will have some adjustments peculiar to its construction, and it would be tedious to the reader, to follow us through a detail of didactic rules, that are applicable to each individual instrument we will therefore satisfy ourselves with going through a succession of adjustments that apply to the instrument we have selected, as one of the best specimens, and which affords, probably, as great a variety of operations, in performing its adjustments, as any other that we could have chosen.

7 *Vision.*—The first adjustment is that which produces distinct vision, at the eye-piece of the telescope, when directed to a star of the second magnitude, which must be effected by sliding the wire plate, or otherwise the object-glass, as the construction will direct, till the wires and star are all seen with perfect distinctness, and without parallax when the eye is carried across the field of view.

8 *Azimuth Circle.*—Secondly, the azimuth circle must be made horizontal in both direc-

tions, pointed to by the levels, in doing this one half of the error is annihilated by the proper feet-screws, when the circle has been turned half round, and the other half by the screws of the levels, taken successively in the different positions, in the way that a tripod is usually managed (§ LXVIII 7.)

9. *Polar Axis.*—Thirdly, to render the polar axis *perpendicular*, it must be clamped nearly so by estimation, and, when the declination circle has been turned gradually round, till the bubble of its surmounted level remains in the middle of its tube, the upper portion of the instrument must be turned half round on the polar axis, as read by the horary circle, and if the level shows an error, by the displacement of its bubble, one half of it must be corrected by the apparatus on the azimuth plate, acting with the socket of the polar axis, and the other half by giving slow motion to the declination circle. When a repetition of this process has made the error disappear, the declination circle and its appendages must be carried round by the polar axis a quarter of a circle from zero to six hours on the horary circle, in which situation, if the bubble is again displaced, it must be re-instated by the adjustment at the top of one of the supporters of the latitude axis, at right angles to the former position when this is done, the azimuth and equatorial circles will have their planes parallel to each other, and their axes also perpendicular to the horizon; and, of course, the level near the telescope will retain its bubble in the middle of its tube, as will also both the other levels, while either one or both of the circles are made to revolve. In this stage of the adjustments, the verniers of the latitude quadrants must both be put to their zeroes, for indicating the elevation of the polar axis to the latitude of the place, when the remaining adjustments are completed.

10 *Declination Axis.*—The fourth adjustment is that by which the axis of the declination circle is made *level*, or placed at right angles with the polar axis. To do this, the telescope must take a perpendicular position, by clamping the declination circle while its vernier reads 90°, in order that the plumb-line may be suspended down the tube, then by the motion of that point which is *adjustable*, and also by the proper motion of the plumb-line, let the latter bisect both the points under the microscopes, this being done, the telescope must have its position reversed, end for end, by turning the declination circle till the opposite 90° is the point read, when the plumb-line must be suspended by the end now uppermost, and brought by its proper motion to bisect the *fixed* point, then half the error that appears at the adjustable point must be corrected by its adjustment, and the other half by that placed at the top of one of the supporters of the declination axis, when the axis will be level.

11. *Position of the lines.*—Fifthly, while the axis of the declination circle, round which the telescope is carried in giving it altitude, remains perfectly horizontal, after the preceding adjustments, the wire plate must be turned round, if necessary, till the middle vertical wire will continue to bisect a distant point through the field of view, while the elevation is varied; in which case, the azimuth motion being slowly produced, the same point will also run along the horizontal wire, these lines being by construction at right angles to each other. but care must be taken that the vision be not altered by turning this plate.

12. *Line of Collimation*—In the sixth place, the line of collimation must be adjusted as it regards both right ascension and declination. In doing this, the upper end of the polar axis and the object end of the telescope must be both pointed towards some distant fixed object a little above the horizon, the telescope being above the polar axis; the declination circle having

90° at zero, and the horary index placed at twelve hours in this situation the centre of the cross wires must be placed on the chosen object, by raising or lowering the polar axis in conjunction with a slow motion in azimuth, and the degrees, minutes, and seconds read by the verniers of the azimuth circle, and also of the quadrants, marked down, in the next place, set the polar axis nearly vertical, and place the horary circle at the opposite twelve, then point the polar axis again to the object, while the telescope is below it, and give a compound motion as before, by this axis and by the azimuth circle, till the intersection of the wires lies a second time on the object; after which read the azimuth circle and quadrants again, and mark down the degrees, minutes, and seconds under the former readings respectively, and take a mean of each pair: while the parts remain in this situation, the azimuth circle must be moved to the arc denoting the mean of the two azimuths, and the polar axis to the mean of the two inclinations, in this position let the clamps be made fast, and the telescope moved carefully, by altering the screws that fix it to the circle, until the vertical central wire coincide with the original object, and the adjustment for collimation in right ascension will be finished. Again, move the telescope, by altering the declination, till the horizontal wire cuts the same object, and adjust the verniers of the declination circle to the points 90° and 96 parts respectively, on its contrary planes, and the collimation in declination will also be set right. If however the object chosen be not at a great distance, since the telescope is placed alternately above and below the declination circle, the adjustment of the collimation in declination will not be correct, for at a short distance two marks must be put upon the object, at double the distance from each that the centre of the declination circle is removed from the telescope's line of collimation

13. *Horary Index.*—Lastly, to adjust the horary index, let the point 90° on the azimuth circle be turned to an object of small altitude, let the polar axis be placed horizontally, and the telescope set to 0 on the declination circle in this position the horary circle becomes vertical, and the object's altitude may be taken by it, but it will be read off in time from six hours now turn the instrument half round in azimuth, and bring the telescope again to the object, by turning the polar axis round in its socket, take another altitude in time in this position from the opposite six, and then the mean of the two altitudes thus obtained will show the point on the horary circle where its index must be placed, to give the horary angle truly

14. The apparatus for correcting the horary angle on account of refraction consists of a small quadrant and level for taking altitudes, which are moveable round the axis of vision, and therefore determine very readily the elevation of the telescope in any given position, also the eye-piece and wires of the telescope, being moveable by means of a nice micrometer screw, may be set with great accuracy to the refraction corresponding to the altitude so determined.

15. The principal advantages resulting from the construction of the instrument we have here described, are, first, that it affords a firm support to a telescope of very considerable power; and secondly, that the great range of polar distance beyond 90° of declination, renders an observation with the telescope below the polar axis unnecessary. But notwithstanding the convenience with which a star or other body may be found by the equatorial motion, when the right ascension and declination of the body are previously known, astronomers are not generally disposed to place much confidence in the accuracy of observations, for determining unknown right ascensions and declinations, out of the meridian, however well the instrument

may be equipoised It is scarcely necessary to add, that the telescope must be put to the declination of the star in the meridian position, and then turned to the star's horary angle at the moment, which is always the difference between the star's right ascension and the sidereal time then shown by an adjusted clock when the former is greater than the latter, the star will be towards the west, and the contrary when smaller. Various accounts have been published of the uses of an equatorial instrument, by Short, Nairne, Martin, Ramsden, the Dollonds, Sir George Shuckburgh, and the Hon Stuart M'Kenzie, but to an intelligent reader of the present day, the details we have given in this section will convey all the information he can require, without troubling him with specific examples of work that may be better performed with other instruments

16 Sometimes a long radial bar carrying a sector, or arc of a circle, is attached to a telescope, having a motion on a polar axis, by which differences of right ascension and of declination may be determined of any two moving bodies; and when the place of one of them is known, that of the other may be had by comparison. Graham first made an instrument of this kind, which we understand remains out of use at Greenwich, and a more complete one was afterwards constructed by one of the Sissons, both which are described in Professor VINCE's *Treatise on Practical Astronomy*, but as they will not give the place absolutely, like the equatorial instrument, and have ceased to be in use, we merely mention their existence, and where they are described, that our readers may gratify their curiosity by referring to the work. They may be considered as a species of parallactic instrument, which enables a telescope to follow a star, or other body, with convenience, by means of the revolving polar axis. The comet of 1815 was however observed by the five-feet equatorial sector at Greenwich in the months of May, June, and July, as recorded in the last page of the Greenwich observations of that year.

§ LXXVIII ON THE ERRORS OF THE EQUATORIAL INSTRUMENT

1. NOTHING can be more encouraging to an observer than the apparent ease with which an observation may be made with an equatorial instrument, particularly when the only use of the equatorial motion is to follow a star or planet, for the purpose of gaining micrometrical measurements, and even in taking right ascensions and declinations out of the meridian, for which the instrument was originally intended, at first sight it may be inferred, that the instrument is peculiarly adapted for gaining these with facility, since all the measures can be referred immediately to the pole and equator, without reference to the zenith, horizon, or latitude of the place yet considerable difficulties occur in practice, that prove extremely discouraging to observers, and it is a curious fact, that notwithstanding equatorial instruments of various constructions have long been in the hands of astronomical amateurs, no good observations of right ascensions and declinations, taken at a distance from the meridian, seem to have been accomplished, except lately at Armagh by Professor Robinson All the instruments that carry horary circles and telescopes on the middle of a long polar axis, have been found to bend more or less, and even the unique instrument at Armagh is not quite free from this fault; and what is worse, in some of the constructions the yielding takes place differently in different

positions, so that errors so produced cannot be guarded against, nor yet corrected by computation. And even in the best constructions, the adjustments seldom remain perfect for any length of time, owing to the stress laid on certain parts by oblique positions, so that when the instrument has been in use, and is settled after being adjusted, it is desirable to ascertain the number and extent of the errors arising from derangement of position, as well as the effect that refraction has on the polar distance and right ascension, as shown by the instrument. For without a knowledge of these causes of erroneous indications, the observer can deduce no good results from circum-meridian observations, not even when taken differentially. The Westbury circle was at first intended for an equatorial instrument, but was converted into a meridian instrument, having its motions in altitude and azimuth, for which the adjustments are more permanent, and allowance for refraction more easily applied: and even at Armagh it was once in contemplation to mount the circle in a vertical position. It will therefore be rendering some service to practical astronomy, to explain how the errors of an equatorial may be detected, and their amount computed, where they are of a permanent nature. When the horary angle and declination of an observed heavenly body, as read by the instrument, differ from the known apparent horary angle and apparent declination, these differences we consider as the *errors* of the instrument.

2. The deviations to which the several parts of an equatorial may be liable, may be enumerated in the following order; first, the polar axis may not be placed perfectly parallel to the earth's axis of rotation, but may be directed to a point that is near the upper pole of the equator; secondly, the plane of the declination circle may not be parallel to the polar axis of the instrument; or, in other words, may not be perpendicular to its equatorial circle, on which the time is counted; thirdly, the vertical line of collimation may not be parallel to the plane of the declination circle; fourthly, the zero point, from which the measure of declination begins to count, may not be situated in a diameter parallel to the equator; and lastly, the zero point, from which the horary angle is counted, may not be exactly in the plane of the meridian, passing through the centre of the instrument. To represent the influence of all these deviations on the horary angle, and on the declination, as given by the deranged instrument, let

i denote the angular distance of the point in the celestial arc, to which the polar axis is directed, from the true pole;

h the horary angle upon which the said point lies: which horary angle, as well as those that will be mentioned hereafter, must be reckoned from the southern meridian westward, from 0° to 360° in arc;

$90^\circ - \eta$ the angle that the plane of the declination circle makes with the plane of the instrument's equatorial, or horary circle: the quantity η being considered positive when such inclination is westwards, while the position is at $0^h\ 0^m$, or at zero of the horary circle, and negative when eastward;

ξ the angle that the vertical line of collimation makes with the plane of the declination circle; this angle ξ being positive when the middle vertical wire is too far from the plane of the said circle, and negative when too near;

e the error of the zero point upon the declination circle: this error must be taken positive when, the horary angle of the instrument being $0^h\ 0^m$; the southern end of the diameter, on which the zero point is placed, lies above a plane parallel to the equator, or towards the north, and negative when the contrary;

ε the error of the zero point on the equatorial or horary circle, taken positive when this point lies to the west of the meridian, and negative when to the east.

3. If we bear in mind the significations of the preceding characters, and designate by h and δ the horary angle and declination read on the instrument, the apparent horary angle H, and apparent declination D, of the observed body will be given by the two following equations

$$\begin{aligned} H &= h + \varepsilon + \iota \operatorname{tang} D \sin(k - h) + \eta \operatorname{tang} D + \xi \sec D. \\ D &= \delta + e - \iota \cos(k - h). \qquad (a) \end{aligned}$$

In these equations the first powers only of the errors ι, η, ξ, e, and ε have been preserved; the terms, containing the superior powers and products of the same quantities, being small, may for all practical purposes be neglected. In these equations the signs remain unaltered whenever the horary angle of the instrument and of the star are the same, or differ only by the errors of the instrument. but we may observe a star also by counting the declination from the opposite side of the circle, and by placing the telescope at 180° − D, in which case the horary angles of the instrument and of the star will differ by twelve hours; in this mode of observing, tang D and sec D become − tang D, and − sec D, and then some of the quantities contained in the two preceding equations must change their signs, and the equations will become

$$\begin{aligned} H &= 12^h + h + \varepsilon + \iota \operatorname{tang} D \sin(k - h) - \eta \operatorname{tang} D - \xi \sec D. \\ D &= \delta - e + \iota \cos(k - h). \qquad (a') \end{aligned}$$

4. When the six quantities ι, k, η, ξ, e, and ε, denoting the deviations of the several parts of the instrument are unknown, they may be determined by observations of the horary angle and declination of known stars. This determination may be abridged by selecting a proper star with a slow motion, say Polaris, which we will take as affording an example, to illustrate the manner of determining the different errors agreeably to our equations. Let us suppose that this star was observed at its upper and lower passages over the meridian of the instrument at the subjoined times, in the year 1827, and having the declinations registered with those times thus;

18 April, lower passage $12^h\ 59^m\ 53^s$, $\angle = 12^h$, observed dec 88° 23′ 28″
19 April, upper passage 0 57 46; $\angle = 0^h$, observed dec 88 22 30
19 April, lower passage 12 59 6; $\angle = 0^h$, observed dec. 88 23 56

The times of the observed passages are here considered as corrected for the error of the clock, at the different times, and also the declinations corrected for refraction in altitude. Then from the Nautical Almanac we have the apparent right ascension and declination of Polaris, at the given epoch, $0^h\ 58^m\ 27^s\ 5$, and 88° 23′ 13″ respectively. Now for the first two observations, in which the horary angles of the instrument and of the star were nearly alike, we must use the equations (a), but for the last observation, in which the difference was nearly twelve hours, the equations (a') must be applied. By observing that in the first observation we have the hour angle of the instrument ($\angle$) $= 12^h = 180°$, and in the two others $= 0^h = 0°$, and by substituting the values there given, we obtain the six following equations, in which H, H', and H'' denote the horary angles of the star at its different passages, viz.

$$\begin{aligned}
H &= 12^h + \epsilon - i \operatorname{tang} D.\sin k + \eta \operatorname{tang} D + \xi.\sec D\\
H' &= \;0 \;+ \epsilon + i \operatorname{tang} D.\sin k + \eta \operatorname{tang} D + \xi.\sec D\\
H'' &= 12 \;+ \epsilon - i \operatorname{tang} D\;\sin k - \eta \operatorname{tang} D + \xi.\sec D
\end{aligned}$$

$$\begin{aligned}
D &= 88°\;23'\;28'' + e + i \cos k\\
D &= 88\;\;22\;\;30 + e - i \cos k\\
D &= 88\;\;23\;\;56 - e + i \cos k
\end{aligned} \qquad (1)$$

From these equations, by subtraction or addition, we deduce

$$\begin{aligned}
H - H' &= 12^h - 2\,i \operatorname{tang} D.\sin k\\
H - H'' &= 2\,(\eta \operatorname{tang} D + \xi \sec D)\\
H' + H'' &= 12^h + 2\,\epsilon
\end{aligned} \qquad (2)$$

$$\begin{aligned}
0 &= \;\;0'\;58'' + 2\,i \cos k\\
0 &= -0\;\;28 + 2\,e\\
2\,D &= 176°\;46'\;26'',\text{ or } D = \frac{176°\;46'\;26''}{2} = 88°\;23'\;13''
\end{aligned}$$

Hence half the sum of the two declinations, observed above and below on the same day, on a supposition that the horary angle of the instrument's position remained the same, gives the declination of the star. If therefore we find a difference from the known declination, greater than the probable error of observation, it must be concluded that the instrument, from some cause or other affecting its steadiness, has not kept its place. The last equation but one gives

$$e = \frac{28''}{2} = 14''$$

This error, being positive, shows that the diameter, passing through zero of the declination circle, rises on the north side of the equator, towards the upper meridian. The quantities i and k may be determined from the first and fourth of the equations (2); for which purpose the difference ($H - H'$) of the horary angles of the star may be obtained from the difference of the two observed times of the two corresponding passages, and is thus found to be $= 12^h\;2^m\;7^s$. The first equation then will become

$$12^h\;2^m\;7^s = 12^h - 2\,i \operatorname{tang} D.\sin k,$$

or, by reducing the time into space, and dividing by tang D,

$$2\,i \sin k = -\frac{31'\;45''}{\operatorname{tang} D} = -\frac{31'\;45''}{\operatorname{tang} 88°\;23'\;13''} = -53''.6$$

From the last equation but two we have

$$2\,\iota \cos k = -\,0'\;58'',$$

then, by dividing the preceding equation by this, we shall obtain

$$\text{tang}\; k = \frac{53''.6}{-58},\; \log \text{tang}\; k = 9.96573;$$

of the two angles that correspond to this tangent we have the choice of $k = 42^\circ\;45'$, or of $k = 222^\circ\;45'$, and because the value of $2\,\iota \sin k$ is negative, we must take the latter, but if $2\,\iota \sin k$ had been positive, we must have taken the former, or smaller angle, as belonging to the first quadrant. The angle k being now known, we have by a former equation

$$\iota = \frac{-\,53''.6}{2 \sin k} = -\,\frac{53''.6}{2 \sin 222^\circ\;45'}$$

$$\log \iota = 1.5964 \text{ and } \iota = 39''.5$$

Hence it appears, that the polar axis of the instrument was directed to a point in the heavens $39''.5$ distant from the pole, and situated on an horary circle, that makes an angle k of $222^\circ\;45'$ westwards with the upper meridian. By putting the difference of the observed times of passage for $H - H''$ in the second of the equations (2), we have

$$47^s = 2\,(\eta \;\text{tang}\; D + \xi \sec D),$$

or, by reducing the time into space, and dividing by 2,

$$\eta \;\text{tang}\; D + \xi \sec D = 5'\;52''.5 \qquad (3)$$

Now the two errors η and ξ cannot be separated by observations of Polaris alone, because, with respect to any single star, the expression $\eta \;\text{tang}\; D + \xi \sec D$ constitutes a single error. In order to separate the two quantities composing this error, let us suppose that the meridian passage of the sun, as observed by the instrument in the same position, was thus

19 April, $1^h\;46^m\;31^s.7$ sid. time ($\angle = 0^h$) observed dec. $10^\circ\;59'\;52''$,

the sun's right ascension by the Tables being at that time $1^h\;46^m\;33^s$, and his declination $11^\circ\;0'\;5''$ N. By representing the horary angle and declination in this case by $H_{,}$ and $D_{,}$, we get from the first of the equations (a)

$$H_{,} = 0^h + \epsilon + \iota \;\text{tang}\; D.\sin k + \eta \;\text{tang}\; D + \xi \sec D;$$

if we subtract this equation from the second of the equations (1), we obtain

$$H' - H_{,} = i \sin k\,(\text{tang}\; D - \text{tang}\; D_{,}) + \eta\,(\text{tang}\; D - \text{tang}\; D_{,}) + \xi\,(\sec D - \sec D_{,}).$$

For the difference of the horary angles $H' - H$ we may substitute the difference of the times of passage of the two bodies, diminished by the difference of their right ascensions, thus.

$$H' - H = (0^h\;57^m\;46^s - 1^h\;46^m\;31^s.7) - (0^h\;58^m\;27^s.5 - 1^h\;46^m\;33^s) = -\,48^m\;47^s.7 + 48^m\;5^s.5$$
$$= -\,40^s.2$$

Then by reducing this time into space, and by noticing that we have already found $i \sin l = 26''.8$, the preceding equation will give

$$\eta\,(\text{tang}\, D - \text{tang}\, D') + \xi\,(\sec D - \sec D') = -603'' + 26.8\,(\text{tang}\, D - \text{tang}\, D')$$

Let us now substitute in this equation, as well as in equation (3) for D' and D their respective values, and we shall have

$$35.316\,\eta + 34.506\,\xi = 345''.5$$
$$35.511\,\eta + 35.525\,\xi = 352.5$$

from which we deduce . . . $\eta = 18''.9$, and $\xi = 8''.9$,

whence we conclude that the plane of the declination circle, when the instrument was placed at the horary angle 0^h, was inclined to the equator, and made an angle towards its west $= 90° - 18''.9$, or $89° 59' 41''.1$; and also that the middle wire ought to be brought nearer to the plane of the said circle by $8''.9$, that the vertical line of collimation may become parallel to it.

It remains yet that we determine the error ε, or displacement of the zero of right ascension. For this object the third of the equations (2) gives

$$\varepsilon = \frac{H' - H'' - 12^h}{2}$$

the horary angles, H', and H'', being given by the sidereal time of the respective passages of the star diminished by its right ascension, thus,

$$H' - H'' = 0^h\,57^m\,46^s - 0^h\,58^m\,27^s.5 + 12^h\,59^m\,6^s - 0^h\,58^m\,27^s.5 = 11^h\,59^m\,57^s,$$

and therefore,

$$\varepsilon = \frac{11^h\,59^m\,57^s - 12^h}{2} = -1^s.5$$

consequently the zero point of the horary circle was found to be $1^s.5$ towards the east side of the meridian.

5. There is this remarkable difference to be noticed between the determination of the foregoing quantities and of this last quantity; that all the other errors have been determined by employing the *differences* of the horary angles, or of the observed times of the passages, where only the rate of the clock was wanted to correct their values; whereas in the last determination the sidereal time itself is introduced as an element of computation, which cannot be correctly used without previously knowing the clock's error. We might however spare the introduction of sidereal time, in determining the error ε, by having a meridian mark properly adjusted for the situation of the instrument; for then, when all the other errors have been detected, and the instrument finally adjusted by their means, the index may be put to the point zero, while the middle wire is brought to bisect the said meridian mark. Whatever may be the construction of the equatorial, these principles will apply to the detection of its errors, and consequently to rectify its rough adjustments.

6. But as it is very difficult to adjust all the parts of so complicated an instrument exactly, it will generally in practice be better, to be satisfied with the approximate mechanical adjustments, such as we specified in our last section, and then apply the computed errors to the observed horary angle and declination, as given by equations (a) and (a'). As the most usual application of this instrument is, to determine the right ascension and declination of an unknown, by comparing its place with that of a known body, at their successive passages over the same horary angle, it may be proper to remark here, that when the difference of their declinations is but small, as 1° or 2°, the errors of the instrument will have less influence upon their relative positions the greater is their distance from the pole. To show what this difference will be, let us represent by H and D the horary angle and declination of one star, and by H' and D' those of any other to be compared, substitute the values of H and D in the equations (a), and take the difference between the resulting equations, then as the horary angle h is the same in both cases, it will be found that the differences will be expressed in the following manner $H-H'=(\sin D-\text{tang } D') \,.\, \iota \sin(h-k)+(\text{tang } D-\text{tang } D') \,.\, \eta+(\sec D-\sec D') \,.\, \xi$, and $D-D'=\delta-\delta'$, where δ and δ' denote the two declinations read on the circle. The second of these equations shows, that, whenever the deviations of the parts of the instrument are so small that the squares and products of their values may be disregarded, the difference of the declinations, given by the instrument, will be the same as that of the true apparent declinations of the two stars; but the difference of the two horary angles, should be nothing, when the instrument is perfectly adjusted, though it will amount to a sensible quantity when the deviations remain uncorrected. To form an idea of the value of this influence, let us suppose $D=62°$ and $D'=60°$, and the horary angle $h=90°+k$, a supposition that will render the effect of the deviation ι the greatest, then according to the values of ι, η, and ξ above determined, we shall have,

$$H-H'=(1.881-1.732)\,.\,(39''.5+18''.9)+(2.130-2.000)\,.\,8''.9=9''.8\,,$$

so that the error affecting the difference of the horary angles, in a zone from 60° to 62° of declination, is to the total value of the deviations themselves, by which it is produced, nearly in the ratio 3 : 20, and this ratio expressing the influence of the deviations would be still less in zones of smaller declination. An approximate knowledge only is therefore wanted of the values of the deviations, to enable us to compute, with sufficient accuracy, the corrections due to the differences of the observed right ascensions and declinations. In higher zones where the effect of the deviations is greater, it must be observed, that the errors in right ascension have also a little influence on the position of the stars.

7. But though we suppose the equatorial perfect in its construction, and correctly adjusted, yet the observations made by it, out of the meridian, are charged with the influence of refraction both in polar distance (Δ) and right ascension (Æ), which influence depends on the refraction in altitude, or which is the same thing, in zenith distance. The refraction apparatus is intended to correct these effects, by altering the equatorial wire in the eye-piece, according to the observed altitude of the body, but this correction, depending on the quantity of mean refraction in altitude, will not be the true correction in all states of the atmosphere, nor at any time in low altitudes. Professor Robinson observes, in his paper read before the Royal Irish Academy on Jan. 10, 1825, that Ramsden's refraction apparatus "cannot be employed with

any regard to the permanence of the collimation." The effects of refraction may however be easily computed, or Tables might be constructed for showing them by inspection. If we put r for the refraction in zenith distance, R for the corresponding refraction in polar distance, and ϱ that in right ascension; and if we put v for the angle of variation formed at the star, by the vertical and horary circles passing through it, we shall have,

$$R = \sin r . \cos v$$
$$\varrho = \frac{\sin r . \sin v}{\sin \Delta}$$

and when v is computed by any of the formulæ we have given in page 459, the corrections or quantities R and ϱ will be easily obtained, as may also the parallaxes in polar distance and right ascension from knowing the horizontal parallax, for if we call the horizontal parallax of the sun or moon h, the parallax in altitude h', in polar distance p, and in right ascension π, then we shall have,

$$\sin h' = \sin h . \sin z$$
$$\sin p = \sin h . \sin z . \cos v$$
$$\sin \pi = \frac{\sin h . \sin z . \sin v}{\sin \Delta}$$

where z denotes the zenith distance of the body, which in a known latitude is easily computed from the hour angle and polar distance, by No. 25 of our formulæ. The refraction in Δ is negative when Δ is less than its auxiliary arc ϕ, which is the arc cut off by a perpendicular falling from the zenith upon the horary circle; and the refraction in Æ is negative when the star is west of the meridian.

§ LXXIX. GRAHAM'S ZENITH SECTOR. [Plate XXVII.]

1. The zenith sector, like the transit instrument, can be used only on the meridian, and its measures are referred to the zenith point of the place of observation. Its principal uses are to determine the latitude of the place of observation by a star of known zenith distance, to measure the zenith distance of a star in a known latitude, and to ascertain the zenith point, and by comparison to transfer it to the arc of another instrument, such as the astronomical quadrant, that is not capable of being reversed in position. The first zenith sector was contrived and constructed by the ingenious Dr. Hooke, with an intention of determining whether or not a fixed star has a measurable annual parallax, as Galileo had suggested. An object lens of thirty-six feet focal length was fixed in a vertical tube in the year 1669, with a plumb-line suspended from the superior, and a graduated arc placed near its inferior end. We learn from the Cutlerian lectures of the Gresham professor, that on the 6th of July of the said year γ Draconis was observed to pass the meridian of Gresham College at 2′ 12″ to the north of the zenith, on the 6th of August at 2′ 6″; and on the 21st of October at 1′ 48″, or 1′ 50″, whence it was concluded that the instrument gave uncertain

results, for there appeared to be an error of about 24″, which, the natural causes of the difference in the measures not being then known, was imputed to the instrument. The Hon. Samuel Molyneux, conceiving that a sufficient trial had not been made of Hooke's instrument, availed himself of the mechanical skill of George Graham, who in the year 1725 put up a zenith sector of twenty-four feet and a quarter focal length at Kew, by the assistance of Bradley, about the end of November, and γ Draconis was observed by it for the first time on the 3d of December following. On the 5th, 11th, and 12th, no sensible difference appeared in the zenith distance of this star, and it was concluded that the instrument proved perfect, but Bradley being still at Kew, on the 17th, and still more on the 20th of the same month, perceived a change towards the south, which, being in a contrary direction to that which would have been produced by parallax, caused various conjectures as to the true cause of this apparent change of declination. About the 26th of March, 1726, the star was found more towards the south by 20″ than on the 3d of the preceding December, about the middle of April the change began to be retrograde; at the beginning of June the star had returned to its original position, and in September following the motion towards the north had increased to 20″ nearly. In December it was found again in the original situation. To ascertain what might be the real cause of this apparent change in the place of γ Draconis, and of other stars that had been observed to have changes, but not of the same quantity, nor yet contemporaneously, Bradley determined to have a second instrument constructed by Graham, to be set up for his own convenience at Wanstead, which was accordingly done on the 19th of August, 1727. The dimensions of this second instrument were guided by the situation that Bradley had fixed upon for its erection, which limited the focal distance to twelve feet and a half, and its total arc to 12½°, which enabled the observer to take Capella into the field of view, and to see about 200 stars contained in the British Catalogue. Bradley gave a Paper to the Royal Society of London, entitled "A new Apparent Motion of the Fixed Stars discovered" (No. 406), which is contained in Vol. VI. p 149, of the PHILOSOPHICAL TRANSACTIONS abridged by Eames and Martyn, in 1734, in which he has detailed the circumstances that led to the discovery of the *aberration of light* by means of Graham's second zenith sector, which, at Bradley's appointment to the Royal Observatory at Greenwich, most fortunately for astronomy, was taken thither, and is the identical instrument that is still there, and yet held in high estimation. The discovery of *nutation*, though previously suspected to exist, was afterwards discovered by a series of twenty years' observations.

2. This sector is represented by fig 2 of Plate XXVII, in which all the essential parts are exposed to view, and therefore may be easily described. The long iron tube denoted by AB is equal to the focal length of the object-glass, which was not originally achromatic, but which was made so in Dr. Maskelyne's time by Mr. Dollond, it has two cylindrical steel pins, a, and another behind not seen, projecting at right angles, a little below the superior end, which constitute the axis of motion, when suspended by a pair of *Y*s, of which there are two, one attached to a wall facing the north, and the other to that which faces the south, to either of which pairs of *Y*s the sector may be applied at pleasure, and when both are used in succession, the collimation in zenith distance may be readily obtained. The horizontal bar CD is fixed to each wall, in the vertical line under the *Y*s, at a determined distance, and carries a sliding adjustable cock, to which the micrometer screw E is fixed, that measures the frac-

tional portion of a minute on its divided head *b*, and also the second screw *c*, having no other use than to relieve that of the micrometer. The ends of these two screws rest against a pair of studs made fast to the tube, and the load *F* pulls the side of the tube towards the points of the screws, by means of a flexible cord going under the fixed pulley *H*, and over that at *G*, before it takes hold of the suspended weight The metallic arc *I K*, attached to the back of the tube, contains the divisions of 12½°, subdivided into 5′ spaces. This arc was originally of brass, but Sisson was employed to change it for one of steel, into which gold pins are inserted to receive the points of division. A plumb-line is suspended from the object end of the tube, by a stud that is adjustable by the screw *e*, and falling over and close to the face of the divided arc, indicates the distance from zero, at the middle of the arc. When a star passing the zenith makes the plumb-line fall on one of the points on a gold pin, or on a division, the arc itself will show the zenith distance, but otherwise the remaining minutes and seconds are afterwards obtained by the screw, which, when zero on its head is put to the index *i*, must be turned till the plumb-line bisects the next nearest dividing stroke or dot, when the sum given by the arc and micrometer jointly will be the whole measure. The head of the screw is divided into thirty-four equal parts, which were intended to indicate so many seconds; but by measuring the degrees on the arc, taken at different distances from zero, the value of one revolution of the screw was found to be only 33″.6328, so that a division on the head, instead of measuring 1″, measures only 0″.9892, according to which value a Table is easily constructed to exhibit the value of any number of revolutions and parts.

3. If this zenith sector had been made to reverse in position, without being detached from the wall, as is the case with the Oxford sector made by Bird, it would have been not only much more convenient in use, but less liable to errors from derangement, and from the difference of distance, of the plate *C D* on which the screw acts, from the points of suspension or axes of motion, which can hardly be expected to be precisely alike in the two positions. The error from the latter cause may, however, be ascertained by experiment, and allowed for.

4. The adjustment for collimation is effected by an apparatus that moves the wires in the common focus of the eye-piece and object-glass, as in the transit-instrument, in a manner that may be thus explained, let the graduated arc first face the east, and place any star passing the meridian near the zenith on the horizontal line, supposed to be adjusted to the direction pointing east and west, then when the water vessel below the instrument has brought the plummet to rest, note the number of degrees and 5′ spaces that the line lies to one side of zero, N. or S. as the case may be, then completing the measure by the micrometer, put down the sum of the two quantities for the *eastern* measure; remove now the sector to face the west, and on the following night, if possible, repeat the same operation; then half the sum of the two measures will be the zenith distance, and half the difference the error of collimation, arising from this pair of observations, that requires to be adjusted by the proper screws.

§ LXXX RAMSDEN'S ZENITH SECTOR [Plates XXVI. and XXVII]

1. When the trigonometrical survey of England was undertaken, it was found necessary to have a portable zenith sector to be carried to certain stations, to compare the celestial arcs,

intercepted between those stations, with the terrestrial corresponding distances, as determined by actual measurement of the connecting lines, and for this purpose Ramsden agreed to finish a zenith sector that had been ordered by the Duke of Richmond, as Master-General of the Ordnance, and that was in a state of progress at the time (1800), but he did not live to see it complete, and his successor, Berge, perfected it in April, 1802. It was first tried at the Tower, and then at Greenwich, before it was removed to the Isle of Wight, where the trigonometrical operations began, and were continued northwards from station to station with the great theodolite, as described by General Mudge, the director. The theodolite has recently had the addition of a third microscope by Troughton, and is now used in Ireland, under the direction of Colonel Colby (who succeeded General Mudge in the appointment), after having extended the survey into Scotland, and united the triangles of the three divisions of the kingdom. In Mudge's account of the sector (Trig. Survey of England and Wales, Vol. II.) it is stated, " that Mr. Ramsden has obviated the inconveniences attendant on the use of former sectors; and has also diminished, in a very considerable degree, the errors unavoidably resulting from their imperfect construction." " The principles", he adds, " on which he has founded the several improvements, consist in the means of uniting the sectorial tube to its axis, so as to ensure the permanency of the length of its radius, when erected for observations, more accurate methods of adjusting the instrument vertically, and an easy way of placing the face of its arc in the plane of the meridian." Our Plate XXVI contains engravings of the different parts of Ramsden's zenith sector when taken to pieces, and fig. 1. of Plate XXVII shows the instrument in its entire state, which is too complex to be well comprehended from this one figure. We will first endeavour to convey a general idea of the construction, as exhibited in the latter Plate, and then describe more particularly the separate parts given in their detached state in the former.

2. This instrument consists of three principal portions, a stand; an interior frame, and the sector comprehending the vertical tube, horizontal axis, and various appendages. the first two are made of mahogany, and the last chiefly of brass. The stand consists of the exterior frame, formed into the shape of an obtruncated pyramid, having a base of six feet square, and its vertex three; these are formed of four straight bars each, attached to the corresponding ends of the four upright double bars, that are each formed of two pieces united by seven strong screws, to prevent their warping. The height is about twelve feet, and on the upper end is made fast a square frame, or open table, crossed by two bars in each contrary direction, so as to leave a square hole in the middle, to receive the pivot of the inner frame across the middle of the base a strong board is fixed, strengthened by an edge-bar along its lower surface, on which board is fixed an horizontal circle of brass, containing a conical hole, adjustable in two opposite directions, in which the point of an axis, fixed to the lower end of the inner frame, turns in azimuth. When the four feet of this stand are placed on a horizontal plane, a line demitted from the middle of the upper end, down to the conical hole below, will be vertical; and to this line the tube of the sector must be adjusted when in its place. The inner frame also is formed of four strong bars, ascending from their framed basis, parallel to one another for about one fourth part of the whole height, as far as the eye piece of the suspended telescope, and then converging towards a point at their upper end, where they are united by cross cylindrical rods of brass. Both the stand and inner frame are moreover strengthened by

four horizontal bars, connecting the upright strong rods above the middle of each; and the stand has also a bracing diagonal rod at each side, meeting one another at the upper and lower ends, so that lightness and strength of materials are equally kept in view. Four of the brass cylinders, that unite the upper sides of the inner frame, project the whole length of the axis, and carry the *Y* plates, on which the pivots of the axis rest, having the proper apparatus for the horizontal adjustment, and to the middle of the upper pair of the same cylinders, within the frame, metallic bearers ascend that carry a piece of strong tube, five inches in diameter, which, entering the central hole of the table or cover, constitutes the upper pivot of this frame's azimuthal motion. This pivot therefore is not connected with the telescope, otherwise than as the axis of the latter rests on the *Y* plates carried by the revolving frame. The upper end of the telescope's tube is inserted into the middle or thick part of the axis, which is therefore weakened by the perforation; but a pair of lateral braces, ascending from the sides of the telescope's tube, support the two ends of the axis at some distance from the central hole, and prevent its sinking. The pressure of the telescope is in a great measure taken from the pivots, by a pair of weights, suspended from levers carried by the upper end of the frame, and lifting the axis by a pair of friction rollers at the end of each lever. The object-glass of the telescope, which has a focal length of nearly eight feet, and a diameter of four inches, is held by a separate piece of tube that is adjustable for vision, and one of the eye-pieces is supplied with a diagonal reflector. The plumb-line is suspended from a cock having a tail-piece that is acted on by a screw, turned by a long rod having an universal joint, and can thus be reached from the floor. A lantern, suspended by the middle of the revolving or inner frame, is so contrived, that an adjustable reflector with a concave curve will throw the light of a lamp either into an aperture in the side of the tube, to illuminate the wires in the eye piece, in the usual way, by means of an internal elliptical reflector, or, by an alteration in its position, will throw it upwards upon the face of an inclined plane reflector, at the end of the horizontal axis immediately above it, which small reflector transmits it through the axis, and illuminates a disc of mother-of-pearl in the middle of a solid circular plate within the axis, which disc forms the upper or fixed point, to which the plumb-line must be adjusted. The plumb-line therefore has its point of suspension perpendicularly over the face of this disc, so as to hang nearly in contact with it. But the light transmitted into the axis is not suffered to disperse, it is caught by a second reflector, at the opposite end of the axis, which again sends it down a long tube of small calibre, having a lens within it, and a diagonal eye-piece at its lower extremity, at the opposite side of the frame to that holding the lantern, but lower down, near the handle of the rod that reaches up to the adjustment of the plumb-line. At this eye-piece the disc in the axis and the plumb-line are viewed together, by means of the long microscope, and it is here that the adjustment of the plumb-line is effected by the observer, while standing on the floor. This is altogether a beautiful contrivance, and convenient as beautiful. When the inner frame is adjusted vertically, and the axis of the telescope made horizontal by a long level that is adapted to its pivots, the plumb-line, brought nearly into contact with the disc, will descend to the face of the graduated sectoral limb, near the eye-end, which by construction is in the same plane with the face of the disc. An edge-bar fixed parallel to the graduated sector forms a frame with it, containing the end of the tube, and standing at right angles to the axis, thereby guarding the position of the scale, and the micrometer stands in its proper situation for acting on a steel stud, fixed in the tube of the telescope, after the manner of Graham's screw. But here the distance between the centre of the telescope's axis and the centre of the stud is invariable, except so far as temperature is concerned, ensuring uniformity in the readings, in the reversed positions of the scale. Lastly, a set of counterpoising weights will retain the tube of the sector in any position that the zenith distance of the star may require, while a slight preponderance against the point of the screw prevents a false indication on its divided head by its being always in close action.

3 After this general account of the instrument as a whole, and as seen in fig. 1. of Plate XXVII, we may now turn to Plate XXVI, and examine the separate parts that constitute the construction, in describing which it will become necessary to use letters of reference Fig. 1 gives a section of the whole instrument in a direction that exhibits many of those parts in their places that could not be so well referred to in the former figure, without injuring the perspective appearance. The letters *a b* and *c d* point out two of the upright double bars of the stand, *b c* the side of the cover, *a d* the edge bar of the cross plank at the bottom; and *b d* one of the diagonal braces. The telescope occupies the middle of the stand, and its axis is seen near the top formed of two cones, terminating with bell-metal pivots, and resting on the *Y* plates, that here present their edges to the eye, as carried by the two projecting cylinders that lie parallel to the axis, one above and the other below it It is here clearly seen how the weights at *f* and *g*, suspended by their levers on their adjustable fulcra, press their little pulleys against the rings surrounding the axis, the fulcra being within the frame and the weights without. The oblique braces are also seen near the fulcra, as is also the universal joint of the rod at *i* that adjusts the cock of suspension, seen detached in fig 3, which we will examine presently. The concave reflector of the lamp is at *k*, and the dotted lines, seen at right angles to each other, are in the two directions that the light of the lamp can be turned by it; that which proceeds horizontally arrives at the eye-piece of the telescope, and that which is sent vertically is again three times reflected, first into the telescope's axis at *l*, then from *m*, at the other end of the axis, down to *n*, the eye-piece of the long microscope *m n*, which has a lens at *o*, and a diagonal rectangular prism in the eye-piece, that at the same time reflects the light and magnifies, in consequence of one of its faces being ground and polished into a convex curve thus rendering the disc in the axis and its contiguous plumb line visible in a magnified state, and therefore capable of nice bisection. The vessel that holds the water, for the immersion of the plumb-line, is also seen at *p*, near the telescope's bent eye-piece, and parallel to its lower end is a section of the end of the sectoral scale, with the contiguous microscope for viewing the plumb line, and reading the divisions. The mechanism for adjusting the frame vertically is seen above the plank *a d*, where *q* shows the situation of the graduated azimuth circle, and *r* the fixed circle for clamping the frame in any given position, indicated by the vernier, rising from the lower to the upper circle. Under the lower circle are two flat plates adjustable by screws, one from east to west, and the other from north to south. These parts are seen in their place in figure 6, which gives a better idea of the relative positions of the screws, clamp, and vernier than words alone can convey The application of the clamp is however better seen at the bottom of fig. 2, where the solid part appears that connects it with the frame. The smaller circle in fig. 6 is the graduated moving azimuth circle, and the larger one the fixed circle holding the clamp when screwed fast. The horizontal top

frame, or table, is represented by fig. 7., having its central hole so shaped as to touch the tubular pivot of the inner frame in three points only, one of which is pressed inwards by a helical spring, to allow of its adapting itself to the situation that the adjustment at the inferior solid pivot may command.

4. Fig. 2 shows an elevation of the interior frame alone seen sidewise; which is the position given it by turning it through 90° of azimuth. Here the manner in which the brass cylinders connect the upper end, and carry the upper pivot and *Y* plate, is distinctly seen, and it appears, by a comparison with fig. 1, how the tube of the microscope is fixed by cross-bars and pillars to the sides of the revolving frame. In this view the microscope's tube lies parallel to the tube of the telescope, and hides the long pipe in part, down which the plumb-line descends. The pulleys round which the cords pass, that sustain the counterpoising weights, seen in this figure at each side of the telescope, have their axles better seen in the perspective general figure in Plate XXVII., but the ratchet wheels of the two lowest are seen in this figure, and the upper pair are given in dotted lines, by reason of their being concealed the lower ends of the cords are made fast to pins adapted for receiving them, and attached to the tube at opposite sides. The situation of the sector, and of its parallel bar, are likewise seen above the eye-piece of the telescope, as well as the screw of the micrometer, the axis of which is in a plane parallel to the face of the sector, and consequently at right angles to the telescope's axis, a small circular section of it only is seen in this figure, which is the end of one of its pivots. An enlarged view of the micrometer, and a section of the field of view of the telescope, are given in fig. 5, where the point of the screw is resting against its stud near the lower extremity of the large tube. The screws of adjustment for collimation and system of wires are here distinctly seen in their places, and the manner in which the micrometer is held by a bar, parallel to the scale, braced by an edge-bar along its middle, on any part of which it may be made fast by the clamping screws, so that the screw may always stand at right angles to the tube of the telescope at the middle of the stud.

5. It remains yet, that we give an account of fig 3, to which we referred in passing, and also of the figure connected with it. Fig. 3 shows the adjustable cock of suspension, carrying the plumb-line, with a section of the axis where the luminous disc is placed, and fixed, after being adjusted to its place, by the screws that act in two directions on the plate that holds it. The screw *t* is that which the long rod with an universal joint takes hold of with its hollow squared end, and by its action against the tail-piece *s* alters the situation of the part *l*, in which is a fine notch or deep line cut to guide the direction of the plumb-line, suspended from the pin above, and fixed by a screw and collet. This piece is made fast to two of the brass rods or cylinders bearing the *Y* plates. The helical spring opposed to the ascending motion of this tail-piece is strong enough to keep it steadily in its place, and it is easy to perceive, that a motion given to this tail-piece will bring the plumb-line to any required situation, before the face of the disc below it, over the centre of which it now lies. This adjustment regulates the position of the plumb-line, only as it regards the central dot of the disc, but not its distance from it, which is important to prevent parallax, this is done by a screw behind, acting on the piece *l*, as shown edgewise in fig. 4, the upper part of which, through which the screw passes, is elastic, and may be brought forward just so much as to allow the plumb-line to hang close to the disc without touching it. The axis is of course perforated above and below, to

allow the plumb-line to pass across at the place of this section, when loaded with as great a weight as it will bear out of water. Besides the preceding parts, a telescope of twenty-nine inches focal length may be attached to a frame carried by the large tube, with its axis in the plane of the divided sector, as seen at the middle of fig. 2, by means of which the instrument may be brought into the meridian, and the bearing of any object determined by the azimuth circle. From this description it will be obvious, that, if the axis of the telescope is suffered to have motion in its *Y*s lengthwise, the disc and divided scale of the sector will be liable to have their distance from the plumb-line varied occasionally, which change would be detrimental to the accuracy of the readings, therefore the *Y* plates carry each a roller, one of them urged by a spring, which, pressing against the ends of the pivots near the circumferences, prevent all play in a longitudinal direction. The *Y*s are adjusted exactly as in a transit instrument, and sustain the diminished weight of the whole instrument The sector is divided into sixteen degrees by fine points made on gold pins, and each degree subdivided into 5′ spaces, which have been found extremely accurate The object-glass of the telescope lies just above the axis, and so nearly in the same horizontal plane with a plate made fast above it, that a slider moved by two strings, passing over fixed pulleys, will close the aperture at any time, when the instrument is not in use.

6. *Adjustments*—The arrangements of the wires in the eye piece of this instrument are supposed to be finally put right by the maker, but may be examined by propping the long tube in a horizontal situation in several places on a long table, while the pivots of its axis rest in a pair of detached *Y*s, and by bisecting a distant mark near the horizon, both with the vertical and horizontal wires, then if the tube be turned so as to place the axis in the reversed position of the same pair of *Y*s remaining unmoved, it will appear whether the optical centre is bisected by either or both wires, which then may be adjusted accordingly, by doing away one half of each error by the proper ocular screws, and the other half by the altered position of the *Y*s But to do this, the sector will require to be dismounted with extreme care, and had better be done by a skilful artist, who will have superior means of managing so delicate a business. If the crossing lines are not in vertical and horizontal positions, which may be seen by altering the elevation of the telescope slowly on a distant mark, when the axis is level, the wire plate will require to be turned round till it is right, before the adjustment for collimation takes place. Supposing the ocular wires to be properly adjusted, and the instrument otherwise in a state fit for being used, and brought to a station, the first measure is to prepare the ground on which it is destined to stand, by driving stakes, or strong posts, as deep into the earth as may seem to be required, at such distances from each other as will adapt them to hold the four feet of the stand, when cut so as to make a level plane, care being taken that zero of the azimuth circle may stand nearly in its place. As a temporary building is generally erected round the station, with an opening in the middle of the floor, this opening must include the stakes, so as to insulate them and the stand from the floor, before the latter be screwed fast to the former. When the inner frame, with the sector in its place, has been introduced carefully into the middle of the stand, with its lower pivot in the conical hole, and the telescope erect, the cover of the stand must be placed over it, so as to receive the piece of tube forming the upper pivot, and then screwed down to its place, when in all probability the adjustments will all be within the command of their proper screws. The plane of the divided arch may now

be brought parallel to one of the sides of the stand, the inner frame may be clamped to the fixed horizontal circle, and the plumb-line be suspended from its pin, and its weight put on within the water-vessel, to avoid a fracture it will now be seen, when the plummet comes to rest, whether the line hangs parallel to the tube or not, by observing its distance from the scale, and also whether the telescope suspended by its pivots hangs vertical, as the plumb-line regards the disc above and zero of the scale below This previous examination of the existing position being made, it will appear in what respect it is most faulty, and what is first required to be done. In all probability the screws of the adjustable plates under the azimuth circle will require to be first used, to bring the inner frame into a vertical position, in the two opposite directions, approximately, so that when the point zero of the sector is brought to the plumb-line it may not depart far from it, while the frame, being unclamped, revolves. During the first revolution it will be remarked in what direction the inclination of the revolving frame is greatest, and the screws of the plate acting in that direction must be applied, to diminish the preponderating error. When the position of the inner frame has been thus brought roughly to a situation approaching to verticality, it may now be made truly so by dividing the error, in each of the two directions, between the proper screw below, and the indication of the plumb-line on the sector, by means of the screw's motion. When this has been done in the reversed eastern and western positions, the same must be done at 90° distance in the north and south positions; and when the plumb line will keep its place exactly at four equidistant points of the horizon, it will do so at every part of a slow revolution, and the frame will be placed in its true vertical position, which will complete the first important adjustment, when the instrument may be left for some hours to settle in its new abode, to show if the foundation be good. If after a convenient interval no alteration has taken place in the vertical position of the frame, the work of adjustment may proceed. The next operation will be to place the telescope's axis level, or at right angles with the vertical line that forms the axis of the revolving frame, this is done by the riding level, placed on the pivots of the axis, when the bubble of the level must be brought to its zero by the screws of the *Y* plates, that elevate or depress one end of the axis, when this is done the level ought to have its bubble in the same place, when turned end for end into a reversed position; but if this will not take place, one half of the error must be attributed to the level, and the other half to the axis, and if the level has an adjustment, the error must be so divided between them, by their respective adjustments, that the bubble will keep its place in both positions of the level, but if this is not the case, and the level has no mechanical adjustment by screws or otherwise, that *Y* must be scraped or filed a little towards which the bubble ascends, until the desired effect is produced; when the tube of the telescope, being by construction at right angles to its axis, will become vertical, and therefore parallel to the axis of the frame that holds it, and will revolve round a vertical line, though its optical axis may not be quite parallel to the tube. In this situation both the plumb-line and level ought to preserve their relative positions unchanged, during an entire revolution of the frame within the stand, provided the stand continue insulated, and on a stable basis. It yet remains to adjust the point of suspension of the plumb-line to the central point of the disc within the axis, and likewise the distance of the line as close to the face of the disc, and also of the sectoral scale, as can be effected without actual contact; which may be done, the former by moving the tail-piece of the suspension cock by the handle of the long rod, and the latter by the screw acting on the elastic small bar behind the pin of suspension; and if the disc and scale are found at the same distance from the suspended line, it will be an additional proof that the preceding adjustments have been correctly made, and also will show that the relative positions of these delicate parts of the instrument have not been deranged by carriage or rough usage, and the work may be commenced with corresponding confidence. The micrometer must now be adjusted on its bar till the plumb-line indicates 0 on the scale, while the index shows 0 on the micrometer's head, and the only deviation from the zenith point of the heavens in this position will be that which will be occasioned by the difference between the optical and mechanical axes of the telescope's tube, which will constitute the double error in measuring with the sector alternately facing the east and west, for half the sum of the two measures, will here, as in other instruments, give the collimated mean, and half the difference the error of collimation in zenith distance. Nothing now remains in the shape of an adjustment, when the value of the micrometer's scale is known, but to place the plane of the sector in the meridian, by means of the attached small telescope and the azimuth circle, which may be done by observing the two greatest elongations of the pole star, or other circumpolar star, during an interval differing but little from twelve hours, when the middle point on the azimuth circle will show the required position with sufficient accuracy, without computation. It is hardly necessary to add, that the zenith star to be observed will pass the telescope, directed to its zenith distance on the scale, at the time when its apparent right ascension is indicated by a good sidereal clock showing proper time; but that, if a solar clock or chronometer be used, the sidereal time corresponding to such solar time on the given day must be computed, as we have directed in our first volume, or still more conveniently by means of the Supplement to the Nautical Almanac, published for the first time in the current year.

7. *Observations* —When the sector is adjusted in all respects, the time of observing a given zenith star will depend on its right ascension, compared with the sidereal time given by the clock, or computed from the solar time, as above directed, move the object end of the tube towards the north or south, accordingly as the star passes with respect to the zenith of the place, which may be known by comparing its co-latitude with the star's polar distance, until it is found in the field of view, and nearly at the central horizontal wire, the sector being first adjusted till the plumb line indicates some point on the scale exactly, which is called *the point on the limb* at the head of every series of observations then adjusting the micrometer till its screw just bears against its stud, put zero on its head to the index, and turn it forwards till the star is bisected by the wire, along which it will run, as a bead on a thread, if the instrument be properly adjusted Now the divisions read on the head together with the entire revolutions of the screw, converted into arc, and added to the reading on the sector, will give the total arc of measure from the apparent zenith in that position, which may be either facing east or west, but, before the observation is complete, the star must be observed with the sector in both positions, and the oftener the more desirable, as collimation is concerned. If the measure be small, it may be taken by the screw alone, beginning at zero of the sector in both positions. In the temporary erection in which the observations of the survey were made, the air in the day time was usually warmer near the upper end than at the lower end of the sector, and the contrary at night; so that the divided arc would not expand and contract in the same ratio as the different parts of the tube, and the correction for the error thus produced was determined

by placing a thermometer above and another below, the mean state of which had been found equal to the state of a thermometer placed at the middle, according to some experiments that were made. Hence the following table was computed to give the required correction of the observed zenith distance of a star, on account of the expansion or contraction of the tube by a variation of 1° of heat.

Zenith distance observed	Correction for 1° of heat	Zenith distance observed	Correction for 1° of heat
1° 0′	0″.018	4° 30′	0″084
1 30	0 .028	5 0	0 .093
2 0	0 .037	5 30	0 102
2 30	0 .046	6 0	0 111
3 0	0 .056	6 30	0 121
3 30	0 .065	7 0	0 .130
4 0	0 .074	7 30	0 139

In the use of this table, the corrections must have a negative sign when the upper thermometer denotes the air to be hotter above than below, but positive if the contrary.

The table, for showing the value of any number of divisions on the head of the micrometer, is founded on an experimental measurement of the whole arc, at a mean temperature, from which it appeared that a 5′ space was run over by five revolutions and forty-five divisions, making one division = 1″.002 nearly, and 60 = 60″.10. As examples of the method of registering and reducing the observations made by this instrument, we will give those that were made on the stars β and γ Draconis at Greenwich, Dunnose, and Beacon Hill, near Clifton in Yorkshire, successively, by the last pairs of which the long arc was determined, so far as these individual stars were concerned.

8. OBSERVATIONS MADE AT THE ROYAL OBSERVATORY WITH THE ZENITH SECTOR April, 1802.

Observed Zenith Distance of β Draconis.

Point on the limb 0° 55′ North.

Day of the month		Face of the arch	Plumb line		Observation of the star		Zen dist in rev. and parts				Zen. dist. reduced			Bar	Thermometer.	
			Rev	Div	Rev	Div			Rev.	Div					Above.	Below.
April	16	W.	9	57 .80	6	44 .2	0°	55′	3	13 .60	0°	58′	10″.92	29 .9	40 .0	40 .0
	23	E.	8	35 .49	11	40 .9				5 .41			2 .71	30 .1	38 0	38 .0
	25	W.	10	7 .84	6	53 0				13 .84			11 .16	29 .8	44 .0	44 .0
	26	W.	9	24 .63	6	11 .5				13 .13			10 .45	29 .5	42 .0	42 .0

Reduction of β Draconis.

April 16	0°	58′	37″.66	Mean of W and E	0°	58′	32″.14
25			36 .02	Sum of five corrections, including refraction, precession, aberration, and nutation, solar and lunar .		+	0 .99
26			34 .98				
Mean of W.	0	58	36 .22				
April 23, E.	0	58	28 .07				
½ diff. or collim. . . .			4 .07	Apparent zenith distance =	0	58	33 .13

Observed Zenith Distance of γ Draconis

Point on the limb 0° 0′, North.

Day of the month		Face of the arch	Plumb-line		Observation of the star		Zen dist in rev and parts				Zen dist reduced			Barom	Thermometer	
			Rev	Div	Rev	Div			Rev	Div					Above	Below
April	16	W.	10	21 73	8	18 3	0°	0′	2	3 .23	0°	2′	1″ 43	29 .9	45 .0	
	19	W	9	9 .40	7	4 .1				5 .30			3 .50	31 1	53 0	
	22	E.	8	14 .48	10	9 .5			1	54 .02		1	53 .21	29 .9	55 0	
	23	E	9	21 .79	10	18 .5				55 .71			54 .90	30 .1	38 0	
	25	W.	9	39 .52	7	34 .4			2	5 .12		2	3 .32	29 .0	44 .0	

Reduction of γ Draconis.

April 16 .	0°	2′	28″.37	April 22	0°	2′	19″.05	Mean of W. and E	0°	2′	24″.30
19 .			29 92	23			20 .54	Sum of five correct.		+	0 .03
25 .			28 .55					Collimation 4″ 57			
Mean of West	0	2	28 .94	Mean of East	0	2	19 .79	App. zen. distance	0	2	24 .39

9

OBSERVATIONS MADE AT DUNNOSE ON β DRACONIS.

Point on the limb 1° 50′, North.

Day of the month		Face of the arch	Plumb-line		Observation of the star		Zen dist in rev. and parts			Zenith distance reduced.		Barom	Thermometer	
													Above	Below
1802			Rev	Div	Rev	Div		Rev	Div					
May	11	W.	9	4 .82	9	17 9	1° 50′	0	13 .08	1° 49′	46″.90	28 .85	43 .5	43 .5
	13	E	9	16 .95	8	56 0			19 .95		40 .02	28 .85	36 .5	38 .0
	14	W.	9	34 .25	9	47 .5			13 25		46 .73	28 .92	34 5	34 .5
	16	E.	8	32 .16	8	14 .0			18 .16		41 .81	28 .82	35 .5	34 .5
June	5	W.	6	23 .00	6	30 0			7 .00		52 99	28 .45	51 .5	51 .5
	8	E.	8	14 .02	8	2 .0			12 .02		47 .96	28 .49	52 .0	51 .8
	11	W.	6	57 .40	7	2 6			4 .20		55 .79	28 .54	52 .5	52 .0
	13	E.	9	39 .50	9	29 .5			10 .00		49 .98	28 .79	53 .0	52 .7
	14	W.	8	19 .29	8	23 .7			4 .41		55 .58	28 .86	54 .2	53 .0
	16	E.	3	56 .61	3	47 .0			9 .61		50 .37	28 .75	59 .5	60 .0
	17	W.	8	38 .52	8	41 5			2 .98		57 .02	28 82	56 .1	58 .0
	18	E.	11	31 .87	11	21 .5			10 .37		49 .61	28 .81	52 .0	51 .0
	20	W.	8	53 .27	8	54 .2			0 .93		59 .07	29 .03	57 .5	58 .0
	21	E.	10	27 .05	10	19 .7			7 .35		52 .64	28 .99	56 .5	55 .5

OBSERVATIONS AT DUNNOSE ON γ DRACONIS.

Point on the limb 0° 50′, North.

Day of the month.		Face of the arch	Plumb-line		Observation of the star		Zen dist in rev and parts			Zenith distance reduced		Barom.	Thermometer	
													Below	Above.
1802.			Rev	Div	Rev	Div		Rev	Div					
May	10	E.	10	15 .52	13	48 .1	0° 50′	3	32 75	0° 53′	30″ 10	29 . 0	——	45 .0
	11	W.	9	38 .66	5	56 .4			41 .26		38 .62	28 .85	43 .9	43 .5
	13	E.	8	47 .30	12	81 .4			34 10		31 .45	28 .85	36 .5	38 .0
	14	W.	7	32 38	3	49 .2			42 .18		39 .54	28 .92	34 .5	34 .5
	16	E.	9	40 .00	13	15 .2			34 .20		31 .55	28 .82	35 .5	36 .5
June	11	W.	7	20 .70	3	29 .5			50 .20		47 .58	28 .34	53 .5	52 .5
	13	E.	9	36 .35	13	20 3			42 .95		40 .31	28 .79	52 .5	52 .3
	14	W.	8	25 .26	4	33 .4			50 .86		48 .24	28 .26	54 .3	53 .0
	16	E.	9	48 .33	14	37 .4			45 .07		43 .44	28 .75	59 .5	60 .0
	17	W.	8	32 .66	4	39 .4			52 26		49 .64	28 .82	56 .0	58 .0
	18	E.	11	32 .77	15	17 .9			44 .13		41 .50	28 .80	52 .0	51 .0
	20	W.	8	9 .48	4	17 .0			51 .48		48 .86	29 .97	58 .6	57 .0
	21	E.	11	52 .92	15	40 .0			47 .08		44 .45	28 .83	56 .0	55 .5

REDUCTION OF β DRACONIS.

May	11	1°	50′	7″.65	May	13	1°	50′	0″.23	Zen dist. mean of W. & E.	1°	50′	3″ 53
	14			7 .00		16			1 .10	Mean refraction, &c. . .		+	1 .83
June	5			6 .11	June	8			6 .12	Temperature		–	0 .05
	11			6 .99		13			0 .55	Expansion of telescope .		+	0 .00
	14			5 87		16			0 .01	Collimation 3″.35 . . .			
	17			6 .32		18		49	58 .59				
	20			7 .43		21		50	0 .68				
Mean of W.		1	50	6 .89	Mean of E.		1	50	0 .18	Zenith distance . . .	1	50	5 .31

REDUCTION OF γ DRACONIS

May	11	0°	54′	0″ 34	May	10	0°	53′	51″ 66	Mean of West and East	0°	53′	55″ 75
	14			0 12		13			52 .31	Refraction, &c . .		+	0 91
June	11		53	59 .45		16			51 .54	Temperature . . .		–	0 .02
	14			59 .14	June	13			51 53	Expansion . . .		+	0 .00
	17			59 .44		17			53 .58	Collimation 3″.64 . .			
	20			57 .83		18			51 11				
						21			53 .07				
Mean of W		0	53	59 .39	Mean of E		0	53	52 .11	Zenith distance . .	0	53	56 .61

10. OBSERVATIONS MADE AT BEACON HILL ON β DRACONIS.

Point on the limb 1° 0′ South.

Day of the month		Face of the arch	Plumb-line		Observation of star		Zen. dist. in revolutions				Zenith distance reduced			Barom.	Thermometer	
															Above	Below
1802.			Rev	Div	Rev	Div			Rev	Div						
July	20	W.	12	1 .04	12	14 8	1°	0′	0	13 76	1°	0′	13″78	28 8	58 .0	56 .0
	22	W.	7	53 .33	13	12 .0				17 .67			15 .52	28 . 7	54 .0	54 .5
	26	E.	13	27 .55	13	6 8				20 .75			20 .78	28 . 8	64 .2	64 .3
	28	W.	9	21 .91	9	32 .3				10 .36			10 .38	28 . 8	59 5	58 .5
	29	E.	9	3 .13	8	44 .1				18 .03			18 .06	28 8	56 .5	57 .5
	31	W.	9	34 .59	9	44 .1				9 .51			9 .52	29 . 0	57 .2	56 .5
Aug.	1	E	8	36 .00	8	18 .5				17 .50			17 .53	29 . 2	59 .5	57 .2
	3	W.	8	57 .87	9	8 9				10 .03			10 .05	29 .16	68 0	64 .5
	5	E.	8	11 .26	7	53 8				16 .46			16 .48	29 . 0	71 .5	73 .2
	7	W	8	51 .74	9	1 .6				8 .86			8 87	28 . 9	67 .2	66 .1
	8	E.	8	14 84	7	57 .9				15 .94			15 96	28 . 9	65 .1	65 .1
	12	E.	11	7 .98	10	50 .6				16 38			16 .41	29 .15	58 .1	58 .0
	13	W.	8	20 00	8	30 4				8 .40			8 .41	29 3	61 .2	61 .1
	17	E.	8	30 33	8	15 8				14 .53			14 .55	29 1	70 .5	71 .0
	18	W.	8	46 62	8	54 .7				8 .08			8 .09	28 . 8	70 .1	70 .3

Reduction of β Draconis.

July 20	1° 0′ 13″ 82	July 26	1° 0′ 22″ 41	Mean of W. and E . .	1° 0′ 16″ 89
22	13 15	29	20 .26	Refraction, &c . .	+ 0 .95
28	12 38	August 1	20 39	Collimation 3″.78 .	
31	12 15	5	20 16		
August 3	13 .29	8	20 15		
7	12 .87	12	21 25		
3	13 42	17	20 11		
18	13 80				
Mean of W.	1 0 13 .11	Mean of E.	1 0 20 68	Zenith distance . . .	1 0 17 .84

Observations at Beacon Hill on γ Draconis.

Point on the limb 1° 55′ South.

Day of the month	Face of the arch	Plumb-line		Observation of star		Zen dist in revolutions			Zenith distance reduced	Barom	Thermometer	
											Above	Below
1802		Rev	Div	Rev	Div		Rev	Div				
July 20	W.	11	49 24	13	12 8	1° 55′	1	22 56	1° 56′ 21″ 69	28 .9	56 5	55 0
21	E	7	23 .81	5	53 .7			29 .11	28 .26	28 5	53 0	52 2
22	W.	7	54 31	9	17 .1			21 79	20 .92	28 7	54 .5	54 .5
23	E	8	46 15	2	18 9			27 .25	26 .39	29 0	56 1	56 .1
26	W.	9	8 .47	10	29 5			21 03	20 .16	28 .8	64 .0	64 0
28	E	9	35 .56	8	9 .6			25 .96	25 .11	28 .8	56 .2	57 .3
29	W.	8	44 .41	10	4 .5			19 .09	19 .03	29 0	56 .5	56 5
Aug 1	W.	8	41 .22	10	3 .0			20 .78	19 91	29 2	59 5	57 .0
3	E	9	7 .59	7	40 .3			26 .29	25 .43	29 .1	68 .0	64 .5
5	E	7	50 50	6	25 .0			25 50	24 .64	29 0	73 0	71 0
7	W	9	7 .55	10	24 .6			17 .05	16 .18	28 9	64 .2	65 .2
12	E.	11	7 .56	9	42 .7			23 .86	23 .00	29 .1	57 .5	57 .5
13	W	8	12 .48	9	29 4			16 .92	16 .04	29 .3	63 .0	61 .2
17	E.	8	10 .32	6	46 .0			23 .32	22 .46	29 .0	69 .5	70 5
18	W	8	32 97	9	48 5			15 .53	14 65	28 .8	70 .0	70 1

Reduction of γ Draconis S

July 20	1°	56′	21″ 63	July 21	1°	56′	28″ 50	Mean of W and E	1°	56′	24″ 86
22			21 .47	23			27 .14	Refraction, &c		+	1 .78
26			21 72	28			27 15	Collimation 3″ 30			
29			21 67	August 3			28 86				
August 1			22 90	5			28 51				
11			20 .45	12			28 28				
13			21 .55	17			28 66				
18			21 03								
Mean of W.	1	56	21 55	Mean of E	1	56	28 .16	Zenith distance	1	56	26 64

Hence we deduce the following differences of latitude from Greenwich without reference to the known polar distance of either of the two stars, which differences, applied to the known latitude of Greenwich, will give the latitudes of the other two stations, and also we obtain the total arc between Dunnose and Beacon Hill, near Clifton, in the following manner, viz.

	By β Draconis				By γ Draconis			
Zen dist at Greenwich . .	0°	58′	33″ 13	N. .	0°	2′	21″ 39	N
at Dunnose . . .	1	50	5 31	N	0	53	56 .64	N.
at Beacon Hill .	1	0	17 84	S .	1	56	26 .64	S
Diff. of Dunnose and Greenwich .	0	51	32 18 .		0	51	32 .25	
of Beacon Hill and Greenwich	1	58	50 97	.	1	58	51 .03	
Whole arc from Dunnose to Beacon Hill	2	50	23 15	.	2	50	23 28	
Taking the latitude of Greenwich at .	51	27	39 00	.	51	27	39 .00	
We have lat of Dunnose	50	36	6 82	.	50	36	6 75	
And also lat of Beacon Hill	53	26	29 97		53	26	30 .03	

By a mean of eight stars the diff of Dunnose and Greenwich is	0°	51′	31″ 39
Beacon Hill and ditto	1	58	51 59
And a mean of seventeen stars gives the total arc between the extreme stations	2	50	23 38

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§ LXXXI ZENITH SECTOR BY TROUGHTON [Plate XXVII]

1 After the ample account we have given of Ramsden's zenith sector, we shall not have occasion to dwell on an instrument by Troughton of a much more simple construction, and more easily conveyed from one station to another, as well as more readily put up and adjusted for observations In all his instruments Mr. Troughton seems to have avoided a bearing at the upper

pivot of a long axis, if we except the Armagh equatorial, of which construction he never made a second and his reason is obvious, when the superior end of a long axis is supported by metallic or wooden supports of considerable dimensions, which they must be if they ascend from the ground, the former will derange the vertical adjustment by expansion by heat, and the latter by the absence or presence of moisture affecting the dimensions of the material. On this account, we generally find his instruments turning on axes at or near the bases, and guarded from sudden changes of temperature by surrounding sockets. Sometimes these axes ascend, as in the South Kilworth circle, but more frequently they descend, as in the Westbury circle, and in one of larger dimensions now in hand for Edinburgh (1828). A zenith sector founded on this principle is represented in perspective by fig 3 of Plate XXVII., in which all the parts are so clearly presented to view, that the description may be very short, and will be easily comprehended. It was constructed for the use of the American States, but in its passage was taken in the war by a privateer, which carried it to Sainte Croix, where it has been of little use to astronomy. Professor Schumacher borrowed this sector, not long ago, to assist him in his trigonometrical operations, but it did not arrive at Copenhagen in time to be serviceable.

2. The stand of this sector is a strong mahogany tripod, braced at every side by a cross, and having a solid top, as seen in the figure; it forms a detached support to sustain the weight of the instrument, on the top of three stakes driven into the ground, at the angles of an equilateral triangle of similar dimensions. The base of the instrument itself is formed of three strong edge-bars connected by a solid circle, and having feet-screws at their extreme ends, very similar to those of our new circular instrument made by Fayrer. (§ LXXI.) The feet screws of this base stand exactly over the strong feet of the tripod, two of them in metallic centre-pieces, and the third on a triangular small stool, two legs of which form the fulcrum round which a slow motion of adjustment is given by a fine screw forming the third leg. This screw stands in the line forming north and south, when the instrument is used, in which direction the vertical position of the axis requires to be nicely adjusted. The axis of motion is a solid cone descending into a deep vertical socket, carried by the centre of the tripod, to which it is firmly screwed; and in the solid central part of the wheel, or graduated circle, made fast to the axis, are inserted six upright conical pillars, that surround a water-vessel resting at the centre. On the tops of these pillars the lower end of a large central tube of brass is made fast, which may be called the body of the instrument, as it has several appendages attached to it that constitute the more delicate parts of the mechanism. This large tube, which is longer as well as of greater diameter than the telescope's tube, is always kept in a vertical position by a plumb-line, that descends down its central part, and is viewed near the lower end in two directions, at right angles to each other, by a ghost-apparatus, composed of two compound microscopes, carrying each a disc of mother of-pearl as an object, the images of which are brought to coincide with the plumb-line, so as to have no parallax, when viewed in either of the two directions; whereas the disc and plumb-line in Ramsden's sector are obliged to be so contiguous as to form but one image in the microscope, the two parts of which cannot be equally distinct. This plumb-line is adjusted by the suspension apparatus, at the superior end of the tube, and by the feet screws, by the method of halving the error, till it will keep its position during

reversion, and, swinging at the centre of motion, is but little disturbed while the circle turns. The azimuth circle has stops that give notice when a semi-revolution has been performed, and reads by three equidistant verniers carried by the arms of the base, which have also pins for receiving the eye-piece, forming a simple microscope, for reading the verniers successively. The arc of the sector has three divided scales on it, one extending the whole length, and the other two a little more than one half, beginning at opposite ends. It is made fast across the exterior face of the large tube, above the ghost apparatus, and at right angles, on an intermediate fixed piece filed quite flat to receive it: this piece projects far enough to allow the observing telescope, which has its axis of motion at the head of the large revolving tube, to embrace it while it slides before the divisions. The axis on which the principal telescope moves is the centre, from which the curved lines of the arc were struck, and is fixed so as to make the telescope move in zenith distance in the same plane that the face of the arc occupies. The eye-piece of this telescope has a reflector placed at 40° of inclination, of the diagonal construction, that enables the observer to direct his eye horizontally as he stands in his erect position, the illumination being through the side of the tube, and a pair of reading-microscopes, one at each side of the telescope's optical axis, read separately the distance from the stationary plumb-line, which constitutes the zero, hence the two separate divided scales are appropriated to their respective microscopes, one or other of which can always read every part of the sector, which could not so well be done with one alone. The whole line connects the divisions of the two halves. These microscopes appear in the figure above the diagonal eye-piece, and the thumb screw with a milled head near the lower end of the main tube, above the microscopes, to the left, is the clamping screw, giving slow motion for adjustment of the star to the horizontal line during its passage. The plumb-line descends down an inclined plane to the angular notch from which it is suspended, which prevents its liability to fracture at that point, when loaded to its full extent. The axis of the telescope's motion is not a joint but a long cylinder, extending diametrically over the upper end of the large tube, or body of the instrument, and is made horizontal by a riding level, which may be seen in its place. The arc of the sector is read by its microscopes, in the same way that the limb of a large circle is read, and the microscopes adjusted in a similar manner, as well as the plumb-line, and therefore require no further directions for their adjustment. A frame carrying a small telescope, which is mounted on the pivots of its axis, like a small transit instrument, is clamped by a ring embracing the circular body of the instrument, in such way as to be adjustable to any vertical plane that it may be required to move in: its use is to find the meridian of the place by the help of the azimuth circle, and to place the plane of the scale exactly in the meridian. This is best done by observing the points of greatest elongation of Polaris, or other circumpolar star, in the manner before directed. (§ LXXVI.)

3. An inspection of this beautiful instrument is sufficient to convince an experienced observer that its convenience and accuracy must be indubitable, when all the parts are completed in the way that its inventor is accustomed to finish his instruments in general. We have not given the dimensions, for this obvious reason, that no practical operations are recorded as performed in actual service, that we know of, but it is quite clear, that the plan of the construction is capable of being executed on any scale of dimensions that accurate results can

require. The Zone of Stars published at page 280 of our first volume, and explained in pages 414 and 415, will be found an useful companion to this instrument, as well as to its two predecessors, and also to the zenith micrometers, which follow.

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§ LXXXII. ZENITH MICROMETERS. [Plates XI. XII. and XIII.]

1. The zenith micrometer is an instrument of recent invention, being simply the application of a good wire micrometer to the eye end of a zenith telescope. It differs from the zenith sector chiefly in the extent of its scale, which in this instrument is limited by the field of view; since the measure is always taken within the tube of the telescope, and when the magnifying power is great, this field is proportionally small, but what the scale wants in extent, it compensates in accuracy. The first zenith micrometer was probably the companion of the Greenwich mural circle, which we have already described (§ LXXII.), where a reflecting telescope forms the principal portion, and was the contrivance of Mr. Troughton.

2. The next instrument for measuring zenith distances by means of a micrometer, was made by Mr. G. Dollond, from, as we understand, a design by Mr. Pond, where a refracting telescope was used in a vertical position. This was one of the instruments ordered by the English government for the use of Dr. Tiarks, when he was sent to co-operate with Mr. Hassler in determining the line of division on the American lakes, which point of dispute is not yet finally settled. Dr. Tiarks speaks in terms of great commendation respecting this instrument, but as his coadjutor and he had duplicates of a good repeating circle, they employed them chiefly to determine the latitudes, and used the zenith micrometer less than otherwise it would have been employed. The different parts of this zenith micrometer are represented by figures 4, 5, 6, 7, and 8, of Plate XI., drawn from a scale of two inches to the foot of its real dimensions. Figures 6 and 7 give different views of the vertical telescope, the object-glass of which is two inches and three quarters in diameter, and forty-two inches in focal length, the tube screws into the horizontal axis of motion, which resembles that of a transit-instrument, so as to be detached at any time, and used as an ordinary telescope. The axis is eighteen inches long, exclusive of the pivots, and when mounted falls into Ys on the upper face of a circular ring of brass, seen in fig. 8, which forms the upper end, of a mahogany stand, braced at its sides, and standing on feet-screws, which are omitted in the plate, as being now easily comprehended without particular description. The upper ends of the four upright bars of the stand appear at equal distances, two at each side of the Ys, upon the face of the ring, and a similar ring at the bottom receives the lower ends of the same bars, and holds the feet-screws. The pivots of the axis may also rest on Ys made in brackets, carrying Y plates, attached to a pier or wall, substituted for the mahogany frame, when opportunity will permit such application. The axis is formed into a telescope, by having an eye-lens at one end, and an object-glass at the other, for the purpose of viewing two objects placed in the same line, one to the east and the other to the west side of the meridian, and exactly at right angles to it, which marks are very convenient for bringing the instrument into its due situation for making zenith observations correctly, in the reversed positions. The reversion is effected by lifting the axis out of its Ys, and changing

it end for end, before it is replaced. A plumb-line, seen in fig. 6, descends through holes made in the axis, from its point of suspension, where the milled head of the fixing screw appears above the axis, and is viewed by a ghost apparatus fixed to the main tube, just above the double micrometer, seen over the diagonal eye-piece at the lower extremity. A disc of mother-of-pearl having an excentric point forms the object here, as in like cases, of the compound microscope, which point, by turning the disc, may be brought to lie coincident with the plumb-line seen in the focal point of its ocular lenses. This arrangement may be easily understood from the appearance of these parts in fig. 6, but in fig. 7 the end of the microscope points towards the eye, and both the fixing screws are seen. The plummet, or perforated vessel holding the shot, is immersed in a water pot borne by the frame not exhibited. The two screws, in contact with the main tube in fig. 7, one at each side, above the double micrometer, act in a pair of cocks made fast to the frame just alluded to, or to the pier when that may be used instead, their use being to fix the telescope exactly perpendicular, by bringing the point on the luminous disc to the plumb-line, and holding it there during an observation: for as the screws of the micrometer alone measure the distance of a star from the zenith point, or in other words from the plumb line, it is necessary that it should keep its station unaltered, as it regards the disc, in both of the reversed positions. If the right hand screw is made to measure the distance in the first position of the telescope's axis, then the left hand one, taking the right hand situation, measures the same a second time, and the sum of the two readings will give the apparent zenith distance, for one half of their difference, which will be the telescope's error of collimation in zenith distance, will merge, as in a sector, in the collimated mean result. The field of view is stated to extend to 2°, so that the ocular lenses must be large in diameter and the magnifying power small, to give such a field: this limits the greatest measure to one degree. The light, that renders the lines in the micrometer visible, enters through the axis, as in a transit-instrument, and the axis itself is made perfectly horizontal by means of the riding level, shown in fig. 4, having Ys below, one of which is seen in fig. 5. The value of the micrometer's screws may be found by any of the usual methods, as the telescope will turn upon its axis to any elevation, when the plumb-line is displaced, and means are provided in the frame for giving azimuthal motion, by a pinion acting on the circular plate. As the accuracy of an observation depends on the relative positions of the telescope and plumb-line, the two eye-pieces are so placed just above one another in this instrument, that the star and plumb-line can be viewed in immediate succession, as in Troughton's sector, which is the great advantage of the construction.

3. Besides the two preceding constructions of a zenith micrometer, and the instrument made for us by Fayrer (§ LXXI.), which in many respects resembles Troughton's zenith sector, we have had two common telescopes of different dimensions mounted on ordinary stands, in such a way as to admit of adjustments for their vertical position, with each a double micrometer attached, one adjusted by a level, and the other by a plumb-line and level, by methods that greatly extend the utility of these telescopes, without interfering with their portability and usual applications. As cheapness is a quality, that often recommends an instrument, competent to its purpose, to amateur astronomers, we will include an account of those zenith telescopes in our present section. Fig. 2 of Plate XIII. shows the larger of these telescopes in its zenith position. In general a claw-feet pillar of this appearance consists of a hollow tube, made fast at the lower end to the junction of the three legs, and having a compound joint at the top, to give motion in azimuth and altitude, in which joint the bearing is very short, and after some time the mo-

tions become too free and ultimately very unsteady, the only use of the joint being, to allow of motion taking place, without regulating or sustaining the position of the telescope when moved. In this stand or tripod pillar, a strong tapering tube of brass nearly eighteen inches long, is first made fast to the central solid part, where the legs are united, its flanch and both ends being nicely turned in the lathe round this fixed axis an external tube, of the same shape, is nicely fitted at the top and bottom, and revolves in a perpendicular position without the least perceptible shake, as the fixed tube descends about three inches downwards below the flanch, where it is fixed by a strong nut, the outer tube is only fifteen inches long at its superior end it carries a solid frame, screwed fast to a circular flanch, which of course revolves with it, the upper end of this frame bends outwards from the centre by a regular curve given to both of its sides, and an horizontal axis four inches and a half long, made fast to a strengthened part of the main tube, passes through pivot-holes at the rounded summit of both sides of the frame, where the axis is kept to its place by collets, and screws passing into its ends. The two sides of the frame are screwed to its solid bottom, and one of them was adjusted by a fine file till this horizontal axis lies at right angles to the vertical axis. To the lower end of the fixed tube a circular graduated plate, of seven inches diameter, is made fast, close upon its flanch, and a vernier bar attached to the revolving tube, just over the graduated plate, indicates minutes of azimuthal motion. By this mode of mounting the telescope, its motions in altitude and azimuth are very steady, and as each is regulated by a screw of slow motion when clamped to the approximate position, a star can be brought to the middle line in the eye-piece with the greatest exactness. The focal length of the object-glass is upwards of three feet and a half, and its diameter two inches and three quarters, so that the magnifying power and light will command a star of small magnitude. A short piece of strong tube is so attached to a cock, made fast to the vertical revolving tube, that it will turn on a strong pivot and adapt itself to any position for receiving a propping tube, that descends in an inclined position, from a cock fixed to the main tube of the telescope, near the eye-end, before it enters the holding piece of tube above described. This propping tube, however, is not in one piece, but consists of a single system of three tubes, sliding within one another, and capable of being clamped fast to one another in any given relative positions, the inner tubes being set fast by the next outer ones, so that the telescope can take any elevation from a horizontal to a vertical position, and be fixed there by the clamps of the three tubes as firmly as by a single one a thumb screw, entering the solid end of the innermost tube, and held by the cock at the eye-end, will turn round and add to or take from the total length of the tubes, thereby giving a small elevation or depression to the approximate position of the telescope. A quadrant of six inches radius, reading minutes with a single vernier, is screwed fast to the side of the telescope's tube, and when adjusted, indicates the altitude, by means of a delicate level affixed to the vernier bar. When the zero of the vernier is clamped at 90° of the quadrant, and the telescope placed by estimation in its vertical position, and clamped by the propping tubes, now retiring into one another, and assuming an horizontal position, as seen in the figure, the vertical axis may easily be adjusted to its zenith position.

4. The first step in the adjustment is to make the axis truly vertical in all directions by the feet screws, according to the method before recommended, in all cases where a tripod is used (§ LXVIII. 7.) this is done partly by the feet screws, and partly by the bubble of

level, which will keep its place in its tube while the axis revolves, when the latter is truly vertical, and this part of the operation may be performed in any position of the telescope, as the vernier that carries the bubble is moveable along the arc of the quadrant After the axis has been made vertical, and the vernier put to 90°, the telescope may easily be brought to its zenith position by the thumb screw, acting on the propping tubes, while in their horizontal position, for, assuming that the quadrant has no index error, bringing the bubble to its zero will put the tube of the telescope into its vertical position, where it will remain during a revolution of the vertical axis As the ocular end of the telescope lies over the face of the azimuth circle, a diagonal adapter is applied to receive the double micrometer, and the observer looks in a horizontal direction, as in Dollond's instrument, while he views a zenith star One of the two screws measures the zenith distance in the first position, and the other in the second, and as in other instances of collimated observations, the mean of the two is the true measure, without reference to the collimation or index errors. When a stop is applied to the azimuthal circle at 180° from the first position, a star can easily be observed in both positions while it is passing the field of view, the bubble of the level being immediately before the eye, and requiring no time to wait for its settling, when the axis has been properly adjusted By means of this instrument the zenith distance of a star, passing near the zenith, and consequently the latitude of any station may be known to the accuracy of a second, allowing for the error of vision, provided that the polar distance of the star be known This is one of the five telescopes for which we computed tables to give the values of an attached micrometer in pages 103 and 104 of this volume, being that which is designated by No. 2. A lamp suspended from the frame gives light to the micrometer's lines, through a circular hole made in the side of the tube, opposite the internal reflector.

5. Fig 3. of the same plate exhibits another position of the same instrument, in which it becomes an excellent equal-altitude instrument, by the help of the level and quadrant, which not only assist in placing the axis truly vertical, but watch the position given to the telescope, as the instrument moves in azimuth. We need not repeat the account we have given of a similar property, (in § LXXI) possessed by our portable circular instrument made by Fayrer, the powers of the two instruments being similar, both as zenith and equal-altitude instruments, but the present instrument having the recommendation of comparative cheapness, on account of the absence of the vertical circle and plumb line apparatus.

6 A smaller zenith telescope, of a still cheaper construction, is represented by fig. 1 of Plate XII, in which the telescope is our No. 1, having an object-glass only 2 15 inches in diameter, and thirty inches and a half focal length. The pillar and feet were originally made by a clock-maker for some experimental purpose, and were purchased at the price of old metal for the express purpose of forming a zenith-stand for this telescope This stand had formerly several wheels and pinions, intended to show minutes and seconds of arc, by a kind of clock-work, but had been rejected on trial. Feet-screws have been applied to it, with a small triangular stool for slow motion placed under one of them, as well as other appendages added, which will appear from our description. The stem *a* of the stand is a perforated solid pillar, nine inches long, through the middle of which a cylindrical steel axis descends, that carries round with it a circular thick plate, *b*, racked at the edge all round, and having a circle of entire degrees engraved on its face. A plate of similar dimensions is carried by the upper end

of the fixed pillar *a*, which is nearly in contact with the other beneath it, and which carries a horizontal endless screw pressed against, and acting with the adjoining circular rack, from the teeth of which it may be turned back at pleasure round a joint: on the axis of this screw is a hand pointing to a small fixed circle divided into sixty, showing minutes of a degree. This side-work acts as a clamp and tangent screw when in action, but allows quick motion when detached. A pair of triangular brass open frames, made fast to the plane of the racked circle, at equal distances from the centre, ascend parallel to one another about seven inches high, and support the horizontal axis of the telescope, which is only three inches long, but very strong. At each end of this axis a wheel of six inches diameter is fixed by collets and screws, close to the outer faces of the frames, one of which wheels is racked all round the edge, and gives slow motion in elevation by another endless screw, lying under it with a milled head, and capable of being discharged for quick motion: the face of the other wheel is divided, and has two opposite verniers reading minutes, seen at *c*, and carrying a short level, only three inches and a half in length. The level keeps its position in all elevations of the telescope, and is adjusted by a thumb screw, *d*, entering an upright lever, attached to the diametrical bar of the opposite verniers, on which also is suspended the level: as the divided circle is fast to the telescope's axis, it turns in elevation, and brings the divisions to the fixed verniers. This axis has no adjustment for horizontality, otherwise than by filing the under side of one of the supporting frames to make it right, which was done by the maker, as was the case with the preceding instrument; and it may be proper to mention here, that in both instruments a very small inclination of this axis towards the east in the first position, will make a similar inclination in the second position towards the west, the effect of which will be, to prolong the time of a star's continuance in the field, without sensibly affecting the measure of zenith distance, since the passage is very nearly in a horizontal line; and such a slight inclination will be rather desirable than otherwise when a plumb-line is used, that requires some time to come to rest. The head only of the remote circle's tangent screw, that acts with its racked edge, is seen, and only a portion of this second circle, the concealed portion of the circumference being parallel to the double dotted line made on the tube, near its screw of adjustment for distinct vision. As this instrument would not allow of a long level without concealing the verniers, a plumb line is suspended from an adjustable point of suspension at *e*, which is clamped to the upper end of the main tube, and a water vessel is attached to the lower end of the same by means of a suspending rod reaching down below the junction of the feet at *f*, which it escapes in moving round. The bed of the telescope is supported by a strong bar, the lower end of which surrounds the middle of the axis, and its sides are screwed fast to the wheels over which it lies, so that all these parts are firmly united, and turn on the same axis with the telescope. This bed has two circular holes at several inches distance from each other, and any adjacent pair of the tapped brass cylinders, made fast to the tube, may be fixed in those holes, that the position of the telescope may require. When the telescope is pointed to the zenith, as in our figure, the two cylinders next the eye end are inserted into the bed, and held fast by a pair of milled nuts, one of which is seen above the wheels, so that more than one half of the main tube is above the axis, in order to allow the smaller tube, at the eye-end, to escape contact with the racked edge of the divided plate, and also that one screw of the double micrometer may lie under the said plate. Above the upper end of the bed is a circular aperture in the tube for the admission of the light of a lamp, which may be closed by turning a broad ring round, that embraces the tube. The eye-piece has two lenses with a thin plate of polished speculum metal interposed, at an angle of 45°, which together constitute a diagonal eye-piece. This eye-piece is formed of two tubes into a cross, the vertical one being cylindrical, and the horizontal one conical, having the broad end nearly in contact with the plumb-line. the lens next the eye has a longer focal distance than the one within the small tube of the telescope, and singly views the plumb-line through a small oval hole, made through the centre of the speculum, while at the same time the star is viewed in the common focal point of both lenses, together with the lines of the micrometer, which, when laid over one another at zero of the micrometer's scale, may be made to coincide with the plumb-line, by sliding the micrometer forwards or backwards, till this coincidence takes place In order that the plumb-line may be well seen, a horizontal slit is made across the end cover of the conical part of the eye-piece, which in our figure is seen bisected by the plumb-line at *g*, and under it is a small round perforation also bisected by the same line, over which the plumb-line is laid by turning the eye-piece when the axis of azimuthal motion is made correctly vertical in all directions, which adjustment is performed partly by the level, and partly by the feet-screws, as in other instruments. This mode of viewing a plumb-line, at the same time with the star and micrometer's lines, is an original contrivance, that answers very satisfactorily, except during the short time that the star disappears in passing the centre of the field, where no reflection takes place; but when the instrument is reversed in position on the same evening, the time of this disappearance will arrive during the act of reversing, and the star will be seen again in the remainder of the field. This telescope, as well as the preceding one, is packed into a very portable case, and when mounted at the telescope's centre of gravity, by a pair of the upper cylindrical pins of the tube inserted into the bed, it becomes also an equal-altitude instrument, by substituting the level for the plumb-line. Indeed rough observations to the accuracy of a minute in arc may be made, with either instrument, both in altitude and azimuth, which on many occasions in travelling may prove very useful. These instruments have all the properties of a common telescope, and may have any of the micrometers applied at pleasure. If it should be objected, that the plumb-line suspended from one side of the object-end of this telescope will have an influence on its vertical position, it must be remembered that this side will be to the east and west alternately, and that the only effect will be, as we have said, to prolong the continuance of the star in the field of view, which is a property rather to be desired than avoided. The axis is first made vertical by the level only, and then the tube of the telescope is made parallel to it by the plumb-line, so that in this instrument these appendages have each a separate use.

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§ LXXXIII. ASTRONOMICAL QUADRANT [PLATE XXIX.]

1. THOUGH Graham, Bird, and Ramsden successively constructed astronomical quadrants for Greenwich, Blenheim, Oxford, and several continental observatories, with some of which the declinations of the stars were regularly observed for upwards of fifty years in succession, by Doctors Bradley, Maskelyne, and others, yet the recent introduction of circles has so com

pletely superseded them, that it is not probable that a large fixed quadrant will ever be again constructed, nor the present ones used, where the means exist of substituting a circle · we may therefore be fairly excused from contributing to perpetuate the use of an instrument, that the present state of practical astronomy has condemned. Except in those instances where the quadrant turns half round on a vertical axis, and has a small arc of excess, the zenith point can only be determined by the aid of a zenith sector, or zenith micrometer, and thus the construction is not independent of adventitious aid. We will not, consequently, take any further notice of a fixed or mural quadrant, than refer our readers, who may be dissatisfied with our silence, to the particular account of quadrants that is given in the Cyclopædia before referred to, as matter on record. The portable quadrant, however, made by Troughton, and sent to Bilboa in Spain, about the time that circles began to be in use, is exempt from the defects of former quadrants, and seems to deserve a place in our collection of modern instruments.

2 Figure 6 of Plate XXIX. exhibits a perspective view of Troughton's astronomical quadrant, such as will enable us to describe it without letters of reference, in a way that will now be intelligible by such readers, as have perused our accounts of the different circles. The body of this quadrant is made of duplicate corresponding parts united by cross pillars, to make lightness and strength consist with each other, which adaptation seems to be a favourite plan, in the construction of all Troughton's instruments. The radius of the quadrantal arcs is just three feet, affording divisions similar to those of the Greenwich mural circles. The manner in which the bracing bars cross one another at right angles, in each of the duplicate portions, is sufficiently apparent without description, and also the small pillars that unite the two parallel arcs and faces of the quadrantal frames. A strong cylindrical tube, forming a column, is included between the two interior faces of the double quadrantal frame, and is made fast to it in a vertical line that passes through the centre of gravity, when the telescopes are mounted. The pedestal is formed into a tripod of strong mahogany bars, each foot of which is furnished with a fine screw, and a Hooke's joint on a long handle; the tripod, besides being braced by crosses at the three sides, has three strong edge-bars proceeding from the feet, and uniting at the centre, so as to hold a piece of brass having a conical hole to receive the lower end of the vertical axis, which passes down through a socket made fast to the circular top of the tripod, that may be considered as a small table; so that the great distance between the pivot and socket, at which places the axis is in contact, assures a steady and easy motion in azimuth to the revolving quadrant, without deranging the verticality. An horizontal or azimuthal circle is attached to the vertical axis, above but nearly in contact with the table, and has a contrivance for allowing an adjustment of about two degrees, for the purpose of placing the zero of the circle right, when the plane of the quadrant is truly placed in the meridian This adjustment is effected by means of the tangent-screw, seen in front of the stand. The other tangent-screw, seen at one side, clamps the solid vernier plate, having opposite readings, to the circular plate containing the azimuthal divisions; to the latter of which it will impart slow motion, when clamped and gradually turned; and consequently will move the whole quadrant with its appendages in an azimuthal direction. This horizontal circle is graduated into spaces of 10′, and will read by each of the opposite verniers, a quantity as small as 10″.

3 The vernier plate, which is circular, has considerable depth, and terminates below with a chamfered edge, containing within it a concealed triangular frame soldered to it below, and opposed, for the sake of strength, by a similar frame visible above it Three small pillars ascend from the concealed triangular frame, and support the upper frame at the three corners, through which the tapped ends of the pillars pass, and are fixed by as many milled nuts, that appear in the figure, above the table These milled nuts are useful in adjusting the plane of the quadrant parallel to the vertical axis of its azimuthal motion , and it is by their means that the quadrant is dismounted from the stand, for package A short but strong conical tube soldered fast to the upper triangular frame, supports the long ascending column, that terminates with a supplemental cone A couple of braces ascend, from nearly the lower end of this column, as high as twenty inches, and reach above the central part of the quadrantal frame : by being firmly attached to the column, these complete the steadiness of the structure in an admirable manner. The observing telescope has a focal length of forty-two inches, and, being achromatic, is competent to bear powerful eye-pieces , the tube tapers from the object-glass to prevent flexure, and has a system of adjustable spider's-lines at the ocular end . its horizontal axis of motion is four inches and a half long, and passes through the thick pillar, that unites the two quadrantal portions of the double frame, at the centre of motion, which is also the centre of the divided arc. Beyond this centre a counterpoise is fixed to the object-end of the tube, which sustains the telescope in any elevation ; and to the ocular end a vernier is attached, that divides the 5′ spaces of the quadrantal arc into readings of 5″, which again are subdivided by a reading micrometrical microscope into single seconds.

4. A nicely-ground spirit-level hangs constantly on an horizontal bar of the quadrant, having the usual adjustments, by which a single second of inclination can be detected , besides which a plumb-line is suspended from the top of the included column, forming an extension of the vertical axis of motion down which it descends, protected from dust and agitation by the wind , which situation being in the centre of azimuthal motion, the plummet immersed in a water vessel, within the short hollow cone above described, is not liable to experience vibrations from a motion in azimuth. Two microscopes, of the ghost kind, view the plumb-line at right angles to each other, across the lower end of the vertical column, the four ends of which are visible in the figure, just above the table. This application of the plumb-line not only serves to put the vertical axis truly perpendicular in all directions, but watches the permanence of this adjustment, and thus checks the indication of the level in variations of temperature · and when one of the two arbiters is displaced or injured, the other will be sufficient for performing the work of the adjustment, and of keeping watch over the position of the axis

5. Besides the preceding appendages there is an auxiliary telescope lying loosely at the back of the frame, in a horizontal position, in a pair of Ys in which it will turn round, and also reverse end for end, in a manner similar to that in which the elder Sisson mounted his spirit-levels and attached telescopes, for viewing two objects, one before and the other behind the station. This second telescope supplies the means of adjusting the horizontal line in the eye-end of a quadrant moving in azimuth, and, by comparison with the usual adjustment of the zenith point, by a zenith star, affords the opportunity of determining the total length of the quadrantal arc, which is an essential determination, where opposite readings cannot be ob-

tained, nor yet a reversion into the eastern and western positions. The graduated arc has consequently several divisions in excess at each end, to assist the adjustment of the index error, and measurement of the total arc. The radial bars that bear the principal weight are made tapering from the centre towards the arc, thereby giving strength to the part most liable to alter its figure by weight; as it is supposed that Bird's quadrant did in a sensible degree, from want of this precaution. The four inches and a half of bearing, that the telescope's axis has through the strong pillar, make the plane in which it moves in elevation a true vertical, which is not rigidly the case when the direction is guided solely by the face of the graduated limb, according to the older constructions. A clamp and tangent screw, applied at the eye-end of the telescope, regulate the motion in altitude as in other modern instruments.

6. *Adjustments.* Many of the adjustments of this instrument are performed in the same way, that has been explained in Sections LXVII and LXVIII, where circles are described that have, the former a plumb-line, and the latter both a plumb-line and spirit-level, applied in a similar manner to adjust the vertical axis, which therefore we need not repeat. The adjustments for distinct vision, and for the true position of the wire-plate for due collimation, by a zenith star, are likewise the same; as is also the method of bringing the plane of the vertical arc into the meridian. But as we have alluded to a method of determining the horizontal point on the arc by means of a second telescope, and of comparing it with the zenith point on the same arc, as determined by a proper star, and of thence examining the *total arc*, it may be proper to explain this double process more minutely, for the satisfaction of observers into whose hands such quadrant may fall. As the two operations are performed, one by means of a terrestrial object requiring day light, and the other by the passage of a zenith star, which, generally speaking, will be best observed in the night, we will begin with the former, and conclude with the latter, though such order is not necessary. When the back telescope, which may be called the adjusting telescope, has been directed to a fine mark chosen in or near the horizon, and brought to bisect it, after the vertical axis has been truly adjusted by the level and plumb-line, its cylindrical tube must be turned half round in the Ys, when it will appear whether the horizontal wire, or spider's-line, lies in the optical axis or out of it; if the mark is not bisected in both positions, a new mark must be chosen at the distance of one half of the deviation above or below, as the case requires, and the other half must be adjusted by the proper screws acting on the wire plate; when the mark will appear very nearly in the same place within the field in both positions; and a repetition of the adjustment with a second or third new mark, will produce this effect; and then this telescope will be adjusted for collimation in altitude. In the next place, fix zero on the vernier, attached to the observing telescope, to zero of the quadrant, and direct this telescope to the last mark made use of, and if it also bisect the same mark, while the level and plumb-line indicate the true adjustment of the vertical axis, this shows that both telescopes are in the same state, with respect to collimation, but it does not yet appear, that the line connecting the Ys lies parallel to the horizontal line, passing through zero of the quadrantal arc; turn now the quadrant half round in azimuth, and reverse the position of the adjusting telescope, end for end, that it may again view the same mark, and if the bisection is exact in this position also, the Ys are in their parallel situation, but if not, one half of the apparent error must be rectified by the screw of one of the Ys, and the other half, by the use of a new mark, till the adjusting telescope will bisect the last chosen

mark in all the positions; and when this is the case, the observing telescope, being placed still at zero, must be made also to bisect the same mark by its ocular screws, when both telescopes will have their collimations truly adjusted. This is the first operation. The second operation requires but little explanation, as it consists in observing a zenith star with face east on one evening, and face west on the following, or as soon after as may be practicable; for the star will pass the point 90° on the arc in both positions, if the axis is still truly perpendicular, and the quadrant exact; but if half the sum of the two readings shall exceed or be less than 90°, one half of the excess or defect will be the error of the total quadrantal arc, one-ninetieth part of which will belong to each successive degree, with its proper sign, accordingly as the measure commences from the horizon or zenith. If after a rigid trial the arc is found to be truly quadrantal, the adjustment for collimation in altitude may in future be made by either of the two methods, that may be most convenient at the time. In thus determining the total arc of the quadrant, it is taken for granted that the maker of the instrument took care to place the plane of its arc parallel to the axis of its motion, as well as the line of collimation parallel to the said plane; which he has the mechanical means of doing, but which an observer ought not to attempt.

7. From the account we have now given of Troughton's portable quadrant it is obvious, that it is competent to determine altitudes with extreme accuracy in skilful hands, and that it is peculiarly fitted for giving either equal altitudes, or zenith distances of any extent; but has not the advantage of being confidently used in taking transits, though it may be otherwise used as an altitude and azimuth instrument out of the meridian.

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§ LXXXIV POLAR INSTRUMENT BY RAMSDEN [Plate XII.]

1. When the late Mr Aubert established a private observatory at Highbury, Ramsden made for him various astronomical instruments, and among the rest an instrument for observing the pole star only, which was fixed by lead so fast to a solid stone, that, after his death, it was difficult to disengage it without injury. Indeed the tube of the telescope was so decayed, by constant exposure to the air, that it crumbled to pieces by the slightest pressure between the fingers. The more solid parts, however, were found perfect, and after replacing the tube of the telescope, when it came into our hands, we mounted it on two strong stones, forming part of a new wall that faces the west, and that allows of the proper position for viewing the pole star. Figure 2 of Plate XII. represents the polar instrument, as attached to a wall facing the east, the draftsman having omitted to reverse the position for the engraver; but it will make no difference with the description. The two stones projecting from the wall are seen at *a* and *b*; *c* is a cast-iron cock made fast to the lower stone, by a pair of iron bolts, passing through a pair of cast-iron plates inclosing the stone between them, and screwing into the back plate. The cylindrical bearer *d* is screwed upon this cock, and terminates above with a solid cube of cast-iron, cut away at one side into an inclined plane pointing to the pole. A square bed of solid brass, *e*, lies upon and is screwed fast to the inclined plane, a section of which is seen in fig. 3,

denoted by the same letter, within this bed lies the square base of a cock, *f*, which is adjusted and fixed by several screws that will be hereafter described. The arbor of a circular plate passes through a round hole in this cock, near *g*, and is kept in its place by a screw and collet, which appear. The chamfered edge of this circular plate is divided, and figured into twenty-four hours and parts, that read with a vernier to a quarter of a sidereal minute of time. The upper plane of this hour circle is seen in fig. 3, having a small three-sided frame screwed to it, the parallel sides of which form a dove-tail, in which the adjustable piece *i* slides, carrying a thick piece near *i*, containing a pivot hole, that is drawn by the screw seen entering it. Into this hole a pivot of the cylinder *k* enters, as appears in fig. 2, the telescope *l* is made fast at both ends to the cylinder *k*, in a position that is made parallel by the maker. Through the upper stone *b* the stem of another cast iron bearer descends, and is screwed fast by a nut below, the stone being inclosed between two circular collets, the upper part of this bearer is similar to that of the bearer *d*, having an inclined plane also pointing to the pole: upon the face of this inclined plane is made fast a brass plate, *n*, having two small bearing cocks above *n* screwed to it; upon the upper end of this inclined plate rests a strong cock, containing within it an universal joint, with two axes at right angles to each other, into the central hole of which cock the upper end of the cylinder *k* enters, and is made fast above by a collet and screws. In consequence of this arrangement of the parts, it is not difficult to comprehend, that whatever motion in altitude, or azimuth, may be given to the lower end of the cylinder *k*, by any mechanical means, the universal joint at the upper end will allow it freedom in either direction. When the pivot of the cylinder *k* is inserted into its pivot hole *i*, which is adjusted to an excentric position as it regards the horary circle *h*, whenever this circle is turned round on its axis, the pivot-hole *i* carries with it the pivot of the cylinder *k*, and also the attached parallel telescope, and its object end, being beyond the universal joint, near the middle of the tube, has a contrary direction to that of the eye-end; so that when the excentric pivot, carried round by the hour circle, carries the eye-piece round a small circle in one direction, the object-glass is also carried in a circle but in the contrary direction. Now the excentricity of the pivot is so adjusted, that when the telescope is directed to the pole-star, turning round the horary circle with the slow motion of a revolution in twenty-four sidereal hours, will make the star remain in the centre of the field, during the whole of its apparent revolution, and, when the adjustments are properly made, the vernier of the hour circle will indicate the time that is measured by a sidereal clock keeping true time. therefore turning the hour circle, till the sidereal time at any moment is indicated, will bring the pole-star into the middle of the field, whenever it is required to be seen.

2. The two principal conditions in adjusting the instrument are, first, that the telescope and its parallel cylinder *k* should be elevated in an angle equal to the latitude of the place, when viewing the star at six hours after the meridian passage, and secondly, that the distance of the excentric pivot hole, from the centre of the horary circle, should bear the same proportion to the whole length of the cylinder *k*, that the sine of the star's polar distance does to radius. There are several pairs of double screws, for bringing the telescope into the meridian, and for regulating its elevation. When we speak of a double screw, we wish it to be understood, that in each instance a longer screw, having a fine thread, passes through the arbor of the shorter and coarser screw, and when two contiguous plates are to be separated a little, and kept con-

stantly at that distance, the inner screw acts with the lower plate only, by screwing into it, and the outer screw acts only with the upper plate, but presses upon the lower plate with its end, and, by tightening the lower screw by its pressure, fixes both plates at the regulated distance; so that unscrewing the outer screw, and screwing the inner, will bring the plates into contact, but unscrewing the inner one, and screwing the outer one, will separate them, and yet fix them, when the number of screws, which thus act in pairs, are sufficient for the purpose. In this way the cocks that bear the horary circle and universal joint are brought nearer to, or removed farther from the brass plates attached to the inclined planes, accordingly as the latitude of the place may require. There are besides single pressing screws acting at the right and left of the universal joint, for putting the instrument in the meridian, as well as others below the lower cock, for keeping it up to its place, and holding the horary circle close to the pivot of the cylinder *k*, when it is adjusted to a right angle with it. These different adjustments can only be performed by means of the star itself. There is also a small screw at *o*, under the eye-piece of the telescope, and a corresponding one at the opposite side, the use of which is, to place the telescope exactly parallel to the cylinder *k*, not as it regards their distance from each other, but their crossing one another, for it is necessary that the axis of motion of the cylinder should lie in all respects parallel to the telescope's optical axis. When the instrument came into our possession, it had simply a common positive eye piece with a pair of cross hairs in the focal point, from which it should seem, that no measures had been taken of the daily or weekly variations of this star, which exceed those of any other star visible by day-light; but its telescope has an aperture of 2.85 inches, and a focal length of forty-five inches, with a power of ninety-two, and is quite competent to command a view of Polaris by day-light: we have therefore applied a spider's line micrometer to it, magnifying 117 times, which is an useful addition, since the variations in north polar distance and in right ascension, arising out of precession, aberration, and nutation, become measurable quantities, when the instrument is adjusted to the mean place. This instrument is we believe unique, and as it is of an expensive construction, may probably remain so.

§ LXXXV. ON FIXED TELESCOPES. [Plate XII.]

1. The question of the existence or non-existence of annual parallax, determinable in the places of certain stars, has produced a new mode of applying an achromatic telescope to celestial observations. There seemed to exist a discrepancy of about a *second* between the results of the two best circles that have been in use, and a laudable desire on the part of the astronomers of Greenwich and Dublin, to reconcile this unexplained difference, suggested not only different methods of observing, but more complex investigations of the constants of aberration and nutation, with a view of obtaining a quantity greater than the probable error of observation, that could not be accounted for but on the supposition of a parallax. That certain stars, such as γ Draconis, Polaris, and some others have no appreciable parallax, seems admitted by both the parties opposed to each other in this investigation. Where the quantity in dispute is so small, the greatest nicety is required in making the observations, as well as a proper

choice of constants in the corrections, at Greenwich the former has occupied the attention of the Astronomer Royal, and at Dublin attention has been paid more particularly to the latter, as they can be corrected by a scientific examination of registered observations. The greatest difficulty seems to consist in reconciling the comparative observations of γ Draconis with α Lyræ at the two places. "According to the observations of Mr. Pond," says the Bishop of Cloyne, "there is no difference between the relative places of these stars in summer and winter; and it is from a relative change of place I find in these two stars, that I adduce what appears one of my strongest arguments for the parallax of α Lyræ In this instance the two instruments are completely at variance, and one of them must give an erroneous result." (*Phil. Trans.* of London, 1821) In another part of his excellent paper the learned prelate says, "some of the results that I have found, although in themselves in no manner inconsistent with parallax, will, justly perhaps with many, add to the difficulty of admitting the explanation of parallax. They will be unwilling to admit that many of the smaller stars are nearer to us than many of the brighter That in a certain part of the heavens of considerable extent, many of the stars exhibit a sensible parallax. This however must be admitted, (continues the Bishop) if my discordances result from parallax. If it be admitted, then several of the difficulties that have occurred, by comparing my observations and those of Mr. Pond, will be done away." How far the observations, made by the union of two circles, at Greenwich, may ultimately affect the question at issue remains to be seen, when multiplied observations afford extended data. A frequent determination of the horizontal point, and of the index errors, will bring the Greenwich and Dublin observations into a state admitting of more satisfactory comparison, than when the observations at the former place, being differential, were mixed up with the changes of refraction, that variable temperature produced in long continued series. In gaining the same latitude from a great number of observations at the four quarters of the year, separately considered at Dublin, it appears that reductions, made from the internal thermometer alone, agree among themselves much better, than if they were made from the external thermometer, which is an important fact in practical astronomy.

2. That the apparent small discrepancy between the Greenwich and Dublin circles might be brought to the test of a comparison with a different instrument, two telescopes, of ten feet focal length each, were erected at Greenwich, on the faces of the walls bearing the quadrants and first circle, one for comparing the declination of α Cygni with β Aurigæ, and the other for comparing that of α Aquilæ with *l* (55) Pegasi. The mode of fixing the telescopes was nearly the same for both, for which reason we have given a representation of the latter in figures 4 and 5 of Plate XII, which will suffice for an explanation of both. The mural clamp *a*, in the shape of a cross, is made fast to the face of the wall, which, by reason of its original destination, is built in the direction of the meridian; this holds the eye-end of the telescope a little above the ground, but so that an observer may place himself under it; the upper end of the telescope required the clamp to be cranked at *b*, to suit the bed that is made at the upper end of the wall, the quantity of inclination having been previously computed to suit the meridian altitude of the star, and the clamps having a small adjustment by means of elongated screw-holes. A double spider's line micrometer is substituted for an eye-piece, which measures the diurnal deviations of the two stars, compared with each other; and provided that no alteration take place in the firm position of the wall, this method of observing the changes,

it was expected, with a proper choice of stars opposed to each other in the best situations for giving a double result, would have settled the question. The light is admitted at a circular hole made in the tube at *c*, and reflected in the usual way down the telescope, without interfering with the cone of rays coming from the object-glass, which is covered by a lid, and uncovered by means of a pulley and string acting near the hinge, as seen in the enlarged view in fig. 5, under these the rollers of the clamping ring are also seen enlarged, that admit of the telescope elongating gradually by an increase of temperature. The observations made with both of the fixed telescopes are published at the end of the volume of the Greenwich Observations for the year 1816, and Mr. Pond's papers on the subject of parallax are contained in the volumes of the Philosophical Transactions of the years 1817 and 1818, which afford data for inferring, that no appreciable quantity of annual parallax exists, as deducible from the observations made by the fixed telescopes. On the contrary Dr. Brinkley undertakes to explain, in his last paper above referred to, why no parallax is obtained by the Greenwich telescopes. "The fixed telescope, used by Mr. Pond," says he, "for the comparison of *α* Cygni and *β* Aurigæ, shows no relative changes of place that can be explained by attributing a parallax to *α* Cygni. This star formerly appeared to have a less parallax than others I had observed. My new observations give a much smaller quantity for it, but I am inclined to think the true quantity lies between my present and former results. Now admitting it to be half a second, no contradiction to this can be drawn from the observations by the fixed telescope, when those observations are carefully examined with a reference to the visible effects of the change of temperature. The fixed telescope used for *α* Aquilæ made the comparison by *l* (55) Pegasi. Now, the same maxima of parallax in declination of this star and of *α* Aquilæ occur within a few days of each other, so that it is completely the difference of parallax that is ascertained by comparing this star and *α* Aquilæ; and my results show, that in this part of the heavens we cannot conclude any thing, as to the absolute parallax of one star by its relative parallax to that of another."

3. Hence the field is still open for future observations by fixed telescopes, when stars affording the most favourable results are selected for a series of observations, which is the reason why we have explained the manner in which the telescopes may be advantageously fixed for the purpose. It may afford some assistance to the observer, who is disposed to employ a good refracting telescope in a fixed position, to point out some circumstances that may direct his choice of suitable stars, for rendering the difference between their heliocentric and geocentric places determinable by observations It has been generally asserted, that the diameter of the earth's annual orbit may be considered as a *point* if it could be seen from a star, and the question is, whether any sensible angle is there subtended or not? Monsieur Biot has instituted a computation from the known velocity of light, from which it appears, that, taking the greatest angle, subtended by the diameter of the earth's orbit, at 3″ centesimal, which is but a little more than 1″ sexagesimal, light will occupy upwards of three years in coming from a star to the earth. This result affords us some idea of the immense distance at which even the nearest star must be placed from us, and renders almost hopeless every attempt to compute it, by means of any angle that is measurable by our best instruments. The rules of trigonometry however will apply to data of the smallest dimensions, and thus give a stimulus to curiosity. If we suppose a star of the first magnitude placed in the pole of the ecliptic, the rays of light coming

from it to each point of the earth's orbit, as the year advances, would constitute a cone of rays having the diameter of the earth's path as a subtending line, that would appear, if seen at the star, of a constant magnitude, but place this star in some other situation, as it has reference to a line joining the sun and earth, and the said diameter would be seen more obliquely, at some parts of the year, than at others, and would consequently subtend a variable angle. As the longitude and latitude of a star have reference to the equinoctial point, and to the plane of the ecliptic, these ought to be both affected by the variations of the earth's distance from the star, but differently according to the obliquity of the direction in which it is seen If we put P for the pole of the ecliptic, the sun being in the centre of the sphere, Z for the point cut by a line passing through both the sun and earth, and call the star S; then PZ will be an arc of 90°, and if we call the longitude of the star l, the arc ZPS will be equal to the longitude of the earth seen from the sun, diminished by the longitude of the star, if we put $\ominus$ for the longitude of the earth, then we have

$$ZPS = \ominus - l,$$

and the parallax will take place in the great circle ZS passing through the earth and star. Let p denote the greatest parallax, or greatest difference between the heliocentric and geocentric places of the star, when $ZS = 90°$ and we obtain

$$\text{Parallax in } \textit{longitude} = p \cdot \frac{\sin(\ominus - l)}{\cos\lambda} \qquad (1)$$

$$\text{Ditto in } \textit{latitude} = -\, p \cdot \sin\lambda \quad \cos(\ominus - l) \qquad (2)$$

where λ denotes the latitude of the star.

The parallax in latitude can never exceed p; and is on that account less proper for finding p from observation, than the parallax in longitude. Now let the star's longitude be found when $\ominus - l = 90°$; then if we put l for the star's longitude seen from the sun, the apparent longitude (geocentric) will be

$$= l - p \cdot \sec\lambda$$

and at the expiration of six months $\ominus - l$ will become $= 270°$, and the apparent longitude now

$$= l + p \cdot \sec\lambda;$$

so that if we put d for the difference of the star's longitude, at these times, we shall get

$$d = 2\,p \cdot \sec\lambda$$

$$\therefore p = \tfrac{1}{2}\, d \cdot \cos\lambda.$$

These conditions will enable us to fix on two stars that will give on the same day, or nearly so, their greatest parallaxes with contrary signs, and enable the instrument to detect the sum of them, instead of their difference: and if the computed parallaxes, put into a tabular form, correspond with the observed parallaxes, it will be a presumption in favour of the success of the observer. The parallax in latitude will become a maximum when $\cos(\ominus - l) = \pm 90°$, or $= -p \cdot \sin\lambda$, and out of all the stars that one will have the greatest parallax in latitude, which has its latitude $= 90°$, where it becomes $= p$; but cannot exceed this limit. With respect to the parallax in longitude, this having cos λ for the denominator of its equation, may a little exceed p, and it increases in proportion as the parallax in latitude diminishes. Hence it appears that in selecting stars suitable for the purpose of being observed, it will be advantageous to fix on stars that are distant from the ecliptic, such as *α* Lyræ and Polaris, though it has been found that the latter gives no sensible parallax. Of course stars too near the horizon will always be avoided, as involving the difficulty attendant on low altitudes.

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§ LXXXVI. MICROMETER BY AMICI. [PLATE XXI.]

1. AT the time when we printed our descriptions of the various micrometers, we had not seen the double-image micrometer of the ingenious Amici of Modena, which we have mentioned at page 194, but the distinguished inventor has since been in England, and has supplied us with the identical instrument, which he had used in his observations of double stars. We have therefore now an opportunity of introducing an account of this micrometer. Figures 4 and 5 of Plate XXI. represent this appendage to a telescope; the former gives a view of its external appearance, facing the eye when applied to a telescope, and the latter shows a section of its parts, as they would be seen edgewise. The same letters of reference apply to the corresponding parts of both. In the lower figure *a* is a short piece of tube, having a female screw which receives an adapter, for applying the micrometer to any particular telescope, its upper end carries a large flanch to which the graduated circle *c* is screwed fast, and round the centre of this circle the plate *b* revolves which bears the cranked vernier *d*, lying nearly in contact with the circle, on the upper side of which a dove-tail appears, into which a reading lens is occasionally fitted by sliding. Upon this plate, *b*, is fixed, by four small pillars, a brass frame having for its bottom a plate *e*, two inches and seven-eighths long, and two inches and a quarter wide, the end *e* being the narrower: the covering plate, *ff*, of this frame is six inches and three-eighths long, and two inches and six-tenths wide at both ends, but a little wider in the middle, to receive the four screws entering the upper ends of the pillars, as seen in both figures: the milled heads, *g* and *h*, have each an arbor, passing down through the long plate, and resting on small cocks, *i*, with each a pinion formed at their lower ends, of which one only is seen near *i*, in fig. 5. Immediately under the long plate, or cover *f*, lies another narrow bar, *k k'*, indented at the edge, with which the contiguous pinion acts, this narrow plate is 5·4 inches long, and 1·1 wide, and is indented through 4·2 inches, leaving 0·6 at each end without teeth. The divided scale *l l'*, which lies above the long covering plate, and parallel to the subjacent indented bar, is made fast to the latter by two strong steel screws, that pass through a long perforation running parallel to the side of the long plate, and as the pinion *g* is turned, and moves the indented bar, the scale, thus connected with it, moves also, the parallel direction being preserved by the stems of the said screws, sliding without shake along the opening with parallel sides, made in the covering plate *ff'*, under the scale. Below each end of the indented bar, at *k* and *k'*, are attached small cocks, in which the ends of a long bar of glass *r r'* are fixed, with its sides cemented to the under face of the indented bar, so that when this bar and attached scale are moved by the pinion, the glass also moves with them. The scale is

seven inches long, but is divided only about 6·55 of that length, into one hundred and sixty divisions and subdivisions, numbered 0, 10, 20, &c. at the longer strokes. Another plate $m\,m'$, cut away except at the two ends, contains a pair of verniers, one or the other of which always reads with the divisions on the graduated scale, whatever may be its position, as it regards right and left. This vernier plate, $m\,m'$, is fixed in the same manner as the scale $l\,l'$, by two screws, passing through a second long parallel opening in the covering plate, to a second indented bar, in every way like the former bar $k\,k'$, and also carrying a second bar of glass cemented, and screwed to it underneath. A part of the second opening is seen at n, and at o a portion of the second indented bar, which is turned by a second pinion on the lower end of another arbor, carrying the milled head h. so that when the head h is turned, the verniers and attached subjacent bar of glass are moved by it, in a line parallel to the scale. The two bars of glass, which are each almost five inches long and 0·8 wide, are nicely ground to a straight edge, and placed with those edges in contact, as seen within the central circular hole p, made at the middle of the long cover. The extreme end of the second glass bar is visible at q, near the end of the second indented bar, holding one of its cocks visible in fig. 4, and also in fig. 5, where it appears as a continuation of the first glass bar $r\,r'$, though at the remote side of the centre These glass bars are about two tenths of an inch thick at one end, and something more at the other, the faces deviating a very little from parallelism, and as the ends are reversed they refract in opposite directions, and consequently produce two images of an object in all relative positions, except when their ends coincide, which is their zero position, where they give but one image. The head g will carry the scale over fifty one divisions, and the head h will move the vernier plate over forty-seven, so that the whole effective portion of the scale is measured by ninety-eight divisions and as the pinions will carry the scale and verniers in either direction, to the right or left, a measure may be taken to the whole extent of ninety-eight divisions, on either side of zero, which mode of application makes the index error vanish.

2. On applying this micrometer to an achromatic telescope of 67·5 inches focal length, (our No. 3.) we found that it affected the place of distinct vision only about the tenth of an inch, and as it elongated the focal distance, the curve of the glass bars must be concave. When this telescope was directed to a distant object, and a spider's-line micrometer applied, as its eye-piece, to a piece of tube that screws into the central hole of the covering plate, the greatest angle, that the double image micrometer, at its position ninety-eight on the scale, would measure, was found equal to 7·17 revolutions of the other micrometer's screw, the value of which angle with this telescope is 3′ 31″·37, (pages 103 and 104,) which, being divided by ninety-eight, gives 2″·157 for the value of a single division, or 0″·2157 for the value of unity on the vernier. The length of this eye tube is about four inches and a half, but it contains an inner tube holding the eye-piece in its cell, which draws out upwards of three inches, to increase the distance of the focal point from the pair of glass bars of the micrometer; and when this inner tube is drawn out just two inches, so as to make the whole distance, from the focus of the positive eye-piece to the nearest face of the glass bars, =7·85 inches, the greatest measure is increased to 4′ 43″·31, or 9·61 revolutions of the spider's-line micrometer, in which position the value of one division becomes $\frac{283''\cdot 31}{98} = 2''\cdot 891$, showing that the value of the

scale is different at each position of the inner sliding tube To ascertain practically the nature of this change in the values of the scale, the tube was put back one inch, just half the distance it was before drawn out to, and the greatest measure was then found 8·39 revolutions only, or 4′ 7″.34, giving the value of one division now only =2″.524, which is just a mean between the other two determinations; this proves that the differences are constant, and that therefore the graduations on the inner tube, if formed into a scale of positions, would be equal divisions. This elongation of the eye-tube gives a property to the micrometer that we have called polymetric, and which the inventor in conversation had not mentioned.

3 At first sight there appeared to be a great resemblance between this micrometer and Ramsden's dioptric micrometer, as made by Mr. G. Dollond (§ XXXI), but on closer inspection this resemblance disappeared. In the instrument before us the cone of rays is divided at the distance of nearly six inches before the focal image is formed, and the eye-piece is of the celestial kind, whereas in the older construction the separation takes place close to the third lens of a terrestrial eye-piece. This micrometer has a circle for measuring angles of position, which the other has not; and will measure angular distances equally well at both sides of zero, which the other has not the means of doing. In this the scale and vernier are moved separately by their own pinions, from the position in which the instrument is packed, in the other one pinion turns both racked bars in contrary directions, by giving motion to the second pinion through the intervention of a wheel, acting with both; and its vernier is fixed so as to have zero of the scale at the index, when the two slips of glass, forming a portion of a concave lens, are drawn out to their full extent, the measure being taken in their return to the situation for packing: which is just the reverse of Amici's glass bars. In the Italian instrument a little more than one-half the length of the scale is available at one measure, the verniers being at opposite ends; but in the English instrument the whole scale passes before the vernier, which has its station at the middle. In one the value of the scale is fixed, in the other it is variable. Hence, though there is a strong resemblance in principle, the constructions are dissimilar in various respects. They are, however, both liable to three inconveniences in use, first, their weight is too great for a telescope of ordinary dimensions, when mounted on the centre of gravity, secondly, they require the object to be seen near the centre of the field of view, to exhibit both images of the object, and with equal distinctness, and thirdly, the central portion of the large lens from which the bars are cut, are partially taken away in grinding them straight, thereby preventing exact superposition of the two images, which disjunction, in taking the distance of close stars, is objectionable, as something is left to estimation. But this indeed is the case, more or less, with all double-image micrometers depending on divided lenses, except perhaps the object-glass micrometer, in which sometimes an interposed slip of brass separates the semi-lenses, and supplies the place of the vitreous matter removed from the centre of the curve, by grinding the edges into a true shape. But the inconvenience peculiar to the micrometer we have here described is, that it cannot be applied to a common telescope till the total length of its tube has been shortened by five or six inches; yet when it has been so shortened, the spider's line micrometer may be substituted for an eye-piece, and then the measure of angular distance may be taken by either of the two micrometers, or indeed by both at the same time; and the parallactic line of the second micrometer will serve admirably to refer the angle of position to, when it is illuminated; for the two micrometers may be turned round together,

from the equatorial position, till the spider's line, commonly called the horizontal wire, just covers all the four images, which is easily effected, when the telescope has a parallactic motion, and the position in which they form a straight line is that in which the angle of position is obtained, on the limb of the divided circle, as heretofore explained (§ XLIV).

4. Mr. Herschel has explained in two very interesting communications to the Royal Society of London, "*On the Parallax of the Fixed Stars*" (Part III. 1826, and Part I. 1827), how the difference of the parallaxes of two apparently contiguous stars, commonly called a double star, may be rendered sensible, by measuring the changes of angular position of any pair of such stars, and it appears probable, that if parallax really exists in an appreciable degree, the method proposed is very likely to detect it; if, for instance, an annual variation of 30′ be discovered in the angle of position, by a good position micrometer, where the measured distance of the two stars is only 3″, the difference of their parallaxes, resulting from such change, will be about one-fortieth part of a second, and where the distance is 15″, the same change in the angle will show one eighth of a second of relative parallax. The most favourable position for observations of this kind is, when the line joining the two stars points as nearly as may be to the pole of the ecliptic, "but," says the learned author, "ten, twenty, or even thirty degrees of deviation, either way from this direction, will not materially vitiate the application of this method to stars near the ecliptic, while, for such as have considerable latitudes, proportionally greater deviations may be allowed, and within thirty degrees of the pole of the ecliptic this element is of comparatively small moment." As the *angle of position* of a line passing through two close stars, may be confounded with the angle formed at a star by the intersection of two great circles passing through a star from the poles of the equator and of the ecliptic, which in astronomical language has also been called *the angle of position*, in which sense it is used in our first volume; Mr. Herschel proposes to call this latter the *angle of situation* by way of distinction, which he considers also more appropriate, and which therefore we will adopt in this section. Let us put

$\odot$ = the longitude of the sun on a given day;
l = the longitude of the double star;
λ = the latitude of the same,
a = the maximum semi-annual parallax;
P = the total effect of parallax on the angle of position,
π = the measured angle of position,
σ = the computed angle of situation,
D = the measured angular distance,
$\cos M = \cos\lambda \,.\, \cos(\pi - \sigma)$ by substitution.

Then the sun's longitude on the days most favourable for observing the angles of position, on which the greatest difference of parallax will depend, may be obtained from the following theorem:

$$\sin\lambda \,.\, \tan(\odot - l) = \tan(\pi - \sigma) \qquad (a)$$

and if we make $P = \frac{2a}{D} \,.\, \sin M$, agreeably to the author's investigation, we may deduce the value of $2a$, the maximum of relative parallax, when the total effect (P) on the angle of position is known, by the subjoined theorem,

$$2a = D\frac{\sin P}{\sin M}, \qquad (b)$$

so that if we take $P = 30'$, we have

$$2a = D\cdot\frac{\sin 30'}{\sin M},$$

and in like manner for any other value of P, that may be determined by actual measurement by a position-micrometer.

5. *Example* 1. Let it be required to determine the two days in the year when the angle of position of 35 Piscium may be observed with most advantage; and also to compute the maximum of relative parallax derivable from such angle, agreeably to the measures of distance and position taken by Messieurs Herschel and South on Nov. 27, 1821, on a supposition that 30′ is the total effect of parallax on the angle of position?

This star is No. 5 in our catalogue of 520 Zodiacal Stars for 1820 (Appendix to Vol. I.), from which we have the longitude, latitude, and angle of situation, thus,

	s	°	′	″		°	′	″		°	′	″
Long. 1820 . . . . =	0	4	26	8.2,	lat. =	6	36	14.6 N.,	N.E ∠ =	66	22	43.1
Add one year . . .				50 1	—			0.1	—			0.9
Nov. 27 (Tab. XV.)				45 3	—			0.1	—			0.8
	0	4	27	43.6		6	36	14.8		66	22	44.8
Angle of situation, or comp. of N.E. ∠ = σ										23	37	15.2

This angle may also be obtained by any of the three methods explained in pages 378 and 379 of Volume I.

Then, to find the days proper for observation, we have the measured angle of position, from Messieurs Herschel's and South's catalogue.

$$\left.\begin{array}{l}\pi = 60^\circ\ 46'\ sf\\ \sigma = 23\ \ 37\ \ 15\end{array}\right\}\text{and } (\pi-\sigma) = 37^\circ\ 8'\ 45'' \ldots \log\text{ tang} \ldots 9\ 8794126$$

(a) $\lambda = 6\ \ 36\ \ 15 \ldots \log\sin\text{ sub.} \ldots 9.0607333$

Tang. $(\odot - l) = 81\ \ 22\ \ 2 \ldots\ldots\ldots\ldots\ 0.8186793$

Add l . . . . . . $4\ \ 27\ \ 44$

$$\left.\begin{array}{l}\odot \ldots\ldots = 85\ \ 49\ \ 46 \text{ or } \overset{s}{2}\ \ 25^\circ\ 49'\ 46''\\ \text{June 17, 1821, } \odot \text{ by Naut. Alm. } 2\ \ 25\ \ 52\ \ 49\end{array}\right\}$$

$$\left.\begin{array}{l}\text{Add six signs} \ldots\ldots\ldots\ 8\ \ 25\ \ 49\ \ 46\\ \text{Dec. 17, 1821, } \odot \text{ by Naut. Alm. } 8\ \ 25\ \ 18\ \ \ 4\end{array}\right\}$$

In the next place to find *M*, we have

Cos λ . . .	6°	36′	15″	log	9 9971086
Cos $(\pi-\sigma)$	37	8	45	log	9 9015134
Cos *M* . .	(37	38	41)		9 .8986220

(*b*) And lastly, we obtain the log of 2 *a* thus

Log *D* = 11″ 186 . . . log	1 .0486748
Sin 30′ . . . . . . log	7 .9408419
	8 .9895167
Sin *M* (37 38 41) sub	9 .7858730
2 *a* = 0″.1598	9 .2036437

Hence 0″.16 is the computed annual parallax corresponding to a periodical variation of 30′ in the angle of position, which quantity is supposed to be within the reach of our position micrometers; and if so, this method of deducing the parallax of a star from the observed periodical change in the angle of position promises to detect a smaller quantity, than can be obtained by direct measurement of the parallax itself. Mr. Herschel has taken the trouble of computing, from approximate data, the annual parallaxes of sixty-nine double stars, together with the days on which they may be most favourably observed, a list of which he gives as a specimen, that may be extended by computation from the theorems (*a*) and (*b*), when the longitudes and latitudes are previously known, which, as arguments only, he observes, may be taken roughly from a globe or map, to avoid the trouble of computation. This mode of taking the data will account for a slight discrepancy in our respective results, in the example we have worked at full length. As the author had occasion to give some corrections of his first specimen, we give it in a following page in its corrected state, and the observer, who devotes himself to such observations, will not fail to extend the list for his own use, which he may do by computing the longitude and latitude of the star by the following formulæ, viz.

$$\tan l = \frac{\tan Æ . \sin(\phi - \omega)}{\sin \phi} . \qquad (c)$$

$$\sin \lambda = \frac{\sin \delta . \cos(\phi + \omega)}{\cos \phi} . \qquad (d)$$

where $\tan \phi = \frac{\sin Æ}{\tan \delta}$, and ω = the obliquity of the ecliptic.

6. *Example* 2. As Mr. Herschel has omitted to give the parallax (2 *a*) of μ Draconis in his list, though he has given the days most proper for observation, let it be required to supply this omission?

In the catalogue of double stars the Æ is given $17^h\ 3^m$ or 255° 45′, the declination 54° 43′

N, the angle of position (π) 61° 39′, and the measured distance (d) 3″.907; let us first determine the longitude and latitude from the formulæ (c) and (d), thus, with five logarithmic figures,

(c)		(d)	
tan Æ 255° 45′	−10 .59522	sin δ 54° 43′ . . .	9 .91185
sin ($\varphi-\omega$) 57° 55′	9 .92802	cos ($\varphi+\omega$) 10° 59′ . . .	9 99197
	− 0 .52324		9 .90382
sin φ (− 34° 27′) sub . . .	− 9 75257	cos φ sub	9 .91625
tan l 80° 23′, or 260° 23′ . .	0 .77067	sin λ = 76° 21′	9 .98757

By the assistance of l and λ we can now determine the proper days, and 2 a, for a variation of 30′ in the angle of position, as before, in the following manner,

		(a)		(b)	
Sin ω . .	9 .60012	Tan ($\pi-\sigma$) . .	11 .17770	Cos λ	9 .37289
Cos Æ	− 9 .39120	Sin λ	9 .98756	Cos ($\pi-\sigma$)	8 .82134
	−18 .99132	Tan ($\odot-l$)	11 .16526	Cos M (89° 6′)	8 .19423
Cos λ sub	9 .37289	($\odot-l$) . . .	86° 5′		
		l.	260 23	Log D 3″ 907	0 .59184
Sin σ . .	− 9 .61843			Sin 30′	7 .94084
σ . . .	− 24° 33′	$\odot$	=346 28		
π . . .	61 39		s =11° 16′ 28		8 .53268
				Sub sin M	9 .99995
($\pi-\sigma$) =	86 12	March 7, 1821	11 16 38		
		Add six signs	= 5 16 28	2 a=0″034	8 .53273
		September 9	5 16 26		

Mr. Herschel observes that the angle of situation, σ, is taken positively, as in our first example, when it falls in the hemisphere that has ♈ for its pole; but that when it falls in the hemisphere that has ♎ for its pole, it will be negative, as in our second example.

7. SPECIMEN OF A LIST OF DOUBLE STARS

FAVOURABLY SITUATED FOR THE INVESTIGATION OF PARALLAX

Numbers in Messrs. Herschel's and South's observations	Stars' Names	AR for 1820	Declination for 1820	Times of the year most proper for observation	Amount of annual parallax indicated by a periodic variation of 30″ in the angle of position	Numbers in Messrs. Herschel's and South's observations.	Stars Names	AR for 1820	Declination for 1820	Times of the year most proper for observation	Amount of annual parallax indicated by a periodic variation of 30″ in the angle of position
		h m	° ′					h m	° ′		
1	35 Piscium	0 0	7 49 N	June 27, Dec 27	0′ 098	188	39 Bootis ..	14 44	19 27 N	Mar 22, Sep 24	0′ 040
20	γ Arietis . . .	1 44	18 25 N	Jan 18, July 21	0 085	193	44	14 58	48 21 N	June 25, Dec 25	0 020
25	α Piscium .	1 53	1 53 N	Jan 16, July 19	0 047	194	Struve 474 . .	14 59	9 55 N	Jan 24, July 27	0 042
38	32 Eridani	3 45	3 30 S	Feb 13, Aug 17	0 071	201	η Cor Bor	15 16	30 57 N	Jan 22, July 25	0 014
39	ε Persei . .	3 46	30 20 N	Feb 13, Aug 18	0 081	205	δ Serpentis	15 20	11 9 N	Feb 3, Aug 7	0 027
46	55 Eridani	4 35	9 9 S	Feb 6, Aug 10	0 103	206	Libræ 178 .	15 30	8 11 S	Feb 12, Aug 16	0 104
47	ω Aurigæ . .	4 47	37 36 N	Mar 6, Sep 8	0 069	211	II 85	15 47	1 39 S	Mar 4, Sep 6	0 085
53	Rigel . .	5 6	8 26 S	Mar 20, Sep 22	0 084	212	III 103 .	15 48	3 50 N	Mar 10, Sep 12	0 132
55	118 Tauri . . .	5 18	25 0 N	Mar 10, Sep 13	0 052	228	ρ 5 Ophiuchi	16 15	23 1 S	Feb 25, Aug 30	0 036
56	32 Orionis	5 21	5 48 N	Mar. 19, Sep 22	0 013	240	.. ..	16 40	19 15 S	Mar 1, - Sep. 2	0 064
59	33 Orionis .	5 22	3 9 N	Mar 22, Sep 24	0 020	242	μ Draconis	17 3	54 43 N	Mar 7, Sep 9	0 034
67	ζ Orionis . ..	5 32	2 3 S	Feb 28, Sep 2	0 025	245	39 Ophiuchi	17 7	24 5 S	Mar 8, Sep 11	0 111
360	41 Aurigæ	5 58	48 44 N	Mar 23, Sep 26	0 077	262	100 Herculis	18 1	26 5 N	Mar 19, Sep 21	0 125
69	8 Monocer. .	6 14	4 41 N	Apr 2, Oct 5	0 136	265	I 86	18 12	25 28 N	Mar 29, Oct 2	0 040
76	38 Geminorum	6 14	13 21 N	Mar 30, Oct 3	0 010	269	39 Draconis	18 21	58 42 N	Mar 22, Sep 24	0 031
80	δ	7 9	22 18 N	Apr 6, Oct. 9	0 064	271	.. . .	18 30	41 7 N	Apr 15, Oct 18	0 053
88	11 Cancri	7 58	28 0 N	Apr 18, Oct 21	0 011	274	.. . . .	18 30	10 39 S	Apr 3, Oct 7	0 040
93	ϕ^2	8 16	27 31 N	Apr 18, Oct. 21	0 051	280	. . . . . .	18 42	10 47 N	Mar 28, Oct 1	0 .042
94	18 Hydræ, Bode	8 26	7 15 N	Apr 29, Nov 1	0 096	287	. ... .	18 58	6 53 N	Apr 14, Oct 18	0 .077
96	141 of 145 .	8 39	71 27 N	Apr 21, Oct 24	0 076	295	III 57.	19 19	20 46 N	Apr 27, Oct 30	0 002
98	57 Cancri ..	8 43	31 16 N	May 4, Nov 6	0 020	306	π Aquilæ	19 41	11 22 N	May 10, Nov 12	0 021
99	17 Hydræ	8 47	7 17 S	Apr 28, Oct 31	0 053	311	ε Draconis ..	19 49	69 48 N	Apr 20, Oct 23	0 028
102	Cancri 194 .	8 57	23 42 N	Apr 30, Nov 2	0 067	312	ψ Cygni	19 51	51 38 N	Apr 18, Oct 21	0 038
114	Leonis 145	10 11	7 22 N	May 21, Nov 25	0 060	313	I 96 ..	19 56	35 32 N	Apr 20, Oct. 23	0 022
128	90 Leonis .	11 25	17 48 N	June 5, Dec 6	0 039	317	II. 96 . . .	20 3	0 19 N	Apr 6, Oct 10	0 045
133	65 Ursæ .	11 40	47 29 N	May 23, Nov 24	0 035	320	I. 95 .. .	20 14	54 18 N	May 12, Nov. 14	0 035
134	2 Comæ Ber	11 55	22 28 N	May 28, Nov 29	0 038	323	ρ Capricorni .	20 20	18 24 S	Apr 23, Oct 26	0 036
147	118 of 145	12 25	75 46 N	June 3, Dec 4	0 053	326	.. . .	20 32	38 5 N	Apr 27, Oct 30	0 085
152	Struve 422	12 40	4 48 N	June 27, Dec 27	0 080	343	Struve 751 . ..	22 16	65 50 N	Feb 14, Aug 18	0 033
155	—— 424 .	12 44	16 0 N	June 30, Dec 30	0 072	349	Aquarii 213. ...	22 31	0 11 S	May 28, Nov 29	0 031
161	54 Virginis	13 4	17 51 S	Jan 11, July 16	0 060	352	.. ..	22 59	31 51 N	June 24, Dec 24	0 076
167	81 . ..	13 28	6 57 S	Jan 12, July 15	0 038	354	94 Aquarii . ..	23 10	14 26 S	May 21, Nov 22	0 133
173	98 of 145 .	14 5	6 11 N	Jan. 20, July 23	0 054	356	107 .	23 37	19 41 N	June 4, Dec 6	0 045
176	Struve 456	14 13	6 56 S	Jan 27, July 30	0 061	359	σ Capricorni. .	23 50	54 45 N	July 16, Jan 14	0 026
177	—— 457 .	14 14	9 16 N	Jan 23, July 26	0 061						

8. In speaking of the position micrometer by Nairne, as applied in the first instance to Sir William Herschel's large telescopes, (§ XLIV. 2.) we omitted to notice, that the plan and section of that appendage are explained by figures 2 and 3 in Plate VIII, containing the forty-feet reflector, we shall therefore avail ourselves of the opportunity, that now offers, of supplying the inadvertent omission. The ring *a b* in fig 2, is an end-section of the wooden box, that holds the brass-work, the interior circle of which includes a brass circular plate, on the plane of which the indented ring, or annular wheel, lies flat, *c c′* is the interior portion of this circular plate, made fast to the upper end of a short tube *d*, that constitutes the interior face of the eye-tube the circular portion *c c′* of the plate is thicker than the rest, that is covered by the annular wheel, and keeps the wheel in its place, while the two bars *e* and *e′* screwed fast to the portion *c c′*, confine it to its bed, but in such a way as not to prevent its being turned round the central part, when acted upon by the pinion *f*, carried by the arbor of the small index plate *g*, partly concealed by the annular wheel. In fig. 3 this wheel is denoted by the parts *h* and *h′* near the two screws, which appear separated, by being shown in section. The other letters denote the same parts as in figure 2, and *i* shows the cock, on which the pivot of the pinion's arbor rests, as made fast to the fixed tube *d*. The short line crossing the centre is a piece of fine wire attached to the face of the plate, between *c* and *c′*, and the long one, crossing it in an acute angle, is carried by the annular wheel, and moves with it, whenever motion is communicated by the pinion. This is done by a single finger laid against the circumference of the index plate *g*, which for this purpose is milled at the edge, to admit of being moved in either direction by the least pressure. The dial, or index plate, was divided into sixty parts, and each of these again subdivided into two; then as the pinion had six leaves and the wheel ninety teeth, the dial must have revolved fifteen times for one revolution of the wheel, i. e. in 360°, and $120 \times 15 = 1800$ were the number of subdivisions measuring the circle, so that $\frac{1800}{360^\circ} = 5$ made the number of divided spaces in the degree, each of which therefore was 12′, the smallest arc that this micrometer would measure without estimation of a fractional part but the play of the pinion in action with the wheel was the greatest objection to this construction, which even the excellence of workmanship cannot remedy. A part of the dial protruded from the side of the wooden box, and a straight line at the side of the opening was in place of an index. It was hardly to be expected, that Sir William would remain long satisfied with the indications of such a coarse contrivance, after a better micrometer was within his reach.

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§ LXXXVII THE SEXTANT [PLATE XXIX.]

1. AFTER having described those astronomical instruments that measure celestial arcs by direct vision, we proceed lastly, to give some account of those instruments that measure by reflection; and that are chiefly used in nautical astronomy. It has been affirmed, that a manuscript, by Sir Isaac Newton's own hand, was found among Dr. Halley's papers after his death, containing an account of a quadrant, or octant, that would measure altitudes from an horizon seen by single reflection, and which, according to Stone, was actually constructed in the year 1672, when

Dr. Halley was preparing to go to the South Seas; but this manuscript was not referred to when Hadley first exhibited his instrument to the Royal Society of London, of which he was Vice President, nor was it made public till the year 1742. Hence some doubt has been occasioned, whether or not both these distinguished men were equally entitled to the honour of this useful invention *. The use of the plumb line and of the level was impracticable on board a ship, as well as the application of mercury or other fluid, as a reflecting surface, so that the altitude of a heavenly body could be referred only to the sensible horizon, or line bounding the sea and sky. Various contrivances had been tried, and successively adopted for giving the measure of an arc, at sea, contained between a celestial body and the horizon, in the form of a cross-staff, quadrant, &c. but none of them with sufficient accuracy. The introduction of a small mirror, as an appendage to the quadrant, was a happy invention that rendered an observation at the same time both easy of attainment and accurate, and to this contrivance, in conjunction with the services of the dividing engine, nautical astronomy is more indebted, than to any subsequent invention.

2. If an incident ray of light be reflected from a perfect plane, revolving round an axis perpendicular to the said plane, the angular velocity of the reflected ray will be *double* the angular velocity of the reflector itself, which catoptric property reduced the quadrant to an octant, or sector equal to one-eighth part of a circle; for when the index is moved over one half only of a degree, the mirror, standing at right angles to its plane, and moving with it round the common centre of motion, occasions a reflected body to move apparently through an arc of an entire degree, and in like proportion for a smaller or larger quantity. Hence an octant is graduated into 90° and its parts, and is therefore usually called a quadrant; and requires a radius of double the length of an astronomical quadrant, that the degree may be of equal extent. If altitudes only had been required in navigation, the quadrant or octant would have been competent to such purpose; but after the practice of measuring the moon's distance from a star became serviceable in determining the longitude of a ship, it was necessary to extend the arc of the instrument's limb to upwards of 60°, measuring 120° by reflection. The name of sextant was then substituted for octant; but as the construction of both instruments is the same, we will describe only the sextant, as comprehending the other within it, and performing all its operations.

3. The divisions of the octant's limb were originally read by means of lines forming a diagonal scale, but the vernier, since substituted, answers the purpose much better, and will subdivide the minute into portions of 10′ each, or even less with a large radius. The sextant which we propose to describe is represented by fig. 8 of Plate XXIX, where the perspective view shows the principal parts, as constructed by Ramsden. The flat bars of brass, forming the sectoral frame of this sextant, are cast in one piece, except the limb, which is screwed upon the ends of the radial bars, they are all strengthened by as many edge bars, screwed

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* In an old instrument which we have, supposed to be similar to Dr. Halley's, the horizon glass is all silvered, and the index glass without silvering, or dark glasses. In this construction the horizon is seen by single reflection, but the sun or star by double reflection, through a long telescope, lying in a radial direction from zero on the limb towards the index glass. The images of the two objects are both seen through the index glass, on the face of the horizon glass, when the telescope is directed to a point between the objects; whereas in Hadley's instrument the telescope lies across the radial bars, and views the horizon, or second object, by direct vision, in a line of sight at right angles to that of Halley's octant.

close to the lower faces respectively by screws from above, of which the heads appear in the drawing. The limb has a radius of nine inches and a half, and has two sets of divisions, of which the outer one only is engraved, this scale, as divided by Ramsden, has the degree divided into only three parts, where a vernier of forty divisions is co-extensive with thirty-nine on the limb, and reads therefore to 30″, but the inner scale, subsequently divided by Troughton, has the degree subdivided into ten parts, and each of the six minutes on the vernier into twelve subdivisions, so that one twelfth of a minute, or 5″, are indicated by this scale, and rendered perfectly legible by the pair of lenses forming a simple microscope, similar to a positive eye-piece of a telescope; it is carried by the short arm and reflector, supported by, and turning upon the long arm of the index, commonly called the vernier bar. This bar is also strengthened by an edge-bar seen above it in the figure, and carries the clamp and tangent-screw of slow motion, which have been already described [§ XLVI 3.] as well as the verniers, which have also been explained [§ XLVII. 2 and 4] its upper end terminates with a circle of two inches and a half in diameter, carrying a perpendicular axis of motion, two inches and a half long, within a conical socket, screwed fast to the back surface, and forming one of three short legs, on which the instrument rests when out of hand. This axis gives a long bearing to the central part of the instrument, over which the frame of the silvered index-glass is fixed by screws, in a direction truly perpendicular to the plane of the graduated limb, and forming by its edge a diameter to the circular portion of the vernier-bar, with which it moves round the long axis as a common centre. The limb contains 139°, exclusively of the arc of excess behind the zero point, which is a greater angle than is computed in the Nautical Almanac for the lunar distance of any star The radial bar of the frame, that is screwed to the extreme end of the limb near 130°, is left broad about the middle, to receive a pair of cocks holding a second glass, called the horizon glass; one half of which only is silvered. This glass stands perpendicularly, with its lower end resting on the face of the broad part of the radial bar, and exactly parallel to the index-glass when the vernier stands at zero on the limb. A small telescope about five inches and a half long, and magnifying four times, screws into a ring of brass attached to an adjustable triangular stem, that will rise and fall by means of a screw, called the up and down piece, the use of which is to hold the line of collimation of the telescope parallel to the plane of the limb, and to alter its distance from the frame till the centre of the object-glass is so divided by the line in the horizon glass, that separates the silvered from the unsilvered part, that the image of any body, seen by reflection from the silvered portion, may appear as bright as the object itself, seen through the unsilvered portion by direct vision; and when this is not the case, the milled head of the screw for this adjustment will move the up and down piece, carrying the telescope, till the object and its image are equally bright; which is an essential adjustment, particularly in taking lunar distances. Besides this telescope there is a shorter one with a concave lens for an eye-piece, magnifying five times, which does not invert the object, and which therefore may be used more familiarly at first, to prevent mistaking one edge of the sun or moon for the other. There is also a tube without glasses to confine the line of sight, when the contact is made without a magnifying power. These three tubes have similar screws, and will any of them fit the female screw of the up and down piece. Mr Troughton supplied a new index glass and telescope of greater magnifying power, when he added the fine divisions When the vernier of the index bar stands at zero, and the two silvered glasses are properly adjusted to parallelism, the rays coming from a star, or other object, and falling on the face of the index glass, are first reflected upon the face of the horizon glass, and a second time from the silvered part of it towards the object-glass of the telescope, and the image seen through the telescope, after this double reflection, coincides with the same star, or object seen also through the telescope by direct vision, where the two will appear as one, provided there be no deviation from parallelism in the relative positions of the two glasses; but if one is seen above the other, when the vernier indicates 0′ 0″, the quantity constitutes an error called the index error, which must always be applied to the measure of an arc with its proper sign, as a constant correction. If the object and its image should not be perfectly coincident in a lateral direction, when the index error has been adjusted, this shows that the glasses are not both perpendicular to the face of the limb, which they ought to be the index glass may first be examined by bringing the vernier to indicate about 45°, more or less, and by looking obliquely into this mirror, so as to view the sharp edge of the limb, by direct vision to the right hand, and by reflection to the left; for if the limb continue in a straight line, at the junction of the reflected portion with the portion seen directly, the mirror is perpendicular, but not otherwise This position however is usually put right by the maker, and is not liable to alter with good usage. The horizon glass has always an adjusting screw below the frame, which will bring the object and its image into a state of lateral coincidence, when properly used. But the less the original adjustments are meddled with the better: these delicate parts of the instrument should never be deranged for the purpose of gratifying curiosity. The handle of this sextant is formed into a crank, the two brass ends of which are made fast to the edge bars, that cross the back part of the frame, where there is room for the hand, and a hole in the middle of the handle allows of its being fixed to a stand, in any vertical or oblique direction.

4. On the external edge of the broad radial bar, between the two glasses, a cock is screwed fast, having four concentric joints carrying each a square brass frame, containing as many circular discs of dark glass of various shades of colour, which will turn forwards, separately or jointly, and intercept the reflected image of the sun coming from the index glass, thereby modifying the quantity of light that the eye can bear, but at other times they will fall back and allow the reflected image to pass without interruption. In the figure, one of these is turned up, and three lie back. Another set of three coloured discs of glass, in round frames, are sustained in like manner by a cock having three concentric joints behind the horizon glass, any one or more of which may be turned up to intercept the direct rays of the sun. Of these one also is seen turned up. Some one of the former set of glasses is used when the sun's altitude is taken, to darken his descending image as the vernier bar advances on the limb, and the horizon is seen through the unsilvered part of the second glass, when this image is brought down into contact with it, at the line of junction separating the silvered from the unsilvered part. When the horizon is not used, a dark glass of each set may be used together, as in the case of measuring the sun's diameter at each side of zero, to determine the index error; or when a double altitude is taken by reflection from mercury, oil, or water, in all which cases the eye will require protection from both the sun and its luminous image, when brought into a state of contact. There is also a set of small coloured glass discs screwing over the eye-piece of the telescope, any one of which alone will secure the eye from both the sun and his reflected image.

In all terrestrial and lunar observations of a star the dark glasses are not wanted, except in a clear sky when the moon is very high, and gives more light than the eye can bear; which, Mr. Fallows informs us, has been the case sometimes at the Cape of Good Hope. It is of the utmost importance that the reflecting glasses should have their faces ground and polished perfectly parallel, to avoid the effects of refraction, and as there are two surfaces that reflect, one a primary and the other a faint or secondary image of a luminous object, the observer must be careful not to mistake one for the other, particularly when the dark glasses are used. Indeed to avoid this effect the dark glasses are usually fixed in an inclined position, as they regard their respective mirrors The dark glasses should also have parallel faces, as well as be placed in parallel positions their correctness in this respect may generally be known by taking a measure of the sun's diameter, with one pair of glasses of corresponding colour; and then, without altering the measure, by trying if it will remain the same when any other pair is used, or when the faces are reversed, as they are sometimes made to do; and also when the single disc is used at the eye-piece of the telescope For want of such examination the best instrument may give a false solar altitude. When the reflecting mirrors have not parallel faces and perfect planes, the error occasioned cannot be remedied without new glasses, which the skilful optician must supply we say skilful, because the French have been in the habit of sending to England for those glasses, as being more perfect than they can make them. If a piece of glass of unequal thickness, of the shape of a thin wedge, be placed on the middle of an object-glass of a good telescope, the object viewed will be displaced by the refraction towards the thicker end; and on reversing the ends the displacement will be in a contrary direction, but a piece of homogeneous glass with parallel faces will not alter the place of the object, when placed before and in contact with an object glass of an adjusted telescope. Hence it will not be difficult to prove whether the index glass, or horizon glass, before it is silvered, be suitable for its purpose: a very small deviation from true parallelism will produce an augmented error, where both reflection and refraction are concerned in creating it.

5. In the elder quadrants, but never we believe in sextants, a second horizon glass was applied for the purpose of gaining a large angle of measurement, as proposed by Hadley himself, that the image of the object observed might be carried to the opposite point of the horizon, when clouds rendered the front view impracticable, it will be sufficient for our purpose to state, that this effect was produced by fixing the second horizon glass behind the other, with its plane exactly at right angles to the plane of the first; and then, when the supplement of any angle of altitude was required to be measured, the observer turned his back towards the object, and brought its image to the opposite point in the horizon. When the angle always exceeded 90°, a little deviation in the adjustments of the glasses, or a small defect in their construction, occasioned a considerable error, and notwithstanding all attempts of scientific men to remedy the evil, and to render the method sufficiently intelligible and convenient in practice, we hear nothing now said about its utility, and therefore may safely suffer it to fall into oblivion.

6. The sextants made by Mr. Troughton have double frames, united face to face by pillars, and are of smaller radius than the one we have above described; but as they unite strength with lightness, they have been in great request in the naval service The advantages that the sextant has peculiarly over former instruments, are, that it requires no steady foundation to stand on, but may be used in the hand while a ship is moving or even tossing about; and that

the measure may be taken in any direction, vertical, horizontal, or oblique, with almost equal ease, which properties render it a most convenient instrument both by sea and land. When on shore, any of the artificial horizons, numbered 2, 3, 4, and 5 in Plate XXIX, being used, a double altitude may be taken with as much ease, as by the natural horizon, and with more correctness, since only half the error of contact will be charged on the observation, while the total arc passed over is doubled. The tangent screw is particularly useful in this instrument, and, what is very convenient, the measure on the arc may be read at any subsequent period, so long as the screw is not altered To succeed in taking an observation, the observer must accustom himself to hold the plane of the instrument parallel to the line that joins the two objects including the arc to be measured, and must also be able to give a slow and steady circular motion of his body to the right or left, considering the axis of the telescope as the centre of motion, that the image of the observed body may just touch the horizon, as it passes in an arc of a circle, when an altitude is taken, or that the limb of one object may just slide by the image of the other in a state of contact. The method of using an artificial horizon will be explained in Section LXXXIX.

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§ LXXXVIII. REFLECTING AND REPEATING CIRCLES [Plate XXVIII]

1 *Mayer's*.—We have already said that Professor Mayer, of Gottingen, was the first person who endeavoured to compensate for the imperfect state of dividing an instrument, by repeating the measure of an arc through a large portion of 360°; and also that Borda was the first astronomer who constructed a repeating circle But the principle of reflection was not at first united with the repeating principle Mayer has described an instrument, in his "*Tabulæ Motuum Solis et Lunæ*, Londini, 1770," in which these two properties are united, and Bird constructed one for the use of Admiral Campbell, of which we have had temporary possession. It was the opinion of the inventor, that a circle could not be divided by the beam-compass to read to a degree of accuracy nearer than 3′, the dividing engine not being then in existence, and he therefore fixed on sixteen inches as the diameter of his circle, to allow of divisions sufficiently large; but Bird exceeded this dimension by nearly an inch and a half, exclusively of the superfluous portion of the limb; and the consequence was, that the bulky instrument required a staff to rest on, and a ball and socket to give its plane the proper direction for measuring. The circle had two radial bars, one carrying the index glass, silvered like that of an octant, and the other the telescope and the horizon glass, partly silvered and partly unsilvered, not far distant behind the index glass, by this adaptation a single arc, requiring two operations, was repeatedly measured round the circle, and the whole amount read off at the last. This circle was divided into 360 parts, and indicated minutes in the usual way by the verniers, of which each bar had one; and as each degree was the measure of two degrees, the amount, as read, was doubled after the observation was finished. The telescope, which magnified four times, was kept parallel to the plane of the bar that carried it, by a pair of jointed pieces that altered the distance between it and the bar, after the manner of a parallel ruler, and it was thus adjusted for good vision of both the object and its image, by being opposed to both portions of the horizon glass the dark glasses were used when required, and the six radii of the

circle were braced by edge bars, which added greatly to the weight. The distance of the two verniers from each other in the instrument we examined is 73° 38′ = 147° 16′ in effect, which is the greatest single angle they will include; and when the index bar stands at zero, and the horizon bar at that place, the two glasses are parallel. Placing the two glasses parallel, that is, where the object and its image coincide, is the first operation; and bringing forwards the index and its glass, to make the contact of the image of the brighter body with the fainter body itself, as seen by direct vision, forms the second operation, for obtaining the measure of a single arc; and when these two operations of moving the index bar and horizon bar alternately, have been repeated round the circle, the last reading, divided by the number of contacts, and doubled for the proper value of the graduations, for the resulting quantity, gave the arc to be measured, either as an altitude or oblique distance, much more correctly than could be obtained from one pair of operations, supposing the circle to be imperfectly divided. But Admiral Campbell found the instrument unmanageable at sea, as a repeating instrument; and on finding that the artist had done justice to the divisions (for there was not an error of more than 1′ in any of the contiguous divisions), he selected that portion of the limb that was most free from errors of division, and used it simply as a sextant, supporting the heavy instrument by a belt; and this mode of using a large divided arc with success introduced the subsequent construction of the sextant, that has been in use ever since, where angles exceeding a quadrant have been required to be measured. We have not encumbered our plate with an engraving of Mayer's circle, now no longer in use; but have deemed it right to give this sketch of its construction and mode of measuring, to explain more fully how Borda's circle, its successor, differs from it in these respects; for this is the instrument which has been generally adopted on the continent, under one modification or another, as the competitor of our naval instruments.

2 *Borda's.*—On contemplating Mayer's construction Borda had observed that much time was lost by having two operations to effect each single measure; and that, when the arc was small, requiring many such pairs of operations to go round the circle, which would otherwise be incomplete, the moving object might require a correction for change of place during the total interval of time, necessary for completing the operations; he therefore conceived the ingenious plan of making *two operations* measure a *double arc*, when the circle was divided into 720 divisions and their subdivisions, to represent so many degrees and parts. The celebrated author published his pamphlet entitled "*Description et Usage du Cercle de Reflexion*," at Paris, in the year 1787, after he had been employed twelve years in perfecting his construction of the repeating circle, and in preparing his useful tables of reduction to the meridian, and other minor tables of correction. He had further remarked that the adjustment for parallelism of the two glasses in one of the operations with Mayer's circle was effected by viewing the horizon at sea, which was frequently productive of error on account of the difficulty of bringing the direct and reflected horizontal lines to be an exact continuation of each other; and also that the perpendicularity of the central mirror, as it regarded the plane of the circle, was disturbed by deviations occasioned at certain parts of the graduated limb; and for these reasons he contrived to dispense with the adjustment for parallelism altogether, except in the beginning of the operations at the zero position.

3 A perspective view of Borda's reflecting circle, as made by Lenoir, is contained in fig.

1 of Plate XXVIII, which will enable us to explain its construction. The graduated limb of the circle is smaller than Mayer's, having a radius of only 5.3 English inches, in the instrument we examined, which therefore is portable by a small handle behind the centre. The index bar *a* terminates with a vernier, that has twenty divisions coextensive with nineteen on the limb, which limb appears to have been divided by an engine, or other mechanical mode of transferring the divisions from a larger circle: it has also the usual clamp and tangent screw of slow motion, and carries, on the rounded central part, the silvered glass parallel to its length, standing at right angles to the plane of the six-armed circle. The horizon bar *b* crosses a large arc of the circle, and swells in breadth at the middle, to admit of a central hole for the common axis of the index bar and mirror to pass through. This long second bar carries the telescope as well as the horizon glass, like Mayer's, and has its second vernier at the eye end of the telescope, where a second clamp and tangent screw are applied. The telescope is shorter than Mayer's, and the horizon glass is removed farther from the centre of the circle, to avoid the interception of an incident ray, coming either from the right or left upon the index glass, the face of which is turned from the observer. This removal of the horizon glass, trifling as it may appear to be, is yet the principal improvement in this instrument, by admitting of the arc of measurement to be doubled. The telescope rests on two cocks attached to the horizon bar, one under each end, and a pair of upright screws with milled heads, *c* and *d*, adjust the position of the line of collimation, both as it regards the middle of the horizon glass *e*, and parallelism with the face of the bar, to effect both which, in rendering an object and its image equally bright, the two screws must necessarily be turned by alternate adjustments, which makes the operation tedious, particularly as each screw has a scale that requires to be read after each turn, to assure the parallelism; the sliding index pieces, that point to the scales, being separately attached to the telescope. This is a more steady contrivance, as a substitute for the up and down piece of the sextant, than Mayer's jointed parallel pieces, that keep their adjusted situation simply by friction of the contiguous parts. There are three pairs of dark glasses occasionally applied by means of fixing screws, at *f*, *g*, and *h*, before and behind the index glass, to be used as in the sextant, which pairs should match one another in colour, except that when a glass is fixed by the two screws seen close to the centre, it must have a lighter shade of colour than the corresponding one at *f*, on account of the rays of light passing twice through it, once before and again after reflection from the index glass: this glass is only used when a ray incident from the left, within the limit from 5° 20′ to 34°, is intercepted by the frame of the glass at *g*. The dark glasses fixed at *f* have their faces inclined in a small angle to the plane of the horizon glass to prevent the interference of a secondary image. Two of these glasses are seen detached, and marked *f* and *h* respectively. The telescope is six inches long, and magnifies only three times, with a field of view of 5° 40′, the space between the two parallel ocular lines being exactly 2°. As usual, it inverts the object. The detached slip of brass *i*, called a *ventelle*, is blackened on both sides, and has a triangular hole, that limits the light coming directly upon the unsilvered part of the horizon glass, when fixed in the socket *f* of the dark glasses, which addition is chiefly useful when observing terrestrial objects, the images of which are thus rendered of equal brightness with the objects themselves. There are also a pair of bent plates, similar to the one seen detached at *h*, called *viseurs*, the use of which is to assist in examining the perpendicularity of the index mirror, they are placed on opposite sides of the divided

limb, where the height of one viseur, seen by reflection, is compared with the height of the other, seen by direct vision. But these pieces may be dispensed with, as the boundary line of the limb will do as well, and is even more convenient.

4. With respect to the mode of using this instrument, we found, on examination, that the constant angle contained between the two vernier bars, when the glasses remained in a state of parallelism, is 167° 35′ instead of looking at the right hand object, and of bringing the index bar towards the vernier of the horizon bar, as Mayer proposed in measuring a single arc, the telescope must in the first place be directed to the left hand object, while the index remains at zero, and its image must be brought to coincide with the right hand object, viewed directly, by carrying the telescope outward, to enlarge the constant angle by a quantity equal to the simple angle to be measured, which we will suppose to be 30°; then 197° 35′ will be the arc intercepted between the verniers, when this coincidence takes place this is the first operation of the pair, when the simple angle might be taken approximately by subtracting the constant angle from the increased angle, as read by the telescope's vernier, if it were thought desirable: the telescope must now be clamped to the limb, while the coincidence remains perfect, and the vernier of the index bar, being unclamped and brought forwards towards the telescope, carries the image again to its original situation, where the glasses become again parallel, with the index standing at 30°, but it is not necessary that the index should stop here, at the new point of parallelism, but must go on crossing this point, and approaching the telescope till the image of the right hand object, in its turn, is found in contact with the left hand object; in which situation the vernier of the index bar will measure 60°, namely 30° at each side of the new point of parallelism, and the motion given to this vernier constitutes the second operation of the observation: the two operations taken together are called the *crossed observation*, giving a double angle Now if the point 60° be considered as a new zero, and the two operations be repeated, as above described, by moving first the telescope and then the index bar in succession, and attending to the clamps and contacts, the next zero point will be found at 120°, and after twelve crossed observations the verniers will have passed over $60 \times 12 = 720$ divisions, representing so many degrees, which in a reflecting instrument fall within one circumference. This instrument has been highly extolled by the Sçavans of Paris, and as a nautical one seems to have superseded the sextant. We have seen some of Gamby's reflecting circles on Borda's principle, that are neatly constructed, and beautifully divided. It can hardly be necessary to add, that the exact times corresponding to the contacts must be observed, and noted down in every series of observations made with the repeating circle.

5. *Mendoza's.*—Joseph de Mendoza Rios, a captain in the Royal Navy, and author of the well known Nautical Tables, presented a paper to the Royal Society of London, which was published in the Philosophical Transactions of 1801, in which he described a repeating and reflecting circle of his contrivance, which accelerated the operations in taking an observation even beyond what Borda's did, which he denominated a *reflecting* and *doubly-multiplying* circle, but he subsequently improved this circle, according to the plan which we purpose in the next place to describe. In all the former circles one limb only was graduated, on which the verniers read the measured angle; but in this instrument there are three graduated circular portions, two moveable round a common centre, and one immoveable, and, what constitutes another peculiarity, the bar, which bears the central mirror, commonly called the index mirror, and which has

usually the vernier, or verniers where more are applied, has here no vernier attached to it, but is used for the sole purpose of conveying the verniers and a circle of 360° alternately to the right and left of their primary position, by a species of vibratory motion, during a series of crossed observations, in which the corresponding clamps are alternately made fast and released Fig. 2, of the same Plate, is a perspective drawing of this instrument in its improved state, in which the upper or graduated face is represented; the disposition of the handles and glasses at the lower face is borrowed from Troughton's construction of the reflecting circle, which we shall describe in our next Section, and which therefore we will pass over at present, as having no connection with Mendoza's principle, but only as rendering it more conveniently effectual. As we had formerly occasion to describe this circle in another work, we shall copy our own description here without hesitation.

6. *C* is the fixed circle, usually called the limb of the instrument, over which is placed a second circle, *D*, and also over that a third one, *E*, the two latter of which move separately and freely round the centre of the instrument, above the last circle *E* the index *F* has its situation, and carries at the low end of its axis the index mirror, which, being at present at the under-side, cannot be seen in the figure. The fixed circle *C* has its inferior surface divided, to the right and left, into two sets of divisions as far as 140°, like two separate sextants; their respective zeroes commencing not at one point, but at the distance from one another of the breadth of the index, so that one of them touches one edge of the index, when the other touches the other at the divided part of the limb. on these two portions of the circular limb slide two small stops, *a* and *a′*, which may be made to remain in any given points. The index mirror and horizon mirror are parallel to each other when the end of the index *F* occupies the situation between the two zeroes; and as it is generally known pretty nearly, what the angular distance of two heavenly bodies is, when a lunar observation is made, these stops may be slided along the right and left divisions of the limb respectively, till their inner edges stand on the supposed degree of angular distance from their respective zeroes, in which situations they will serve as guides, particularly in the night, for fixing the index alternately in a crossed observation, in order to find the places of successive contact, more conveniently than they could be found without some such rough guide: nor will these stops be serviceable for the first crossed observation only, but for every subsequent one; since the successive observations require not the index to have any other than alternate backward and forward motions, between the two stops, how often soever repeated. The circle *D* is nicely divided into 360° and their subdivisions, and the adjoining circle *E* carries two verniers, *A* and *B*, diametrically opposite each other, which read off each to 10″. On the index *F* is the usual tangent-screw for procuring a slow motion, when the index bar is clamped to the limb, which clamping is effected by the action of the small lever *c*. There are, moreover, four other clamps with fixing screws that have milled heads like *l*, which may be called dead clamps, and which open by means of their own springs, when their fixing screws are turned backwards; but which lay hold of their respective moveable circles, when acted on by the finger screws. The clamps *d* and *g* are attached fast to the fixed circle *C* at opposite sides, and the clamps *e* and *f* are attached to and carried by the index *F* the clamps *d* and *f* clamp the upper circle *E* to the limb, and *e* and *g* clamp the lower one *D* Also when an observation is made with the motion of the index and its mirror to the left, *f* and *g* must both be fast, but *d* and *e* both loose. on the contrary, when the mo-

tion of the index is to the right, *d* and *e* must be fast, and *f* and *g* loose. The heads of the clamps *d* and *e* have each a knob, by which they may easily be distinguished in the dark from those on the other side. In making an observation with this instrument, which appears more complex than it really is, the reader may now conceive, that, when the clamp of the index has seized one of the two moveable circles, and carried it to the stop on the right, where it is deposited and clamped fast, and has then taken up the other, and brought it back the same distance to the left, before it is deposited in its turn, which two alternate motions complete two crossed observations, one to the right, and the other to the left, the verniers have departed from their original situations, as they regard a given point on the divided contiguous circles, just as many degrees as are equal to two crossed observations, or four single angular distances; for the verniers moved from the original point, which we will suppose to have been zero, one half, and again zero of the divided circle moved from the vernier, by a contrary motion, the other half.

7. The minutiæ, attending the taking of a series of crossed observations, may be thus more fully explained, in the first place slide the stops to the reputed angle to be measured, which we will suppose to be 50°, as read on the under side of the fixed circle *C*, and fix them there, one at each side of their respective zeroes, let the index for the present remain at the point of parallelism, which we have said is between the two zeroes, on the inferior surface of the fixed circle, in the next place arrange the two moveable circles so that vernier *A* of the circle *E* may be beyond the nearest stop, and may have its zero coincident with zero of the circle *D*, divided into 360°, in which situation fix the two clamps, *e* and *f*, of the index, and carry it with a quick motion to touch the stop on the right, and having fixed it by the lever *c*, complete the contact by the tangent-screw of slow motion, and the instrument will then be in a state of *rectification* (provided the glasses are truly placed) for commencing a series of crossed observations; for, as the clamps, *e* and *f*, were both made fast, while the index was at the point of parallelism, when it moved to the right, it brought both the moveable circles along with it, without altering the respective positions of vernier *A* and of zero of the circle *D*. The index has first to move from right to left, as seen in the figure, therefore the clamps, *d* and *e*, must be both loose, and also that of the lever *c*, and the clamps *f* and *g* fast, but *f* is a clamp of the vernier circle *E*, and *g* one of *D*, the graduated circle; therefore when the index moves the whole space of a crossed observation to the stop on the left, it leaves the graduated circle *D* fast behind, and takes the verniers along with it. Suppose the second contact to be completed by the tangent-screw again as before, then the vernier will read off 100°, more or less, the amount of the crossed observation, but the whole circle is divided into only 360°, instead of 720°, as Borda's is; the observed angle may consequently, though a crossed observation, be considered as the simple angular distance taken as the mean of two successive angular distances, if there had been 720° in the circle, agreeably to the divisions on a Hadley's sextant, but it is not necessary to read off yet. Change now the state of all the clamps, by fastening *d* and *e*, and loosening *f* and *g*, and carry the index back again to the stop on the right; during this motion of the index the verniers, being fixed by *d*, will remain behind, and the graduated circle *D*, being clamped to the index by *e*, will now in its turn move along with it the exact space of the second crossed observation, and the vernier *A* will read off 200°, more or less, which is four times the angle required to be measured. This quadruple angle has been ob-

tained by two crossed observations, made alternately to the left and to the right, without any useless motion of the index, which result, at this stage of the series of crossed observations, explains the reason why the instrument is called not only a reflecting, but also a doubly-multiplying circle, for we have seen that it doubles the simple angle required to be measured, at each crossed observation, taken both backwards and forwards any number of times The quadruple angle however is read off, as being only double, by reason, as we have before seen, of the circle being divided into 360° only instead of 720°, which reflection requires. This process of alternately fixing and releasing the two pairs of clamps, and of moving the index as many times alternately backwards and forwards between the stops, and ending with as many exact contacts, by the help of the tangent screw, will give a final result as read by the vernier *A*, which, divided by the number of crossed observations, exclusive of the angle of primary rectification, will give, as a quotient, the true distance sought, which distance will be the more accurate the greater the number of crossed observations Should the second vernier *B* be also read off, the mean of the two results will be still more accurate, inasmuch as not only the inequalities of simple division will be partly corrected, but also the excentricity of the circle, if there should happen to be any As the interval occupied in taking a series of observations with this instrument is much abbreviated, it may in many cases, where an expert observer is engaged, be sufficient for him to note the times of the first and last contacts only; though it will always be the safest plan to record the exact time of each contact.

8 *Hassler's.*—Professor Hassler, of Philadelphia, republished his "Papers on various Subjects connected with the Survey of the Coast of the United States," extracted from the American Philosophical Transactions (Vol. II. New Series), in the year 1824, with a copy of which we have been favoured. Before this author came to London to obtain proper instruments for completing his undertaking, he had prepared different plans of a repeating and reflecting circle, which he submitted to Mr. Troughton, who undertook to construct an instrument that should preserve the inventor's principle of a revolving circle and two pairs of opposite verniers, one pair fixed, and the other pair moveable, but with such alterations of the frame work, as would render the instrument both lighter and more firm. At the same time Mr. Troughton explained to his American friend the ideas of Mendoza, whose instrument was supposed to have failed of complete success, by reason of the numerous clamps and moving circles, requiring to be clamped and unclamped after a contact had been finished. In the formation of this new instrument Mr. Troughton effected an union of the repeating principle, on Hassler's plan, with the construction of his own reflecting circle, which we have not yet described. The graduated circular border is made moveable round the centre of the frame work, which holds a pair of opposite verniers fixed to it; and a bar, forming a diameter of the circle, carries round the same centre, either jointly or separately, above the plane of the circle, a second pair of opposite verniers, that read with the moveable circle and may be clamped to it, or released from it, at pleasure. The axis of this diametrical bar has a long bearing in the centre work, and carries the index glass at its lower end, beyond the back frame. A graduated semicircle lying under the border, connects the opposite ends of the two fixed verniers of the frame work, to receive a pair of sliding pieces of brass, that act as stops to the indices, when they are properly placed at the rough angle to be measured, by a previous operation, to the right and left of the flying circle's zero, so that when yielding slips of metal, attached to the vernier bars, come in contact with these stops, it is known, even in the dark, that the place, where a contact is to be

made, is nearly ascertained; and the vernier bar may then be fixed, for the screw of slow motion to finish the contact. The double frame, position of the telescope and glasses, and adaptation of the handles, are so nearly the same as those represented in figure 5, not yet explained, that it is unnecessary to give any additional figure, for representing the shape and relative situation of these parts, which constitute the body of the instrument.

9. The principle on which the measurement is effected is this: the revolving verniers move forward from zero of the graduated circle, when the stops are previously set to the rough angle, till the index or vernier bar touches the stop to the left, when the graduated face is uppermost, and is there clamped to the fixed verniers, when the tangent-screw completes the required contact. The two revolving verniers might now give the angle, by two readings, but the repeating principle has not yet been introduced, and consequently no advantage is yet derived from this first observation, over a common circle with a double vernier; the fixed verniers are in the next place unclamped, but as they have no motion, the flying circle and revolving verniers are brought back to the right, in a state of union, across the point zero, till the vernier bar touches the second stop; during this motion the revolving verniers have moved backwards just *double* the rough distance with the attached circle, that they did forwards without it; consequently the fixed verniers will now read the same angle at the right hand of zero, that the revolving verniers did on the left, when the clamping must be again made, and the contact completed; but still this is only a second mode of reading a single measure of the angle, and nearly all that is yet gained in accuracy, is the extermination of the index error and of the error occasioned by the dark glass, if used. These two measures, if separately read, are equivalent to Borda's *crossed* observation, because the index bar in both cases crossed the zero point. It is here presumed that the two objects are equally luminous; but if not, it will be necessary to invert the face of the instrument before each second, fourth, sixth, &c. contact, and then the motions will all become forward from right to left, which otherwise would have been alternate: the second reading may however be omitted, and the revolving verniers, being unclamped, must be moved again to the first stop in the original position of the instrument, where, the contact being complete, the verniers would give each a *double* measure if examined; but the readings are not yet necessary; the fixed verniers are in the next place unclamped, the instrument again inverted, and the contact again completed, when these verniers, if examined, would also give double measures; and thus treble and quadruple measures must be obtained successively, both at the revolving and fixed verniers, or even more, if the circle has not been passed over by both pairs of verniers, before the readings are required to be examined; and then the average of all the measures taken by the fixed verniers, added to an average of all the measures taken by the revolving verniers, will afford the means of getting a mean of all the measures that the operations have effected. It must however be remarked, that as the horizon glass and index glass are parallel to one another, by construction and adjustment, when their bars include an angle of just 90°, the zero of one will be at 0°, and of the other at 90°, consequently ninety degrees must be subtracted from the sum of the measures of this pair of verniers, before their average is taken: otherwise if neither pair of verniers commence at zero, the two numbers from which they start must be respectively deducted before their averages are taken, and then it will be of no importance at what part of the flying circle the operations begin. The subjoined exemplar will explain the Author's mode of registering and reducing an observation.

STATION NEAR CHATEAUGAY RIVER

DOUBLE ALTITUDE OF THE SUN WITH THE REFLECTING REPEATING CIRCLE FOR DETERMINATION OF TIME

29 Sept P M 1818 Time by Chronom No 50, Hardy	Mean of times	Readings of the mirror		Readings of the Circle		Results		Reduction	Mean Alt
		A	*B*	*C*	*D*				
H M S	H M S.	° ′ ″	° ′ ″	° ′ ″	° ′ ″		° ′ ″		° ′ ″
3 52 40 0	.....	300 0 0	300 0 0						
3 53 52 0		...	....	47 32 25	47 32 18				
3 54 17 5						By A	177 59 25	$\frac{177\ 59\ 24}{10} =$	17 47 56
3 54 50 0	3 55 12	.	. ...	... .	....	B,	177 59 23		
3 55 12 0									
3 55 31 0									
3 55 57 0									
3 56 15 0						" C	177 13 7	$\frac{177\ 13\ 26\ 5}{10} =$	17 43 20
3 56 33 0	3 55 40	117 59 25	117 59 23	. ...	.. ..	D	177 13 40		
3 56 52 0									
3 57 20 5	...	...	......	224 45 32	224 46 4				

DETERMINATION OF TIME [*Vide* § XCI]

1818 29 Sept P M	Time of Chron No 50	Zen dist of ☉ observed	Declination South,	True time	Mean time	Difference of Chron from		Mean time
						true	mean	
	3h 55m 12s 0	72° 12′ 3″ 6	2° 25′ 3′ 6	4h 6m 41s 46	3h 57m 10s 0	0h 11m 39s 46	−1m 58s 01	−1m 58s 44
	3 56 40 0	72 16 39 4	2 25 14 95	4 7 22 33	3 57 38 88	0 11 42 33	−1 58 88	

10. *Fayrer's.*—When Fayrer was recently applied to by a naval officer, to construct him a reflecting circle on Borda's principle, with such improvements as might occur to him, he so far deviated from the French instrument, as to remedy several of its imperfections without affecting its general structure and mode of operating. Fig 3, of our Plate of reflecting circles, gives a perspective view of Fayrer's instrument, which will not require a general description of its various parts, that are nearly the same as Borda's, but only its improvements to be pointed out. In the first place the up and down piece *A* is made fast to the long bar carrying the telescope and horizon glass, and rises about two inches perpendicularly, with the milled head of the screw, that moves the telescope up and down, at its superior end, conveniently situated for adjusting the illumination suitable to the two objects without affecting the parallelism, secondly, the telescope has a higher magnifying power than in Borda's circle, thirdly, a thin bar of brass stands parallel over the bar of the horizon glass, and forms with it a light frame; one end of which additional bar is screwed fast to the up and down piece, and the other end is supported by the two arms of a piece cranked below and made fast to the block of the horizon glass, the two screws above are visible over the horizon glass, and a single pillar ascending between the index glass and up and down piece gives further strength to the frame fourthly, the two sets of dark glasses are carried by this second bar of the frame, as clearly shown in the

figure, and never intercept the incident rays till after reflection from the index glass; for when they are turned down, the distance between the two bars allows of their falling in the way of the reflected and direct rays respectively, as in the sextant, and when turned up, they are out of the way; they are inclined a little in opposite directions from a right angle to the bar that carries them fifthly, the horizon frame *B* has a long hollow axis of motion, descending through the central strong conical tube, that receives the handle below the back of the instrument, and the diametrical bar *C* of the index glass, which carries a pair of opposite verniers, has a long steel axis descending two inches and a quarter through the said bell-metal axis of the frame, as far as into the middle of the wooden handle, that contains a brass tube screwing upon the fixed central tube, the broad flanch of which tube is screwed to each of the nine radial edge-bars, that bear the graduated limb sixthly, the end of the lower bar of the horizon frame, opposed to the single or third vernier, has a plain clamp and thumb screw that assists in fixing the frame fast in any given position. Lastly, the strong collar receiving the screw of the telescope is attached to the ring of the up and down piece by four screws opposite each other, by means of which the line of collimation of the telescope is adjusted parallel to the plane of the circle, an adjustment which Doctor Maskelyne had recommended in one of the Nautical Almanacs, and which is known to be right, when the moon, being brought into contact with a star at about 120° distance from her, at the exact centre of the field, will show equal errors in the measure at equal distances, to the right and left, from the central line, lying parallel to the face of the circle In all these respects it will be allowed, that this construction is an improvement on Borda's original instrument, being an union of it and of Troughton's circle, in some degree, and that it may be used in precisely the same manner; but still it wants the advantage of auxiliary handles when inverted, and used in all the oblique positions. From this account of the several different constructions of the reflecting and repeating circle, that we have had occasion to examine, it is manifest that various other modifications of the instrument might be easily devised, without compromising the repeating principle. The chief difference between Hassler's and Fayrer's constructions, for instance, consists in this, that in the former the circle moves and a pair of verniers are fixed, in the latter the verniers move and the circle is fixed, the frame work being a little different, and Hassler's having the advantage of Troughton's handles.

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§ LXXXIX TROUGHTON'S REFLECTING CIRCLE [Plate XXVIII]

1 After Mr. Troughton had made several of Borda's reflecting circles, and found various imperfections in the construction, notwithstanding his improvements, he contrived a circle, now introduced into the English navy, which is free from those imperfections, and gives a result at one crossed observation, that banishes almost all the errors of the instrument, without the assistance of the repeating principle. He had observed in the first place, that the two index bars of Borda's circle turn round a centre upon bearings equal only to their own thickness, and consequently want that steadiness which is derived from a long axis, to make the telescope and glasses reverse in the same plane through every part of the circle · secondly, the adjustment of the telescope for equalization of the light, being performed by two screws alternately used,

agreeably to the indications of their scales, is very inconvenient, and liable to produce errors, by displacing the telescope from exact parallelism with the face of the limb thirdly, the dark glasses are inconveniently placed, and require much time to change them for others, when too dark or too light for the object observed. fourthly, the want of a handle on the upper side, renders the observation in the inverted position very difficult, and in some positions almost impracticable fifthly, the necessity of making a previous computation, to determine the points on the limb where the verniers must stop, at every stage of the operations, occasions great embarrassment to the observer; this, indeed, was remedied in the English construction, by attaching a divided arc to one of the index bars, having two stops, that arrested the verniers at the points of the limb, where the object and reflected image were nearly in coincidence; so that an observation might be made in the dark sixthly, the correction of the error of excentricity will not be perfect with single verniers, if the repeated angle be not carried all round the circle, and, if carried much further, the error will regenerate and lastly, there is a great waste of time in a long series of observations, as well as trouble in the reduction, to which may be added, that frequent screwing and unscrewing, together with a multiplicity of contacts for gaining one result, can hardly fail of being productive of errors. To avoid these various inconveniences and sources of error in making an observation, the following instrument was contrived, the construction of which, in our opinion, cannot be too much admired, as approaching perfection as near as human ingenuity and workmanship can effect

2. Figures 4 and 5 of Plate XXVIII exhibit two representations of Troughton's reflecting circle, the first shows the divided limb and three equi-distant verniers; on one of which are fixed a clamp and tangent-screw for regulating the contacts, and another carries the magnifier with its reflector for reading the verniers, which may be successively applied to all the vernier bars, the second shows the telescope and all the glasses, as well as the up and down piece and handles. In both these figures the frame work of edge bars is so clearly seen, as to require no particular explanation. Its connection with the circular border at the under side is also visible This form of the body of the instrument was found by experiment to resist the pressure of the weight in every position, and to assure a coincidence of the body and image of the objects to be observed, at all angles, when held by the different handles, and from trial of many other figures was found the only one In the middle of the frame a hollow centre, *a*, is fixed upwards of two inches long, having its broad base, or flanch, close to the back of the frame, in which the axis revolves freely. At one end of this axis the triple vernier bar is fixed, and at the other the index glass, silvered as in the sextant, which therefore revolve together at the contrary faces This is the only central motion that the instrument has, and the plane of the divided limb is generated from the long hole in which the axis turns, which completely removes the first objection to the French construction of Borda's circle. A secondary frame, *b b'*, is erected on the back surface of the principal frame, which may be seen in fig. *5*, at the distance from it of the whole depth of the hollow centre, and is supported by five pillars at equal distances from one another: both the square and round frames of the dark glasses turn on hinges, remote enough from the limb to allow of their being turned, one or all, beyond the secondary frame when used, as far as to face the mirror and half silvered horizon glass; but at other times to a situation between the two frames, without interrupting an observation not requiring their use, so that they may be immediately turned up or down at pleasure, as in the

best sextants. The distance between the frames allows a length of barrel, *c*, in which is contained the contrivance called the up and down piece, consisting of a squared hole, and square arbor notched at the four corners, and turned by a tapped milled nut, seen under the telescope: this squared axis holds the adjustable ring into which the telescope screws, parallel to the face of the limb, by which means the telescope preserves its parallelism, while its distance from the limb is varied, to adjust the brightness of the direct object and reflected image of the second object, when compared together, which adjustment is performed while the eye is at the telescope: thus the second inconvenience is obviated. The handle *d*, seen in both figures, and used on the divided side of the instrument, is fixed to the central strong tube on the side of the glasses, it is attached to a brass tube that is bent over the edge of the circle, and allows the index bars to revolve under it, but as the bent tube covers the limb at a place where a vernier may require to be read, it is readily removed by taking out the finger screw that fixes it. A second handle, *e*, is seen in both figures, the extreme point of which enters the central tube as a steady pin, and a finger screw, passing through a hole at the junction of two cross bars, sets it fast. A third handle, *f*, screws into the cranked part of a strong cock, *g*, made fast to the frame, and stands on the side of the glasses at right angles to the plane of the limb: it will also screw into the hole at the thick end of the bent handle *d*, in both which situations it stands in a line with the central axis. This handle is very convenient when the line of position, of the objects to be observed, is horizontal, or nearly so, and when applied to the side of the glasses, which may be called the lower side, affords the best support to the instrument in reading the verniers. Thus this instrument presents to the observer a convenient hold for either hand, in every possible position, and therefore removes the fourth, or principal objection to the construction of Borda's circle. The fifth objection cannot occur here, as the verniers do not proceed by steps along the limb, but only move forwards and backwards over the same parts of the arc, during a series of observations, as will be explained presently: there is therefore no need of making a preliminary computation for ascertaining the length of a step.

3. The diameter of the circle, for the sake of lightness, is only ten inches, and the limb is divided into 720 spaces, making so many degrees, which are again subdivided, so that each vernier may read 20″, which was thought sufficient for the seaman's use; but for some observatories it has been made on larger dimensions, to read at each vernier 10″. The zero of the circle is at the point of parallelism of the mirrors, and the divisions are figured each way to 160°, which is the largest angle that can be measured, or required, by this instrument. The three verniers are intended to be a substitute for the repeating principle, and as a *crossed observation* is taken, the first operation with the divided limb up, and the second with it down, there are six readings on as many different parts of the circle, at each complete observation; three of which are first taken and read to the right of zero; and then the remaining three to the left, namely, again to the right with the face inverted, so that $\frac{1}{6}$″, or 1″.66, is the probable mean error of the readings. This property of giving six readings on different parts of the graduated circle after two contacts, is considered to be a practical compensation for the repeating principle, for, as circles are now divided by the best engines, the inequality of the divisions and also the excentricity are reduced to very small quantities, for the errors arising from both these mechanical sources, will be nearly annihilated by the number of collimated readings. Indeed the errors of excentricity will be entirely obviated as often as any one of the vernier bars

lies in the direction of the conjugate diameter of the excentric circle, and will be very nearly so in the other situations. Another very important advantage of this application of a crossed observation is, that all the errors occasioned by the want of parallelism in the faces, or in the position of the silvered, and also of the dark glasses, are counteracted by the reversion of their position, in the second operation. In this respect the instrument with three equidistant verniers has greatly the advantage over the repeating instrument, where an existing error may remain, not only uncorrected, but undetected. The readings of this circle, as at present divided and used, divide the minute into smaller portions than the optical part of so small an instrument can appreciate in making the observation, and it seems quite unnecessary to attempt greater accuracy than the human eye is capable of discriminating. The inventor has circulated a printed paper containing directions for observing with his reflecting circle, of which the following is a copy, viz.

" Prepare the instrument for observation by screwing the telescope into its place, adjusting the drawer to focus, and the wires parallel to the plane, exactly as you do with a sextant: also set the index forwards to the rough distance of the sun and moon, or moon and star, and, holding the circle by the short handle, direct the telescope to the fainter object, and make the contact in the usual way. Now read off the degree, minute, and second, by that branch of the index to which the tangent-screw is attached, also the minute and second shown by the other two branches, these give the distance taken on three different sextants; but as yet, it is only to be considered as half an observation; what remains to be done is, to complete the whole circle, by measuring that angle on the other three sextants. Therefore set the index backwards nearly to the same distance, and reverse the plane of the instrument, by holding it by the opposite handle, and make the contact as above, and read off as before what is shown on the three several branches of the index. The mean of all six is the apparent distance, corresponding to the mean of the two times at which the observations were made.

" When the objects are seen very distinctly, so that no doubt whatever remains about the contact in both sights being perfect, the above may safely be relied on as a complete set, but if, from the haziness of the air, too much motion, or any other cause, the observations have been rendered doubtful, it will be adviseable to make more, and if, at such times, so many readings should be deemed troublesome, six observations and six readings may be conducted in the manner following. Take three successive sights forwards, exactly as is done with a sextant; only take care to read them off on different branches of the index; also make three observations backwards, using the same caution; a mean of these will be the distance required. When the number of sights taken forwards and backwards are unequal, a mean between the means of these taken backwards, and those taken forwards, will be the correct angle.

" It need hardly be mentioned, that the shades, or dark glasses, apply, like those of a sextant, for making the objects nearly of the same brightness; but it must be insisted on, that the telescope should on every occasion be raised or lowered by its proper screw, for making them perfectly so.

" The foregoing instructions for taking distances apply equally for taking altitudes by the sea or artificial horizon, they being no more than distances taken in a vertical plane. Meridian altitudes cannot, however, be taken both backwards and forwards the same day, because there is not time: all, therefore, that can be done is, to observe the altitude one way, and use the index-error; but even here, you have a mean of that altitude, and this error, taken on three different sextants. Both at sea and land where the observer is stationary, the meridian altitude should be observed forwards one day, and backwards the next, and so on alternately from day to day; the mean of the latitudes, deduced severally from such observations, will be the true latitude; but in these there should be no application of index-error, for that being constant, the result would in some measure be vitiated thereby

" When both the reflected and direct images require to be darkened, as is the case when the sun's diameter is measured, and when his altitude is taken with an artificial horizon, the attached dark glasses ought not to be used · instead of them, those which apply to the eye-end of the telescope will answer much better, the former having their errors magnified by the power of the telescope, will, in proportion to this power, and those errors, be less distinct than the latter.

" In taking distances, when the position does not vary from the vertical above thirty or forty degrees, the handles which are attached to the circle are generally most conveniently used; but in those which incline more to the horizontal, that handle which screws into a cock on one side, and into the crooked handle on the other, will be found more applicable.

" When the crooked handle happens to be in the way of reading one of the branches of the index, it must be removed for the time, by taking out the finger-screw, which fastens it to the body of the circle

" If it should happen that two of the readings agree with each other very well, and the third differs from them, the discordant one must not on any account be omitted, but a fair mean must always be taken

" It should be stated that when the angle is about thirty degrees, neither a distance of the sun and moon, nor an altitude of the sun with the sea horizon, can be taken backwards, because the dark glasses at that angle prevent the reflected rays of light from falling on the index-glass, whence it becomes necessary, when the angle to be taken is quite unknown, to observe forwards first, where the whole range is without interruption, whereas, in that backwards, you will lose sight of the reflected image about that angle. But in such distances where the sun is out of the question, and when his altitude is taken with an artificial horizon (the shade being applied to the end of the telescope) that angle may be measured nearly as well as any other, for the rays incident on the index-glass will pass through the transparent half of the horizon-glass, without much diminution of their brightness.

" The advantages of this instrument, when compared with the sextant, are chiefly these: The observations for finding the index error are rendered useless, all knowledge of that being put out of the question, by observing both forwards and backwards. By the same means the errors of the dark glasses are also corrected, for, if they increase the angle one way, they must diminish it the other way by the same quantity. This mode also perfectly corrects the errors of the horizon-glass, and those of the index-glass very nearly. But what is still of more consequence, the error of the centre is perfectly corrected, by reading the three branches of the index; while this property, combined with that of observing both ways, probably reduces the errors of dividing to one-sixth part of their simple value. Moreover, angles may be measured as far as one hundred and fifty degrees, consequently the sun's double altitude may be observed when his distance from the zenith is not less than fifteen degrees; at which altitude, the head of the observer begins to intercept the rays of light incident on the artificial horizon; and, of course, if a greater angle could be measured it would be of no use in this respect.

"This instrument, in common with the sextant, requires three adjustments. First, the index-glass must be perpendicular to the plane of the circle: this being done by the maker, and not liable to alter, has no direct means applied to the purpose: it is known to be right, when, by looking into the index-glass, you see that part of the limb which is next you reflected in contact with the opposite side of the limb, as one continued arc of a circle: on the contrary, when the arc appears broken, where the reflected and direct parts of the limb meet, it is a proof that it wants to be rectified. The second is, to make the horizon-glass perpendicular. This is performed by a capstan-screw, at the lower end of the frame of that glass; and is known to be right, when, by a sweep with the index, the reflected image of any object will pass exactly over, or cover the image of that object seen directly. The third adjustment is for making the line of collimation parallel to the plane of the circle. This is performed by two small screws, which also fasten the collar into which the telescope screws, to the upright stem on which it is mounted: this is known to be right, when the sun and moon, having a distance of one hundred and thirty degrees, or more, are brought into contact, just at the outside of that wire which is next to the circle, and then moved across to the outside of the other wire: the contact being the same in both positions is the proof of adjustment."

4. When this circle is used with a reflecting horizon, a tripod stand is found very convenient for holding it in any required position, when the bent handle is taken off, and the maker usually packs one in a separate small box, containing also a wooden bottle of pure mercury, and an oblong wooden vessel for receiving the mercury, when substituted for an horizon, with a glass roof to protect it from the wind. The tripod is composed of three legs, carrying a vertical tube of brass screwed fast to it, which has an horizontal axis surmounted over it, in a frame that has an azimuthal motion, so that the two motions form an universal joint, that admits of the attached circle having its plane put into any position, horizontal, vertical, or oblique, in which it is sustained by a pair of counterpoising weights, on levers made fast to the ends of the horizontal axis, by thumb-screws that increase the friction at the pivots. In using this tripod for taking an altitude, the vessel of mercury is placed in front of it, at such a height and at such a distance, on a firm pillar, stand, or three-legged table, that the eye of the observer may command the reflected image, by looking in the direction that the small telescope will require to be pointed, when the tripod has taken its place also on a firm support, then, the dark glass of the eye-piece being put on, the image of the sun, if that be the object to be observed, as reflected from the mercury, must be brought into the field of view, by depressing the telescope by a handle, and using the two motions of the universal joint, when the plane of the circle is nearly vertical by estimation, by a little practice the telescope is easily adjusted to keep the image in view, which will be at first tremulous, but will soon become steady when the mercury has settled and remains undisturbed. When this preparation is gone through, the index must be turned from parallelism, till its mirror facing the sun, by being reclined, brings his image down to the mercury, which may be done by first moving the reading vernier through double the computed rough angle, and then, if the plane of the circle is vertical, a motion in azimuth, and another in altitude by the screw of slow motion, will complete the contact, at

which instant the time must be noted by a clock or watch well regulated, which may be done to a second, when the object is at a distance from the meridian; but when the greatest or meridian altitude only is wanted for giving the latitude, the time is of no importance. The continuance in contact for a minute or two, without over-lapping or separation of the two images, will be a proof that the *greatest* altitude has been obtained. When a single observation is taken out of the meridian, it is usually to determine time; which may be also still better done, in a stationary situation, by corresponding observations at opposite sides of the meridian, in the form of equal altitudes, but if two observations of the altitude be taken on the same side of the meridian, by knowing the time elapsed between them, the latitude may be determined by reducing the observations to the meridian by the nautical methods. When a body, having a sensible diameter, has a contact of the borders of its images made, those which first come in contact will require a semi-diameter to be subtracted from the single angle, as being that of the upper limb, but when they are made to cross one another before contact, it is the lower limb, when the telescope inverts, and the semi-diameter must then be added, but in all cases of doubt, the two images may be brought to coincide, by using both the tangent-screw of the vernier and the third foot-screw, which should stand at a right angle with the line of vision, these screws may be used either successively or at the same time, while the beats of the time-piece or clock are counted, which they should always be at the conclusion of the contact. Otherwise dark glasses of different colours may be used with the sun, which will distinguish the image reflected by the mercury, from that reflected twice in succession from the mirrors. When the lower limb forms the contact, the images will begin to overlap when noon is past, but when the upper limb is the one in contact, they begin to separate after the highest altitude, which appearance is perhaps the best criterion, and will also show within a minute, or two at the most, the time of noon, when the telescope magnifies considerably and inverts the object, which we here suppose it to do. Examples of the use of this instrument will be given in the two following Sections.

, ON reperusing the preceding paragraph after it was printed, we find that by inadvertence the words "subtracted" and "upper limb" have been improperly substituted for "added" and "lower limb," and the contrary when a concave eye-lens or a plain tube is used, the moving image descends before the contact, but when an inverting eye-piece is applied, it apparently ascends

§ XC TO DETERMINE THE LATITUDE OF A PLACE FROM OBSERVATION

1. The terrestrial latitude of any station is its distance, in geographical degrees and parts of a degree, from the equator, which distance, being considered parallel to a corresponding arc in the heavens, is assumed to be equal to a celestial arc extending from the zenith of the station to the celestial equator or equinoctial line: any observation therefore of a heavenly body, which will afford data for determining this arc, will be competent to determine the latitude required. The nature of the observation will in general depend on the properties and powers of the instrument made use of, and on the opportunity of using that instrument with advantage. In an observatory supplied with fixed instruments, meridian altitudes or zenith distances of the sun, or, what is better, of a known star, afford the readiest as well as the most correct means of deducing the latitude, by direct measurement of the arc in question, or of its complement; but at sea, where there is a natural horizon, an instrument that measures by reflection, and that is also of a portable construction, such as the sextant and reflecting circle, is the only species that can be employed with success. When the celestial equator and observed object are on the same side of the zenith, the measured arc denoted by the zenith distance, when corrected for refraction, and for parallax if necessary, ± the declination, accordingly as it is north or south, will give the latitude, or arc contained between the zenith point and celestial equator, or, which is the same thing in other terms, the observed altitude, properly corrected, by changing the signs of the declination, will give the co-latitude: this may be called the solar method, as being applicable in all cases, both by land and sea, when this luminary is observed. It is however equally applicable to observations of stars that do not pass beyond the zenith, towards the elevated pole, provided their declinations be known.

2. The second method of determining the latitude by observation is, by means of a known star passing the meridian between the zenith and elevated pole, in this case we have the latitude = the declination — the meridian zenith distance; or = the altitude — the polar distance (co-declination). but when the star is observed in passing the meridian below the pole, the co-latitude = the zenith distance — the polar distance; or = the altitude + the polar distance. And thirdly, when a circumpolar star is observed on the meridian both above and below the pole, at an interval of twelve sidereal hours, half the sum of the two altitudes, respectively corrected for refraction, will be the latitude, without reference to the declination, which circumstance renders this an independent method, except so far as refraction is concerned, and may be relied on in an observatory. If these direct methods be repeated with different stars, and give the same result, when compared together, it may be safely concluded, that the instrument is good, and that the latitude is correctly deduced. When the star is observed according to any of these methods, at a short distance from the meridian, either before or after, as well as on the meridian, for the extermination of errors, the additional observations must of course be reduced to the meridian, by the proper tables, before a mean is taken for the true meridian observation. All these direct methods are founded on the consideration, that the arc contained between the pole and nearest horizon is equal to the arc extending from the zenith to the celestial equator, which are each equal to the latitude of the place; and that therefore the

arcs from the pole to the zenith, and from the equator to the other horizon, are each equal to the co-latitude hence it becomes a matter of indifference, as to accuracy, which of those four arcs be determined by observation; for any one of the four may be measured, that may be found most convenient to be observed.

3 There are moreover other methods of ascertaining the latitude of a place, which are either differential or indirect, but which under certain circumstances may be resorted to, as substitutes for those that are direct. first, when the latitude of one station has been well determined, the latitude of another within sight may be inferred by a trigonometrical operation secondly, the pole star may be observed at any point of its diurnal arc, and may be reduced to the meridian by some one of the tables we have given in our first volume for this purpose thirdly, when the horary angle of a star is known, a single observation of its zenith distance, taken out of the meridian, will enable the observer to compute the co-latitude from our formula 17, given in page 459, of this volume; and when the exact time and line of the meridian are not previously known, the hour angle may be obtained from equal circum meridian altitudes, by taking one half of the interval fourthly, the true latitude, as well as the horary angle, may be computed from two observed altitudes and the interval of time elapsed, provided the latitude be nearly known fifthly, the latitude may be determined from observing the arc contained between two known stars, when they have the same azimuth, or vertical position. But with respect to the fourth and fifth indirect methods, we must refer for the former to the different authors who have published horary tables or regular treatises on navigation; and for the latter, to Dr O Gregory's *Elements of Plain and Spherical Trigonometry* (pp. 172—174.) Lastly, the latitude may be computed from the sidereal interval of a known star's passage, from the eastern to the western prime vertical, as observed by an altitude and azimuth circle; or by a portable transit-instrument, with its axis placed exactly north and south, instead of east and west, but perfectly level. The observation depends solely on the optical part of the instrument, and when the star passes the meridian not far from the zenith, and has its apparent declination well ascertained, the method is capable of as much accuracy as any other that gives the measure of an arc from an observation of time. The following formula, which is very simple, will give the latitude, viz $\cot L = \cot \delta \,.\, \cos \frac{1}{2}(t-t')$, where $(t-t')$ denotes the true sidereal interval, and δ the apparent declination of the star This mode, first suggested in Schumacher's *Nachrichten*, requires previous preparation, and great care in placing and adjusting the instrument, for making the observation but when it is once adjusted, distant marks to the east and west might be fixed, to facilitate a renewal of the same position

4 After the examples which we have already given on this subject, to illustrate the use of the various instruments we have described, it will be sufficient to add here an example, to explain the application of a double altitude taken by reflection from mercury, by Troughton's reflecting circle, above referred to.

Example.—The double altitude of the sun's upper limb was observed, at a station near London, by means of reflection from mercury, on the noon of September 19, 1828, with the face looking towards the west; and on the next noon the lower limb was observed with the face towards the east, at which times the barometer and thermometer were noted; these observations afford data for the following operations, to determine the latitude; viz.

SEPTEMBER 19.

A	80°	20′	40″	Log refrac 1.8385
B. ..		20	40	Bar. 29.95 9 9987
C.....		21	0	Ther. 67° 9 9846
Mean ...	80	20	46.66	66.″30 ... 1 8218
				6 75 parallax
Half ..	40	10	23.33	
Refrac —par			—59.55	
	40	9	23.78	
Semi-diam.		—15	57.90	
Centre ..	39	53	25.88	
N. Dec. sub	1	24	4.00	
	38	29	21.88 = co-lat.	
	51	30	38 12 = lat.	

SEPTEMBER 20.

A . ..	78°	28′	20″	Log refrac. 1.8531
B . ..		28	20	Bar 30.0 0 0009
C . .		28	20	Ther. 64° . 9 9873
Mean ..	78	28	20	69″40 ... 1.8413
				6.75 parallax.
Half .	39	14	10	
Refr.—par.	— 1		2.65	
	39	13	7.35	
Semi-diam.		+15	58.20	
	39	29	5.55	
N. dec .	1	0	43 00	
	38	28	22 55 = co lat.	
	51	31	37.45 = lat	

From the mean of these two observations, the resulting latitude is 51° 31′ 7″.78, with an index error of 29″.66; and very nearly the same result would have been obtained, if the semi-diameters of both days had been omitted, since the opposite limbs were observed. also either of the two observations would have given the same latitude, if the index error had been previously taken, and applied with a negative sign on the 19th, and a positive one on the 20th; which error would have appeared on previously measuring the sun's diameter before and behind zero. The tripod stand was placed on a pier, and sustained the circle in both these observations, which rendered them perfectly easy, when both of the images had been brought into the field of the telescope, either by previous computation of the double angle, or by the plain tube without computation · the dark glass was in both instances applied at the eye-end of the telescope. On the 19th it was known that the upper limb was observed, because the two limbs overlapped before the contact was made perfect, and receded after; but on the 20th the appearance was just the contrary, which shewed that the lower limb was then observed. In cases of doubt, this criterion will prove, by waiting four or five minutes after the contact, which of the limbs should be considered in contact, at the moment of completing the observation. When an altitude is taken at sea, by either a direct or inverting eye-piece, or by a plain tube, to limit the line of sight parallel to the plane of the graduated face, the lower limb of the sun's image is usually brought down to the marine horizon, by moving both the index and body forwards together, with the image in the field of the telescope, till the contact is correct, and then a correction for the *dip* of the horizon depending on the height of the eye from the sea's surface, must be included with the corrections for refraction, parallax, and semi-diameter, the angle thus nautically measured, is at once the single angle required. The accuracy will mainly depend on the clear delineation of the horizon used, which will always be concealed when the body observed is surrounded by clouds that descend with that body. The varying state of the atmosphere near the horizon will, however, greatly affect the tabular correction for the dip, and render the observation less correct than when a reflecting horizon gives a double altitude. When a natural horizon is used for taking altitudes, the single angle must be corrected for the index error, before the refraction is applied; but when an artificial horizon is substituted, the double angle must be the one so corrected, before one half of it is taken for the single measure. When the inverting telescope is used, the limb of the sun or moon that appears the lower, must always be considered as the upper, in making the contact by either of the two methods of observing. It may be further remarked, that when two images are brought into contact, one of them twice reflected from the glasses of the instrument, and the other once only from the horizontal reflector, the double angle measured is that which is contained between the upper limb of one and the lower limb of the other, and therefore the whole diameter of the body observed might be applied with its proper sign to the double angle, as a reduction to the centre; but it is better to apply the semi-diameter to the single angle, when corrected for refraction.

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§ XCI. TO DETERMINE THE TIME BY MEANS OF AN OBSERVATION OF A HEAVENLY BODY TAKEN AT A DISTANCE FROM THE MERIDIAN.

1. We have already explained how the exact time, either solar or sidereal, may be ascertained in an observatory, by means of a transit instrument, or other instrument that is capable of measuring equal circum-meridian altitudes; but occasions frequently occur, both at sea and on the land, where such instruments cannot be used, or are not at hand; the altitude of the sun, or of a known star, may notwithstanding be conveniently taken, by a sextant or reflecting circle, with considerable accuracy, from which the time, at the moment of observation, may be deduced by computation. The problem is that in which the three sides of a spherical triangle are given, to find an angle opposite one of them; the usual mode of solving which is given in all the elementary books of spherical trigonometry, and of navigation: it is necessary however that the latitude of the place of observation should be known, and also the declination of the object observed, as well as its altitude; for the complements of these three arcs constitute the sides of the given triangle, of which the angle at the pole, subtended by the zenith distance, is required to be found. The process of computation by the common method is long, and perplexing to those who do not understand all the steps, referred to the principle, as they proceed; and therefore we will continue our plan of computing from formulæ, that require not the aid of didactic rules, and that shorten the logarithmic calculus. On reverting to page 459 of our present volume, which contains a series of formulæ relating to the triangle in question, our readers will find, that No. 1 is applicable to our present purpose, where z Δ λ are given, to find h, the hour angle, which formula has been exemplified in page 463, in the case of α Aquilæ, observed out of the meridian, on the 20th of May, 1828. We shall therefore add an example in which the sun is the object observed by reflection from a protected vessel of mercury, similar to the one represented by fig. 4 of Plate XXIX.

2. *Example.*—At $9^h\ 28^m\ 45^s$ A.M. mean time, by a Hardy's chronometer, on September 20,

1828, in latitude 51° 31′ 8″ N., the double altitude of the sun's centre was found, by a Troughton's reflecting circle, $=62° 16' 46''.66$, or the single angle $=31° 8' 23''.33$, from a mean of the three verniers, when the two images were rendered coincident, to avoid an allowance for the semi-diameter, and when the index error was nothing; the correction for refraction was 1′ 33″.6, due to the zenith distance 58° 52′, with the barometer at 30 .0, and thermometer at 64°, also the parallax was taken from the proper table at 6″.7; so that the true altitude was 31° 6′ 56″.43, making the zenith distance $=58° 53' 3''.57$; while the co latitude was 38° 28′ 52″, and the sun's polar distance (Δ) 88° 56′ 45″, agreeably to the Nautical Almanac, when allowance was made for the time before noon: according to these data, and subjoined formula, the operation will stand thus, viz.

$$\text{Formula, } \tan \tfrac{1}{2} h = \sqrt{\frac{\sin \frac{1}{2} (z+\Delta-\lambda). \quad \text{Sin } \frac{1}{2} (z+\lambda-\Delta).}{\sin \frac{1}{2} (z+\Delta+\lambda). \quad \text{Sin } \frac{1}{2} (\Delta+\lambda-z).}} \qquad (a)$$

OPERATION.

$$\begin{array}{lllll}
\text{Sin } \frac{1}{2} (z+\Delta-\lambda) & 54° \; 40' \; 28''3 & \ldots\ldots\ldots & \log & 9.911626 \\
\text{Sin } \frac{1}{2} (z+\lambda-\Delta) & 4 \;\; 12 \;\; 35.3 & \ldots\ldots\ldots & & \underline{8.865748} \\
 & & & \text{Sum} & \underline{8.777374} \\
\text{Sin } \frac{1}{2} (z+\Delta+\lambda) & \left.\begin{array}{l} 93° \;\; 9' \;\; 20''.3 \\ \text{or } 86 \;\; 50 \;\; 39.7 \end{array}\right\} & \ldots\ldots & \log & \left\{\begin{array}{l} 9.999341 \\ \underline{9.750594} \end{array}\right. \\
\text{Sin } \frac{1}{2} (\Delta+\lambda-z) & 34 \;\; 16 \;\; 16.7 & \ldots\ldots\ldots & & \\
 & & & \text{Subtract} & \underline{9.749935} \\
 & & & 2) & \underline{9.027439} \\
\text{Tang } \frac{1}{2} h = 18° \;\; 4' \;\; 32'' & & \ldots\ldots\ldots\ldots & \surd = & 9.513719
\end{array}$$

Multiply by 2

$$\begin{array}{lr|l}
\text{App. } h = & 36 \;\; 9 \;\; 4 & \because 9^h \; 28^m \; 44^s = \text{time by the observation.} \\
= & 2^h \; 24^m \; 36^s.3 & \;\; 9 \;\; 28 \;\; 45 = \text{time by the chronometer.} \\
\text{Equation} & + \; 6 \;\; 39.7 & \quad \text{Or, in astronomical language,} \\
\text{Mean } h = & \underline{2 \;\; 31 \;\; 16} & \text{Sept. 19, } 21^h \; 28^m \; 44^s = \text{time by observation.}
\end{array}$$

Mr. Maddy, in his *Elements of Plane Astronomy*, (Cambridge, 1826,) has given an equivalent formula, consisting of sines only, in a still more convenient form, thus,

$$\text{Sin } \tfrac{1}{2} h = \sqrt{\frac{\sin \frac{1}{2} (z+\Delta-\lambda). \quad \text{Sin } \frac{1}{2} (z+\lambda-\Delta).}{\text{Sin } \lambda \quad \text{Sin } \Delta,}} \qquad (b)$$

According to this formula the work will stand as follows, viz.

Sin $\frac{1}{2}(z+\Delta-\lambda)$ 54° 40′ 28″.3 . . . . log 9.911626
Sin $\frac{1}{2}(z+\lambda-\Delta)$ 4 12 35.3 . . . . . . 8.865748

Sum 8.777374

$\left.\begin{array}{l}\text{Sin } \lambda \ 38^\circ \ 28' \ 52'' \text{ log } 9.793969 \\ \text{Sin } \Delta \ 88 \ 56 \ 45 \quad\quad 9.999926\end{array}\right\}$ . . . Sum sub. 9 793895

2) 8.983479

Sin $\frac{1}{2}h$ = 18° 4′ 32″ . . . . $\sqrt{}$ = 9.491739
Multiply by 2

36 9 4
= 2^h 24^m $36^s.3$ app. h.
Equation + 6 39.7

2 31 16 = mean h, as before.

3. When a star is the object observed, the horary angle, h, is that arc of the equator which is intercepted between the star's vertical and plane of the meridian, and may lie either to the east or west of the meridian, it may be denominated, either by hours, minutes, and seconds of sidereal time, or by degrees, minutes, and seconds of arc, at the rate of 15° to each hour, and this angle added to or subtracted from the star's known right ascension, or sidereal time of its meridian passage, accordingly as it lies to the west or east of the meridian, will give the sidereal time of the observation, a sidereal day being measured by the earth's absolute rotation round its axis. This time, so far as we know, is equable, and is that astronomical time, of which the right ascension of a star or other heavenly body consists, when counted from the true vernal equinox, as a zero When, therefore, we want to ascertain the *apparent* solar time, from a stellar observation, we must have regard to the distance from the star to the sun's centre, which is obtained from a comparison of their apparent right ascensions, at the moment of the observation, to which distance, or difference, must be applied the retardation of solar compared with sidereal time, and also the equation of time taken from the preceding noon, given by the Nautical Almanac, as directed at pages 333 and 334 of our first volume, in the work of which example there is an error, in taking out the tabular quantities (pointed out in the errata), but which fortunately does not affect the method of working. Perhaps it may not appear superfluous to some of our readers, if we remark here, that the equation of time is a compound quantity, depending partly on the excentricity of the earth's orbit, that produces an equation of the centre, and partly on the inclination of the earth's axis, that renders a reduction from the ecliptic to the equator necessary, for their want of parallelism, occasioned by the obliquity of the ecliptic; of which circle unequal arcs pass the meridian in equal intervals of time

4. But in an observatory, that has two or more clocks, one of them usually indicates *mean* solar time, which is an artificial measure, not founded in natural appearances, but which

gives also apparent solar time when the *equation* is applied to it. When this equation, as given in the Nautical Almanac, is applied with its proper sign to twenty-four hours, it gives the time of noon, or of the sun's meridian passage, indicated by the solar clock in due regulation; and, *vice versâ*, when the equation is applied with a contrary sign to the mean time indicated, and corrected for rate, it will give the apparent noon in mean time. To facilitate the reduction of sidereal to mean solar time on any day, without reference to the equation of time, which often puzzles the nautical observer, we have already said, that a *Supplement* to the Nautical Almanac has lately been published, in which the sidereal time, for mean noon at Greenwich, is given for every day in the year; which will answer equally well for any other place, when $9^s.8565$ are added for every degree of western longitude, and subtracted for every degree towards the east; and in like proportion for parts of a degree When the time specified in the second column of this Supplement is indicated by the sidereal clock, the solar clock ought to indicate 0^h 0^m 0^s of mean solar time, provided both the clocks be properly timed, and their respective errors allowed for: if, therefore, we take the difference between the time shown by the sidereal clock, and the tabulated time of mean noon immediately preceding, we get the interval in sidereal time, which, converted into solar, by the table of retardation, given in page 110 of our first volume, will give the mean solar time corresponding. For instance, if we take the example of α Aquilæ, observed on the evening of the 20th of May, at a station 4^m 25^s west of Greenwich, when the sidereal time was 15^h 42^m 26^s, we have

From the Supplement, May 20, 1828 . .	3^h	52^m	$26^s.88$
Proportional part for 4^m 26^s			+ .03
Sidereal time of mean noon	3	52	26 .91
Sidereal time of the observation (page 463)	15	42	26
Sidereal interval	11	49	59 .09
Retardation in this interval		−1	56 .31
Mean solar time of the observation . . .	11	48	2 .78

According to this simple method of reducing sidereal into solar time, a mistake can hardly occur; and the reverse is equally easy, as will thus appear, viz.

Mean solar time	11^h	48^m	$2^s.78$
Acceleration of sidereal time (p. 463) . .		+1	56 .31
	11	49	59 .09
Sidereal time at mean noon	3	52	26 .91
Sidereal time of the observation . . .	15	42	26 .00

If the question had been to determine the sidereal time when the sun was observed on the 20th of September, as above stated, the solar time being first determined, the additional operation would have been as follows; viz.

Mean solar time from mean noon of Sep. 19	21^{h}	28^{m}	44^{s}
Acceleration of sidereal time		+3	31.71
	21	32	15.71
Sidereal time at mean noon Sep. 19 .	11	53	26.57
	33	25	42.28
Subtract	24		
Sidereal time of the observation . .	9	25	42.28

The same result may be obtained, in this case, more readily from the horary angle reduced into sidereal time, and subtracted from the sidereal time of noon on September 20th, as the sun was then to the east of the meridian, in the following manner,

Horary angle observed	2^{h}	31^{m}	16^{s}
Acceleration			24.85
Sidereal interval subtract	2	31	40.85
Sidereal time at mean noon Sep. 20 .	11	57	23.13
Sidereal time of the observation . . .	9	25	42.28

The nautical methods of obtaining time by means of more prolix computations, and of horary tables, are exemplified in the different Treatises on Navigation, which may be easily obtained.

§ XCII AN ACCOUNT OF THE DIFFERENT METHODS OF DETERMINING THE LONGITUDE

1. It has long been agreed among astronomers, that the equatorial diameter of the earth, taken in any direction, exceeds the polar diameter, though the quantity of excess remains yet to be finally settled, by measurements of long terrestrial arcs, or by the comparative vibrations of the pendulum, suspended in different and distant stations. Hence measured distances taken from the earth's equator towards either pole, or in the direction of its breadth, have long been denominated *latitude*, and a distance measured along, or parallel to, the equator, has been in like manner called *longitude*, as being taken lengthwise. As an arc on the equator subtends a corresponding horary angle at the pole, the subtense of this angle varies in every degree of latitude as it approaches the pole, so that the absolute length of a terrestrial degree of longitude diminishes in each successive parallel of latitude in the ratio of the cosine of that latitude to radius. Now, as the angle of the pole and its subtense are convertible terms in a known latitude, the longitude of any given place, as it regards the first or given meridian, may be determined either by geodetic measurement of the subtending arc, or by celestial observations that ascertain the horary angle, and various methods have been proposed, and practised with dif-

ferent degrees of success, to determine the longitude of one place on the globe, as it respects another, assumed as the point of no longitude, which may be fixed wherever common consent shall appoint In all our English maps and charts, as now constructed, the Royal Observatory at Greenwich is fixed upon as the first meridian, and all places, east and west of that, are said to have east and west longitude To an insular and commercial country, like England, the exact determination of the longitude has long been a matter of the utmost importance, as it respected her navy, and an Honourable Board of Longitude was therefore appointed under legal authority, to promote the laudable endeavours of any individual, whose aim might be to facilitate the attainment of this object. Hence various premiums have been awarded for improved chronometers, lunar tables, and dividing engines, that have greatly improved the former state of nautical astronomy, and rendered the problem of the longitude feasible within limitations, that can now be dispensed with. Under this feeling the English Government, at the recommendation of a Committee of Finance, has lately dispensed with the meetings and labours of this scientific board, and some new arrangement will probably be made for the continuance of our national subsidiary work, the Nautical Almanac, either in its present state, or on an enlarged and modified scale, that shall comprehend all the wants of astronomy, as well as of navigation

2 The different methods of determining the longitude of a given place, may be divided into *terrestrial* and *celestial*, which we will consider in succession. The first terrestrial method is by means of trigonometrical measurement, and as the longitude of various church steeples, and of elevated stations, have already been ascertained by the general survey of the united kingdom, which is still in operation, these determined land marks afford facilities, in every county, of obtaining the longitude of any observatory, or temporary station that may be within sight of one or more of them. We have given an example of this method, at pages 331 and 332 of our Volume I., which will render any further notice of it here unnecessary. The second of these methods that we propose to notice is that in which signals are used, which may be practised with considerable success, when the distance from the known station, or point of departure, does not exceed twenty or thirty miles, accordingly as the face of the country may be favourable to such mode of transmitting time from one place to another. When this plan is adopted, it is necessary that the exact time should be known at both stations, the longitude of one of which is supposed to be known, for ascertaining which two portable transit-instruments may be recommended, for previous or subsequent regulation and correction of the clocks or chronometers used. A rocket, or succession of rockets, sent up at certain instants previously agreed upon, will be proper signals when they burst at one station, and are observed also at the other the absolute time of such bursting will be the same at both stations, within a quantity depending on the velocity of light, not appreciable at a practicable distance. If a rocket cannot be seen at the whole distance, an elevated spot may be chosen between the two stations, where it may be viewed from the extreme stations at the same time, or a repetition of signals from several intermediate eminences may be observed; and then the difference of the observed times, taken at the two extreme stations, or the mean of the several differences will give the difference of longitude in solar time without regard to the bearing of the stations, or other computation. Of course a clear evening must be mutually agreed upon, and the

bearing of one place from the other must be so far known, as to guide the position of the telescopes, to be used on the occasion, which should have large fields of view, and consequently but small magnifying powers. This method, we understand, has recently been used with satisfactory success. The third and last terrestrial method, of comparing the longitude of one station with that of another, is by the conveyance of a chronometer, or, what is much better, of several chronometers, the rates of which are known for when the time taken at the first station is conveyed to the second, and compared with the exact time there obtained, the difference between the two, when due allowance is made for the rates during the interval, will, as in the case of signals, give the difference of longitude, and at any unlimited distance. This method has been practised with success by Dr. I. L Tiarks, who, under the employ of our government, conveyed several chronometers from England to the Isle of Madeira, the longitude of a station in which he thus determined, as a convenient place for checking the marine rates of chronometers, when vessels on long voyages touch at that island; which rates frequently differ from those previously ascertained on shore As the vessel which conveyed the chronometers was actuated by steam, that occasioned frequent shocks in its motion, the ingenious German contrived a table that was suspended in gimbals, two out of the three parallel frames of which turned on pivots at right angles to each other, and were loaded by a heavy weight, hanging down below the table, and lashed to it by diagonal ropes. Four hollow tubes were screwed down to the floor of the cabin, which contained each a spiral spring, on which the four feet of the outermost frame rested, and where the lower ends of the four diagonal ropes were fastened, that held the feet in their places This contrivance kept the surface of the table so level, and at the same time so free from the effect of the shocks from the engine, as well as from the heavings of the vessel by the waves, that the compartments, for holding the chronometers, might have been dispensed with, for their boxes had no tendency to slide from the positions given them in the fixed compartments; and as the vessel never turned suddenly round from its course, the vibrating balances were not acted upon, in a way that lengthened or shortened their vibrations, or that otherwise affected their going, which we have known to happen in a chariot, that stopped or turned suddenly. One of these chronometer tables was used on board his Majesty's steam vessel *Comet* by the inventor, in his three voyages to Denmark, Norway, Sweden, Hanover, Heligoland, &c in the year 1824. When several chronometers are used for the purpose here specified, an individual one is chosen as the standard, by which the indications of the rest are compared, in obtaining a mean of the whole, when they do not differ very considerably from each other, but in making these comparisons method and some practice are indispensable, not only to prevent confusion, but to assure accuracy. When the comparison of the standard chronometer is made with a sidereal clock, at either station, a small fraction of a second may be taken into the account by the periodic coincidences of the solar oscillations with the sidereal vibrations, when the exact value of an oscillation is known. The chronometers in our possession make just four oscillations each in the second, which train of wheel work renders them extremely convenient, in making comparisons of time, as well as in noting that which belongs to a contact in any celestial observation, where there is no clock. But the chronometer is of most general use as a nautical machine, to preserve the time given to it at the first meridian, for the solar time at the ship, determined by a single observation of a celestial body, as explained in our last section, and compared with the corrected time of the chronometer, will give a difference, equal to the longitude expressed in solar time, east when the time at the ship is the greater, but west when smaller, than the Greenwich time.

3 The celestial or astronomical methods of obtaining the longitude may be divided into six; viz. (1.) by observing an immersion or emersion of one of Jupiter's satellites, (2.) by a lunar eclipse, (3.) by the moon's passage over the meridian compared with that of a known star, preceding or following her, (4.) by measured lunar distances, (5.) by an occultation of a star by the moon, (6.) and by an occultation of the sun, or solar eclipse. With respect to the first mode by Jupiter's satellites, though it is not so accurate as some of the following methods, it is often used in observatories, and as these phenomena frequently occur, they are previously computed for Greenwich time, and published annually, three or four years in advance, in the Nautical Almanac, and when several of these occurrences have been observed with a good telescope, at any particular place, the mean of the differences between the computed Greenwich times and the corresponding times of observation will give the longitude, east or west of Greenwich, accordingly as the observed times are greater or less than the computed times given in the almanac. Two persons, however, having telescopes with different magnifying powers and apertures, will seldom agree within a second of each other, but will frequently differ much more, even at the same station. But the principal objection to this method is, that an immersion or emersion cannot be observed at sea, where a knowledge of the longitude is of daily importance. The second astronomical method can seldom be practised, from the rare occurrence of a lunar eclipse; and when an opportunity does occur, the exact instant of commencement, or of termination, is rendered uncertain by the penumbra, which gradually increases into a perfectly defined umbra, and leaves the observer in doubt as to the exact time he is to record. This method has been attempted with most success, when the times are noted, at which the lunar spots are successively observed, where the umbra makes its appulses: several observations of this kind are recorded in the earlier volumes of the Philosophical Transactions of London; and if either the luminous or dark lune were measured by a micrometer, or even by a sextant, from time to time during the eclipse, and noted both at Greenwich and at the distant station, it would answer the same purpose, for, like an immersion of Jupiter's satellites, a lunar eclipse takes place at the same instant of absolute time, at both places, and the difference of the observed times will consequently, in both cases, be the difference of longitude The remaining four methods deserve to be more particularly noticed, as being better adapted for giving correct determinations of the longitude, when properly applied. We will therefore treat these methods more fully in separate Sections.

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§ XCIII TO DETERMINE THE LONGITUDE BY THE MOON'S PASSAGE OVER THE MERIDIAN.

1. The simplest and perhaps the most accurate way of determining the longitude of an observatory from actual observation, is by ascertaining the increase of the moon's apparent right ascension, during the interval of her passages over the two meridians to be compared; for

practising which method opportunities frequently occur in every month, when the state of the atmosphere is favourable. Before we proceed to explain the different processes, it may be acceptable to some of our younger readers, if we give a familiar illustration of the nature of the moon's motion, as it has regard to the apparent motion of the sun, occasioned by that of the earth, and to the fixed position of the stars. Ferguson long ago furnished an easy means of illustrating this subject, which on this occasion we may adopt with some additions. If we suspend an ordinary watch by its chain before a mirror, so that its face, showing hours and minutes only, may be seen by reflection, its hands will revolve in a direction from right to left, as there seen, and the reversed Roman figures will follow one another in the same order of succession, as the hands appear to move. let these figures represent the ecliptic numbered into twelve signs, conceive the extreme end of the hour hand to be substituted for the sun's place, projected on the ecliptic through the centre, from a point at the opposite side of the circle, which will be the earth's place in her annual orbit; and let the end of the minute hand be substituted for the moon, while the dividing strokes or dots are considered as so many equidistant stars, near the ecliptic. On such a supposition the revolution of the hour hand will represent a solar year, and that of the minute hand a tropical revolution of the moon, which is a shorter period than a lunar month, or lunation. Now as the velocity of the moon's mean motion is to that of the sun, or rather of the earth, as 13.37 : 1, we may take 12 : 1 as sufficiently near for our purpose; which is exactly the ratio between the velocities of the minute and hour hands, let us suppose that Aries begins at the point XII of the dial, Taurus at I, Gemini at II, and so of the rest; then, if a conjunction of the sun and moon be assumed to take place exactly at the first point of Aries, that is, when both hands point to XII hours exactly; the moon (minute hand) will soon precede the sun (hour hand), and travel from one star (stroke) to another, round the circle by an uniform motion, till it has arrived at its original position, at the vernal equinox again, and during this period will have passed by the sixty stars, supposed to be situated near her path, some one or more of which she may have concealed from sight, as she passed before them, if her apparent latitude was equal to that of such stars. but the sun in the mean time has advanced, on our supposition, an entire sign, in the same forward direction, to I, the beginning of Taurus, and the moon must yet proceed something more than this sign, before she comes again into conjunction. in doing this she may conceal the same star a second time within the period of her lunation, which is longer than her absolute period, by more than two days. If we suppose the sun, for the sake of round numbers as before, to pass one degree in each day, and the moon twelve (two spaces on the dial), then one twelfth of the year will be thirty days, for the period of the moon's tropical revolution, but if we conceive a graduated circle carried round by the sun, or hour hand, and the moon travelling round such circle, the lunation, or synodic period, would be pointed out on the 360 day-spaces, viz. $\frac{360^d}{12-1} = 32^d\ 17^m\ 26^s$; whereas in fact the true tropical period is $27^d\ 7^h\ 43^m\ 4^s.7$, and her lunation $\frac{360^\circ}{13^\circ\ 10'\ 35''.027 - 59'\ 8''.33} = 29.530588$, or $29^d\ 12^h\ 44^m\ 2^s.8$. Hitherto we have considered the motions equable, or mean, as they are in the watch, but if we conceive the centres of motion displaced, that of the sun, or hour hand, but a little, and that of the moon, or minute hand, above three times as much, then some of the arcs of the ecliptic, or dial, will be passed over more rapidly than others, and it will become a matter of some complexity to compute the exact points where, after a given interval of time, the respective bodies, or hands, may be situated. and if we conceive moreover that the excentric point of the lunar orbit, represented by the circle of the minute hand has itself a motion in a forward direction in the anomalistic revolution, or $8^y\ 309^d.5$; it is obvious that the point of quick motion in the ecliptic will fall successively in different arcs of that circle, so that not only first but second differences will become necessary, to reduce the place of mean motion to the apparent place, produced by the irregularities of such lunar motion No one day's motion can be assumed as equal to that of another, and therefore the determination of the time due to an observed arc of the moon's motion in right ascension, which is the object of our inquiry, becomes a matter of some computation and when we consider also, that her orbit is not parallel to the ecliptic, but crosses it at 500 different points, in the course of 18 6 Julian years, the period of the retrograde motion of the nodes, the computation of her declination, at a given time, becomes also a subject of some difficulty, and yet it is necessary to be known, before the improved method of computing can be gone through.

2. Before the present state of theoretic and practical astronomy rendered an attention to small second, and sometimes even third differences necessary in computations of the moon's place, the old method of computing the longitude, from a change in the moon's right ascention, was a simple process, that required but two direct proportions, after the observation was made and reduced, but in bringing out the determination it was necessary, that the transit instrument, with which the moon's enlightened limb was observed, should be truly placed, and in perfect adjustment, that the passage of the centre should be thence deduced; that the error and rate of the clock should be known, that the lunar tables should be perfect; that the computations given in the Nautical Almanac, for noon of each day, should be quite correct, that the arc, contained between the moon's centre and neighbouring star, previously or subsequently observed, should be considered parallel to the equator, and that it might be taken as a direct proportional part of the daily movement of the moon's right ascension, without second differences. If these assumptions could be safely granted, the method would be attended with but little trouble in the execution. We will therefore give a short account of this as an introduction to the new method, that requires but few concessions beyond the accuracy of the observations, though it introduces more numerous computations. If we put t for the sidereal time, by the clock, of the transit of the moon's centre at any given meridian, t' for the time of the star's passage over the same; and $24 + v$ for the interval by the same clock, between two successive transits of the same star, x being the error of the clock during such interval; then the first proportion is this,

$24 + v : t - t' :: 360°$: difference of Æ of ☾ and ✱;

which difference, added to the known apparent Æ of the star, will give the moon's apparent right ascension at the moment of her meridian passage: now if we put A for the observed right ascension of the moon, and a for the computed right ascension at Greenwich, and call her daily increment of right ascension I, considering the change of Æ uniform, we shall have

$I : a - A :: 360°$: the required longitude in arc.

This method has two advantages, that are desirable, it gives the result independently of any other corresponding observation, and requires not any previous knowledge of the longitude sought, but as it can only be considered as an approximate method, and has always been found incorrect in practice, its principal use now is, to determine approximately the longitude of any given place, as an argument for determining the true longitude by the new and more correct method, that requires a previous knowledge of the approximate longitude hence it may still be useful for this preparatory purpose, particularly as the same observation, or set of observations, will supply data for both methods, of which one may be considered as introductory to the other, whenever the longitude of the place of observation is not previously known within a minute of time.

3. The new method of determining the difference of longitudes between two meridians takes no unobserved data for granted, except the moon's hourly motion in Æ, taken from an Ephemeris, and proceeds altogether upon the principle of differences, requiring corresponding observations at both the stations, or observatories, where the angular distances of the moon's enlightened limb from the same star, or stars, are observed by successive transits, taken on the same evening Here no dependence is placed on the exact position and adjustments of the instrument, or on the going of the clock, except for the short interval between the transits; nor does the accuracy of the determination depend on the lunar computations given in the Ephemeris, except so far as the moon's declination is concerned, which, being only an argument, is not necessary to be perfectly correct. Neither is the question of the earth's compression, affecting the parallaxes, involved in the computation, nor yet is an exact knowledge of the star's place indispensably necessary. Besides which advantages that this method possesses, it is not limited to time or place, but may be considered as universal. M Nicolai, a distinguished astronomer of Manheim, was the first observer who practised this method with success, which was announced and explained by Professor Schumacher in the earlier numbers of his *Astronomische Nachrichten* already referred to. Mr. Baily soon afterwards modified the process, and gave an account of it, first in a paper read before the Astronomical Society in 1824, and published in their second volume, and since in his octavo volume of Tables and Formulæ printed in 1827, which forms a most useful manual for the computer and practical astronomer. We cannot do more justice to the subject than by giving Mr. Baily's explanation in nearly his own words.

For the solution of the problem, let us make

t = the difference (in sidereal time) of the transit of the moon's *limb*, and of the star previously agreed on, at the observatory situated most *westerly*, which will be *positive* when the star precedes the moon, or when the Æ of the moon exceeds that of the star, but on the contrary *negative*

τ = the similar difference at the observatory situated most *easterly*.

$(t-\tau)$ = the true observed difference in the Æ of the moon's *limb*, for the time elapsed between the two observations·

c = the *apparent* time (as shown at Greenwich or other first meridian) of the culmination of the moon, at the *western* observatory

κ = the *apparent* time (as shown at the first meridian) of the culmination of the moon at the *eastern* observatory.

a = ☾'s Æ in *space* }
d = ☾'s true declination } computed for the time c
r = ☾'s true radius or semidiameter }
α = ☾'s Æ in *space* }
δ = ☾'s true declination } computed for the time $\varkappa$
ϱ = ☾'s true radius or semidiameter }
s = the length of the true solar day, expressed in seconds of sidereal time:
m = the moon's motion in Æ in *half* that interval, expressed in seconds of space.
χ = the *assumed* difference of longitudes in time, *plus* when west, and *minus* when east;
$(\chi+e)$ = the *correct* difference of longitudes.

Here the *true* declination and *true* semidiameter of the moon mean such as they would appear, if seen from the centre of the earth, and the Roman and Greek characters, denoting the same things, apply respectively to the two stations or observatories: the Roman characters always to the *western* station, and the Greek to the *eastern*. Let the times, c and $\varkappa$, of the moon's culmination *to the nearest minute* be found by the help of the Nautical Almanac, or other Ephemeris for the first meridian, in order to compute d, r, and δ, ϱ, for those approximate times respectively, and then make

$$\Delta^{*}=(t-\tau)\pm\frac{r}{15\,.\cos d}\mp\frac{\varrho}{15\,.\cos\delta}, \qquad (a)$$

which is the true observed difference in the Æ of the moon's *centre*, for the time elapsed between the two observations; where the upper sign is to be taken when the first or western border of the moon is observed, and the lower sign when the second or eastern border is observed. Then, by assuming χ equal to the presumed difference of longitude, and knowing the apparent (Greenwich) time at the other observatory, by the following equation

$$c=\varkappa+(\chi+\Delta)\frac{86400}{s}, \qquad (b)$$

compute a and α for the respective times c and $\varkappa$; correcting the moon's motion for third differences, if required, and the formula for the correction of the assumed difference of longitude will be

$$e=[15\,\Delta-(a-\alpha)]\,\frac{s}{2\,m}, \qquad (c)$$

which, being added to χ, will give $(\chi+e)$ for the true difference of meridians required. It is evident that, if $15\,\Delta-(a-\alpha)=0$, the value of χ has been assumed sufficiently accurate, and does not require correction. In fact, the difference will in general be very small: and when this is not the case, we may justly suspect some error in the steps of the process.

4. The example which Mr. Baily has given, in his *Astronomical Tables* and *Formulæ*, is well adapted to illustrate the subject of our present Section, but as he has given the operations in an abridged form, we propose to lay before our readers all the steps of the arithmetical process, at a sufficient length to render the computation familiar. In gaining the right

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* We have before used the character Δ, taken singly, to denote *polar distance*, but in this section it is put for *difference*, to preserve the author's notation

ascension and declination of the moon, due to the times of the observations of her meridian passage at the respective places, the approximate quantities are first found, on a supposition of equable motion, by means of logistic logarithms, or of an appropriate table giving natural numbers, or otherwise by working out the direct proportion, and then the correction from a table of second differences will convert the approximate into the true quantities. This correction has usually been obtained from the rules laid down by Dr. Maskelyne in the Nautical Almanac, but Professor Lax's new method is much shorter, and perhaps less liable to be misapplied: when the time at Greenwich is given, his rule may be thus expressed, viz.

Take out of the Nautical Almanac four successive right ascensions and four corresponding declinations for noon and midnight, two of each immediately preceding, and two immediately following the Greenwich time of each observation, and, applying + to the northern, and − to the southern declinations, add algebraically the first and fourth quantities respectively into one sum, and the third and fourth into another, then half the difference of these two sums, will at once be the mean of the two second differences, whether four right ascensions or four declinations be the quantities under consideration The same abbreviated process will apply also to the determination of correct longitudes and latitudes. Then if the sum of the first and fourth quantities be *less* than the sum of the second and third, the correction is *positive*, but if *more*, then it becomes *negative*.

5 *Example* —"On December 5, 1824, Lieut. Foster, R. N., observed the differences in the culmination of the moon's first limb, and of the two stars 62 and 95 Tauri, at Port Bowen, the station where the Expedition, for the discovery of a North-West passage, under the command of Captain Parry, passed the winter of 1824-25, while similar differences were observed at Greenwich in sidereal time as follow, viz.

	At Greenwich.	At Port Bowen.
62 Tauri	$\tau = +9^m\ 45^s.58$	$t = +24^m\ 53^s.98$
95 do	$\tau = -9\ \ 25.98$	$t = +\ 5\ \ 42.90$

What was the longitude of the latter place?"

Before this query can be solved, it is necessary to assume an approximate longitude for Port Bowen; and it will be convenient to take a mean of each pair of observations, as two single observations, to shorten the computations. Agreeably to some occultations observed by Lieut. Foster, the longitude of his station may be considered about $5^h\ 55^m\ 40^s$ west of Greenwich, and the means of the respective observations give $t = +15^m\ 18^s.44$, and $\tau = +0^m\ 9^s.80$, therefore $(t-\tau)$ will be $= +15^m\ 8^s.64$, which interval added in this case to the assumed longitude, $5^h\ 55^m\ 40^s$, gives $6^h\ 10^m\ 48^s.64$ of solar time, or $6^h\ 9^m\ 47^s.9$ sidereal time, when diminished by the acceleration $1^m\ 0^s.74$. From the Nautical Almanac of 1824 it appears that the moon's centre passed the meridian of Greenwich at $11^h\ 35^m$ P.M. on December 5, and therefore we may take $11^h\ 34^m$ as the approximate time of the first limb's passage at Greenwich, and $17^h\ 44^m$ Greenwich time for that at Port Bowen, for which times the semidiameters and declinations of the moon may be calculated, for a few seconds more or less, in these arguments, will not sensibly affect the quantities wanted. In the Nautical Almanac above referred to the moon's semidiameter is given $= 15'\ 37''$ for noon, and $15'\ 42''$ for midnight, of the day in question, and her declination 23° 0′ 3″ and 23° 40′ 27″ respectively at the said times: with respect to the former, the proportional parts for 0h 26m and 6h 10m from midnight, are −0″.18 and +2″.57 to be applied to 15′ 42″, whence we have

$$\begin{array}{ccc} \text{at } 11^{h}\ 34^{m} & & \text{at } 17^{h}\ 44^{m} \\ \varrho = 0^\circ\ 15'\ 41''.82 & \text{and} & r = 0^\circ\ 15'\ 44''.57. \end{array}$$

The declinations however undergo considerable alterations in twelve hours, being not exactly proportionate to the time; and require, besides the proportional parts, equations for second differences, according to the subjoined process,

		°	′	″	
1 Midnight December 4	. .	22°	0′	46″ dec. N	
2. Noon 5	. . . .	23	0	3	1st diff. = 40′ 24″
3 Midnight 5	. . . .	23	40	27	
4. Noon 6	. . . .	24	0	25	
Sum of 1. and 4.	. . . . . .	46	1	11	
Sum of 2. and 3.	. . . . . .	46	40	30	
Difference of the sums	. . . .		39	19	
Half ditto, or mean of second diff.			19	39 .5	

Here the sum of 1. and 4. is less than the sum of 2. and 3. and therefore the correction arising from the second difference will be positive, and the result will stand thus;

		°	′	″
Declination at noon December 5, 1824	. . . .	23°	0′	3″
P.p. as 12h . 40′ 24″ diff. in 12h ·· 11h 34′	. . . .	+	38	56 .46
Correction from Lunar Table 11 (Vol. I.) for 19′ 39″.5	.	+	0	20 .58
Moon's declination, δ, when observed at Greenwich	. .	23	39	20 .04
According to Mr. Baily's computation	. . . .	23	39	20

The proportional part might have been taken from our Solar Table 7 (Vol. I. p. 165) by doubling the first difference in twelve hours, since that table is adapted for an interval of twenty-four hours, in the following manner, without the work of a direct proportion, or reduction into the lowest terms, thus;

40′.4 (=40′ 24″) ×2=80′.8, then in the column headed 8 (for 80)

$$\begin{array}{llll} \text{We have at} & 11^{h}\ 36^{m}\ \ldots & 38'.67 & \multirow{2}{*}{\Big\}\ \text{mean} = 38'.55500} \\ & 11\ \ 32\ \ldots & 38.44 & \\ \text{And for } 0'\ 8 & \ldots\ldots\ldots & & .38555 \\ \text{Whence the proportion for } 11^{h}\ 34^{m} \text{ is} & & \ldots & 38\ .94055 \\ & & \text{Or} & 38'\ 56''.43. \end{array}$$

In the next place we proceed to find the moon's declination for December 5 at 17^h 44^m thus,

1. Noon December 5	23°	0′	3″ N	
2. Midnight of do.	23	40	27	} 1st diff. = 19′ 58″
3. Noon December 6	24	0	25	
4. Midnight do.	23	58	48	
Sum of 1. and 4.	46	58	51	
Sum of 2. and 3.	47	40	52	
Difference of the sums		42	1	
Half, or mean of 2d diff.		21	0 5	

Here we have declin at midnight of Dec. 6 5 . . .	23°	40′	27″
P.p. as 12^h 19′ 58″. 5^h 44^m ·	+	9	32 .38
Correction for second diff. 21′ 0″.5	+	2	37 .15
Moon's declination, *d*, at 17^h 44^m p. m. December 5	23	52	36 .53
According to Mr. Baily	23	53	30

We may now find the difference of the moon's right ascension in the elapsed time by the formula (*a*) thus

r = 15′ 44″.57 or 944″.57 .	log	2 9752341	ϱ = 941″.82	log	2 .9739679
Cos d 23° 52′ 36″.5 Ar. Co. . .		0 .0388554	Cos δ (23° 39′ 20″) Ar. Co. . .		0 .0381168
15 Ar. Co. . .		8 .8239087	15 Ar. Co. . .		8 .8239087
68^s 865		1 .8379982	68^s.548		1 .8359934

Now the small difference of these two numbers, = 0^s.317, being added to the observed interval $(t-r)+15^m\ 8^s.64$ will give $\Delta = 15^m\ 8^s.957$, or according to Mr. Baily 15^m 8^s.938, and the correct value of *c* may now be obtained from the formula (*b*) in the following manner; viz.

Sun's right ascension December 6, 1824 . . .	16^h	52^m	13^s.7
Ditto 5, . . .	16	47	51 .7
Difference in a solar day		4	22

so that we have $s = 24^h\ 4^m\ 22^s$, or 86662 seconds, $\chi = 5^h\ 55^m\ 40^s$, $\Delta = 15^m\ 8^s.957$; and $\varkappa = 11^h\ 34^m$ sufficiently near for our present purpose, to determine *c*, the apparent time of the moon's culmination at Port Bowen, by the following computation;

$\frac{86400}{86662}$ of 6^h 10^m 48^s.957 (or of $\chi+\Delta$) . =	6^h	9^m	40^s.54
To which add $\varkappa$	11	34	0
And there will be *c*, the app. time, . . . =	17	43	40 .54
According to Mr. Baily also	17	43	41 .67

The moon's right ascensions a and α must in the next place be computed for the times $\varkappa$ and c, by taking proportional parts and applying thereto the second differences as before, thus;

α for $11^h\,34^m$

1. Midnight Dec	4	51°	59′	58″	
2. Noon	5	59	5	59	diff 7° 16′ 25″
3. Midnight	5	66	22	24	
4. Noon	6	73	47	6	
Sum of 1. and 4 .		125	47	4	
Sum of 2. and 3. .		125	28	23	
Difference of sums			18	41	
Half, or mean 2nd diff.			9	20 .5	

a for $17^h\,44^m$

1. Noon Dec.	5 .	59°	5′	59″	
2. Night	5 .	66	22	24	diff. 7° 24′ 42″
3. Noon	6 .	73	47	6	
4. Night	6 .	81	17	25	
. . . .		140	23	24	
. . . .		140	9	30	
. . . . . .			13	54	
. . . .			6	57	

Noon December 5 . .	59°	5′	59″
As 12^h . 7° 16′ 25″ :: $11^h\,34^m$.	7	0	39 .43
Cor. for 2d diff. 9′ 20″ . .		–	9 .60
α	66	6	28 .83

Midnight December 5 . .	66°	22′	24″
As 12^h : 7° 24′ 42″ : · $5^h\,43^m\,40^s.54$.	3	32	16 .047
Cor. for second diff. 6′ 57″ .	–	0	51 .80
a	69	53	48 .247
α	66	6	28 .830
Diff. = or $a-\alpha$	3	47	19 .417
15 Δ, or $15^m\,8^s.944$ in arc .	3	47	14 .355
$15\,\Delta-(a-\alpha)$	= –		5 .062

Lastly, to find e, the error in the assumed difference of longitude by our formula (c) we have $m=7^\circ\,24'\,42''$, the first difference with which a was found, and $\frac{s}{2m}=\frac{24^h\,4^m\,22^s}{2\times7^\circ\,24'\,42''}=\frac{86662}{53364}$ $=1.624$, which multiplied by $-5^s.062$ will give $e=-8^s\,22$; consequently the correct longitude, $\chi+e$, according to this computation, where first and second differences only are used, will be $5^h\,55^m\,31^s.78$; which Mr. Baily makes $5^h\,55^m\,31^s.56$ by having used third differences also. He remarks however, in a note, that from a mean of twenty-one eclipses of Jupiter's satellites, the longitude was found to be $5^h\,55^m\,29^s$. When 15 Δ differs materially from $a-\alpha$, the operation should be repeated with χ corrected by the first operation.

In this example it will be remarked, that the exact times of the moon's limb passing each equidistant wire of the transit-instrument is not stated for either place, which statement would have facilitated the computation of the declinations and right ascensions due to those respective times, and ought always to be given as a part of the observation.

6. The principal inconveniences that attend this accurate mode of correcting the approximate longitude of a place, as compared with another place, are first, that it requires a corresponding observation at the second place of the same star on the same evening, and secondly, that the longitude cannot be computed till both observations are laid before the computer, and

therefore can be of no use till the comparison can subsequently take place. Collections of stars lying near the moon's path in each lunation, denominated *moon-culminating* stars, are now selected and arranged for several periodic astronomical works, that tend to promote the frequency of observations of this kind, and it is presumed that the arrangement which we have given of 520 zodiacal stars liable to suffer occultations, in the appendix to our first volume, will be found convenient for this purpose. The Supplement to the Nautical Almanac also gives a monthly list that, if continued, will be found very useful to the different astronomers, who now employ themselves, in various public as well as private observatories, in registering the transits of the moon-culminating stars, as well as of the moon's enlightened limb

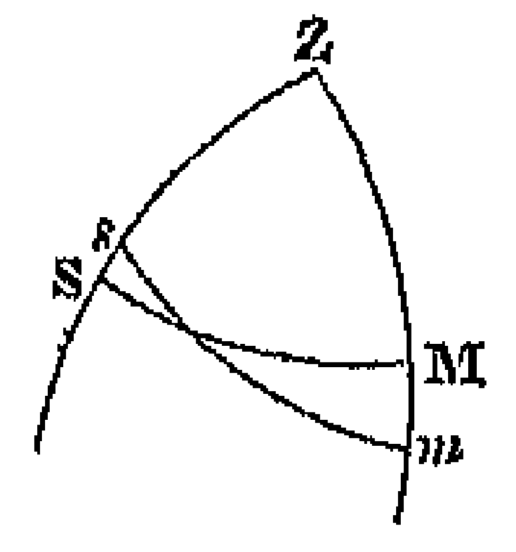
§ XCIV ON LUNAR DISTANCES

1. According to the new method of determining the difference of the longitudes of two places, by means of the differences of the moon's and star's right ascensions, as explained in our last section, it is necessary to employ a transit instrument to determine the times of their meridian passages respectively at both stations; hence that method can be of no use at sea, where the knowledge of a ship's longitude is essential in long voyages, for the correction of what is called the dead reckoning, and consequently for the safety of the crew But the measure of an oblique distance from the moon to the sun, or to one of the stars, that has its distance from the moon computed for every three hours in the Nautical Almanac, or other Ephemeris, may be taken by a sextant, or still better by a reflecting circle, either by land, or on board a ship, which measure furnishes one of the best methods of obtaining, with tolerable accuracy, the absolute longitude of the place of observation, provided that the true time of the observation be known, and also the apparent altitudes of both the bodies observed, as well as the distance, that the three apparent sides of the spherical triangle may be known, for determining the true sides by computation.

2 In the practical application of the *lunar method* of determining the longitude, the mode of making the necessary observations claims our first attention As an oblique arc and two altitudes are required to be measured at the same moment, to ensure complete success; and as a repetition of such cotemporary measures taken at successive intervals, and as near as may be equidistant, will conduce to accuracy; it has been found most convenient to employ four persons, the first to observe the oblique arc, or apparent lunar distance, with the sextant or circle; the second to measure the altitude of the moon, which may be with an octant, the third to measure that of the sun, or star, with a similar instrument; and the fourth to note down the exact time by the chronometer, at each instant when the observer taking the distance gives the notice; or, what will be more correct, the equidistant times may be announced by the person watching the chronometer, while the observers keep their respective contacts correct by the tangent screws. But if so many skilful persons are not present, one or even both of the altitudes may be taken by a single observer, both before and after the oblique distance is measured, and the corresponding times put down; from which the altitudes, due to the mean time

of all the distances, may be inferred by computation. When the sun, or star, is at a proper distance from the meridian, the time may be computed from its observed altitude, provided the latitude be known; with which the time shown by the chronometer may be compared as a check on the resulting longitude otherwise the time must be determined by a preceding or subsequent observation. It will not be necessary to give directions about taking the altitudes, by means of either a natural or artificial horizon, after what we have said in the eighty-seventh and following sections, but it is very important that the contact, in measuring a lunar distance, be, as nearly as can be estimated, at that diametrical line of the telescope's field of view, that lies parallel to the face of the sextant or circle, for at any other part, to the right or left of that line, the measured distance will be always too great, however properly the adjustment for parallelism of the telescope's collimation may have been previously made. The measure taken at the centre of the field will therefore always be the safest to adopt; and as this is the smallest measure that can be taken of a long arc, a practised observer will seldom have occasion to entertain a doubt of its accuracy; but without such precaution, there may be an error of some minutes in the measured distance, which, in spite of all the Nautical Tables, will be charged on the longitude. The apparent altitudes of the sun and moon are those which are corrected for the effects of refraction, parallax, dip, index error, and semi diameter, by the tables in the usual way, but that of a star requires not the correction for either parallax or semi-diameter · the apparent distance is the measured arc corrected for the index error, and for the moon's semi-diameter at the time when a star is the second object, but when the sun is the object, the sum of the semi-diameters of the sun and moon must be applied with the index error *.

3. Since the effect of refraction is to elevate a heavenly body, and that of parallax to depress it, the sun is more elevated by the former, than depressed by the latter, but the moon, on the contrary, is more depressed by the latter than elevated by the former, and these effects vary at different altitudes, the first question therefore that arises, which is indeed the only difficult one to a common observer, is how the apparent distance of the moon from the sun, or star, is affected by a reduction of their apparent to their true altitudes, as seen in a right line from the centre of the earth, instead of from a point on its surface through a refracting medium? To reduce the apparent to the true angular distance, is a problem that has employed the pens of many eminent mathematicians, and various tables have been computed and published, for the express purpose of clearing the apparent distance from the effect of parallax and refraction, by such rules and computations as have appeared best adapted for nautical usage. Our business however is, to consider the question in its original form, as a direct spherical problem capable of as easy solution, by means of known formulæ, as by any of the approximate or indirect tabular methods. In the annexed figure let Z represent the zenith of the place of observation, s the *apparent* place of the sun, or star; and m the *apparent* place of the moon. then, as the effects of both refraction and parallax in altitude operate in a vertical line tending to the zenith, S the *true* place of the sun or star will be lower than s the apparent place, but M, the true place of the moon, will be higher than m the apparent place, and from the measured distance $m\ s$, we want to deduce the true distance $M\ S$. Now as the apparent altitudes of m and s are known from the observation, and as their elevation or depression does not alter the azimuthal angle at Z, we can first determine this angle opposed to the apparent distance, from having the three apparent sides of the triangle given, which may be called the first operation, and when this



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[*] In strictness, a small allowance should be made for the obliquity of the measured line

angle is known, and the sides $Z\,m$ and $Z\,s$ are reduced to $Z\,M$ and $Z\,S$ by the proper tabular corrections, then, as a second operation, we may determine the true distance $M\,S$ by means of the two true sides $Z\,M$ and $Z\,S$ and included angle, which is the thing wanted

4. If we refer to our page (459) of formulæ, we shall find that Nos 1, 5, and 12, which are analogous to each other, may be applied to find the angles h, v, and α opposed to the three given sides, z, λ, and Δ, and by a change of characters any one of these three formulæ will constitute a rule for the first operation, though the triangle is not formed on the same part of the sphere also the formulæ 20 and 25, which are analogous, will either of them afford a rule for finding a side, opposed to an angle included between two given sides. In the first operation if we put α for the angle at Z, Δ for the side $m\,s$, z for the side $Z\,s$, and λ for the side $Z\,m$, formula 12 will apply as it now stands, viz.

$$\operatorname{Tan}\tfrac{1}{2}\,\alpha=\sqrt{\frac{\sin\frac{1}{2}\,(\Delta+\lambda-z)\,.\,\sin\frac{1}{2}\,(\Delta+z-\lambda)}{\sin\frac{1}{2}\,(\Delta+\lambda+z)\,.\,\sin\frac{1}{2}\,(z+\lambda-\Delta)}}\ \text{for the first operation,}$$

and for the second operation, if in like manner we put α for the angle at Z, Δ for the side $M\,S$, λ for the side $Z\,S$, and z for the side $Z\,M$, with z, λ, and α given to find Δ, formula 20 will be the proper one, where an auxiliary angle $\tan\varphi=\cos\alpha\tan z$ is introduced to simplify the computation, which formula stands thus

$$\operatorname{Cos}\Delta=\frac{\cos z.\ \cos(\lambda\sim\varphi)}{\cos\varphi}.$$

But as the arithmetical complement of $\cos\varphi$ is $\sec\varphi$, the formula becomes $\cos\Delta=\cos z.\ \cos(\lambda\sim\varphi)\ \sec\varphi$. which shortens the computation a little.

5. *Example.*—On the 14th of March, 1818, in latitude 37° 56′ N., and longitude by account 67° 0′ W., the following observations were made for determining the true longitude of a ship at anchor, when the moon was nine days old, and to the east of the meridian; the observer's eye being twenty-two feet above the surface of the sea, the time corresponding to the mean of all the observations $0^{h}\ 29^{m}\ 45^{s}$; and the index errors of the two quadrants and sextant as stated in the subjoined register, let it be required to determine the longitude from these data, by means of our formulæ?

OBSERVATIONS.

Observed alt of ☉'s lower limb			Observed alt of ☾'s upper limb			Observed distance from limbs of ☉ and ☾			
48°	51′	45″	24°	40′	30″	86°	23′	45″	
	49	15	24	52	45		24	15	
	47	30	25	1	45		24	45	
	45	30	25	12	30		25	0	
	43	45	25	21	30		25	30	
	237	45	125	9	0		123	15	Sums.
48	47	33	25	1	48	86	24	39	Means.
	− 3	13		+ 1	21		− 1	15	Index errors
	− 4	37		− 4	37		.	.	Dip of the horizon
	+16	6		−14	55		+31	1	Semi-diameters and sum.
48	55	49	24	43	37	86	54	25	Apparent altitudes and distance.

☉'s semi-diameter .	16′	6″
☉'s horizontal parallax . .		8″.95
☾'s ditto	54′	17
☾'s horizontal semi-diameter	14′	49″
Augmentation . .		6
☾'s augmented semi diameter	14	55

From these observations we have given

$Z\ s$ or $z = 90^\circ - 48^\circ$	$55'$	$49'' = 41^\circ$	$4'$	$11''$	From which we have to compute the angle α formed at the zenith Z.
$Z\ m$ or $\lambda = 90 - 24$	43	$37 = 65$	16	23	
$m\ s$ or Δ		$= 86$	54	25	

First operation, by formula 12 (p. 459).

$\text{Sin}\ \frac{1}{2}(\Delta + \lambda - z)$	55°	$33'$	$18''.5$	. . . . log	9 .9162806
$\text{Sin}\ \frac{1}{2}(\Delta + z - \lambda)$	31	21	6 .5	. . . .	9 .7162467
				Sum	19 .6325273
$\text{Sin}\ \frac{1}{2}(\Delta + \lambda + z)$	96	37	29 .5 or $83^\circ\ 22'\ 30''.5$		9 .9970904
$\text{Sin}\ \frac{1}{2}(z + \lambda - \Delta)$	9	43	4 .5		9 2273662
				Subtract this sum	19 .2244566
				Extract the root of remainder 2)	0 .4080707
$\text{Tan}\ \frac{1}{2}\alpha$	57°	$59'$	$22''.5$	. . . .	0 .2040353 = ✓
α .	$= 115$	58	45 or $-64^\circ\ 1'\ 15''$ from the triangle $Z\ m\ s$.		

We prefix the negative sign to the angle in the second quadrant, agreeably to the rule given in page 446, where the sign of a tangent in the second quadrant is $-$. Before we proceed to compute the true distance $M\ S$, in the triangle $Z\ M\ S$, with the same angle α unchanged, we must know the true sides $Z\ M$ and $Z\ S$ corrected for parallax and refraction thus,

$z\ (Z\ s)$	41°	$4'$	$11''$	65°	$16'$	$23'' = \lambda$	
Refract.			$+50$		$+2$	5	
Paral.			-6		-49	21	
z corrected, or $Z\ S =$	41	4	55	64	29	$7 = \lambda$ corrected, or $Z\ M$.	

Second operation, by formula 20.

$\text{Cos}\ \alpha$	-64°	$1'$	$15''$	. . .	$-$ 9 .6415180
$\text{Tan}\ z$	41	4	55	. . . .	9 .9404173
$\text{Tan}\ \varphi = $	-20	54	4 .3	. .	$-$ 9 .5819353
$\lambda =$	64	29	7		
$\text{Cos}\ (\lambda \sim \varphi)$	85	23	11 .3	. . .	8 .9054411
$\text{Cos}\ z$	41	4	55	. . . .	9 .8772391
$\text{Sec}\ \varphi$	20	54	4 .3	. . .	0 .0295615
$\text{Cos}\ M\ S$	86	16	44 .1	. . .	8 .8122417

6. The true distance therefore between the centres of the sun and moon was 86° 16′ 44″.1, and the Greenwich time corresponding to this distance lies between the computed distances for three hours and for six hours, as given in page 34 from "Sun West of the Moon," in the Nautical

Almanac on the given day and year and we have only to compute at what intermediate time this distance will take place, which may be done generally by direct proportion only, but if great precision be required, the approximate time may be corrected by the mean of second differences.

At 3 hours we have the distance . . .	85°	25′	20″
At 6 ditto	86	46	37
Difference in 3 hours	1	21	17
Observed distance corrected	86	16	44.1
Difference from the distance at 3 hours . .	0	51	24.1

Then as 1° 21′ 17″ 3^h :: 0° 51′ 24″ : . . .	1^h	53^m	$49^s.7$ beyond 3 hours.
Therefore add to this proportional part . . .	3	0	0
Greenwich time deduced from ☾	=4	53	49.7
Time at the ship by a celestial observation . .	=0	29	45.0
Approximate longitude	4	24	4.7 = 66° 1′ 10″.5 W.

The longitude is *west* because the time at the ship is slower than at Greenwich.

7. As the table of corrections by means of second differences is usually constructed for an interval of twelve hours, and our interval is only three hours, we may multiply the second and fourth terms of the direct proportion by 4, viz. $3^h \times 4 = 12^h$, and $1^h\ 53^m\ 49^s.7 \times 4 = 7^h\ 35^m\ 18^s.8$, in which altered terms the ratio will remain the same; and therefore the common table may be used, if $7^h\ 35^m\ 18^s.8$ be substituted, as an argument, for $1^h\ 53^m\ 49^s.7$ in the following manner:

March 14, 1818, noon	84°	4′	7″
Hour 3 . . .	85	25	20
6 . . .	86	46	37
9 . . .	88	7	59
Sum of 1 and 4 . . .	172	12	6
2 and 3 . . .	172	11	57
Difference			9
Mean of second differences			4.5

Then with the arguments $7^h\ 35^m$ at the side, and mean between 4″ and 5″ at the bottom of Lunar Table 11 (Vol. II. p. 201), we find the correction 0″.5, and as the sum of the first and last distances exceeds the sum of the third and fourth, this small correction is to be subtracted from the approximate longitude which will then be $4^h\ 24^m\ 4^s.2$. Professor Lax, who has worked this question at page 270 of his *Tables to be used with the Nautical Almanac*, has made the true distance 86° 16′ 45″ by his tabular operations, and consequently the corresponding longitude $=4^h\ 24^m\ 7^s$ or 66° 1′ 45″. Since a small difference in the distance produces a considerable difference in the longitude, obtained by this method; it is of consequence that the observations be correctly taken, as well as the computations carefully performed.

8 In No. XXXVII. of Mr. Brande's *Journal of Science* (April 1825) Mr. Charles Blackburne has published a convenient Rule for clearing the Lunar Distance from the effects of parallax and refraction, which is derived from the preceding trigonometrical method, but in which he has introduced the use of natural versed sines in conjunction with logarithmic tables. If we put D for the true distance, D' for the apparent distance, S and S' for the true and apparent altitudes of the sun or star, M and M' for the true and apparent altitudes of the moon, d for the difference of the true altitudes, and d' for the difference of the apparent altitudes, then the rule alluded to may be expressed by the following equations, of which the author has given a demonstration viz.

$$N \text{ (a natural number)} = \sin \tfrac{1}{2}(D'+d') \,.\, \sin \tfrac{1}{2}(D'-d') \,.\, \cos M \,.\, \sec M' \,.\, \left(2\,\frac{\cos S}{\cos S'}\right)$$

And secondly, Nat ver sin $D = N +$ nat ver sin d.

in which expressions there is no distinction of cases.

The operation is simple, and for our example will stand thus,

$$\begin{aligned} D' &= 86^\circ\ 54'\ 25'' \\ d' &= 24\ \ 12\ \ 12 \\ D'+d' &= 111\ \ 6\ \ 37 \\ D'-d' &= 62\ \ 42\ \ 13 \end{aligned}$$

Sin $\frac{1}{2}(D'+d')$	55°	33′	18″.5	log	9.9162807
Sin $\frac{1}{2}(D'-d')$	31	21	6.5		9.7162467
Cos M	25	30	53		9.9554850
Sec M'	24	43	37		10.0417652
Cos S	48	55	5		9.8176564
Sec S'	48	55	49		10.1824499
2					10.3010300
N	852833				9.9308639
Ver sin d	82268 (=23° 24′ 12″)				

Ver sin of D = 935101

$D = 86^\circ\ 16'\ 44''.2$

When the apparent distance consists of degrees, minutes, and any number of seconds not divisible, or not exactly given in the tables used, such seconds may be increased or diminished to a convenient even number, and the rejected or borrowed quantity, may be added to, or subtracted from the true distance, after the operation has been completed.

9. A third method of computing the true distance of the moon from the sun, or a star, by a logarithmic process, is that which depends on Borda's Theorem *, viz.

$$\operatorname{Sin}^2 F = \frac{\cos S \,.\, \cos (S-D) \,.\, \cos h \,.\, \cos h' \,.\, \sec H \,.\, \sec H'}{\cos^2 \frac{1}{2}(h+h')}$$

then, when F is known, the theorem becomes

$$\sin \tfrac{1}{2} d = \cos \tfrac{1}{2}(h+h') \,.\, \cos F.$$

Dr. O. Gregory has demonstrated this theorem in his *Elements of Plane and Spherical Trigo-*

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* Vide *Connaisance des Tems* for 1775.

nometry, (pp. 175–179,) and as the work is not liable to any ambiguity, and as the terms become all additive by adding the secant of an angle where its cosine is to be subtracted, the operation is sufficiently simple to be comprehended by mariners, who may have a difficulty in comprehending the steps necessary for performing the preceding method. We will take the same example again for illustrating this method, in which the terms representing the altitudes and distances, apparent and true, are taken thus, viz.

H = the apparent height or altitude of the sun's centre, or of a star,
H' = that of the moon's centre,
h = the altitude of the sun, or star, corrected for refraction, and for parallax when the sun is the object,
h' = the altitude of the moon, when corrected for parallax and refraction;
D = the measured distance between the centres of the two objects,
$S = \frac{1}{2}(D+H+H')$,
d = the true distance.

When the computation is made by logarithms, for which the theorem is well adapted, the sum of the various indices, for finding F, must be diminished always by 60.

Operation for determining F.

$\cos S$	80°	16′	55″.5 . .	log. 9 2273662
$\cos(S+D)$ . .	6	37	29 .5 . .	9 .9970904
$\cos h$	48	55	5 . .	9 .8176564
$\cos h'$	25	30	53 . .	9 .9554350
$\sec H$ . . .	48	55	49 . .	10 .1824499
$\sec H'$ . . .	24	43	37 . .	10 .0417652
$2 \sec \frac{1}{2}(h+h')$.	37	12	59 . .	20 .1977844
				79 .4195475
			Subtract	60
			Divide by	2)19 .4195475
sin	30	50	$12.4 = F$.	$9.7097737 = \checkmark$

Second operation.

$\cos F$ . . .	30	50	12 .4 . .	9. 9338065
$\cos \frac{1}{2}(h+h')$.	37	12	59 . .	9 .9011078
$\sin \frac{1}{2} d =$ . .	43	8	22 .1 . .	9 .8349143
and $d =$ . .	86	16	44″.2	

10. The true distance, derived from this method, is less than Professor Lax's computed distance by his tabular method, as much as the preceding logarithmic methods give it in defect and that the comparison might be fairly made, the same difference between the refrac-

tion and parallax has been adopted, that was used by the Professor himself. A great variety of methods, instrumental and tabular, have been proposed, and partially adopted, for clearing the measured distance of the moon from the sun or from a star, which are more or less convenient according to the length of the computation, and number of tables consulted in gaining the solution. Many of them depend on the small triangle, of which the arc *M m*, or *S s*, forms the base, or side. Dr. Thomas Young has given a comparative view of the principal methods of correcting lunar observations, in No. II. of NAUTICAL COLLECTIONS, in Vol. IX. of the QUARTERLY JOURNAL of SCIENCE, LITERATURE, and the ARTS, (pp. 350–371,); which the nautical student may read with advantage Among the methods explained, or alluded to, are those of Maskelyne, Lyons, Witchell, Dunthorne, Kelly, Mendoza, Brinkley, Borda, Rossel, Adams, Shepherd, and Kraft, to which may be added the names of Margetts, Mackay, Inman, Turner, Garrard, Wiseman, Thompson, Lynn, and others, who have successively devoted their attention and time to this interesting subject· but to enter into a detail of all the various processes, would enlarge our Volume much beyond the prescribed limit. For the exercise of those readers, who may wish to practise the logarithmic computation of lunar distances, we subjoin the data and resulting true distances of six different series of observations, including the example we have worked, together with the distances derived from Professor Lax's and Capt. Thompson's Tables.

☾'s hor par		App alt of ○ or *.				☾'s app alt			Apparent distance			True distance according to Lax			True distance according to Thompson.		
54′	17″	○	48°	55′	49″	24°	43′	37″	86°	54′	25″	86°	16′	45″	86°	16′	44″
58	25	○	7	38	29	46	9	28	40	36	5	41	18	54	41	18	55
53	59	*	28	24	59	61	36	50	33	30	21	33	56	49	33	56	49
58	36	*	36	36	39	6	10	10	30	52	15	30	1	34	30	1	34
55	2	⊙	52	38	40	31	13	50	56	4	55	55	32	42	55	32	42
58	30	*	32	5	51	43	23	33	96	3	26	95	30	4	95	30	5

In this list of examples, the refraction and parallaxes are supposed to be correctly computed by means of the proper corrections, depending on the state of the barometer and thermometer for the former, and for the latter on the latitude for the horizontal parallax used in gaining the parallax of the moon in altitude, and on the sun's anomaly for his horizontal parallax, the horizontal parallax of the moon, given in the Nautical Almanac, being that at the equator, may be reduced to any latitude by our LUNAR TABLE 4, (Vol. I p. 183.)

A REFERENCE TO THE WORKS OF THE DIFFERENT AUTHORS WHO HAVE BEEN MENTIONED.

BORDA'S method .	Connaissance des Tems, 1775. Dr O. Gregory's Elements.
MASKELYNE'S . .	Appendix to the Requisite Tables.
WITCHELL'S . .	Ditto.
LYON'S	Selections from the Nautical Almanac, (p. 151.) 1813.
DUNTHORNE'S .	Ditto, (p. 35)

Shepherd's . .	Tables for correcting the Apparent Distance; 4to, Cambridge, 1772.
Margett's . .	Linear Longitude Tables, 4to, and Horary Tables, folio, 1790.
Krafft's . . .	Novo Acta Petropolitanæ, 1791
Mendoza's . .	Complete Collection of Tables, &c. 4to, London, 1805.
Kelly's . . .	Practical Introduction to Spherics, 1805.
Mackay's . .	Theory and Practice of finding the Longitude, 2 vols. 8vo, 1810.
Rossel's . .	Addition to Traité Elementaire d'Astronomie, by Biot, 1811.
Turner's . . .	Longitude in Rees' Cyclopædia.
Garrard's .	Ditto ditto.
Brinkley's . .	Two practical Rules and Tables, in the Nautical Almanac, 1819.
Adams's . . .	Brande's Journal of Science, Vol. IX. p. 362.
Blackburne's .	Ditto. Vol. XIX. p. 117., 1825.
Wiseman's . .	Ditto. No. III. new series.
Inman's . . .	Treatise on Navigation, and Navigation Tables, 1821.
Lax's	Tables to be used with the Nautical Almanac, 1821.
Thompson's . .	Lunar and Horary Tables, 1825.
Lynn's . . .	Nautical and Astronomical Tables, 8vo, 1825. Horary Tables, 4to, second edit. 1828.
Connaissance des	Tems, 1775, 1785, 1796, 1807, and 1808.

It may be mentioned here, that Dr. Brinkley gave a convenient practical rule, in the Nautical Almanac of 1825, for computing the latitude at sea from the observations made in the lunar method of finding the longitude, and that Professor Lax has shown how both the latitude and time may be computed in like manner, in the Nautical Almanac of 1829.

§ XCV ON THE OCCULTATIONS OF STARS BY THE MOON.

1. When we computed several new tables of lunar parallaxes and published them in our first volume, we exemplified the methods of obtaining their quantities under different circumstances, and in the Appendix rectified some errors, but deferred the application, till we should have occasion, in our second volume, to treat of the computation of occultations of the zodiacal stars by the moon, which subject now claims our attention. As different authors have proposed peculiar methods of computing the immersions and emersions of stars, and as we are not aware that any of them have compared the respective results obtained by all the methods, we have imposed on ourselves the labour of working the same example by the different methods, that the reader may judge for himself which is entitled to preference, on the score of both conciseness and accuracy. In performing this important and difficult part of our work we have been favoured with the valuable assistance of a foreigner*, whose skill in mathematics, and whose habits in computing astronomical phenomena, peculiarly qualify him for the undertaking; and we trust that the English reader will participate with ourselves, in the feeling of gratitude,

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* Mr Mossotti, formerly of the Observatory of Milan, but now Professor of Mathematics at Buenos Ayres

with which we acknowledge the service that he has done to the astronomical computist. The best way of arranging the different methods of computing the occultations of stars by the moon, will be by that sort of classification, which arises out of their reference to particular great circles of the sphere respectively. The great circles commonly employed are 1. the horizon and meridian of the place of observation, 2. the equator and the colure of the equinoxes, 3. the ecliptic and the colure of the equinoxes; and 4. the moon's orbit and a circle perpendicular to it. Our plan is, to explain and exemplify the four methods of computation that have reference to these pairs of great circles, in successive sections, both in the case when the time of immersion or emersion of a star in a visible occultation is required to be computed, and also, on the contrary, when the exact time of a conjunction is required to be known from an *observed occultation*. Both in explaining the methods, and in working the examples, care has been taken to render the formulæ employed correct, and to repeat the operations, till the results come out truly, within the precision of a single second, for though the repetitions may appear tedious, yet they become necessary, where great accuracy is professed for the purpose of comparing the final results of the different methods.

2. *The First Method of computing Occultations of the Stars, viz by the Altitudes and Azimuths.*

When the places of the two luminaries have reference to the horizon and meridian, they are denoted by their *altitudes* and *azimuths*. Lalande in his *Traité d'Astronomie*, (Tome II. p. 390) and Mr. Herschel in the *Memoirs* of the Astronomical Society of London (Vol. I. p. 325) have each given a method of computing the occultations of stars, by employing the parallaxes in altitude and azimuth; as the latter method is the more simple in its principles, and also requires fewer data, we will adopt it in preference, and give an example, to illustrate its rules, at full length. This method is grounded upon the following rules:

I. From the moon's right ascension, as given in the Nautical Almanac, (if for Greenwich,) and the apparent right ascension of the star on the day of occultation, compute the moment of true conjunction in right ascension to the nearest minute:

II Compute the real apparent zenith distances of the moon's centre and of the star at the instant of conjunction in R. A., applying to the moon the corrections for parallax on account of the earth's ellipticity by the usual formula, or from Table 20 in page 256 of our volume I.

III. Compute the apparent azimuths of the moon's centre and of the star for the moment of conjunction, correcting that of the moon for the earth's ellipticity, by the proper formula, or by Table 25 in the page above referred to The azimuths must be reckoned from the north eastward, and if the object be west of the meridian, its azimuth must be regarded as greater than 180°

IV. Determine, from the azimuths and zenith distances thus found, whether the occultation is likely to be accelerated or retarded by the effect of parallax, if doubtful, leave it undecided

V. Repeat the computations of zenith distances and azimuths for an hour *before* the moment of conjunction, if parallax be likely to *accelerate* the occultation, but for an hour *after* that moment, if it be likely to *retard* it, or if it be dubious, which may be the case

VI. The zenith distances and azimuths of both luminaries being computed for two instants of time, at an interval of an hour from each other, let the earliest of these instants be assumed for an epoch. Let Z and z denote the respective zenith distances (apparent) of the moon and star, and A, a, their azimuths at this epoch, and let the same letters accented denote their values at the beginning of the subsequent hour:

VII Compute α and β in seconds of space by the following formulæ,

$$\alpha = Z' - z' - (Z - z)$$
$$\beta = (A' - a') . \sin z' - (A - a) \quad \sin z$$

VIII. Compute P and Q thus

$$P = (Z - z) . (A' - a') . \sin z' - (Z' - z') . (A - a) . \sin z.$$
$$Q = \frac{\alpha\ (Z - z) + \beta\ (A - a) \sin z}{\alpha^2 + \beta^2}$$

Then will P be an arc expressed in seconds and equal to the least distance of the star from the moon's centre, and Q will be the time, in the fraction of an hour, from the given epoch, when the minimum distance is attained, and this time added to or subtracted from the epoch, according to its sign, will give the moment of nearest approach, which, in ordinary cases, is the same as the middle of the occultation.

IX. Compute, from the Nautical Almanac, the moon's semi-diameter at the last determined instant, and correct it for the augmentation, then, when so corrected, call it ϱ, and if ϱ be less than P there will be no occultation.

X. The semi-duration of the occultation, in all ordinary cases, will then be expressed, in the fraction of an hour, by

$$\frac{\sqrt{\varrho^2 - P^2}}{\alpha^2 - \beta^2}$$

and this added to, and subtracted from the time of the middle of the occultation, will give the moments of emersion and immersion respectively. When extreme precision is required, the computation must be repeated, assuming for the new epoch the moment of the nearest appulse now found, and, instead of an hour's interval between the first and second instants, allowing only ten minutes. In this re-computation the value of Q will be a fraction of only ten minutes, and therefore if the value given by the formula be multiplied by 10, Q will be expressed in minutes, which must be applied, with the proper sign, to the first, or approximate time of the nearest appulse.

3. *Example.*—For the purpose of exemplifying this method of computing an occultation, let it be required to compute the first occultation that was visible in the year 1826 at Greenwich, agreeably to the data given in the Nautical Almanac of that year, viz. that of δ ♏?

According to the elements given at page 181 of the Nautical Almanac of 1826 we have the following computations registered,

☽'s conjunction with δ ♏ in R. A. Jan. 31, 1826	17^h	27^m	26^s	
☽'s and ∗'s right ascension	15	50	4	
☽'s declination	21°	0′	48″	S.
∗'s declination	22	7	1	S.

The moon's meridian passage as given at page 6 is at $18^h\,56^m$ at the moment of true conjunction, therefore she will not have reached the meridian, and will be in the east; so that the effect of parallax towards the east will be to depress her and probably to accelerate the occultation, we must then, according to the rules IV. and V., compute the moon's second place answering to an hour earlier. By interpolating the moon's places taken from the Nautical Almanac we find for an hour before, or

Jan. 31.	$16^h\,27^m\,26^s$	☽'s R. A.	$15^h\,47^m\,36^s$
		☽'s decl. S. . .	20° 56′ 17″

Also the sun's right ascensions deduced from the Nautical Almanac (pages 2 and 4) are

Jan. 31.	$16^h\,27^m\,26^s$ . .	⊙'s R. A. . . .	$20^h\,57^m\,18^s.7$
	17 27 26 . .	⊙'s R. A. . . .	20 57 28 .9

If from these right ascensions we subtract the corresponding ones of the moon and star we shall obtain

Jan. 31.	$16^h\,27^m\,26^s$	☽ − ⊙	18 50 17 .3
		* − ⊙	18 52 45 .3
	17 27 26	☽ − ⊙ and * − ⊙ .	18 52 35 .1

Which quantities subtracted from the two apparent times, or sun's horary angles, give the moon's and star's horary angles thus;

Jan. 31.	$16^h\,27^m\,26^s$	☽'s horary angle . . .	$2^h\,22^m\,51^s.3$ E.
		*'s horary angle . . .	2 25 19 .3 E.
	17 27 26	☽'s and *'s horary angle	1 25 9 .1 E.

These horary angles in time may be converted into space by the Time Table at page 109 of vol. I., thus,

$2^h\,22^m\,51^s.3$			
2^h	30°	0′	0″
22^m . .	5	30	0
51^s .		12	45
0 .3			4 .5
☽'s hor. angle	35°	42′	49″.5

$2^h\,25^m\,19^s.3$			
2^h	30°	0′	0″
25^m . .	6	15	0
19^s .		4	45
0 .3			4 .5
*'s hor. angle	36°	19′	49″.5

$1^h\,25^m\,9^s.1$			
1^h . . .	15°	0′	0″
25^m . .	6	15	0
9^s . .		2	15
0 .1			1 .5
☽'s & *'s hor. an.	21°	17′	16″.5

With these horary angles, the declinations above given, and the latitude of Greenwich 51° 28′ 40″, the following zenith distances and azimuths have been computed by the well known formulæ, viz.

Jan. 31 . .	16^h 27^m 26^s	☽'s Z. D . .	78°	53'	19".3
		azimuth	146	14	51 3
		*'s Z. D. . .	80	11	47 .2
		azimuth .	146	9	9 .6
	17 27 26	☽'s Z. D. . .	74	51	34 .9
		azimuth .	159	26	44 .1
		*'s Z D. .	75	55	57 .8
		azimuth .	159	42	43 .4

According to the rules II. and III., we must apply the corrections for parallax, both to the moon's distances and azimuths, having regard to the earth's ellipticity.

The two equatorial horizontal parallaxes given in the Nautical Almanac of 1826 are

Jan. 31 . .	16^h 27^m 26^s . .	equatorial hor. par. . . .	58'	46".7
	17 27 26 .	ditto	58	45 .9

These parallaxes, reduced to the radius of the parallel of latitude by Lunar Table 4 (page 183) with the compression $\frac{1}{309}$ and latitude 51° 29', have the tabular corrections 7" and 6".9 respectively, which being subtracted from the equatorial parallaxes given above, will leave for

Jan. 31 . .	16^h 27^m 26^s . .	hor. par.	58'	39"7
	17 27 26 . .	hor. par.	58	39 .0

To find the moon's parallaxes in altitude corresponding to those horizontal parallaxes we must enter the Lunar Table 8 (page 188) with them as arguments at the top, and with the altitudes as arguments at the side, and we shall have

Tab. 8	Top argument	58' 39".7	parallax in altitude	57'	33".7
	Side argument	11° 7'			
Tab. 8	Top argument	58' 39".0	parallax in altitude	56	37 .0
	Side argument	15° 8'			

4. As the argument of the table is the apparent altitude of the moon, as viewed from the surface of the earth, and we have employed the true altitudes as seen from the centre, we must reduce the apparent altitudes, by means of the parallaxes now found, and repeat the computation of the corresponding parallaxes, or take them again from the table with the altered altitude thus;

Tab. 8	Top argument	58' 39".7	parallax in altitude	57'	44".7
	Side argument	10 9 .0			
Tab. 8	Top argument	58 39 .0	parallax in altitude	56	51 .5
	Side argument	14 11 .0			

These parallaxes of the sphere must be again reduced for the spheroid on account of the earth's ellipticity, which produces also a small parallax in azimuth to be yet applied. The Zodiacal Tables 20 and 25 (page 256) supply these reductions in the following manner;

Tab. 20	Top argument	33°.8	reduction of paral. in alt.	$-1''.7$*
	Side argument	10 .1		
Tab 20	Top argument	20 .6	reduction of paral. in alt.	$-2''.6$
	Side argument	14 .1		
Tab. 25	Argument .	33 .8	paral. in azimuth .	$-6''.2$
	Argument .	20 .6	paral. in azimuth . .	$-3''.9$

These parallaxes in azimuth are computed in a parallel which passes through the point indicated by the moon's altitude, and should be divided by the sine of the moon's zenith distance in order to reduce them to the horizon.

5. By applying all the reductions hitherto found we shall now have

$$\text{Jan. 31}\quad 16^h\ 27^m\ 26^s \left\{\begin{array}{lllll} \text{☽'s apparent Z. D.} & 79^\circ & 51' & 2''.3 & \\ \text{app. azimuth} & 146 & 14 & 51.3 & -\dfrac{6''.2}{\sin z} \end{array}\right.$$

$$17\quad 27\quad 26 \left\{\begin{array}{lllll} \text{☽'s apparent Z. D.} & 75 & 48 & 23.8 & \\ \text{app. azimuth} & 159 & 26 & 44.1 & -\dfrac{3''.9}{\sin z'} \end{array}\right.$$

These are the moon's apparent zenith distances and azimuths which, together with those of the star above found, must now be employed in the expressions of α, β, P, and Q, and we shall then have

$$Z-z = 79^\circ\ 51'\ \ 2''.3 - 80^\circ\ 11'\ 47''.2 = -20'\ 44''.9 = -1244''.9$$

$$A-a = 146\ \ 14\ \ 51.3 - \frac{6''.2}{\sin z} - 146^\circ\ 9'\ 9''.6 = 5\ \ 41.7 - \frac{6.2}{\sin z}$$

$$Z'-z' = 75^\circ\ 48'\ 23''.8 - 75^\circ\ 55'\ 57''.8 = -7\ \ 34.0 = -454''.0$$

$$A'-a' = 159\ \ 26\ \ 44.1 - \frac{3.9}{\sin z'} - 159^\circ\ 42'\ 43''.4 = -15'\ 59''.3 - \frac{3''.9}{\sin z'}$$

and likewise

$$\begin{array}{lr|lr} \text{Log } 5'\ 41''.7 = \log 341''.7 \ \ldots = & 2.53364 & \text{Log} -15'\ 59''.3 = \log -959''.3 \ . = & \bar{2}.98195 \\ \text{Log sin } 88^\circ\ 11'.8 \ \ldots\ldots = & 9.99361 & \text{Log sin } 75^\circ\ 56' \ \ldots\ldots = & 9.98678 \\ \hline \text{Number } 336''.7 \ \ldots\ldots & 2.52725 & \text{Number} -930''.5 \ \ldots\ldots & \bar{2}.96873 \end{array}$$

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* When we reckon the azimuths according to the Rule III., the reduction of parallax in altitude must be added to the spherical parallax when the azimuth is in the first and fourth quadrants, or when the moon is north of the horizon, and subtracted in the second and third quadrants, or when she is south of the horizon The parallax in azimuth is positive in the last two quadrants, or when the moon is in the west, and negative in the first two quadrants, or when she is in the east

Therefore

$$\left.\begin{array}{l}(A-a)\sin z=\ \ 336''.7-6''.2=\ \ 330''.5\\(A'-a')\sin z'=-930.5-3.9=-934.4\end{array}\right\}\text{ and }\left\{\begin{array}{l}\alpha=-454''.0+1244''.9=\ 790''.9\\\beta=-934.4-\ 330.5=1264.9\end{array}\right.$$

From whence we deduce

$$P=\frac{1244.9\times934.4+454.0\times330.5}{\sqrt{\{(790.9)^2+(1264.9)^2\}}}$$

$$Q=\frac{790.9\times1244.9+1264.9\times330.5}{\{(790.9)^2+(1264.9)^2\}}$$

The computation of these expressions may be performed by logarithms in the following manner, viz.

2 log 790 .9 = 5 .79624 number 625520

2 log 1264 .9 = 6 .20411 number 1600000

$(790.9)^2+(1264.9)^2$ . . . 2225520 . log = 6 .34743

½ log = 3 .17376

log 934 .4 =	2 .97053	log 790 .9 =	2 .89812
log 1244 .9 =	3 .09513	. . . .	3 .09513
1163200	6 .06566	984580	5 .99325
log 454 .0 =	2 .65706	log 1264 .9 =	3 .10206
log 330 .5 =	2 .51917	. . . . . .	2 .51917
150050 .	5 .17623	418050	5 .62123
1313250	6 .11835	1402630 .	6 .14694
ar. co.	6 .82624	ar. co.	3 .65257
log P =	2 .94459	Q = 0 .63025 .	9 .79951

The time Q, which is expressed by the fraction of an hour, being reduced into minutes and seconds, and added to the epoch, will give the moment of the nearest approach, according to rule VII., thus

$$\begin{array}{rrrr} Q= & 0^h & 37^m & 48^s.9\\ \text{Epoch}= & 16 & 27 & 26\\ \hline \text{Time of the nearest approach}= & 17 & 5 & 14.9\end{array}$$

The moon's semi-diameter corresponding to this time, as given by the Nautical Almanac, will be 16′ 0″.7, and by the Lunar Table VII. the augmentation corresponding to the proper arguments will stand thus

$$\left.\begin{array}{l}\text{Top Argument } 16' \ \ 0''7 \\ \text{Side Argument } 12^\circ \ \ 41'\end{array}\right\} \text{Augmentation } 3''.6$$

We shall now have $\varrho = 16' \ \ 4''3$

$$\begin{array}{llrr}
\varrho = 964''.3 & 2 \log \varrho = 5.96842 \ \ldots\ldots & \varrho^2 = 929870 & \\
 & 2 \log P = 5.88918 \ \ldots & P^2 = 774780 & \\
 & & \overline{\varrho^2 - P^2 = 155090} \ \ldots & \log 5.19058 \\
 & & \text{Ar. Co. log } (\alpha^2 + \beta^2) \ \ldots & 3.65257 \\
 & & & \overline{2)8.84315} \\
 & & \text{Number } 0.26398 \ \ldots & 9.42157
\end{array}$$

This last number reduced into minutes and seconds, and subtracted from and added to the time of the nearest approach just found, will give the times of immersion and emersion nearly as follows;

Semiduration $= 0.26398$ . .	$=$	15^m	$50^s.3$	
Time of nearest approach . .	17^h	5	14.9	
Time of the immersion . .	16	49	24.6	
Time of the emersion . . .	17	21	5.2	

6. These times of immersion and emersion, however, are only approximate, and must be corrected by taking a short interval, as five minutes, instead of an hour, and the operation must therefore be repeated with this new interval. If we begin with the immersion, and interpolate for the times $16^h \ 44^m \ 24^s.6$ and $16^h \ 49^m \ 24^s.6$, we shall have the moon's right ascensions and declinations as follow, viz.

16^h	44^m	24^s6 . .	☽'s R. A. . .	15^h	48^m	$17^s.81$
			☽'s Declin. .	20°	$57'$	$34''.7$
16	49	24.6 .	☽'s R. A. . .	15^h	48^m	$30^s.14$
			☽'s Declin. .	20°	$57'$	$57''.4$

From these right ascensions, and from those of the sun, which are

16^h	44^m	$24^s.6$ . .	⊙'s R. A.	20^h	57^m	21^s58
16	49	24.6 .	Ditto	20	57	22.43

we deduce the moon's and star's horary angles in space at the same times,

16^h	44^m	$24^s.6$ . .	☽'s hor. angle	31°	$37'$	$54''4$
16	49	24.6 . .	Ditto	30	25	46.6
16	44	24.6 . .	*'s hor. angle	32	4	27.3
16	49	24.6 . .	Ditto	30	49	14.3

With these horary angles, the latitude of the place, and the declinations above given, the following zenith distances and azimuths are now obtained, and must be substituted for those before used, thus

16h 44m 24s.6	☽'s Z. D. .	77°	33′	52″.0
	Azimuth	149	53	57.2
	*'s Z. D. .	78	47	33.7
	Azimuth	149	54	0.7

16 49 24.6	☽'s Z. D. .	77	12	3.7
	Azimuth	150	59	15.6
	*'s Z. D.	78	24	28.1
	Azimuth	151	1	4.5

The parallaxes now to be applied have been taken from the same tables as before, and are as follow, viz.

16h 44m 24s.6	Spherical paral. in alt. . . .	57′	28″.99
	Reduction for the earth's ellipticity .	−	1.95
	Parallax in azimuth . . .	−	1.57
16 49 24.6	Spherical parallax in altitude .	57	24.56
	Reduction for the earth's ellipticity	−	2.02
	Parallax in azimuth . .	−	3.70

We shall now have the apparent zenith distances and azimuths of the moon corrected as follow, viz.

16h 44m 24s.6 . .	☽'s apparent Z. D. .	78°	31′	19″.0
	apparent azimuth . .	149	53	$57.2 - \frac{5.7}{\sin z}$
16 49 24.6 . .	☽'s apparent Z. D. . . .	78	9	26.2
	apparent azimuth .	150	59	$15.6 - \frac{5.7}{\sin z'}$

Hence we have

$$Z - z = -974''.7 \qquad {}^*A - a = -\ 3''.5 - \frac{5.7}{\sin Z}$$

$$Z' - z' = -902.2 \qquad A' - a' = -108.9 - \frac{5.7}{\sin Z'}$$

and then,

$$(A-a)\sqrt{\sin z\ \sin Z} = -9'.1 \qquad \alpha = 72.5 \qquad \beta = -103.2$$

$$(A'-a')\sqrt{\sin z'\ \sin Z'} = -112.3$$

And at last $P = \frac{974.7 \times 112.3 - 902.2 \times 9.1}{\sqrt{\{(72.5)^2 + (103.2)^2}}$

$$Q = \frac{72.5 \times 974.7 - 103.2 \times 9.1}{\{(72.5)^2 + (103.2)^2\}}$$

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[*] In these formulæ we have substituted the product $\sin z\ \sin Z$ for the factor $\sin z$, employed in the formula of Rule III, because the former gives a more accurate result in some cases, though in the present instance the difference is very trifling.

By the computation (in logarithms) of these last two quantities we obtain $\log P = 2.90460$, and $Q = .43841$.

The value of Q multiplied by 5, the interval in minutes, gives

$Q =$	0^h	21^m	$55^s.23$
Epoch	16	44	24 .60
Middle of the occultation	17	6	19 .83

7. The moon's augmented semi-diameter ought to be computed, in this repetition, for the approximate time of the immersion, but when great exactness is the principal object of the computation, the augmentation given in Lunar Table 7, being computed from the first term of an equation, will require a small correction depending on the second term, for giving which the small Tables subjoined to the work of this first method, were computed, as being supplemental to the Lunar Table 7.

Thus in the present case we have the moon's true semi-diameter at 16^h 47^m from the Nautical Almanac	$= 16'$ $0''.90$
Augmentation for the altitude 11° 29′, from Table 7	3 .27
Reduction from the subjoined supplemental Table (page 631)	0 .14
Moon's apparent semi-diameter	16 4 .31

From this semi-diameter and the values above stated we deduce $\sqrt{\frac{\rho^2 - P^2}{\alpha^2 + \beta^2}} = .42339$, which number being multiplied by 5, and expressed in minutes and seconds, becomes 21^m $10^s.77$, and this time being subtracted from that of the middle of the occultation gives 16^h 45^m $9^s.1$ for the true time of the immersion.

The approximate time of the emersion, viz. 17^h 21^m $5^s.2$, being only 6^m $20^s.8$ less than the time of conjunction in R A., which we have seen is 17^h 27^m 26^s, we will, for the purpose of finding the time of emersion, use the interval 6^m $20^s.8$, instead of 5^m as we did for the immersion, and making the former of these times the epoch, we obtain the moon's right ascension, declination, zenith distance, and azimuth corresponding thereto, agreeably to the numbers computed below, viz.

Jan. 31. . .	17^h 21^m $5^s.2$	☽'s R. A. . .	15^h	49^m	$8^s.3$
		Declination	21°	0′	19″.9
		Z. D . .	75	11	49 .3
		Azimuth .	158	0	44 .9
		✱'s Z. D. .	76	17	18 .4
		Azimuth .	158	14	16 .5

The parallaxes that must be applied to the moon's place are

The spherical parallax of altitude from Lunar Table 4	56′ 56″.5
Reduction for the earth's ellipticity from Table 20 . . .	− 2 .5
Parallax in azimuth from Table 25 (page 258)	− 4 .4

With these parallaxes we obtain the moon's apparent zenith distance and azimuth thus,

☽'s apparent Z. D.	76°	8′	43″.3
apparent azimuth .	158	0	$44.9-\frac{4''4}{\sin z}$

And therefore the differences will be

$$Z-z=-8'\ 35''1=515''.1 \qquad (a'-a)\sqrt{\sin z}\sin Z=-792''.6$$

8. These values and those above found in the first approximation, viz

$$Z'-z'=-454''.0 \qquad (A'-a')\sqrt{\sin z'}\sin Z'=-934''.2$$

substituted in the expressions of P and Q, give

$$P=\frac{515.1\times934\ 2-454\ 0\times792\ 6}{\surd\{(61.1)^2+141\ 6)^2}$$
$$Q=\frac{61\ 1\times515.1-141.6\times792.6}{(61.6)^2+(141.6)^2}$$

From which we deduce log $P=0.55853$ and $Q=-.339203$

The interval between the epoch and second instant being $6^m\ 20^s.8=380^s.8$ we must multiply the value of Q here found by 380.8 to obtain the proper number of seconds to be subtracted from the epoch, and then we shall have

Epoch	17^h	21^m	$5^s.2$
Q .		21	31.7
Middle time of the occultation	16	59	33.5

The moon's true semi-diameter 16′ 0″.7, having been augmented by the tabular quantities taken from Lunar Table 7, and the small Supplemental Table subjoined (page 631) becomes 16′ 4″.8, and by this value of ϱ and that of P just found, we lastly obtain

$$\sqrt{\frac{\varrho^2-P^2}{\alpha^2+\beta^2}}=.36185$$

This number, being also multiplied by $380^s.8$, gives $22^m\ 57^s\ 9$ to be added to the middle time above written, and the sum brings out the correct instant of emersion $17^h\ 22^m\ 31^s.4$ in apparent time at Greenwich

9 *Supplement to Lunar Table* 7, *Vol. I. p* 186.

Let D denote the moon's semi diameter; Δ D its augmentation, ϵ a constant number expressive of the ratio of the moon's parallax to her semi diameter when divided by the radius expressed in seconds, and A the moon's altitude, then the Lunar Table 7 affords the value by the formula $\Delta D = D^2 \varepsilon \sin A$ to the nearest hundredth part, but this formula is only the first portion of the whole formula that is necessary to give the value truly, even to the nearest tenth. it is requisite therefore to use one of the two following formulæ, viz.

$$\Delta D = D^2 \varepsilon \sin a + \frac{D^3 \varepsilon^2}{2} (3 \sin^2 a - 1)$$

$$\text{Or, } \Delta D = D \varepsilon \sin A + \frac{D^3 \varepsilon^2}{2} (\sin^2 A + 1)$$

in which a denotes the *true* altitude of the moon, and A the *apparent* altitude. The two following small tables give the values of the last term of each of the two formulæ respectively The first table gives the correction to be applied to the number taken from table 7, when we make use of the *true* altitude, and the second when we make use of the *apparent* altitude.

10. TABLES *containing the second term of the Augmentation of the Moon's semi-diameter*.

TRUE ALTITUDE.

☽'s Alt	☽'s true semi-diameter	
	14′ 30″	17′ 0″
0°	−0″.10	−0″.17
10	−0 .09	−0 .15
20	−0 .07	−0 .11
30	−0 .03	−0 .04
40	0 .02	0 .04
50	0 .08	0 .13
60	0 .13	0 .21
70	0 .17	0 .28
80	0 .20	0 .32
90	0 .21	0 .34

APPARENT ALTITUDE.

☽'s Alt	☽'s true semi-diameter	
	14′ 30″	17′ 0″
0°	0″.10	0″.17
10	0 .11	0 .17
20	0 .12	0 .19
30	0 .13	0 .21
40	0 .15	0 .24
50	0 .17	0 .27
60	0 .18	0 .29
70	0 .21	0 .32
80	0 .21	0 .33
90	0 .21	0 .34

11. *The Inverse Problem, or computation of an observed occultation by the Altitudes.*

We come now to consider the inverse problem, by the method of altitudes, and to show how the time of the true conjunction, or nearest approach of the moon to a star, may be deduced from an *observed* occultation. A solution of this problem, in which the parallax in altitude only is employed, has been given by Dr. Thomas Young, in the Nautical Almanacs of the years 1826 and 1827, which for its conciseness merits our attention, and may in several instances be usefully employed. As an example for illustrating this method, we will give the same occultation that furnished data for our preceding computations. The immersion of δ ♏, as we have already seen, is computed to happen at Greenwich on Jan. 31, 1826, at $16^h\ 45^m\ 9^s.1$. now supposing this to be an observed occultation, let us determine the instant of the moon's nearest approach.

The author supposes that the difference of the apparent altitudes at the moment of immersion, or at least that the moon's altitude has been observed. In our example this last altitude, corrected for refraction, ought to be 11° 31′ 58″. The following are the other elements which must be either prepared, or taken from the Nautical Almanac.

Apparent time of the observation	16^h	45^m	$9^s.1$ -
☉'s right ascension at the same time	20	57	21 7
Right ascension of the mid-heaven	13	42	30 .8
δ ♏'s right ascension	15	50	4
δ ♏'s horary angle . . .	2	7	35 .2
δ ♏'s declination	22°	7′	1″ S
☽'s parallax in altitude	0	57	27
☽'s apparent semi-diameter . . .	0	16	4 3
Diff. of declin between moon and star	1	6	13 N
Polar orbital angle	S 82	40	0 E

These elements being prepared correctly,

RULE I Compute the altitude of the star by means of the horary angle, the declination above given, and the true latitude of the place, which may be done by the following process, viz

H. A. = 31° 53′ 18″ . . log. $\sin^2 \frac{1}{2}$ H A. = 9 .178749
Dec. = −22 7 1 . . . . . . log cos dec . = $\bar{9}$ 966807
Lat. = 51 28 40 . . . . log cos lat. . = 9 .794361

Nat. Number . . . . −0 .087080 . $\bar{8}$ 939917
Sum of lat. and dec 73° 35′ 41″ cos 0 .282430

Sin alt. . . . . = 0 195350 ✱'s alt = 11° 15′ 55″
☽'s observed altitude . 11 31 58

Difference of the altitudes . . 16 3
☽'s parallax in alt. add . . . 57 27

Difference of the true altitudes 73 30

RULE II. Having found the difference of the true altitudes from the difference of the apparent altitudes combined with the parallax of the moon in altitude, add together the squares of the semi-diameter properly augmented, and of the difference of the true altitudes, and subtract the square of the difference of the apparent altitudes, the remainder will be the square of the true distance Thus

The semi-diameter . . . . 16′ 4″.3 = 964″ 3 squared . . = 929875
Difference of true altitudes . . 73 30 = 4410 squared . . 19448100
Difference of apparent altitudes . 16 3 = 963 ar. comp of square 99072631

True distance . . . . squared . . 19150606

RULE III From the difference of declinations at the conjunction, reduced in the ratio of the radius to the sine and cosine of the orbital angle, we obtain the nearest distance of the star from the orbit, and the distance of the nearest point of the orbit from the point of conjunction In the Nautical Almanac for 1827, and for the succeeding years, we find these arcs ready computed The square of the nearest distance subtracted from that of the true distance, gives the square of the orbital distance from the point of nearest approach, which, being converted into time by means of the moon's hourly motion, and applied to the time of immersion or emersion, will show the true time of the nearest approach, to be compared with that of the Nautical Almanac

12. In our example we have, from the elements of the Nautical Almanac,

Nearest distance 66′ 13″ sin. 82° 40′= . . .	3940″.5
Time of the nearest approach	17^h 41^m 51^s
Consequently nearest distance squared	15527540
True distance squared	19450606
Diff^r from the nearest point 1980 .7 squared . . .	3923066

Now the hourly motion being 34′ 51″, the distance 1980″.7 = 33′ 0″.7 becomes equivalent to 56^m 50^s, which time, added to the time of observation, viz. 16^h 45^m $9^s.1$, gives the time of the nearest approach = 17^h 41^m $59^s.1$, in perfect accordance with that deduced from the Nautical Almanac Should a difference be found in any case, it must be considered as an error of the tables, or of the observation.

13. *Remark.*—With respect to this method we ought to notice, that it should never be employed when the moon is very near the zenith, and when extreme accuracy is required, it is necessary to subtract from the square of the difference of the true altitudes, the reduction given by the subjoined formula,

$$sp\,(d^2-(a-\alpha)^2)\,\text{tang}\,a+sq\sqrt{d^2-(a-\alpha)^2}$$

where d denotes the apparent semi diameter of the moon,
a her apparent altitude,
p her parallax in altitude,
q the parallax of Azimuth as given by Zodiacal Table 25.
α the altitude of the star,
s the sine of a second.

§ XCVI THE SECOND METHOD OF COMPUTING OCCULTATIONS OF THE STARS, VIZ BY THE RIGHT ASCENSIONS AND DECLINATIONS

1 IF we suppose any two great circles to cross one another at right angles, in any direction, at the place of a star to be occulted, and if we call m and n the cosines of the arcs which unite

the poles of these two circles with the true centre of the moon, and μ and ν the cosines of the arcs which unite the same poles with the geocentric zenith of the observer, we shall have at the moment of the immersion or emersion, the equation

$$(m-\pi\mu)^2-(n-\pi\nu)^2=\sin^2 d \tag{1}$$

where π denotes the sine of the horizontal parallax, and d the true semi-diameter of the moon This equation has been deduced by analysis, expressing the condition, that the moon, considered as a sphere, should at the moment of immersion or emersion, have as a tangent the straight line proceeding from the eye of the observer to the star, and as no limitation was introduced in the analysis, the above equation may be considered as mathematically accurate.

If we were to construct a figure to represent the result of our equation, by assuming for the plane of the coordinates, a plane passing through the moon's real centre, and perpendicular to a straight line proceeding from the centre of the earth to the star, and for their axes the intersections of this plane with the planes of the two great circles that we before supposed to be crossing one another, we may easily perceive,

First, that m and n represent the coordinates of the centre of the moon in parts of her geocentric distance from the earth taken as unity;

Secondly, that $\pi\mu$ and $\pi\nu$ represent the coordinates of the point where the observer projects the star upon the said plane,

And thirdly, that therefore $m-\pi\mu$ and $n-\pi\nu$ are the coordinates of the moon's centre, relative to the point where the star is projected, and that, according to our equation, at the moment of an immersion or emersion, these relative coordinates are the legs of a right-angled triangle, of which the moon's true semi-diameter is the hypothenuse.

From this reasoning it clearly appears, that our analysis has led us to the same result produced by Cassini's method, commonly called the method of (or by) projections *.

In the formula we have given there is a peculiarity that may require some explanation, which is, that the computation of parallaxes is performed by employing the plane of the star, instead of that of the moon. As the reason of this we may observe, that, at the moment of the immersion or emersion, the point of the moon's limb which appears in contact with the star, has the same altitude as the star, so that if we apply the parallax to it with a contrary sign, we shall obtain the true position of that point of the moon's limb, which will be distant from her centre by a quantity that is just equal to her semi diameter. Du Sejour was, we believe, the first person, who by his analysis arrived at a process, which has the advantage of employing the true semi-diameter of the moon, requiring no augmentation on account of her altitude, and of rendering the variations of parallax such as depend on the horary angle of the star, the place of which may be considered immutable during an occultation. By choosing, for one of the great circles above-mentioned, the circle of the star's altitude, declination, or latitude, or a circle perpendicular to the moon's true orbit, we may obtain different solutions of the problem of an occultation of a fixed star by the moon, in which the parallaxes of alti-

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* Vide Astronomie par M De Lalande, Tome II. p. 491

tude, of right ascension and declination, or of longitude and latitude, &c. are used according to the circles chosen[*].

The example we propose to give, agreeably to one of the most approved methods, is that which employs the circle of declination, from the work of which the reader will afterwards be enabled to apply the formula to cases depending on other circles, to which the same equation is applicable.

2 Let A denote the right ascension of the moon, B her declination, a the right ascension of the star in question, and b its declination, and it will be easy to deduce by spherical trigonometry, or by the more general method of the coordinates,

$$(2)\quad m = \cos B . \cos (a-A)$$

$$n = \sin b \; \cos B . \sin (a-A) - \cos b . \sin B = \sin (b-B) - 2 \sin b . \cos B . \sin^2 \tfrac{1}{2} (a-A)$$

As the arcs $a-A$ and $b-B$ are generally small, we may substitute them for their sines, and we shall then have in seconds of space

$$(2')\quad m = (a-A) . \cos B$$

$$n = b - B - \frac{1}{2r} . \sin b . \cos B \; (a-A)^2$$

where r denotes the radius expressed in seconds.

Likewise if we represent the right ascension of the mid-heaven by θ, and the reduced latitude of the place, using lunar Table VI for gaining the reduction, by φ, we have

$$(3)\quad \mu = \cos \varphi . \sin (a-\theta)$$

$$\nu = \sin b . \cos \varphi . \cos (a-\theta) - \cos b . \sin \varphi$$

These formulæ and the equation marked (1) afford us the means of laying down rules for the computation of an occultation.

When the object we have in view, in computing an occultation, is to ascertain only the *approximate* time of an immersion or emersion, in order that the observer may be prepared for his observation in due time, it may be sufficient to adopt a short computation instead of a more correct one. Before, therefore, we proceed to exemplify our method at full length, we will give an example of the computation of an occultation by means of an approximation, in which we will employ the data already prepared in the Nautical Almanac.

3. *Rules for computing the Approximate Times of the Immersion and Emersion of a Visible Occultation.*

RULE I. Take from the TABLE of ELEMENTS contained in the Nautical Almanac of the year in question the horary motion of the moon in her orbit, the orbital angle, the difference of declination between the star and the moon corresponding to the time of true conjunction, and the right ascension of the star. Find also for the same time the logarithm of the equatorial parallax, and reduce it to the parallel of the equator by applying the reduction for the earth's ellipticity; and lastly take out of the same book the moon's true semi-diameter.

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[*] Properly speaking they are not the parallaxes of right ascension, of longitude, &c that are used in our formulæ, but the parallaxes in a circle passing through the star and perpendicular to its circle of declination, of latitude, &c which parallaxes indeed differ but little from the former ones, when reduced to the parallel of the star.

Example.—For computing the occultation of δ ♏, on Jan. 31, 1826, to which we have applied our first method, by ALTITUDES, we find in page 181 of the Nautical Almanac of that year the following elements, viz.

Time of the true conjunction	17ʰ 27ᵐ 26ˢ
Hourly motion of the moon in her orbit . .	3′ 51″
Orbital angle	S. 82° 40′ 0″ E
Difference of declination of the sun and moon	66′ 13″ N.
Declination of the star (say)	22° 7′ 1″
Right ascension of the star (say)	15ʰ 50ᵐ 4ˢ

The logarithm of the equatorial parallax, expressed in seconds of space, is obtained by adding to the arithmetical complement of the proportional logarithm, given in the Nautical Almanac, the constant number 3 0334*. We shall now have

Equatorial parallax at 17ʰ 27ᵐ 26ˢ Ar. Co. . . .	0 .5319
Constant number	3 .0334
Logarithm of the equatorial parallax	3 .5653

To reduce this logarithm on account of the ellipticity of the earth we may use Lunar Table 5, or otherwise a small table by Dr. Young which we have inserted in our *fourth method*, under Rule I, which answers the same purpose.

The latitude of Greenwich, which is 51° 28′ 40″, being comprehended between 51° and 56°, according to the latter table we have to subtract 9 from the logarithm of the equatorial parallax, in order to obtain the logarithm of the horizontal parallax of the place, which will therefore be . . . . 3 .5464
The moon's true semi diameter deduced from the Nautical Almanac corresponding to the time of the true conjunction is . . . . 16′ 1″.

RULE II. Find for the time of true conjunction, and for an hour before, if the occultation is likely to be accelerated by the effect of parallax, or for an hour after, if it is likely to be retarded, the values of the cosines, which we have denominated m and n in equation (1) For this purpose let it be observed that at the first period, or time of true conjunction, in this case, we have $m=o$ in seconds of space, and $n=$ the difference of declination between the moon and star, taking it negatively when it is marked N in the Nautical Almanac. With respect to the cosines corresponding to the second time, and which we will distinguish by the accented letters m and n', multiply the hourly motion of the moon in her orbit by the sine, and after by the cosine of the orbital angle when it is marked $N-E$ in the Nautical Almanac, or of its Supplement when it is marked $S-E$. The first product taken with its own sign, will be the value of m' for an hour before, and with the contrary sign for an hour after, and the second product algebraically added to the value of n, or subtracted from it, will give the value of n' likewise, for an hour before or an hour after.

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* We have preferred employing the direct logarithm of the parallax, as it is found in an ordinary table of logarithms, to the *proportional* logarithm, which requires a particular table not contained in our work, but those who like the use of such table may apply to it, and spare the reduction by the constant logarithm

Example.—The moon being at the moment of true conjunction in the east, in our case, the depression of parallax will carry her apparently sooner towards the star, and the occultation will be accelerated we must therefore compute m and n for an hour before, and we shall have

$m=0$			
$n=-66'\ 13''=$ . . .	$-3973''$		
☽'s hourly motion $34'\ 57''=2097$		log	3.3204
Suppl. of the orbital angle $S-E$ 97° 20′		log sin	9 9964
		log cos	$\bar{9}$.1060
$m'=34'\ 34''=2074''$ . .		log m'	3.3168
Natural number	-267		$\bar{2}$.4264
$n'=-70'\ 40''=$	-4240		

RULE III. Compute now for the two times for which we have found the values of m, n, and m', n', and according to the formulæ (3), the values of $\pi\mu$, $\pi\nu$, and $\pi'\mu'$, $\pi'\nu'$, employing the reduced or geocentric latitude of the place $=\phi$, which will be 51° 17′ 43″, if we take 10′ 57″ as the reduction from either of the tables before specified, and subtract it from 51° 28′ 40″.

The horary angles of the star corresponding to the times for which the values of m, n, and m', n' have been found, viz. $17^h\ 27^m\ 26^s$ and $16^h\ 27^m\ 26^s$ are to be computed thus;

Times chosen		17^h	27^m	26^s	16^h	27^m	26^s
Sun's corresponding R. A. from the Nautical Almanac		20	57	29	20	57	19
Right ascension of the mid-heaven	$\theta=$	14	24	55	13	24	45
Right ascension of the star	$a=$	15	50	4	15	50	4
Horary angle in time	$a-\theta=$	1	25	9	2	25	19
Horary angle in space	$a-\theta=$	21°	17′	15″	36°	19′	45″

With these horary angles, the reduced latitude, the declination of the star, and the horizontal parallax above given, the computation of $\pi\mu$, $\pi\nu$, $\pi'\mu'$ and $\pi'\nu'$ may be thus performed,

Times as before		$17^h\ 27^m\ 26^s$	$16^h\ 27^m\ 26^s$
π = parallax	$\log \pi =$	3 5464	
φ = reduced latitude 51° 17′.7 . . .	$\log \sin \varphi =$	9 .8923	
	$\log \cos \varphi =$	9 .7961	
	$\log \pi . \sin \varphi =$	3 .4387	
	$\log \pi \cos \varphi =$	3 .3425	
$a-\theta$ = horary angle . . .	$\log \sin (a-\theta) =$	9 .5600	9 .7726
	$\log \cos (a-\theta) =$	9 .9693	9 .9061
	$\log \pi \cos \varphi . \sin (a-\theta) =$	2 .9025	3 .1151
	$\log \pi . \cos \varphi \cos (a-\theta) =$	3 3118	3 2186
(3) .	$\pi\mu = \pi \cos \varphi . \sin (a-\theta) =$	799	1 .303
	$\log \sin b =$	$\bar{9}$ .5758	
	$\log \cos b =$	9 .9668	
	$\log \pi \cos \varphi . \sin b . \cos (a-\theta) =$	$\bar{2}$ 8876	$\bar{2}$ .8244
	$\log \pi \sin \varphi . \cos b =$	3 .4055	
	$\pi \cos \varphi . \sin b . \cos (a-\theta) =$	-772	-667
	$\pi \sin \varphi . \cos b =$	2544	2544
	$\pi\nu = \pi \cos \varphi . \sin b . \cos (a-\theta) - \pi \sin \varphi . \cos b =$	-3316	-3211

Rule IV Form the values of the differences $m-\pi\mu$, $n-\pi\nu$, both corresponding to the first and second times respectively. These differences, as we have before observed, represent the co-ordinates of the moon's centre relatively to the star upon the plane of the projection, and in this case they are counted upon the axes, the first of which is perpendicular to the circle of the star's declination, and the second is parallel to it. Divide by 12 the difference which arises from subtracting the relative co-ordinates that correspond to the second time, from those that correspond to the first, and, by adding the quotient successively to the latter, compose a table of the values of the relative co ordinates from five to five minutes, and extend it as far as may appear necessary. The examination of this table will disclose the two successive times, in which the sum of the squares of the two co-ordinates will pass from being greater than the square of the moon's semi-diameter, to being smaller, or vice versâ.

Compute now the excess and defect, given by the table, and by proportion find the time in which the sum of the squares of the two co ordinates will be equal to the square of the moon's true semi-diameter, and that will be the time of the immersion or emersion, as the case may be.

Example.—We have, for the first time,

$m=0$	$n=-1^\circ\ 6'\ 13''$
$\pi\mu=13'\ 19''$	$\pi\nu=-0\ \ 55\ \ 16$
$m-\pi\mu=-13\ \ 19$	$n-\pi\nu=-0\ \ 10\ \ 57$

and for the second time,

m' . . . $= 34'\ 34''$ $\qquad$ n' . . . $1^\circ\ 10'\ 40''$

$\pi'\mu'$ . . $= 21\ \ 43$ $\qquad$ $\pi'\nu' = -0\ \ 53\ \ 31$

$m' - \pi'\mu'$. $= 12\ \ 51$ $\qquad$ $n' - \pi'\nu' = -0\ \ 17\ \ \ 9$

The second differences will therefore be,

$-26'\ 10''$ $\qquad$ and $\qquad$ $6'\ 12''$

$\frac{1}{12} = 2\ \ 10.83$ $\qquad\qquad$ $\frac{1}{12} = 0\ \ 31$

With the twelfth parts of these second differences the following Table was constructed by perpetual addition, in which the values of the co-ordinates $m - \pi\mu$ and $n - \pi\nu$ are expressed in minutes and hundredths of minutes.

4. *A Table of Co-ordinates with their corresponding Times*

Times	Values of $m - \pi\mu$	Values of $n - \pi\nu$	Moon's Semi-diameter
$16^h\ 27^m\ 26^s$	12′, 85	−17′, 15	16′, 02
32 26	10, 67	−16, 63	
37 26	8, 49	−16, 11	
42 26	6, 31	−15, 60	
47 26	4, 13	−15, 08	
52 26	1, 95	−14, 57	
57 26	− 0, 23	−14, 05	
17 2 26	− 2, 41	−13, 53	
7 26	− 4, 59	−13, 02	
12 26	− 6, 77	−12, 50	
17 26	− 8, 95	−11, 98	
22 26	−11, 13	−11, 47	
27 26	−13, 32	−10, 95	

A simple examination of this Table will show that the immersion must take place between $16^h\ 42^m\ 26^s$ and $16^h\ 47^m\ 26^s$; for we have

At $16^h\ 42^m\ 26^s$

$6', 31 \times 6, 31$ . . . $= 39, 81$

$15, 60 \times 15, 60$ . . $= 243, 36$

Sum . . . . . . $= 283, 17$

$16, 02 \times 16, 02$ . . . $= 256, 64$

Excess . . . . $= 26, 53$

And at $16^h\ 47^m\ 26^s$

$4, 13 \times 4, 13$ . . . $= 17, 06$

$15, 08 \times 15, 08$ . . $= 227, 41$

Sum . . . . $= 244, 47$

$16, 02 \times 16, 02$ . . $= 256, 64$

Defect . . . . $= 12, 17$

Now the interval between the two times being 5^m or 300^s, we may say, as $26', 53 + 12', 17 \cdot 300^s :: 26', 53 \,.\, x = 206^s = 3^m\ 26^s$.

This time being added to $16^h\ 42^m\ 26^s$ gives, for the first approximation, the instant of immersion $16^h\ 45^m\ 52^s$.

To correct this first approximation we may now repeat the computation for the two minutes between which the time of immersion just found is comprised, thus,

At $16^h\ 45^m\ 26^s$	
$m-\pi\mu = 5,00 \times 5,00$.	$= 25,\ 00$
$n-\pi\nu = 15,25 \times 15,25$.	$=232,\ 56$
Sum	$=257,\ 56$
Semi diam. 16, 02 × 16, 02 .	$=256,\ 64$
Excess	$=\ 0,\ 92$

At $16^h\ 46^m\ 26^s$	
$m-\pi\mu = 4,57 \times 4,57$	$= 20,\ 88$
$n-\pi\nu = 15,19 \times 15,19$.	$=230,\ 73$
Sum	$=251,\ 61$
Semi-diam 16, 02 × 16, 02	$=256,\ 64$
Defect .	$=\ 5,\ 03$

Hence we have to obtain a second correction by the following proportion thus, as $0', 92 + 5', 03$ 60^s :: $0', 92$ $x = 9^s$, and the immersion will take place very nearly at $16^h\ 45^m\ 26^s + 9$ or $16^h\ 45^m\ 35^s$.

In like manner it appears from the Table, that the emersion is likely to happen between $17^h\ 22^m\ 26^s$ and $17^h\ 27^m\ 26^s$, and we have

At $17^h\ 22^m\ 26^s$	
11, 13 × 11, 13 . .	$=123,\ 88$
11, 47 × 11, 47 . . .	$=131,\ 56$
Sum	$=255,\ 44$
16, 02 squared . .	$=256,\ 64$
Defect . . .	$=\ 1,\ 20$

And at $17^h\ 27^m\ 26^s$	
13, 32 × 13, 32 .	$=177,\ 42$
10, 95 × 10, 95 .	$=119,\ 90$
Sum . . .	$=297,\ 32$
16, 02 squared . .	$=256,\ 64$
Excess	$=\ 40,\ 68$

and from hence the following proportion, as $1', 20 + 40''.68$ 300^s · $1', 20$ $x = 9^s$

So that $17^h\ 22^m\ 26^s + 9^s = 17^h\ 22^m\ 35^s$ will be the Greenwich apparent time of the emersion, and a repetition of the same computation would increase the time here given by about a second.

5. *By Construction.* Instead of computing a Table, it will be convenient in practice to find the times of immersion and emersion by constructing a figure Let SP, SO, fig 1 of Plate XXXI, represent two straight lines parallel to the axes above mentioned, and crossing in the point S, which is the projection of the star upon the plane of the co-ordinates. To those lines as axes apply the co-ordinates Sp, pm equal to the values of $m-\pi\mu$ and $n-\pi\nu$, corresponding to the first time, and the co-ordinates Sp', $p'm'$ equal to the values of $m'-\pi'\mu'$ and $n'-\pi'\nu'$; then join the points m and m' by the straight line mm', which will represent the apparent path of the moon relative to the star. From the point S as a centre, and with a radius equal to the moon's true semi-diameter, describe a circle, then the lines $m'i$, me, comprehended between the point m, or moon's place at the first period, and the points e and i, where the circle cuts the line mm', will represent, in parts of mm', as an hour, the times between the first epoch and the instants of immersion and emersion respectively.

6. *By Projection.* We may also obtain the values of $\pi\mu$ and $\pi\nu$ from a drawing on which the ellipses are delineated that rise out of projection of the observer's parallel upon planes perpendicular to straight lines directed to different degrees of declination. It is not our intention however to enter into the particulars of the construction and use of such drawings, which the reader will find explained in Lalande's *Traite d'Astronomie,* Tome II., p. 491, et seq

Mr Baily, the late President of the Astronomical Society of London, whose zeal for the promotion of both the theory and practice of astronomy is too well known to require our commendation, has caused some of these drawings to be engraved and printed, suitable for the latitude of Greenwich, and on a scale of one foot to the degree The values of $\pi\mu$, $\pi\nu$, and $\pi'\mu'$, $\pi'\nu'$ were taken from one of these, and employing the construction above described, we found the time of immersion thereby $16^h\ 45^m\ 46^s$, and that of the emersion $17^h\ 22^m\ 32^s$, which values are sufficiently correct for the purpose of giving notice to an observer, who is preparing to make the observation

When, in the computation of a visible occultation, we aim at the greatest accuracy, we must have regard to the second differences, as well of the moon's motions in right ascension and declination, as of the variations of parallax, which object may be accomplished by the following rules.

7. *Rules for Computing with Accuracy the exact Times of Immersion and Emersion of a Visible Occultation.*

By Right Ascensions and Declinations.

Rule I. Find the moment of the true conjunction in right ascension of the moon with the star in question, and the difference of declination between them corresponding to that moment, or take these data from the Nautical Almanac, where they may be found already computed. Determine the horizontal parallax and semi-diameter of the moon for the moment of true conjunction, and compute her hourly motions of the first and second order, as well in right ascension as in declination, and also the hourly variations of parallax and semi-diameter, after which determine the right ascension of the sun, and his hourly variation, all which will be illustrated by the following work.

Example —We will again take the occultation of δ ♏, which we have already computed, and then we shall have

The time of true conjunction in right ascension		17^h	27^m	26^s
Right ascension of δ ♏		15	50	4
Declination of δ ♏		22°	7′	1″ S.
Difference of declination between ☽ and * .	$b-B$	0	6	13
Equatorial horizontal parallax of the moon		0	58	46
Reduction of lat. 51° 28′ for compression $\frac{1}{309}$ (Lun. Tab. 4.)		0	0	7
Horizontal parallax		0	58	39
True semidiameter of the moon		0	16	0.7

The moon's hourly motions of the first and second order, as well in right ascension as in declination, may be deduced by the method of second differences thus,

		Difference.	2d diff.	
☽'s R. A. Jan. 31, 1826, noon	226° 48′ 51″			
midnight	234 9 21	7 20 30	4′ 1″	Mean 3′ 6″
Feb. 1, noon	241 33 52	7 24 31	2 12	
midnight	249 0 35	7 26 43		

☽'s hourly motion, term of second order $\frac{3' 6''}{288}$	0″.6
From midnight to the time of true conjunction	5h 27m
Complement to six hours	0 33
Twice the complement in the tenth of an hour	1.1

Then the term of first order of the ☽'s hourly motion will be

$$\frac{7^\circ\ 24'\ 31''}{12} - 1.1 \times 0''.6 = 37'\ 2''.6 - 0''66 \quad \ldots\ldots \quad = 37'\ 1''.9$$

		Difference.	2d diff.	
☽'s Dec. Jan. 31, noon	−19° 23′ 7″			
midnight	−20 34 33	−1° 11′ 26″	18′ 59″	Mean 19′ 24″
Feb. 1, noon	−21 27 0	−0 52 27	19 50	
midnight	−21 59 37	−0 32 37		

☽'s hourly motion in Dec. term of second order $\frac{19' 24''}{288} = 4''.0$

Term of first order of the moon's hourly motion in declination

$$= \frac{-0^\circ\ 52'\ 27''}{12} - 1.1 \times 4''.0 = -4'\ 22''.2 - 4''.4 = \quad \ldots\ldots \quad -4'\ 26''.6$$

Hourly variation of the parallax	−0″.7
Hourly variation of the ☽'s true semidiameter	−0.2
⊙'s right ascension	20h 57m 29s
Hourly variation of his right ascension	0 10.2

Rule II. Examine whether the effect of parallax be likely to hasten, or impede the occultation, or whether it be doubtful what effect it may produce. If we perceive that the occultation will in all probability be accelerated, we must compute for half an hour, and also for an hour before the time of true conjunction, if the probability is, that the occultation will be delayed, we must compute for similar intervals after the said time, but in doubtful cases, it will be better to compute, for half an hour before the time of conjunction and again for half an hour after, the differences of right ascension and of declination between the moon and star. The differences of right ascension must be reduced by multiplying them by the co-sine of the moon's declinations respectively, and the differences of declination must also be diminished by subtracting from them the small quantity arising out of the term

$$\frac{1}{2r}(a-A)^2 \cos B.\sin b.$$

8. *Example.*—According to the Nautical Almanac the moon will pass the meridian at $18^h\ 56^m$; she will therefore not have arrived at the meridian, but will be in the east, and the depression of parallax towards the east will certainly accelerate the occultation. Hence we must compute the differences of right ascension and of declination for half an hour, and for an hour, before the true conjunction

Differences of Right Ascension.

$17^h\ 27^m\ 26^s$. $a-A=0$

16 57 26 . . $a-A'=\frac{1}{2}(37'\ 1''.9)-\frac{1}{4}(0.6)=18'\ 30''.8$

16 27 26 . . $a-A''=37'\ 1''.9-0.6$ . . $=37\ \ 1.3$

Differences of Declination.

17 27 26 . . $b-B=$ . . . . $-66'\ 13''$

17 57 26 . $b-B'=-66\ \ 13-\frac{1}{4}(4'\ 26''.6)-\frac{1}{4}(4''.0)=-68\ \ 27.5$

16 27 26 . . $b-B''=-66\ \ 13-4'\ 26''.6-4''.0$. $=\ 70\ \ 43.6$

To reduce these differences to the values of the cosines of m and n we have $b=-22°\ 7'\ 1''$ and the work as follows; viz.

$16^h\ 57^m\ 26^s$		$16^h\ 27^m\ 26^s$	
$B'=-20°\ 58'\ 34''$ log cos B . .	$=9.97022$	$B''=-20°\ 56'\ 17''$ log cos B . .	$=9.97033$
$a-A'=1110''.8$ log $(a-A')$ . .	$=3.04564$	$a-A''=2221''.3$ log $(a-A'')$ . .	$=3.34661$
$(a-A')\cos B'=1037''.2$	3.01586	$(a-A'')\cos B''=2074''.7$	3.31694

Then the values will be thus $m=0$, $m'=17'\ 17''.2$, and $m''=34'\ 34''.7$

Now to get the values of n, n', n'' we must first compute the following term thus

$\frac{1}{2r}=$ constant . . $\log\frac{1}{2r}$ $=4.386$

$B''=-20°\ 56'$ . . $\log\cos B''=9.970$

$b\ =-22\ \ 7$ . . $\log\sin b\ =\bar{9}.576$

$a-A''=2222''$ . . $2\log(a-A'')=6.693$

Nat. num. $=-4''.2$. Sum 0.625

Then the reduced differences, or the values of n, n', n'' will be

$$n=b-B-\frac{1}{2r}\sin b.\cos B\,(a-A)^2=-66'\ \ 13''$$

$$n'=b-B'-\frac{1}{2r}\sin b.\cos B'(a-A')^2=-68\ \ 27.3+\tfrac{1}{4}(4''.2)=-68'\ \ 26''.3$$

$$n''=b-B''-\frac{1}{2r}\sin b.\cos B''(a-A'')^2=-70\ \ 43.6+4''.2\ \ =-70\ \ 39.4$$

RULE III. Compute for the three times chosen the horary angles of the star, and with them the declination of the star, the reduced latitude of the place, and the respective horizontal parallaxes, then find by the Zodiacal Tables 13 and 17 the values of the co-ordinates $\pi\mu$, $\pi\nu$, expressed in seconds, or otherwise compute them directly by the formula (3).

Example.

Times chosen . . .	17^{h} 27^{m} 26^{s}	16^{h} 57^{m} 26^{s}	16^{h} 27^{m} 26^{s}
☉'s right ascension . . .	20 57 29	20 57 24	20 57 19
R. ascension of the mid-heaven	14 24 55	13 54 50	13 24 45
R. ascension of the star δ ♏ .	15 50 4	15 50 4	15 50 4
Horary angles of the star .	1 25 9	1 55 14	2 25 19
Horary angle in space $a-\theta$.	21 17 15	28 48 30	36 19 45
Horizontal parallax of the moon	58 39	58 39 .3	58 39 .7
Latitude of Greenwich			51 28 40
Reduction with compression $\frac{1}{309}$ from Lunar Table 5 . .			10 52
Reduced latitude of Greenwich			51 17 48
Reduced co-latitude of Greenwich			38 42 12
Polar distance of δ ♏			67 52 59

Computation of $\pi\mu$ and $\pi\nu$ by the Tables

Arguments for Table 13		21°	22°	Diff.	
Co-latitude 38° 42′ 12″	38°	13′ 14″.28	13′ 50″.28	36″.00	Mean 36″.39
Horary angle 21 17 15	39	13 31 .91	14 8 .69	36 .72	
Difference		17 .63	18 .41		

60″ . 42′ 12″ :: 18″.02 . . .		12″.67	first p.p.
60 . 17 15 :: 36 .39 . . .		10 .46	second p.p.
First tabular quantity	13′	14 .28	
Sum	13	37 .41	§
Tab. 17* with 60−58 .39 = 1 .21 gives		18 .40	to be subtracted.
$\pi\mu$ =	13′	19″.00	

By a similar process we find for the second and third times

$$\pi'\mu' = 17'\ 40''.6 \text{ and } \pi''\mu'' = 21'\ 43''.9$$

In like manner for the values of $\pi\nu$, $\pi'\nu'$, and $\pi''\nu''$ we have

Arguments for Table 13		67°	68°	Diff.	
Red. Latitude 51° 17′ 48″	51°	42′ 55″.32	43 14 .01	18″.69	Mean 18″.87
Polar dist. δ ♏ 67 52 59	52	43 31 .32	43 50 .27	18 .95	
Diff.		36 .00	36 .26 . . .		36 .13

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* As the parallax is here computed in a line perpendicular to the star's circle of declination, and passing through it, as explained in a preceding note at page 635, and not in right ascension, the reduction given by Lunar Table 18 must not be applied

60′ : 17′ 48″ : 36″.13 . . . . . . 10″72
60′ 52 59 :: 18 .87 . . . . . 16 .66
First tabular quantity . . . . 42′ 55 .32

Sum 43 22 .70
Tab. 17. with 60′ − 58′ 39″ = 1′ 21″ gives 58 .56 to be subtracted

First part of the value of $\pi\nu$ . . . 42 24 .14

And by the same means we obtain the first parts of the values of

$$\pi'\nu' = 42'\ 24''4 \text{ and of } \pi''\nu'' = 42'\ 24''8.$$

The computation of the second part is performed thus

Argument for Table 18. .		68°	69°	Diff.	
Red. co-latitude 38° 42′ 12″	38°	34′ 14″.99	34 29 .17	14″.18	Mean
Co. of hor angle 68 42 45	39	35 0 .38	35 15 .07	14 .49	14″.33
	Diff.	45 .59 .	45 .90 .		45 .74

60′ . 42′ 12″ :: 45″.74 . . . . 32″.16
60′ 42 45 :: 14 .33 . . . . 10 .21
First tabular quantity . . . . 34 14 .99

Sum 34 57 .36
Tab. 17. with arg. 60′ − 58′ 39″ = 1′ 21″ gives . 47 .20 to be subtracted

Argument for Table 19. . . . . 34′ 10″ 16

Argument for Table 19. .		22°	23°	Diff.	
Preceding number 34′ 10″.6	34′	12′ 44″.20	13 17 .09	32″.89	Mean
Declination . . 22 7 .1	35	13 6 .67	13 40 .54	33 .87	33″.38
	Diff.	22 .47 .	23 .45 .		22 .96

60′ : 10″.6 : 22″.96 : . . . . . . 3″.89
60 · 7 .1 ·· 33 .38 · . . . . . . 3 .70
First tabular quantity . . . . 12 44 .20

Second part of the value of $\pi\nu$ . . − 12 51 .79
First part above found . . . . − 42 24 .14

And consequently $\pi\nu$ . . . . . . 55′ 15″.93

By the same means we find $\pi'\nu' = 54'\ 30''.3$ and $\pi''\nu'' = 53'\ 32''.2$.

9. The computation of $\pi\mu$ and $\pi\nu$ by logarithms.

The times for computation	$17^h\ 27^m\ 26^s$	$16^h\ 57^m\ 26^s$	$16^h\ 27^m\ 26^s$
π = parallax . . . log π	3 .54642	3 .54646	3 .54651
ϕ = latitude . . . log cos ϕ	9 .79608	9 .79608	9 .79608
log π. cos ϕ	3 .34250	3 .34254	3 .34259
$a-\theta$ = horary angle . . . log cos $(a-\theta)$	9 .96931	9 .94262	9 .90613
log sin $(a-\theta)$	9 .55996	9 .68294	9 .77263
log π cos ϕ. cos $(a-\theta)$	3 .31181	3 .28516	3 .24872
log π cos ϕ sin $(a-\theta)$	2 .90246	3 .02548	3 .11522
$\pi\mu = \pi$ cos ϕ sin $(a-\theta)$ nat. num.	798″.8	1060″.4	1303″.8
b = declin. of star . . . log. sin b	9 .57576	9 .57576	9 .57576
log π sin b. cos ϕ. cos $(a-\theta)$	$\bar{2}$ .88751	2 .86092	$\bar{2}$ .82448
log sin ϕ	9 .89231	9 .89231	9 .89231
log cos b	9 .96680	9 .96680	9 .96680
log sin ϕ. cos b	9 .85911	9 .85911	9 .85911
log π sin ϕ. cos b	3 .40553	3 .40557	3 .40562
π sin ϕ. cos b . . . nat. num.	2544 .1	2544 .3	2544 .6
π sin b. cos ϕ. cos $(a-\theta)$	−771 .9	−726 .0	−667 .5
$\pi\nu = -\pi$ sin ϕ. cos $b+\pi$ sin b cos ϕ cos $(a-\theta)$	−3316″.0	−3270″.3	−3212″.1

RULE IV With the values of m, n, $\pi\mu$, and $\pi\nu$ form the differences $m-\pi\mu$, $n-\pi\nu$, which will represent the co-ordinates of the moon's centre, relative to the place where the observer projects the star, on the plane of projection. Then having determined the values of these co-ordinates for each of the three times, take the first and second differences between them, and by interpolation extend their values to every five minutes. Insert these values in a small table, with a column of corresponding values of the moon's true semi-diameters adjoining, and the inspection of such table will show the two nearest arguments, or times between which the immersion or emersion must of course happen Take for both these two times the difference between the sum of the two coordinates squared, and the square of the true semi-diameter, and by repeated proportions find the exact time when that difference will be nothing, and that will be the instant of immersion or emersion, as the case may be.

Example.

$m = 0'\ \ 0''.0$ $\qquad m' = 17'\ \ 17''.2$ $\qquad m'' = 34'\ \ 34''.7$

$\pi\mu = 13\ \ 13\ 8$ $\qquad \pi'\mu' = 17\ \ 40.4$ $\qquad \pi''\mu'' = 21\ \ 43.8$

$m-\pi\mu = -13\ \ 13.8$ $\qquad m'-\pi'\mu' = -\ 0\ \ 23.1$ $\qquad m''-\pi''\mu'' = 12\ \ 50.9$

first diff. $12'\ \ 55''.6$ $\qquad$ first diff. $13'\ \ 14''.1$

second diff $18''.5$

$$\begin{array}{lll}
n = -66'\ 13'' & n' = -68'\ 26''.3 & n'' = -70'\ 39''.4 \\
\pi\nu = -55\ \ 16\ \ 0 & \pi'\nu' = -54\ \ 30\ .3 & \pi''\nu'' = -53\ \ 32\ .1 \\
\hline
n-\pi\nu = -10\ \ 57\ .0 & n'-\pi'\nu' = -13\ \ 56\ .0 & n''-\pi''\nu'' = 17\ \ \ 7\ 3
\end{array}$$

$$\text{first diff.} = -2'\ \ 59''.0 \qquad \text{first diff.} = -3'\ \ 11''.3$$

$$\text{second diff} = 12''.3$$

To extend the values of the coordinates $m-\pi\mu$ and $n-\pi\nu$ to every five minutes, we have first for the coordinates $m-\pi\mu$

$$\text{Second difference between the new terms } \frac{18''.5}{36} \ldots\ldots\ldots\ldots = 0''\ 514$$

$$\tfrac{1}{2}\ \text{Second difference} \ldots\ldots\ldots\ldots \quad 0\ .257$$

$$\frac{\text{Half the sum of the two first diff.}}{6} = \frac{12'\ \ 55''.6 + 13'\ \ 14''.1}{6 \times 2} \ldots\ldots = \underline{2\ \ 10\ .808}$$

$$\text{First difference between the middle term and the preceding one} \ldots = 2\ \ 10\ .551$$

$$\text{First difference between the middle term and the following one} \ldots = 2\ \ 11\ .065$$

Likewise for the coordinates $n-\pi\nu$ we shall have

$$\text{Second difference between the new terms} = -\frac{12\ .3}{36} \ldots\ldots\ldots\ldots = 0\ .342$$

$$\tfrac{1}{2}\ \text{Second difference} \ldots\ldots\ldots\ldots \quad 0\ .171$$

$$\frac{\text{Half the sum of the two first diff.}}{6} = \frac{-2'\ \ 59''.0 - 3'\ \ 11''.3}{6 \times 2} \ldots\ldots = -\underline{30\ .858}$$

$$\text{First difference between the middle term and preceding one} \ldots\ldots = -30\ .687$$

$$\text{First difference between the middle term and following one} \ldots\ldots = -31\ .029$$

With these differences, beginning from the middle term, or that corresponding to $16^h\ 57^m\ 26^s$, and going up and down by additions and subtractions, we constructed the following Table, in which the values of the coordinates $m-\pi\mu$, and $n-\pi\nu$, and of the true semi-diameters of the moon for the corresponding times, are given in minutes and decimals of a minute respectively.

10. *Table of Coordinates.*

Times			Values of the coord $m-\pi\mu$	Values of the coord. $n-\pi\nu$	☽'s true Semi-diam
16^h	27^m	26^s	12′.848	−17′.122	16′015
	32	26	10.622	−16 577	16.015
	37	26	8 402	−16.037	16.014
	42	26	6.192	−15.502	16.014
	47	26	3.990	−14.973	16 014
	52	26	1 798	−14 450	16.013
	57	26	−0 387	−13.933	16.013
17	2	26	−2.897	−13.422	16.013
	7	26	−4.730	−12.917	16.013
	12	26	−6 888	−12.417	16.012
	17	26	−9 038	−11.922	16 012
	22	26	−11 180	−11.433	16.012
	27	26	−13.313	−10 950	16.012

By this table we now easily discover that the immersion will happen between $16^{h}\,42^{m}\,46^{s}$ and $16^{h}\,47^{m}\,26^{s}$; for we have

At $16^{h}\,42^{m}\,26^{s}$	At $16^{h}\,47^{m}\,26^{s}$
$m-\pi\mu=6.192$ squared $=38.34$	$m-\pi\mu=3.990$ squared $=15.92$
$n-\pi\nu=15.502$ squared $=240.25$	$n-\pi\nu=14.975$ squared $=224.19$
278.59	240.11
$d=16.014\times 16.014\ \ 256.45$	d squared 256.45
Excess $22'.14$	Defect $16'.34$

The sum of these last two numbers being 38.48 and the corresponding interval of time $5^{m}=300^{s}$ we must say,

$$\text{As } 38'.48\quad 300^{s}.:22'.14\quad x=173^{s}=2^{m}\,53^{s},$$

which, added to $16^{h}\,42^{m}\,26^{s}$, gives the approximate time of the immersion $16^{h}\,45^{m}\,19^{s}$. Now if we suppose ten terms interpolated within the interval of the two nearest times of the table, the immersion will be comprehended between the two terms corresponding to $16^{h}\,44^{m}\,56^{s}$ and $16^{h}\,45^{m}\,26^{s}$, and we shall then have as follows, viz

At $16^{h}\,44^{m}\,56^{s}$	At $16^{h}\,45^{m}\,26^{s}$
$m-\pi\mu=5.091$ squared $=25.92$	$m-\pi\mu=4.871$ squared $=23.73$
$n-\pi\nu=15.237$ squared $=232.17$	$n-\pi\nu=15.185$ squared $=230.58$
Sum $=258.09$	Sum $=254.31$
$d=16.014$ squared $=256.45$	$d=16.014$ squared $=256.45$
Excess 1.64	Defect 2.14

Hence lastly, as $3.78\ (=1.64+2.14)\quad 30^{s}\quad 1.64\quad x=13^{s}$.

The true time of the immersion therefore finally comes out

$$16^{h}\,44^{m}\,56^{s}+13^{s}=16^{h}\,45^{m}\,9^{s}\ \text{apparent time at Greenwich.}$$

The emersion will happen, according to the table, between the two times $17^{h}\,22^{m}\,26^{s}$ and $17^{h}\,27^{m}\,26^{s}$, and to find the approximate time we shall have

At $17^{h}\,22^{m}\,26^{s}$	At $17^{h}\,22^{m}\,56^{s}$
$m-\pi\mu=11.180$ squared $=124.99$	$m-\pi\mu=11.393$ squared $=129.89$
$n-\pi\nu=11.433$ squared $=130.71$	$n-\pi\nu=11.385$ squared $=129.62$
Sum $=255.70$	Sum $=259.51$
$d=16.012$ squared $=256.38$	$d=16.012$ squared $=256.38$
Defect 0.68	Excess 3.13

And lastly, the proportion $3.81\quad(0.68+3.13)\ 30^{s}.:0.68\ .\ x=5^{s}.4$

so that the true time of emersion will be at $17^h\ 22^m\ 31^s.4$ apparent time at Greenwich, exactly as by our first method.

11. *The Inverse Problem, or Computation of an Observed Occultation by the Right Ascensions and Declinations.*

The problem of deducing from observed occultations the true place of the moon, the difference of longitude between any two places of observation, and the corrections of the moon's parallax and semi-diameter, may easily be solved by our equation (1), viz.

$$(m-\pi\mu)^2+(n-\pi\nu)^2=\sin^2 d.$$

Let us suppose that, by taking the moon's right ascension, declination, parallax, and semi-diameter out of the respective tables, we have computed the values of the coordinates $m-\pi\mu$ and $n-\pi\nu$ for the instant of observation, and that, by substituting these values and that of d in the said equation we have found the error $2\,\varepsilon$; then representing the moon's hourly motion in right ascension and declination by m and n, the difference of the correct time of conjunction and that computed by means of the tables by Δt, the corrections of the declination, horizontal parallax, and true semi-diameter, as taken from the tables, by ΔB, $\Delta\pi$, Δd; and putting for the sake of brevity,

$$\begin{aligned}
d^2-(m-\pi\mu)^2-(n-\pi\nu)^2 &= 2\,\varepsilon \\
(m-\pi\mu)\,m'+(n-\pi\nu)\,n' &= (l) \\
\frac{n-\pi\nu}{(l)}\ .\ \ .\ \ .\ \ .\ \ .\ \ . &= B \\
\frac{(m-\pi\mu)\,\mu+(n-\pi\nu)\,\nu}{(l)} &= (\pi) \\
\frac{d}{(l)}=(d)\ .\ \ .\ \ .\ \ \frac{\varepsilon}{t} &= e
\end{aligned}$$

it will be easy to know by the differentiation of the above equation (1), that we have

$$(4)\quad \Delta t-(B)\,\Delta B-(\pi)\,\Delta\pi-(d)\,\Delta d=e$$

In this equation we have neglected, as being inconsiderable, the squares and higher powers of the supposed corrections, as well as some other terms of very little value.

Before we lay down rules for computation arising out of this equation, it will be proper to make a few remarks respecting the circumstances of the observations that furnish the data.

1st. When we have a single observation, and only the immersion or emersion has been observed, since the element most liable to error is the time of the true conjunction, we may neglect all the other errors as they have respect to this, and shall be able to obtain the correction of the assumed time of the true conjunction by the equation $\Delta t = e$.

2dly. If both the immersion and emersion have been observed, then, distinguishing by an accent the quantities belonging to the emersion from those belonging to the immersion, we shall have for the former of these times the equation $(4)'\ \Delta t-(B)'\,\Delta B-(\pi)'\,\Delta\pi-(d)'\,\Delta d=e'$. As the hourly motions of all the elements of the moon's Tables may be considered well known,

Δt, ΔB, $\Delta\pi$, Δd will be the same in both the computed equations, and eliminating Δt from them, we shall deduce the following equation *between* the other corrections, viz

$$(4)' \qquad \{(B)'-(B)\}\,\Delta B+\{(\pi)'-(\pi)\}\,\Delta\pi+\{(d)'-(d)\}\,\Delta d=e-e'.$$

This equation taken alone will give the value of one of the three corrections, if the other two be considered as nothing, and whenever we possess three complete observations made at different places, since each of them will afford a similar equation, we shall be able to determine all the three corrections, and then either of the two equations marked (4) and (4)′ will give the correction for the true conjunction for the corresponding place.

3dly. For an observation made in another place, the equation between the aforesaid corrections will also be of the form $(4)_{\prime}\ \Delta t-(B)_{\prime}\,\Delta B-(\pi)_{\prime}\,\Delta\pi-(d)_{\prime}\,d\Delta=e_{\prime}$, where $(B)_{\prime}\,(\pi)_{\prime}\,(d)_{\prime}\,e_{\prime}$ denote the values of $(B)\,(\pi)\,(d)\,e$ belonging to that place.

4thly When the difference of the meridians of the two places is well known, the correction of the time of the true conjunction must be the same in the equation (4) as in the last equation, and by subtracting one from the other we shall obtain between the corrections ΔB, $\Delta\pi$, Δd an equation of the form

$$\{(B)_{\prime}-(B)\}\,\Delta B+\{(\pi)_{\prime}-(\pi)\}\,\Delta\pi+\{(d)_{\prime}-(d)\}\,\Delta d=e-e_{\prime}.$$

But if we have a doubt respecting the existence of an error in the difference of the meridians of the places of observation, the two times will differ one from the other by this error; and if we call c the correction of the longitude of the latter place, we shall have

$$c=e_{\prime}-e+\{(B)_{\prime}-(B)\}\,\Delta B+\{(\pi)_{\prime}-(\pi)\}\,\Delta\pi+\{(d)_{\prime}-(d)\}\,\Delta d.$$

Which equation, if we suppose ΔB, $\Delta\pi$, Δd to be nothing, or to have been corrected, reduces itself to $c=e_{\prime}-e$.

5thly. It is not our object to illustrate, by rules and examples, all the cases in which observed occultations by the moon may be employed to determine the corrections of her place, of her parallax and semi diameter, as given by the Tables, but those who undertake such researches, being generally well acquainted with algebraical formulæ, will find in what we have above detailed, all that is necessary to guide their computations. The rules and example, which we propose to present to our readers, will explain the process proper to be employed in computing the difference of the geographical longitudes of two places by an immersion or emersion observed at both places, which is a problem at the same time useful and of common occurrence, though seldom correctly solved. In order to propose a case to our purpose, let us suppose the emersion of δ♏, which we have before computed, to have been observed at Paris, $9^{m}\ 21^{s}$ to the east of Greenwich, at $17^{h}\ 31^{m}\ 43^{s}.4$, the same being supposed to have been observed at Greenwich at $17^{h}\ 22^{m}\ 31^{s}.4$, agreeably to our computations from the elements taken out of the Nautical Almanac of 1826, and let us suppose, also, that the longitude of Paris is not exactly known, but considered only $9^{m}\ 16^{s}$, then let it be required to find the error of this difference of longitude?

12.

Rules for finding the Difference of Longitude.

RULE I. Compute according to the rules given before (in our *second method*) the relative coordinates $m-\pi\mu$, $n-\pi\nu$ both corresponding to the observations at Greenwich and Paris, and also the values of the moon's true semi-diameters respectively; then from the square of the latter quantity, belonging to each place, subtract the sum of the squares of the former quantities, and call the arising differences 2 ε and 2 ε' according to the subjoined results of the proper computations, viz.

$$\begin{array}{rrrr@{\qquad\qquad}rrrr}
m= & 2' & 49''.9 & & n= & -1^\circ & 6' & 34''\,8 \\
\pi\mu= & 14 & 3\,.0 & & \pi\nu= & 0 & 55 & 9\;3 \\
\hline
m-\pi\mu= & -11 & 13\;1 & & n-\pi\nu= & -0 & 11 & 25\,.5
\end{array}$$

Also the moon's true semi-diameter $=16''\,0.7$

Hence $m-\pi\mu-11'.218$ squared $=125\,.8$ nearly

$n-\pi\nu=11\;423$ squared $=130\;5$ nearly

Sum $256\,.3$

$d=16\,.0117$ squared $=256\;3$

$2\,\varepsilon=\;0\,.0$

As the difference of longitude between Paris and Greenwich is taken at $9^m\ 16^s$ east, we must now compute the values of m and n as well as of π for the Parisian time of observation, or for $17^h\ 22^m\ 26^s.9$ of true time at Greenwich; but the values of $\pi\mu$ and $\pi\nu$ must be computed according to our former rule III. by employing the time of observation $17^h\ 31^m\ 43^s.4$ and the reduced latitude of the Royal Observatory at Paris $=48^\circ\ 39'\ 11''$ We shall then find

$m_{,}-\pi\mu=-10\,.528$ squared $=110\,.84$

$n_{,}-\pi\nu=-12\,.100$ squared $=146\,.41$

Sum $257\,.25$

$d=16\,.0117$ squared $=256\,.37$

$2\,\varepsilon=\;0\,.88$

RULE II. Find the hourly motion of m and n, and call their values m' and n'; then by these values and by those above found of $m-\pi\mu$, $n-\pi\nu$, and $m_{,}-\pi\mu_{,}$, $n_{,}-\pi\nu_{,}$ complete the two values of the expressions

$$(m-\pi\mu)\,m'+(n-\pi\nu)\,n'$$
$$(m-\pi\mu_{,})\,m'_{,}+(n_{,}-\pi\nu_{,})\,n'$$

Let (t) represent the value of the former of these quantities and $(t_{,})$ the value of the latter, then divide the error ε by (t) and the error $\varepsilon_{,}$ by $(t_{,})$ and subtract the former from the latter, and their difference will represent, in parts of an hour, the correction of the assumed longitude to be always applied with its own sign, when we consider the east longitude as negative.

Example.—The hourly motions m' and n', or the differences between the values of m and n corresponding to the time of the true conjunction, and an hour after, according to our former rule II. (p. 642) are

$$m' = 34'\ 58'' \qquad\qquad n' = 4'\ 44''$$

Seeing the error ε was found$=0$, the quotient e will in this case be$=0$, and we have only to compute $e_{,}$, for which we have,

$$\begin{array}{rr}
\log (m_{,} - \pi\mu_{,}) = \bar{1}\,.0223 & \log (n_{,} - \pi\nu_{,}) = 1\,.0828 \\
\log m' = \underline{\bar{1}\,.5388} & \log n' = \underline{0\,.6474} \\
\log (m_{,} - \pi\mu_{,})\, m' = 2\,.5611 & \log (n_{,} - \pi\nu_{,})\, n' = 1\,.7302 \\
(m_{,} - \pi\mu_{,})\, m' = 364\,.0 & \\
(n_{,} - \pi\nu_{,})\, n' = \underline{-53\,.7} & \\
(t_{,}) \quad 310\,.3 & \text{Ar. co} \log (t_{,}) = 7\,.5082 \\
\varepsilon_{,} = -\ 0\,.44 & \log \varepsilon_{,} \ = \underline{\bar{9}\,.6435} \\
e_{,} = -\ 0\,.001418 & \log e_{,} \ = \bar{7}\,.1517
\end{array}$$

This time e reduced into seconds gives $-5''.1$, which, applied with its own sign to the east or negative assumed longitude of Paris $9^{m}\ 16^{s}$, gives $9^{m}\ 21^{s}.1$, in which the remaining error is only one tenth of a second.

We may remark here, that if the correction thus found should in any case happen to amount to several minutes, and if we are aiming at great accuracy, it will be necessary to repeat the computation, by employing the values of $m_{,}$ and $n_{,}$ which result, to correct the approximate longitude before the repetition, but the values of $\pi\mu$ and $\pi\nu$ already determined, depending on the place of the star, and the time of observation, as reckoned at the place where it was made, do not require any modification.

§ XCVII THE THIRD METHOD OF COMPUTING OCCULTATIONS OF THE STARS, VIZ BY THE LONGITUDES AND LATITUDES

1 The method of computing eclipses and occultations by the parallaxes in longitude and latitude has an advantage over the other methods in this respect, that the elements are given by the Tables, and do not require to be converted into other denominations, such as right ascension, declination, altitude, or azimuth, by computation. Cassini and Mayer gave formulæ

for computing by the parallaxes in longitude and latitude, but as Cagnoli has rendered them more accurate, as well as more easy, we will follow his formulæ, for the computation of parallaxes, which obtained the prize of the Academy at Copenhagen, and which may also be found in his *Trigonometria piana e sferica*, p. 482, et seq.

2. *Scheme of Cagnoli's Formulæ*

for finding the apparent distance between the centres of two heavenly bodies S and L (Sol and Luna), when their longitudes, latitudes, and horizontal parallaxes are given.

A = ○'s right ascension + app. time in parts of the equator.
B = the altitude of the pole − the angle of the vertical.
Tang C = cot B. sin A.
If $A \geq 360°$ take $A - 360°$ instead of A.
D = C + apparent obliquity of the ecliptic.
$$\text{Cos } F = \sin B \, \frac{\cos D}{\cos C}.$$
Sin G = tang D. cot F.

When A is contained between 80° and 100°, or between 250° and 290°, then instead of the last equation make use of the following one, viz.

$$\text{Cot } G = \cot A \, \frac{\sin C}{\sin D}.$$

Take G always in the *ascendant* or descendant signs accordingly as A lies between the former or the latter ones.

H = the true longitude of the luminary $L - G$.
[*]K = radius of the earth (hor. paral. of L − hor equat. paral. of S).
$$\dagger M = K \, \frac{\sin F \sin (H+M)}{\text{cos true lat. } L}$$
$N = K$ (cos F cos app. lat. L − sin F sin app. lat. L cos $(H + \frac{1}{2} M)$.
Apparent longitude of the body L = true longitude + M.
Apparent latitude of L = true latitude ± N.
The sign + is to be used when the true latitude is south, but the sign − when it is north, and vice versâ when N is negative.
Q = app. long. of $L \sim$ true long. of S.
T = app. lat. of $L \sim$ true lat of S.

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[*] The value of K may be found by means of *Lunar Table* 8, employing at the top argument the *difference* of the two horizontal equatorial parallaxes

[†] The computation of the values of M and N may also be performed by the *Zodiacal Tables* 13, 14, &c , but we must observe, that in the example given at page 381 of our Vol I for the computation of the parallax in longitude, the *apparent* latitude of the moon has been employed, whereas her *true* latitude is introduced into Cagnoli's formulæ Likewise the distance from the nonagesimal, in the example for the latitude at page 383, is augmented by the *whole* parallax in longitude, while in Cagnoli's formulæ *one half* only of the said parallax is applied The Tables notwithstanding are equally useful in both cases, as the reader will hereafter see in our example

y = app. lat. of L + true lat. of S.

Tang $u = \frac{Q \cos \frac{1}{2} y}{T}$.

Apparent distance of the centres $= \frac{T}{\cos u}$.

In order to obtain the augmented semi-diameter Cagnoli proposes the following formula, viz.

$$\Delta' = \Delta \frac{\sin (H+M) . \cos \text{app lat} ☾}{\sin H . \cos \text{true lat.} ☾}$$

where Δ denotes the true, and Δ' the apparent semi-diameter of the moon. This formula, which is due to Gertsner, does not, however, give an exact value, when the moon's longitude coincides with, or differs but little from that of the nonagesimal. but in this case the moon's altitude is known, since it is equal to the complement of the difference between the latitude of the nonagesimal and that of the moon.

3. We propose to apply these formulæ to a process devised by Mayer (see his *Opera posthuma*) for determining the times of an immersion and emersion of a star, which is grounded upon the following

Rules.

Rule I. Find for three different times, within which both the immersion and emersion are likely to be included, the differences of the moon's and star's apparent longitudes and latitudes, viz. the values of the quantities designated in Cagnoli's formulæ by Q and T, multiply the former by the cosine of half the sum of the star's and moon's apparent latitudes, or by $\cos \frac{1}{2} y$, and determine for the same times the moon's apparent semi-diameters.

Example.—According to the elements of the occultation of δ ♏ given in the Nautical Almanac of 1826, if we take the apparent obliquity of the ecliptic at 23° 27′ 40″.5, the time of the true conjunction with the moon in longitude will be, at Greenwich,

Jan. 31, 1826	17^h	51^m	54^{s}7
The moon's lat at that time	0°	51′	49″.2 S.
The * 's longitude	240	8	36 .9
The latitude of ditto	1	57	44 .8 S.
The moon's hourly motion in long of the 1st order . .	0	31	43 .4
Ditto term of 2d order . . .			−0 .45
The moon's hourly motion in latitude of the 1st order .		3	2 9
Ditto term of the 2d order . .			+0 .12
The moon's horizontal equatorial parallax		58	45 .7
Its hourly motion			−0 .2
The sun's right ascension	20^h	37^m	31^s 4
His hourly motion			10 .2

As the effect of parallax must in this case considerably accelerate the occultation, both the immersion and emersion may be expected to happen before the moment of true conjunction, we will therefore compute the differences of longitude and latitude of the moon and star for the time of the true conjunction, and for 36 and 72 minutes before it.

For the first of these epochs we have

The sun's right ascension in space . .	314°	22′	51″
Time of the true conjunction in space .	267	58	45
$A=$	222	21	36

Taking 51° 28′ 40″ for the latitude of Greenwich, and 10′ 52″ for the reduction, or angle of the vertical, we have $B=51°\ 17'\ 48''$, or reduced latitude, and according to the preceding formulæ,

Log cot B . $=9.90377$
Log sin A . $=\bar{9}.82852$

Log tang C $=\bar{9}\ 73229$
C . $=151°\ 38'\ 12''$
App. obliq. of eclip. 23 27 40

D . 175 5 52

Log sin B . $=9.89231$
Ar. Co. log cos $C=\bar{0}.05554$
Log cos D . $=\bar{9}.99841$

Log cos F . $=9.94626$

Log tang D $=\bar{8}.93343$
Log cot F $=0.27579$

Log sin G $=\bar{9}.20922$

As the arc A is in the descending signs we shall have $G=189°\ 19'\ 0''$, and the moon's longitude being 240° 8′ 37″, we deduce $H=240°\ 8'\ 37''-189°\ 19'\ 0''=50°\ 49'\ 37''$.

The horizontal parallax of the moon is 58′ 45″.7, and as the star has no sensible parallax we shall have

Log 58′ 45″.7 . . . . . $=3.54725$
Log of the earth's radius, Lunar Tab. 5 $=9\ 99914$

Log K . $=3\ 54639$
Log sin F $=9.67045$

Log $(K \sin F)=3.21684$
Ar. Co. log cos (D's true latitude) . $=0.00005$

Sum, or constant logarithm . . $=3.21689$
Log sin $(H+M)=51°$ by account . . $=9.89050$

Approximate parallax or log M nearly $=3\ 10739=21'\ 10''$

Log constant given above . . $=3.21689$
Log sin $(H+M)=51°\ 0'\ 57''$ . . $=9.89162$

Log M . $=3.10851=21'\ 23''.8$

Log cos F given above . . . . $= 9.94626$
Log K given above . . . . $= 3.54639$

Approximate parallax or log N nearly . $= 3.49265 = 51'\ 49''$
Therefore log cos (☽'s ap. lat. $= -1^\circ\ 43'\ 38''$) $= \bar{9}.99981$

Log first part of the value of N . . . $= \bar{3}.49246 = -51'\ 47''.8$

Log (K sin F) as above . . . . $= 3.21684$
Log cos $(H + \frac{1}{2} M) = 51^\circ\ 0'\ 19''$ . . . $= 9.79881$
Log sin (☾'s app. lat. $= -1^\circ\ 48'\ 38''$) . . $= \bar{8}.47916$

Log second part of the value of N . . $= \bar{1}.49481 = -\ 0\ \ 31.2$

N . . . . $-52\ \ 19.0$

4 Having reduced M and N, the moon's parallaxes in longitude and latitude, by Cagnoli's formulæ, it will be satisfactory to compute them also by the tables of our volume I, to show how they agree, and to give the computist his option of adopting which mode of computation he may deem preferable For this purpose we have $A =$ the right ascension of the mid heaven given above, to find, from Zodiacal Table 7, G the longitude of the nonagesimal, and H its altitude, thus,

		Long. of nonages.	Alt. of nonages.
Zod Tab. 7 Arg (A)	222° 0′ 0″	188° 55′ 54″	28° 3′ 33″
Prop. part for . .	21 36	+ 22 29	− 8 29
Cor. for 19″ 5 dim. of obliquity .		+ 36	− 3
Longitude of nonagesimal G . .		= 189 18 59	Alt. of ditto F = 27 55 1
☾'s true longitude		= 240 8 37	
☾'s true distance from nonag. H		= 50 49 38	

The equatorial horizontal parallax 58′ 45″ 7, reduced by Lunar Table 4, with the radius of Greenwich, and with compression $\frac{1}{310}$, becomes 58′ 38″.7.

If now we enter Zodiacal Table 13 with the moon's distance from the nonagesimal obtained above as the top argument, and with the altitude of the nonagesimal already obtained as the side argument, we shall see at once by inspection that the moon's parallax in longitude will be about 21′, which added to her true distance from the nonagesimal will make her apparent distance by account about 51° 11′, as the corrected argument for obtaining the parallax more exactly thus,

For the parallax in longitude.

Arg. for Tab. 13 . .			51°	52°	Diff.	Mean
Dist. from nonag.	51° 11′ 0″	27°	21′ 10″.14	21′ 27″.90	17.76	18.05
Alt. of nonag .	27 55 1	28	21 53.47	22 11.81	18.34	
		Diff.	43.33	43.91		43.62

As 60′ : 18″.05 : 11′			3″.3
As 60 : 43 .62 : 55 .1			39 .9
First tabular quantity		21	10 .1
Sum		21	53 .3
Hor. par. 58′ 38″.7 Tab. 17. Args. 1′21″.3 & 21′53″.3		−	29 .6
Approximate parallax in longitude . . . =		21	23 .7
Moon's true distance from the nonag.	50	49	38 .
Moon's apparent distance from the nonag. .	=51	11	1 .7

This distance, differing less than 2″ from the assumed one got by inspection of Table 13, requires no further computation.

To obtain the reduction of the moon's latitude we ought, according to the note † in page 653, to employ the moon's true latitude, and then the arguments for Table 15 will be

Top argument . .	21′	24″	− 0′	0″.1
Side argument . .	51	49		
Approximate parallax			21	23 .7
☽'s parallax in longitude . .			=21	23 .8 by the tables.
			21	23 .8 by Cagnoli's formulæ.

For the parallax in latitude.

Zodiacal Table 14, with arg. 27° 55′ 1″ (alt. of nonag.) gives .	53′	1″.0
Reduction by Table 17, and arg. 1′ 21″.3 (as before)	1	11 .8
☽'s true lat. for the top arg. . 51′ 49″	51	49 .2
Side argument for Tab. 15 . 1° 43′ 38″ for diminution		− 1 .4
First part	51	47 .8

The moon's true distance from the nonagesimal being 50° 49′ 38″, if we add to it *one half* of the parallax in longitude according to the note † in page 653, viz. 10′ 42″ we shall have 51° 0′ 20″ for the complement of the top argument of Tab. 13 thus

Top argument	38°	59′	40″	17′ 40″
Side argument	27	55	1	

Then for Table 16 top arg.	17′	40″	. . . . .	Second part.	31 .9
Moon's app. lat. side arg. .	1	44			
Reduction by Tab. 17 arg.	1	21 .3	. . . . .		− 0 .7

Moon's parallax in latitude by the tables	=52′	19″.0
Ditto by Cagnoli's formulæ	=52	19 .0

5. By applying the values of M and N thus found to the moon's true longitude and latitude we shall have

☽'s apparent longitude $= 240° \ 8' \ 36''.9 + 21' \ 23''.8 = 240° \ 30' \ 0''.7$
☽'s apparent latitude $- \ 0 \ 51 \ 49.2 - 52 \ 19.0 = - \ 1 \ 44 \ 8.2$

The differences between the apparent longitude and latitude of the moon and of δ ♏ will now be

$Q = 21' \ 23''.8$ $T = 13' \ 36'' 6$

The first of these quantities multiplied by the cosine of half the sum of the star's and moon's apparent latitudes, viz by cos 1° 50′ 56″, becomes 21′ 23″.1.

To find the moon's augmented semi-diameter, we have according to the formulæ of Genstner,

Log (☾'s true semi-diameter) 16′ 0″.6 . . = 2.98254
Log sin $(H+M)$ above given . . . . = 9.89162
Log cos (☾'s app. lat) 1° 43′ 38″ as before = 9 99981
Ar. co. log sin $(H = 50° \ 49' \ 37'')$ . . = 0.11063
Ar co. log (☾'s true lat) 0° 51′ 49″ as above = 0.00003

Log (☾'s augmented semi-diameter) . . = 2.98463 = 16′ 5″.3

By means of the hourly motions we compute for the second time, or for $17^h \ 15^m \ 54^s.7$

☽'s true longitude . . .	239°	47′	46″.7
☽'s true latitude . . .	0	53	38.9
☽'s horizontal parallax . .	0	58	46.2
☽'s true semi-diameter	0	16	0.7
☉'s right ascension . . .	20^h	57^m	$25^s.3$

By using the same process with these quantities we shall obtain

M . . $= 26' \ 28''.3$ $N = 50' \ 27''.4$
Q . $= 5 \ \ 8.1$ $T = 13 \ \ 38.5$
$Q \cos y = 5 \ \ 7.9$ $\Delta' = 16 \ \ 4.7$

Likewise for the third time, or at $16^h \ 39^m \ 54^s.7$, we have

☽'s true longitude . . .	= 239°	26′	56″ 2
☽'s true latitude	= 0	55	28 5
☽'s equatorial horizontal paral.	=	58	46.6
☽'s true semi diameter . .	=	16	0.8
☉'s right ascension	= 20^h	57^m	$19^s 2$

and by a similar computation

M . . $= 31' \ \ 7'' 9$ $N = 48' \ 22''.6$
Q . . $= 10 \ \ 32 \ 8$ $T = 13 \ \ 53.7$
$Q \cos \frac{1}{2} y = 10 \ \ 32 \ 5$ $\Delta' = 16 \ \ 4.2$

RULE II Put down in a table of four columns the three determined values of $Q \cos \frac{1}{2} y$, of T, and of the moon's apparent semi-diameters, together with the corresponding times; and extend these values by interpolation to every five minutes of the interval comprehended between them. By comparing the values of $Q \cos \frac{1}{2} y$ and of T to the moon's semi-diameters, estimate what difference of latitude will nearly correspond with the beginning or end of the occultation. Call x the estimated value of T and c the moon's apparent semi-diameter corresponding to it in the table, and compute the quantity $\sqrt{c^2 - x^2}$. If this quantity, considered as the value of $Q \cos \frac{1}{2} y$, is found in the table, corresponding to the supposed difference of latitude, then will that be the value of $Q \cos \frac{1}{2} y$ belonging to the beginning or end of the occultation, as the case may be, if not, call dx the difference between the supposed value x and the value of T corresponding to $\sqrt{c^2 - x^2}$, considered as a value of $Q \cos \frac{1}{2} y$, and say, as $\sqrt{c^2 - x^2} : x : dx :$ a fourth term, which added to $\sqrt{c^2 - x^2}$, if the supposed value of x be greater than the right one, or subtracted if smaller, will give the correct value of $Q \cos \frac{1}{2} y$ at the beginning or end of the occultation, consequently if the exact time corresponding to either of these be found by computation, that will be the true time of the immersion or emersion, as the case may require. By employing the process of interpolation, explained under Rule IV. of our *second method* of computing occultations, the following table of times, and of the corresponding values of $Q \sin \frac{1}{2} y$, of T, and of the moon's apparent semi-diameter, was carefully constructed.

6. *The Table of Values.*

Times	Values of $Q \cos \frac{1}{2} y$	Values of T	☾'s apparent semi-diameters.
16^h 39^m $54^s.8$	$10'$ $32''5$	$13'$ $53''.7$	$16'$ $4''.2$
45 54 8	7 49 .0	13 50 .2	16 4 .3
51 54 .8	5 6 .2	13 47 .2	16 4 .4
57 54 .8	2 24 .1	13 44 .4.	16 4 .4
17 3 54 .8	0 17 .3	13 42 .1	16 4 .5
9 54 8	2 57 .9	13 40 .1	16 4 .6
15 54 .8	5 37 .9	13 38 .5	16 4 .7
21 54 .8	8 17 2	13 37 .3	16 4 .8
27 54 .8	10 55 .8	13 36 .4	16 4 .9
33 54 .8	13 33 .6	13 35 .9	16 5 .0
39 54 .8	16 10 .8	13 35 .8	16 5 .1
45 54 .8	18 47 .3	13 36 .0	16 5 .2
51 54 .8	21 23 1	13 36 .6	16 5 .3

From the inspection of this table we may perceive that the immersion will take place a short time before $16^h\ 45^m\ 54^s.8$, for at this time we have

$$c = 16'\ \ 4''3 = 16'\ 072 \qquad 2 \log c = 2.41214 = \log 258.31$$

$$x = 13\ \ 50.2 = 13.837 \qquad 2 \log x = 2.28208 = \log 191.46$$

$$\log (c^2 - x^2)\ 1.82310 \qquad 66.85$$

$$\sqrt{c^2 - x^2} = 8'\ 10''.8 = 8.176 \qquad \log \sqrt{c^2 - x^2} = 0.91255$$

Now considering the value of $\sqrt{c^2-x^2}$ as a value of $Q \cos \frac{1}{2} y$, we find, by proportion, that the value of T, or difference of latitudes, corresponding to it in the table, is 13′ 50″.66, and $dx = 0.46$, so that we shall have

$\log dx = 9.665$

$\log x \ldots = 1.141$

$\sqrt{c^2-x^2} \ldots = 8'\ 10''.6$ $\qquad$ Ar. co $\log \sqrt{c^2-x^2} = 9.087$

Correction . $-\ 0.8$ $\qquad$ log of correction . . $= 9.893$

Diff. . . . . $8\ \ 9.8$

The value of $Q \cos \frac{1}{2} y$ corresponding to the instant of immersion will therefore be 8′ 9″.8, which exceeds the value belonging to 16ʰ 45ᵐ 54ˢ.8 by 20″.8. According to the Table, the value of $Q \cos \frac{1}{2} y$ varies 2′ 43″.5 or 163″.5 in 360ˢ, the interval of time from 16ʰ 45ᵐ 54ˢ.8 to 16ʰ 39ᵐ 54ˢ.8; hence, we may say, as 163″.5 : 360ˢ :: 20″.8 : x = 45ˢ.8, so that the time of the immersion will be 45ˢ.8 before 16ʰ 45ᵐ 54ˢ.8, or at 16ʰ 45ᵐ 9ˢ apparent time at Greenwich.

Likewise we may perceive from the Table that the emersion will take place a short time after 17ʰ 21ᵐ 54ˢ.8; and for this time we have

$c = 16'\ 4''.8 = 16'.080$ $\qquad$ $2 \log c = 2.41257 = \log 258.57$

$x = 13\ 37.3 = 13.622$ $\qquad$ $2 \log x = 2.26848 = \log 185.56$

$\log (c^2 - x^2) = 1.86338$ $\qquad$ 73.01

$\sqrt{c^2-x^2} = 8'\ 32''.7 = 8'.545$ $\qquad$ $\log \sqrt{c^2-x^2} = 0.93169$

The value of T corresponding to this value of $\sqrt{c^2-x^2}$, considered as a value of $Q \cos \frac{1}{2} y$, is 13′ 37″.2, so that we have $dx = -0.1$, and . . . . $\log dx = \bar{9}.000$

$\log x = 1.134$

$\sqrt{c^2-x^2} = 8'\ 32''.7$. $\qquad$ Ar. co $\sqrt{c^2-x^2} = 9.068$

Correction $+\ .2$ $\qquad$ Log of correction $= 9.202$

$8\ \ 32.9$

This last number, considered as a value of $Q \cos \frac{1}{2} y$, corresponds to the tabular time 17ʰ 22ᵐ 30ˢ.5, which, according to this method, will be the time of emersion.

7. *The Inverse Problem, or computation of an observed Occultation by the Longitudes and Latitudes.*

Cagnoli, in his *Trigonometria piana e sferica*, (p 489 of the second Italian edition,) has given the formulæ, illustrated by an example, for determining the corrections of the moon's longitude and latitude, and for finding the difference of geographical longitude between two places, by means of three observations made in each of these places Our object is to work an example of the computation to be employed in finding the geographical longitude of a place, on the supposition that the moon's place given by the Tables is correct, or has been previously corrected.

RULE.

Compute by the formulæ already given (in our third method) the apparent distance of the moon from the star for the moment of observation, employing the assumed longitude of the place, and the moon's longitude and latitude from the Tables, or as corrected: compare this distance with the moon's augmented semi-diameter, and if they are found equal, the assumed longitude of the place will be the right one, if not, call dD the correction of the computed distance, l the apparent latitude of the moon, and r the ratio of the moon's hourly motion in latitude to the hourly motion in longitude, and the formula

$$(1) \qquad dL = \pm \frac{dD}{\sin u \cos l + r \cos u}$$

will give the error of the difference of longitude between the two luminaries; and this error, reduced into time by means of the hourly motion, will be the correction of the assumed longitude of the place.

In this formula, the sign $-$ takes place before the apparent conjunction, and cos u ought to be considered negative, or belonging to an obtuse angle, whenever the moon's distance from the nearer pole of the ecliptic is less than that of the star.

It may be proper to remark on this rule, that if the error dD should consist of several minutes in any case, as the equation given above is only a differential one, it will not give the value of dL quite correctly But when this happens, the value of dL thus obtained may be converted into time, and applied as a first correction to the assumed longitude; then if the computation be repeated with this approximate longitude, we shall obtain another error dD, smaller than the former one, and this dD, employed in the said equation, will bring out the true correction of the assumed longitude. .

8. *Example* —Suppose the moon's place given in the Nautical Almanac of 1826 to be correct on Jan. 31, and the longitude of the observatory at Paris $9^m\ 21^s$ east of Greenwich, likewise that the emersion of δ♏ was observed at Paris at $17^h\ 31^m\ 43^s.4$, Parisian time; then, assuming for the longitude of Paris $9^m\ 30^s$, let it be required to find the error of this assumption?

If we subtract the supposed difference of longitude of the two places from the time of emersion as seen at Paris, we shall obtain the corresponding time at Greenwich, viz. $17^h\ 22^m\ 13^s.4$, and for this time we have, according to the Nautical Almanac,

☽'s true longitude	$239°$	$51'$	$25''.8$
☽'s true latitude	0	53	19 .6 S.
☽'s equatorial hor parallax		58	46 1
☽'s true semi-diameter		16	0 7
☉'s right ascension	20^h	57^m	$26^s.3$

With these data, and the reduced latitude of Paris = 48° 39′ 11″ (by Lunar Tab. 6), we have obtained, by the foregoing rules (of our third method), the following values of Q, T, y, and Δ, viz.

$Q = 7' \; 36''3$ $\qquad T = 0^\circ \; 14' \; 7''.7$

$l = 1 \; 43.37$ S $\qquad \frac{1}{2} y = 1 \; 50 \; 41$

$\Delta = 16' \; 5''.6$

With these values of Q, T, and $\frac{1}{2} y$ we must now compute the apparent distance of the moon's centre from the star, thus;

$\log Q = \log 456.3$	2.65925
$\log \cos \frac{1}{2} y = \log \cos (1^\circ\ 50'\ 41'')$	9.99977
Ar. co. $\log T$, or ar co. log 847.9	7.07176
$\log \text{tang}\, u = \log \text{tang}\ 28^\circ\ 16'\ 50''$	9.73078
Ar. co log cos u	0.05520
Add log T	2.92824
Sum	2.98344

This last logarithm gives the apparent distance of the moon's centre from the star equal to $16'\ 2''.6$, which quantity, compared with the moon's apparent semi-diameter, is found less than it by $3''.0$. The numerator dD in our formula (1) will therefore be $-3''$. For the value of the denominator we shall have as follows;

log sin u	= 9.675
$\log \cos l = \log \cos (1^\circ\ 43'\ 37'')$	= 0.000
$\sin u \cos l = 0.473$ . . log sin u cos l	= 9.675
log (☾'s hourly motion in lat.) 182.9	= 2.262
Ar co. log (☾'s hourly motion in long.) 2083.4	= 6.681
log r	= 8.943
log cos u	= 9.945
$r \cos u = 0.077$ $\qquad$ log (r cos u)	= 8.888
$\sin u \cos l + r \cos u = 0.550$ . . . . Ar. co. log	= 0.260
$\log dD = \log 3''$	= 0.477
$dL = 5''.46$ . . . . log dL	= 0.737

The difference of longitude $5''.46$, at the ratio of $2083''.4$ per hour, gives the time $9^s.4$ to be subtracted from the assumed longitude at Paris, $9^m\ 30^s$, in order to obtain the true one, which, by our computation, will then be $9^m\ 20^s.6$, differing but 4 of a second from what we ought to find; and if the computation should be repeated with this corrected longitude, the exact correction would be obtained.

9. *Remarks.*—Before we conclude this our *third method*, we beg leave to add a few remarks, that may be acceptable to the reader who is interested in this subject.

If in the formula of the *second method* we substitute the longitudes and latitudes of the moon and star for their right ascensions and declinations, and the longitude and altitude of the nonagesimal for the right ascension of the mid-heaven, and co-latitude of the observer, they will afford a new mode of solution of the problem in our present case. The solution thus arising would be analogous to that which Signore Carlini has given in the *Effemeridi di Milana* for the year 1809, and which was afterwards extended to the computation of the eclipses of the sun by Signore Conti, in the *Opuscoli Astronomici di Roma*.

On a trial of this method we have been satisfied with the results, but for greater accuracy we would recommend substituting the cosines n and m for the difference of latitude and difference of longitude, reduced to the parallel of the moon, as was shown at formula (2)' (page 635) and Rule II (page 636).

The computation of the longitude and altitude of the nonagesimal may be avoided when we have a table which gives the angle of position of the star: for in this case if we represent by μ' and ν' the values of μ and ν computed according to the formula (3) of the preceding method, (page 635,) and by P the angle of position, taken positively in the first six signs, and negatively in the last six, the values of μ and ν to be employed in the present case will be

$$\mu=\mu' \cos P-\nu' \sin P$$
$$\nu=\mu' \sin P+\nu' \cos P.$$

§ XCVIII THE FOURTH METHOD OF COMPUTING OCCULTATIONS OF STARS, VIZ BY THE PARALLAXES APPLIED IN THE MOON'S ORBIT AND IN A CIRCLE PERPENDICULAR THERETO

1. The fourth and last method of computing the occultations of stars, which we proposed to explain, is that in which the parallaxes are applied to the moon's place, in the direction of the true orbit, and of the circle perpendicular to it. The contrivance of this method is due to Dr. Thomas Young, the ingenious superintendant and editor of the Nautical Almanac, who published its principles in Vol. X. (1821) of the Quarterly Journal of Science, &c., and who has given practical rules, and worked an example in the Nautical Almanac of 1826, as well as published monthly tables of the elements for computation, as a constituent portion of the almanac, for a few years back and to come. This method will be found short and very convenient, and is grounded on the subjoined Rules, which we will illustrate by the same example which we have worked by our three former methods, in order that the comparison between them may be more complete

Rule I. Find the geocentric or reduced latitude of the place of observation, by subtracting from the true latitude the correction standing opposite to it in the following table, or from our Lunar Table 6, which will answer the same purpose, if the compression be taken at $\frac{1}{309}$, or nearly a mean between $\frac{1}{300}$ and $\frac{1}{317}$.

2. Dr. T. Young's Table for finding the geocentric latitude.—

0°	90°	0′	0″	15°	75°	5′	37″	30°	60°	9′	44″
1	89	0	24	16	74	5	57	31	59	9	55
2	88	0	47	17	73	6	17	32	58	10	6
3	87	1	10	18	72	6	36	33	57	10	16
4	86	1	34	19	71	6	55	34	56	10	25
5	85	1	57	20	70	7	13	35	55	10	33
6	84	2	20	21	69	7	31	36	54	10	41
7	83	2	43	22	68	7	48	37	53	10	48
8	82	3	6	23	67	8	5	38	52	10	54
9	81	3	28	24	66	8	21	39	51	10	59
10	80	3	51	25	65	8	36	40	50	11	4
11	79	4	12	26	64	8	52	41	49	11	7
12	78	4	34	27	63	9	5	42	48	11	10
13	77	4	55	28	62	9	19	43	47	11	12
14	76	5	16	29	61	9	31	44	46	11	13
								45°		11	14

For instance, the latitude of Greenwich being 51° 28′ 40″, the correction is 10′ 57″, and the reduced latitude 51° 17′ 43″ by the table above given.

RULE II. With the latitude thus reduced, find the moon's altitude for the instant of conjunction in right ascension, and for an hour earlier or later, and compute the parallactic angle (*P* ☾ *Z*) fig. 1. of Plate XXXI.), formed by the circle of declination with the vertical circle: this angle, subtracted from the *polar orbital angle,* (*P* ☾ ☾′) found in the Elements, or added to it, according to the relative situation of the angles, will give the *parallactic orbital angle* (*Z* ☾ ☾′) formed by the orbit with the vertical circle

Example.—According to the Elements of the occultation δ♏ taken from the Nautical Almanac of 1826, we have

Moon's conjunction with δ♏ Jan. 31 at Greenwich .		17h	27m	26s	
Diff of declination between the star and moon . . .		1°	6′	13″	
Moon's hourly motion in her orbit		0	34	51	
Polar orbital angle	S.	82	40	0	E.
Moon's and star's right ascension		15h	50m	4s	
Moon's declination		21°	0′	48″	S.
Sun's right ascension		20h	57m	28s.9	

This occultation as we have had occasion to observe more than once before, being accelerated by the effect of parallax, we must compute the moon's altitude for an hour before the time of true conjunction, or at 16h 27m 26s, for which time we have deduced by interpolation, from the Nautical Almanac,

Moon's right ascension	. . . . . .	15^h	47^m	36^s
Moon's declination	. . . .	20	56	17 S.
Sun's right ascension	. . . .	20	57	18.7

The differences of the moon's and sun's right ascensions at the two times will then be

$$\begin{array}{rrr} \text{at } 17^h & 27^m & 26^s \\ 16 & 27 & 26 \end{array} \quad ☾ - ○ \quad \left\{ \begin{array}{lll} 18^h & 52^m & 35^s.1 \\ 18 & 50 & 17.3 \end{array} \right.$$

and subtracting from these differences their corresponding times, we shall have the moon's horary angles, thus;

$$\left. \begin{array}{rrr} \text{at } 17^h & 17^m & 26^s \\ 16 & 27 & 26 \end{array} \right\} ☾\text{'s horary angle} \left\{ \begin{array}{llll} 1^h & 25^m & 9^s.1 & \text{E.} \\ 2 & 53 & 51.3 & \text{E.} \end{array} \right.$$

3. Dr. Young computes the altitudes by means of the *requisite Tables*, or of Professor *Lax's Tables*, but as these Tables may not be always at hand, we will make the computation in our example by the common logarithmic Tables.

Let the reduced latitude be denoted by ϕ, the moon's declination by D, and the horary angle by H, then the altitude and the parallactic angle, or angle made by the vertical with the circle of declination, may be obtained by the following formulæ; viz.

$$\text{tang } x = \cos H \cot \phi$$

$$\sin \text{alt} = \sin \phi \frac{\sin (D+x)}{\cos x}, \text{ and tang paral. angle} = \text{tang } H \frac{\sin x}{\cos (D+x)}.$$

Now for the earlier time we have $H = 1^h\ 25^m\ 9^s.1$, or in space (by our TIME TAB. at p. 109), $H = 21°\ 17'\ 16''$. Then

		log cos H = 9.96931	log tang H . . .	= 9.59066
log sin ϕ .	= 9.89231	log cos ϕ = 9.90379		
		log tang x = 9.87310		
Ar. co. log cos x	= 0.09621	x = 36° 44′ 45″	log sin x	= 9.77689
		D = 21 0 48		
log sin $(D+x)$	= 9.43320	$D+x$ = 15 43 57	Ar. co. log cos $(D+x)$	= 0.01658
log sin alt. .	= 9 42172		log tang $P.A$.	= 9.38413
☾'s alt. . . .	= 15° 18′ 43″		☾'s $P.A$ . . .	13° 36′ 39″

In the same manner, for the second period we have found

☾'s alt. = 11° 15′ 49″ ☾'s $P.A$ 21° 51′ 0″

The moon being to the east of the meridian, the vertical circle (☾Z) passes up to the right, or to the west of the circle of declination (☾P), and within the moon's orbital angle P☾☾′, which is marked SE in the Table of elements, we therefore subtract from

Polar orbital angle (P ☾ ☾')	82° 40'	82° 40'
Parallactic angle (P ☾ Z)	13 37	21 51
Parallactic orbital angle (Z ☾ ☾') . . .	69 3	60 49

Rule III. To the proportional logarithm of the horizontal parallax, properly reduced, add the logarithmic secant of the apparent altitude, obtained from the approximate parallax in altitude, and also first the secant and then the cosecant of the parallactic orbital angle, in each position. the first result will give the parallax reduced to the true orbit, and their difference will give the correction of the hourly motion in the orbit. The difference of the second result will be the tangent of the angular correction, if the corrected hourly motion be made radius.

Table for correcting the proportional Logarithm.

Lat.	0.	11.	19.	25	30.	34	39.	43.	47.	51	56.	60.	65.	71.	79.	90
Add	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	

For instance, the moon's horizontal parallax at midnight of Jan 31, 1826, is expressed by the proportional logarithm 4856, and at noon Feb. 1, by 4868 (Naut. Alm.) consequently at $17^h\ 27^m$ we have 4867, and at $16^h\ 27^m$, 4860, which numbers, corrected by adding 9 for the latitude, become 4870 and 4869.

We then proceed thus,

First Prop. log	4870	Second Prop. log	4869
log sec alt 15° 19' . . .	0157	. . . . 11° 16' . . .	0085
56' 34"	5027 [*]	. . . 57' 31" .	4954 [*]
Alt. 15 18 43		. . . . . . 11 15 49	
log sec 14 22 9 .	0138	. . 10 18 18 .	0071
Prop log cor par. in alt .	5008	. . . .	4940 = 1
log sec par. orb ang. 69° 3'	4467	. . . . 60° 49' . . .	3119 = 2
log cos	0297	. . . .	0590 = 3
Par. on true orbit 20' 18".7	9475	Par. on true orbit 20' 8" 5 . .	8059 = 1 + 2
Par. perpend . 53 3 .5	5305	Paral. of perpend. 50 22 7 . .	5530 = 1 + 3

Differences . { 7 49 .8
2 40 .8

☾'s hourly motion (34' 51") – (7' 49".8) = 27' 1".2 Prop. log . . 8256
2 40 .8 Prop. log . 1 8271

log tang of angular correction 5° 41' . . . . 8 .9985

As the orbital parallax tends towards the western horizon, and retards the moon's apparent motion in her orbit, the difference of the retardation, 7' 49".8, is *subtracted* from the hourly motion.

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[*] The reader must recollect, that if a Table of proportional logarithms should not be at hand, the common logarithms corresponding to the parallaxes expressed in seconds, may be obtained by adding to the complements of these numbers the constant number 3 0334.

With respect to the angular correction, as the parallax in altitude is less an hour before, the visible orbit is diverging from the true orbit in a south-easterly direction, consequently it must be subtracted from the S.E. orbital angles, which will then become 82° 40′ − 5° 41′ = 76° 59′, and 69° 3′ − 5° 41′ = 63° 22′ respectively

Rule IV. By means of the orbital angles thus found, reduce the parallax at the conjunction in right ascension to the visible orbit, and to the direction perpendicular to it; and reduce the difference of declination to the same direction, then the sum or difference of the former results reduced into *time*, according to the visible hourly motion, will give the time of the nearest approach, and the sum or difference of the latter results will give the nearest distance. The hourly motion may be reduced to the visible orbit by increasing it in the ratio of radius to the secant of the angular correction, agreeably to the following work; viz.

prop log cor. paral in alt.	5008	prop. log diff. dec. 66′ 13″	4343
log sec 63° 22′	3485	log sec 76° 59	6474
cosec	0487	cosec	0113
prop. log 25′ 28″	8493	prop. log 14′ 54″.5	1.0817
prop. log 50 47	5495	prop. log 64 31	4456

Differences { $10^{m}\ 33^{s}.5$ = retardation in the visible orbit.
{ 13′ 44″ = nearest distance.

prop. log visible hourly motion in the orbit		8236
log sec angular correction		0021
prop. log visible motion		8215
prop log one hour		4771
		m 3444
prop. log	$10^{m}\ 33^{s}.5$	1.2317
prop. log	23 20	0.8873
conjunc in R. A =	17 27 26	
middle	$17^{h}\ 4^{m}\ 6^{s}$	

Rule V. It may in some cases be necessary to repeat the computation of the effect of parallax at the time thus found, and to correct the visible hourly motion accordingly, or to find the apparent distance at the given time by some of the ordinary methods but as we propose to determine the times of immersion and emersion, such as may be sufficiently accurate for all ordinary purposes, we will in the first place see what results arise simply from an approximation, and will afterwards show how to deduce the respective times to the nearest second.

Rule VI. Find the sum and difference of the nearest distance and of the moon's semi-diameter properly augmented, and subtract from the half sum of their proportional logarithms, that of the visible hourly motion the difference will be the proportional logarithm of the semi-duration.

The moon's semi-diameter, as given in the Nautical Almanac, will be 16′ 0″.7 at 17^h 27^m, and 16′ 0″.9 at 16^h 27^m, with these semi-diameters and the altitudes found above, used as arguments, enter LUNAR TABLE 7, which will give the respective augmentations 4″.2 and 3″.1, and consequently the apparent semi-diameters will be 16′ 4″.9 and 16′ 4″.0, so that by interpolation we shall have the semi-diameter = 16′ 4″.5 at 17^h 4^m, thus

16′ 4″.5		
13 44 .0		
29 48 .5	prop log	0 .7809
2 20 .5	prop. log	1 .8857
	Sum	2 .6666
	Half	1 .3333
prop. log for visible motion	. . . . . *m*	3444
prop. log	. 18^m 28^s semi duration . . .	9889
middle .	17 4 6	
	16 45 38 immersion.	
	17 22 34 emersion.	

4. The times of immersion and emersion thus found are sufficiently accurate for the object of announcing the occurrence of the phenomena, but as it is always desirable to know how to obtain all possible accuracy by the method we adopt, we will proceed with our example to determine the time of the immersion to the nearest second.

A little consideration of the principles of this fourth method of computing an occultation will convince us, that to obtain our object, we have only to repeat the computation according to the same process, by assuming the two times, including the moment of immersion, separated by a short interval of only five minutes, in order that, during this short interval, the corresponding portion of the visible orbit may be strictly considered as a straight line described with uniform motion.

Let us assume the said times at 16^h 42^m 26^s, and 16^h 47^m 26^s, and then we shall find

	at 16^h 42^m 26^s		at 16^h 27^m 26^s
Moon's right ascension	237° 3′ 13″.9	. .	237° 6′ 19″.0
Moon's declination	20 57 25 .8	. .	20 57 48 .5 S
Moon's polar orbital angle	82 31 12		
Moon's hourly motion in the orbit .	34 52 .1		
Star's distance from the circle of dec. passing through the moon . .	25 43 .5		
Star's distance from the circle passing through the moon and perpendicular to her circle of dec.	69 37 .5 *		
Moon's reduced horizontal parallax	58 39 .5		

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* These last two quantities ought to be computed by the same formulæ that we have given in the *second method* for obtaining the values of *m* and *n*, except that the letters belonging to the moon must be changed for those belonging to the star, and vice versâ

5. The computation according to the formulæ, given under Rule II., will afford the following altitudes and parallactic angles, viz

at	16ʰ 42ᵐ 26ˢ		16ʰ 47ᵐ 26ˢ
Moon's altitudes	12° 26′ 40″	. .	12° 48′ 53″
Moon's parallactic angles .	19 53 50	. .	19 30 47
Moon's paral. orbital angles	62 37 22	.	63 17 25

From these numbers, by the process under Rule III., the apparent altitude, visible hourly motion, and angular correction of the orbit, will be found as follow,

log moon's parallax in alt. at 16ʰ 42ᵐ 26ˢ . . .	3 .53769
log hourly motion in the visible orbit	3 .21644
Angular correction	5° 54′

Now according to Rule IV. we must determine the nearest distance of the star from the visible orbit, and the time at which the moon is at this point. As, in the supposition of Rule IV., the star's distance from the circle of declination at the first period, or moment of true conjunction in right ascension, was nothing, no reduction was applied on account of this distance, but in the present assumption, in order to obtain the nearest distance, we must apply to the result given by Rule IV., with the proper sign, the star's distance from the moon's circle of declination multiplied by the sine of the visible orbital angle, and to obtain the retardation in the orbit, the same distance must be multiplied by the cosine of the said angle. The work will stand thus,

1.	log star's dist from moon's circle of declin. .	0° 25′ 43″ .5	.	= 3 .18851
2.	log star's dist. from circle perpendicular to it	1 9 37 .5	.	= 3 .62091
3.	log sin visible orbital angle	76 57 12	.	= 9 .98805
4.	log cos ditto		.	= 9 .36438
(2+3)	. . . . nat. num	4064″.1	.	3 .60896
(1+4)	. . . . nat. num.	357 .2	.	2 .55289
	Diff.	3706 .9		
(1+3)	. . . . nat. num.	1501 .6	.	3 .17656
(2+4)	. . . . nat. num.	966 .7	.	2 .98529
	Sum	2468 .3		

Before we proceed further we may remark here that, as the visible orbit is inclined towards the star, the perpendicular drawn from it upon the orbit is diminished on account of the distance of the star from the circle of the moon's declination, and the number 357 .2 is subtracted on the contrary, the portion of the visible orbit contained on the moon's orbit, between the said perpendicular and the moon, is augmented on account of the star's distance from the circle of the moon's declination, and therefore the number 1501 .6 is added. Then

Log moon's parallax in altitude		3 .53769
Log sin visible paral. orbital angle 56° 13′ 22″		9 .92220
Log cos of ditto		9 .73933
nat. num	2883″.3	3 45989
nat. num.	1892 .4	3 .27702
Nearest distance	823 .6	
Retardation in the visible orbit . .	575 9	

6. Now the moon's augmented semi-diameter being found equal to 16′ 4″.3, the root of the product of its sum and difference taken with and from the nearest distance becomes 501″.5, which number, subtracted from the retardation in the orbit, leaves 74″.4, and this small arc converted into time by means of the hourly motion in the visible orbit becomes $2^m\ 42^s.7$; and the corrected time of immersion will be $16^h\ 42^m\ 26^s+2^m\ 42^s.7=16^h\ 45^m\ 8^s.7$ apparent time at Greenwich, which is very nearly the same as we determined by the preceding methods

7. *The Inverse Problem, or computation of an observed occultation by the parallaxes applied to the Moon's Orbit and in a circle perpendicular thereto.*

The problem of deducing from an observation the time of the moon's nearest approach to a given star, by applying the parallaxes in the orbit itself, and in a circle perpendicular to the orbit, has been treated of by Mr Thomas Henderson of Edinburgh, whose method of computing depends on certain rules which have been published in the Nautical Almanac of the year 1827. This method supposes that we know from observation the value of the true nearest distance of the moon from the star, and also the moment when the moon is in this situation, which quantities will be found already computed in the Nautical Almanac of 1827 and following years. (See Rule III. of the INVERSE PROBLEM by the *first method*)

RULES.

RULE I Compute the altitude and parallactic angle of the star for the time of observation, and for the reduced (or geocentric) latitude of the place, as has been before explained. The parallactic angle ($P * Z$) has the sign + before the star has passed the meridian, and − after. The complement of 90° of the moon's orbital angle (P ☽ ☽′) must have the sign − when the moon's nearest approach to the star and the orbital angle have the same denomination, *N* or *S*, and + when they are of different denominations. The sum of these two angles is the complement of the parallactic orbital angle Z ☽ ☽′, or the complementary angle with its proper sign.

As an *Example*, suppose that the immersion of δ♏ has been observed at Greenwich at $16^h\ 45^m\ 9^s$, let it be required to find the nearest approach?

The right ascension of the sun being at this time $20^h\ 57^m\ 21^s.8$, and the right ascension of the star $15^h\ 50^m\ 4^s$, we shall have the horary angle = $2^h\ 7^m\ 32^s.2$, or in space 31° 53′ 18″, and from these, by means of the formula given under Rule II. (*fourth method*) the star's altitude will be found 11° 25′ 20″, and the parallactic angle + 19° 41′ 38″. Then we proceed thus

Orbital angle	82°	40′	SE — ☽ *N*	
Complement	+ 7	20		
Parallactic angle	+19	41	38	
Comp. paral orbital angle .	+27	1	38	

If the complementary orbital angle should exceed 180°, its supplement to 360° must be taken with the sign reversed.

RULE II. Add together the logarithm of the moon's reduced horizontal parallax, the logarithmic cosine of the star's altitude, and the logarithmic sine of the complementary angle. the sum will be the logarithm of the orbital parallax, which must have the same sign as the complementary angle. To this last logarithm add the logarithmic cotangent of the complementary angle, and this sum will be the logarithm of the perpendicular parallax, which must have the contrary sign to that of the moon's nearest approach, when the complementary angle is less than 90°, but the same sign when it is greater, considering + as belonging to the moon's distance, when she is N of the star, and — when S. The work will stand thus,

log reduced horizontal parallax . . .	0°	58′	39″.5	. .	3.54648
log cos star's altitude	11	25	20	. .	9.99131
log sin comp. paral. orbital angle . . .	27	1	38	. .	9.65745
orbital parallax 26′ 7″.6				Sum	3.19524
log cot. comp. paral. orbital angle					0.29232
perpendicular parallax 51′ 13″				Sum	3.48756

RULE III The sum of the moon's nearest approach and of the perpendicular parallax may be considered as one of the sides, and the moon's semi-diameter, without augmentation, as the hypothenuse of a right-angled triangle, of which the other sine is to be determined It will have the sign + in the case of an immersion, and — in that of an emersion. The sum of this quantity of the orbital parallax, being reduced to time by means of the moon's hourly motion, and then applied to the time of observation by addition or subtraction accordingly as it bears the sign + or —, will give the time of the nearest approach, reckoned according to the meridian of the place of observation, which time being compared with that of the same phenomenon, as given for Greenwich in the Nautical Almanac, will afford the means of determining the difference of longitude of the two places.

Let us assume the value of the nearest distance, and the time of nearest approach, as determined under Rule III. of the *Inverse Problem* of our *first method*, and we shall have the subjoined computation; viz.

Nearest distance		65′	40″.5	
Perpend. parallax . .		+51	13	
Difference		14	27.5	
Semi-diameter		16	0.8	
	Sum . .	30	28.3	log 3.26205
	Diff.	1	33.3	log 1.96988
				2)5.23193
Side		6	53.0	. . 2.61596
Orbital paral		26	7.6	
		33	0.6	

The space 33′ 0″.6 reduced into time at the rate of 34′ 51″ per hour (as before determined), gives $56^{m}\ 49^{s}.9$ to be added to $16^{h}\ 45^{m}\ 9^{s}$, the observed time of the immersion, and makes $17^{h}\ 41^{m}\ 58^{s}.9$ for the time of nearest approach, which differs only by two tenths of a second, from that which we obtained by our *first method* (Rule II.) from the elements contained in the Nautical Almanac *.

8. It will be proper to remark here, that though the result is very satisfactory in this instance, yet such accuracy is not always to be expected from this method without some modification, particularly when the interval between the time of immersion and of the nearest approach, as here, is nearly one hour. The variation of a second, or even of some tenths of a second, in the nearest distance of the moon from the star, may sometimes produce an error of some seconds in the time of the nearest approach, and as all the quantities are considered, in this method, as changing with an uniform motion, and in a straight line, without having regard to the terms of the *second order*, it is probable that slight errors will frequently exist in them. On this account, we will proceed further to show a process of computation, by which an account will be taken of the terms of the second order, so as to render the solution perfectly correct. For in working the *Reverse Problem*, it will seldom happen, that approximation will be so satisfactory as complete accuracy, particularly when the object is, to determine the longitude of the place of observation.

9. *A more correct Solution.*

RULES.

I. Find, as was explained under Rule I. "for computing with accuracy," &c. by our *second method* (at page 641), the moon's hourly motions of the first and second orders, both in right ascension and declination, multiply the hourly motion of the first order in right ascension by the cosine of the moon's declination, and divide the product by her hourly motion of the first order in declination; the quotient will be the tangent of the polar orbital angle to be employed in our case. Then divide the same product by the sine of the polar orbital angle, and this quotient will be the moon's hourly motion of the first order in the orbit. With these data find, as usual, the nearest distance of the moon from the star, and the time of the nearest approach.

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* In the Nautical Almanac of 1830, we find that Mr Henderson has given a method of solution of the approximate method, which renders the ambiguity of the angles less liable than in the preceding rule

Under Rule I. of our second method above referred to, we found, for instance,

Moon's hourly motion of 1st order in R. A. 37′ 1″.9 . . .	log	3 .34672
Moon's declination 21° 0′ 48″.	log cos	9 .97011
	log product . . .	3 .31683
Moon's hourly motion of 1st order in declin. 4′ 26″.6	Ar. co. log .	$\bar{7}$ .57414
Tang. polar orbital angle 97° 19′ 28″	first sum .	$\bar{0}$ .89097
	Ar. co. log sine .	0 .00356
Moon's hourly motion of 1st order in orbit 34′ 51″.2 .	second sum	3 .32039
Diff. of declin. between the moon and star 66′ 13″ . . .	log	3 .59911
	log sine paral. angle	9 .99644
	log cos orb. angle .	$\bar{9}$ .10547
Nearest distance 65′ 40″.5		3 .59555
Distance from the nearest approach − 8′ 26″.5		$\bar{2}$ .70458

The distance in the orbit from the nearest approach reduced into time, at the rate of 34′ 51″2. per hour, produces $14^m\ 32^s.0$ to be added to the time of true conjunction, and the time of the nearest approach thus obtained is $17^h\ 41^m\ 58^s$.

II. By the formulæ and the process given under Rule II. of our *second method*, (Approximate Inverse Problem,) compute the values of m and n corresponding to the time of the observed immersion. From the value of m reduced to the meridian of Greenwich, and that of n, diminished by the difference of declination between the moon and star at the time of the true conjunction, subtract respectively the moon's motions, of the first order, in right ascension and in declination, in the proportion of the interval of time between the immersion (or emersion) and that of the nearest approach. The difference between the first residue multiplied by the cosine of the orbital angle, and of the second residue multiplied by its sine, and reduced into time, at the rate of the hourly motion (of the first order) in the orbit, will give the correction of the time of nearest approach; and the sum of the first residue, multiplied by the sine of the orbital angle, and of the second residue, multiplied by its cosine, will give the correction of the nearest distance.

Having by these means found the proper values of the time of the nearest approach, of the nearest distance, of the orbital angle, and of the hourly motion, determine by these, according to Mr Henderson's method, the exact time of nearest approach for the place of observation.

10. The values of m and n corresponding to $16^h\ 45^m\ 9^s$, according to the directions under Rule II of our *second method*, (page 643,) are $m = 24'\ 22''.0$, $n = 69'\ 20''.84 - 66'\ 13'' =$

3′ 7″.84, and the moon's motions, of the first order, in right ascension and declination, in 42ᵐ 17ˢ, are mot. in R A. = 24′ 21″.6, mot. in dec 3′ 7″.87, hence we have

1st residue = 0 .4		2d residue 0 03	
log 0 .4	9 60	log 0 03	$\bar{8}$ .47
log sin orb ang.	0 .00	. . . .	0 .00
log cos. orb ang	$\bar{9}$ 11	. . . .	$\bar{9}$ 11
0 4 . .	$\bar{9}$ .60	−0 .03 .	8 .47
−0 05	$\bar{8}$ 71	0 00	7 .56
1st part	0″ .4	1st part	0″ 03
2d part	0 .0	2d part	0 05
correc. of nearest approach =	0 .4	correc of nearest distance =	0 .08

The proper time of the nearest approach will now be 17ʰ 41ᵐ 58ˢ.4, and the corrected nearest distance 65′ 40′ .4

Lastly, if we repeat the operation with these last found values of the time of the nearest approach, of the nearest distance, of the hourly motion, and of the orbital angle, we shall find, that the interval of time between the observed immersion and the time of nearest approach, according to Mr. Henderson's method, comes out 56ᵐ 49ˢ.4, which added to the time of immersion makes 17ʰ 41ᵐ 58ˢ.4, which determination nearly accords with the solution we have before given.

11. Having now gone through the four methods of computing the times of an immersion and emersion of a star by the moon, we subjoin a comparative statement of the results, to show that, in point of accuracy, they may be resorted to with equal confidence, when the data for finding the moon's exact place are repeatedly corrected, and all the requisite minutiæ attended to in the computation.

	By the first method	By the second method	By the third method	By the fourth method
Time of immersion	16ʰ 45ᵐ 9ˢ.1	16ʰ 45ᵐ 9ˢ	16ʰ 45ᵐ 9ˢ	16ʰ 45ᵐ 8ˢ.7
Time of emersion	17 22 31 .4	17 22 31 .4	17 22 30 .5	17 22 34 *

It is scarcely necessary to remark, that in observing an occultation with due care, both a good telescope and a good clock are indispensable, where the fraction of a second is a portion of time not to be neglected, and where both the error and rate of the clock are involved A transit-instrument is therefore also necessary for giving the corrections depending on the error and rate. As both the immersion and emersion may be accompanied by circumstances that call for particular notice, according to the part of the limb where the phenomena take place, and to the general appearance of the lunar disc, it will in general be desirable to pay attention to any gradual or sudden change that may take place in the apparent size, colour, or motion of the star, preceding the moment of its disappearance, and subsequently to its re-appearance, and for such purpose, the same telescope should be used in both cases while in the same state

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* This is given only by approximation (p 668)

of adjustment, as it regards the star observed. In fact, any unexpected appearance should be recorded as a part of the observation, in conjunction with the corresponding moment of time. In several total eclipses of the sun, a ring of light has been observed to surround the moon, which phenomenon has been considered as indicative of a lunar atmosphere and indentations have been observed on the edge of the moon's dark limb, at the immersion of a planet or satellite

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§ XCIX ON SOLAR ECLIPSES, OR OCCULTATIONS OF THE SUN BY THE MOON

1. The computation of a visible eclipse may be required either for a given place of observation, or in general for all the places of the earth, where it may be seen As the computation of an eclipse, in the former case, differs very little from that of occultations of stars, which has been fully explained in the preceding sections, we will limit ourselves to the modifications, that each of the four given methods requires, when applied to the eclipses of the sun. The beginning or end of an eclipse takes place when the disc of the moon comes in contact with that of the sun, at which moments the distance between the centres of the sun and moon is equal to the sum of their semi-diameters When the disc of the moon covers a part of the sun's disc, then a phasis takes place, and the magnitude of this phasis is measured by the obscured part of the sun's diameter, which lies in a direction towards the centre of the moon. The phasis, or obscured part, is expressed in twelfth parts of the whole diameter of the sun, which are called *digits*, and it is evident that this part is equal to the difference between the sum of the semi-diameters of the sun and moon, and the distance between their centres. The magnitude *of an eclipse*, in a given place, is determined by the greatest phasis that is seen at that place When all the disc of the sun is obscured, or covered by that of the moon, the eclipse is called *total;* when the disc of the moon appears projected on that of the sun, leaving all round him a ring of light, the eclipse is said to be *annular*, and when the centres of the sun and moon are seen in the same direction, one over the other, then the eclipse is said to be *central*.

2. *First Method.*

From these definitions, it clearly appears, that, in applying the first method, given for the occultations of stars, to the eclipses of the sun, for the beginning and the end of the eclipse, we must substitute the sum of the apparent semi-diameters of the sun and moon for the value of ϱ, which in an occultation of a star represented only the moon's apparent semi-diameter, and that, when we wish to determine a phasis, we must substitute for ϱ the sum of the two apparent semi-diameters, diminished by the phasis, or obscured part of the sun's semi-diameter. It is also to be observed, that, according to Rule II, which employs the apparent zenith distances of the star, these now become the zenith distances of the sun, and will require the correction for parallax.

3 *Second Method.*

The analysis employed for finding the equation (1), in the Second Method for the occultations of stars, when applied to the eclipses of the sun, leads to the following equation —

$$\{(m - n\mu) + (n - \pi\nu)^2\}^{\frac{1}{2}} = \frac{\sin d}{\cos (D-f)} + (o - \pi w) \operatorname{tang} (D-f)$$

where m, n, μ, ν, and d denote the same quantities as before,

D denotes the semi-diameter of the sun,

f the phasis, or obscured part of the sun's semi-diameter, in the direction of the apparent centres,

o the cosine of the true distance of the moon's centre from that of the sun,

ϖ the cosine of the sun's zenith distance,

π the difference of the horizontal parallaxes of the moon and sun.

In this equation, some small terms have been neglected, which, however, can never produce an error exceeding one-tenth of a second of space

We may frequently substitute, in the above equation, 1 for cos $(D-f)$, and for o, and the arcs d and $D-f$ for sin d and tang $(D-f)$, by which substitutions, only the third powers of small arcs are neglected. In this case the equation is reduced to

$$(a) \quad \{(m-\pi\mu)^2+(n-\pi\nu)^2\}^{\frac{1}{2}}=d+(1\pi\varpi)\,(D-f)$$

and at the moments of the beginning and end of the eclipse, to

$$\{(m-\pi\mu)^2+(n-\pi\nu)^2\}^{\frac{1}{2}}=d+(1-\pi\varpi).\ D.$$

This equation shows, that, instead of the true semi-diameter of the moon, which is employed in the occultations of stars, we must now substitute the sum of the moon's true semi-diameter, and of the sun's, when diminished, according to his altitude, by the same quantity, by which the augmentation of the moon's semi diameter, on account of her altitude, is computed. (See *Lunar* Table 7, Vol. I., p. 186.) Several celebrated authors have given false rules about this reduction of the semi-diameters.

4.

Third Method.

The formulæ of Cagnoli's scheme are general, and applicable to eclipses, and we have only to compare the apparent distance of the centres expressed by $\dfrac{T}{\cos u}$ with the sum of the moon's apparent semi diameter and sun's semi-diameter diminished by the phasis f.

5.

Fourth Method.

This method may be applied to the eclipses by employing the difference of the horizontal parallaxes of the sun and moon, instead of that of the moon. In the direct problem, viz that of computing a visible eclipse, we must employ the sum of the moon's apparent semi-diameter and of the sun's diminished by the phasis f, as in the third method; and in the inverse problem, the second member of the equation (a), must be substituted for the moon's true semi-diameter.

6.

Computation of a General Eclipse.

The computation of a general eclipse is performed, to ascertain in what places an eclipse will happen under particular circumstances favourable for observations, to warn travellers, who visit distant countries, of the phenomena that they may there observe or expect; or sometimes for mere curiosity. For these objects great accuracy is never wanted, and astronomers have adopted the plan of making only a rough computation, just sufficient to trace a map, in which are delineated, 1st, the lines passing through all the countries in which the magnitude of the eclipse will be denoted by a certain number of digits, and these lines are called the *lines of the greatest phasis* 2dly, the lines through all the countries in which the beginning or end of the eclipse will happen whilst the sun is rising or setting 3dly, the lines through the countries where the greatest phasis will be seen when the sun is at the horizon. The method most commonly employed for the computation and construction of such maps is that of projection, and we propose to give here an example of this method, by applying it to the construction of a map of the eclipse that will happen on the 15th of May, of the year 1836.

7. In performing this example we will avail ourselves of the elements that Dr Cassian Hallaska has laid down in a work, the title of which is, *Elementa Eclipsium quas patitur tellus lunâ eam inter et solem versante, ab anno* 1816 *usque ad annum* 1860. *Pragæ*, 1816. *Typis Theophyli Haase.* These elements have been computed from Triesnaker's solar and lunar tables*.

Conjunction of the moon with the sun after ☊ at $2^h\ 10^m\ 28^s.1$ true time at Greenwich.

	s			
☽'s true longitude in the ecliptic . .	1	24°	42′	22″.1
☽'s northern latitude		0	25	57 .7
Hourly motion of the ☽ from the ☉ in long.			27	36 .2
☽'s hourly motion in latitude		+	2	54 .8
○'s semi-diameter			15	50 .3
☽'s semi-diameter			14	51 .2
Sum of the ☉'s and ☽'s semi-diameters .			30	41 .5
☽'s horizontal equatorial parallax			54	25 .8
☉'s horizontal parallax				8 .6
Difference between the ☽'s and the ☉'s parallaxes			54	17 .2
Angle of the ☉'s circle of declination, or first meridian, with the ecliptic on its eastern side		75	55	6
○'s declination		18	57	48 N.

These elements being prepared, in order to understand the following computations, we must first imagine a stereographical projection of the path of the moon, and of the surface of the earth, on a plane passing through the centre of the moon, and perpendicular to the line that goes from the centre of the earth to that of the sun; the pole of the projection being situated in the centre of the sun itself. As the lines to be projected are very small in comparison of the distance of the sun from the plane of projection, we may, without any sensible error, substitute an orthographical for the stereographical projection, by only diminishing the lines on the former projection, in the ratio of the distance of the projection from the sun to the distance of the point to be projected from him.

8 In Plate XXX, the radius *C M* of the circle *a L M b*, which represents the projection of the earth, is supposed to be assumed of such a length, as to be proportional to the difference of the

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* Triesnaker's Solar Tables have not yet been published, his Lunar Tables are found in the Ephemerides of Vienna for the year 1803

moon's and sun's parallaxes, and the straight lines WCE, CL, and CM have been traced to represent a portion of the ecliptic, of the circle of latitude, and of the circle of declination, passing through the sun, which is projected in C, when seen from the centre of the earth. The angle ECM must therefore be equal to 75° 55′ 6″, as given in the elements. CQ is supposed to be taken proportional to the moon's latitude, and Qr and rq are drawn respectively parallel to WE and CL; the former being proportional to the moon's hourly motion from the sun in longitude, and the latter to the moon's hourly motion in latitude. The line AB, which passes through Q and q, thus represents the projection of the relative orbit of the moon. On several points of this line have been marked the corresponding times, as reckoned at Greenwich, at which the moon is projected on them, the time of the true conjunction $2^h\ 10^m\ 28^s$ having been placed at the point Q, where the circle of latitude cuts the orbit. CIK is a straight line perpendicular to AB. The elliptical arcs $P\,I$, $P\,II$, $P\,III$, &c., and $P\,XXIII$, $P\,XXII$, $P\,XXI$, &c., are designed to represent the projections of the different horary circles, and the elliptical arcs passing through the points marked 0°, 10°, 20°, 30°, &c., the projections of the parallels of latitude of the earth.* These latter arcs, when stereographically projected, may be considered as formed by the successive intersection of the plain of projection with the lines that, from the centre of the sun, go to the different points of the corresponding parallel, and therefore the same arcs will also represent the places in which an observer, situated on that parallel, successively projects the centre of the sun during the diurnal rotation of the earth. The length of the line ID is supposed to be proportional to the sum of the semi-diameters of sun and moon, and that of the line $12\,D$ to the diameter of the sun. When $12\,D$ is greater than ID, as in the present case, the eclipse will be annular, when $12\,D$ is less than ID, the eclipse will be total, and will continue so for a while, when $12\,D$ is equal to ID, the eclipse will be total, but instantaneous. The straight lines $a\,b$, $\overset{a\ b}{3\ 3}$, $\overset{a\ b}{6\ 6}$, $\overset{a\ b}{9\ 9}$, $\overset{a\ \ b}{12\ 12}$, are supposed to be parallel to AB, and distant from each other by the fourth part of the diameter of the sun. The line $\overset{c\ \ d}{12\ 12}$ ought to stand under AB, at the same distance that $\overset{a\ \ b}{12\ 12}$ stands over it, and the distances from $\overset{c\ \ d}{12\ 12}$ to $\overset{c\ d}{9\ 9}$, from $\overset{c\ d}{9\ 9}$ to $\overset{c\ d}{6\ 6}$, from $\overset{c\ d}{6\ 6}$ to $\overset{c\ d}{3\ 3}$, are likewise the fourth part of the sun's diameter. To know what these lines are destined to represent, let us suppose the centre of the moon in any point whatsoever m of the line AB, and from this point let a straight line be drawn perpendicular to AB, this line will intersect $a\,b$ in some point s, and if we consider it as the projection of a place on the surface of the earth, and therefore as the point in which an observer, situated on that place, projects the centre of the sun, this observer will see a contact of the limbs of the sun and moon. The motion of the centre of the moon on the plane of projection being quicker than that of the projection of the centre of the sun, their distance (if we neglect the small inclination of the parallels of the earth to the line $a\,b$) at the moments before and after the contact, will be greater than $ID = ms$, or than the sum of the semi-diameters of the sun and moon, therefore their discs will appear separated, and the observer will see only a contact of the limbs. The line $a\,b$ ought then to be considered as comprehending approximately the projections of the places, in which the observers see only a contact of the limbs for their greatest phasis In the same manner it may be shown, that the lines $\overset{a\ b}{3\ 3}$ and $\overset{c\ d}{3\ 3}$, $\overset{a\ b}{6\ 6}$ and $\overset{c\ d}{6\ 6}$, $\overset{a\ b}{9\ 9}$ and $\overset{c\ d}{9\ 9}$, will comprehend the projections of the places where the observers see 3 digits, 6 digits, and 9 digits respectively, for their greatest phases The obscuration will be on the north side of the sun for the places projected in the lines *a b*, $\overset{a\ b}{3\ 3}$, $\overset{a\ b}{6\ 6}$, $\overset{a\ b}{9\ 9}$, $\overset{c\ d}{12\ 12}$, and on the south side of him for the places projected in $\overset{a\ b}{12\ 12}$, $\overset{c\ d}{9\ 9}$, $\overset{c\ d}{6\ 6}$, $\overset{c\ d}{3\ 3}$. The places having their projections between the lines $\overset{a\ b}{12\ 12}$ and $\overset{c\ d}{12\ 12}$, are those in which the eclipse will be seen annular, and the observers of those places that have their projections falling on some point of the line *A B*, at the moment that the centre of the moon is found in it, will see the eclipse central.

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* For the manner of tracing all these elliptical arcs, see Rees' Cyclopædia, art. Perspective.

9. When the figure of projection has been accurately drawn, and on a scale sufficiently large, we can find the longitude and latitude of several places where a given greatest phasis will be seen by the following construction Let us, for instance, find the longitude and latitude of the place where the greatest phasis to be seen will be of 6 digits, supposing that the place be at that moment situated under the horary circle *P II*. From the point *t*, where the horary circle cuts the line $\overset{a\ b}{6\ 6}$, corresponding to the phasis of 6 digits, draw a perpendicular to the line *A B*, the time $3^h\ 2^m\ 30^s$, which should be marked at the point *n*, where this perpendicular meets the said line, will give the horary angle of the sun at Greenwich for that moment. From the angle of the horary circle *P II*, viz 2^h, (augmented if necessary, as in the present case, by 24^h), subtract the horary angle of the sun at Greenwich, now found, and the difference, $22^h\ 57^m\ 30^s$, will be the longitude of the place in time counted eastwards from Greenwich. The latitude of the place is found by tracing a diameter passing through the points where the horary circle, *P II* cuts the circle *a*, *L*, *M*, *b*, and by drawing two perpendiculars to it, one through the projection *P* of the pole of the earth, and the other through the point *t*, then the arc *t z*, comprehended between these two perpendiculars, which shall be found $= 52°\ 4'$ will give the co-latitude of the place The construction of which we have just given an example being performed, for each horary angle, and successively for all the lines *a b*, $\overset{a\ b}{3\ 3}$, $\overset{a\ b}{6\ 6}$, $\overset{a\ b}{9\ 9}$, $\overset{c\ d}{12\ 12}$, $\overset{a\ d}{12\ 12}$, $\overset{c\ d}{9\ 9}$, $\overset{c\ d}{6\ 6}$, $\overset{c\ d}{3\ 3}$, will afford the longitude and latitude of several places where the corresponding greatest phasis may be seen, and these places being marked on a stereographical map of the earth, and joined together, will give the lines of the greatest phasis, which we have represented in fig. 3 of Plate XXXI In this figure the two lines of the greatest phasis, which bear the same denomination, (the farthest from the pole *P* being that in which the sun is seen eclipsed in his northern side, and the nearest one that in which the obscuration is observed in the southern side of the sun,) are joined by two other lines that comprehend the places where the same phasis is seen on the east side of the sun when he is rising, or on the west side when he is setting. The longitudes and latitudes of the places of such lines may be found as follows. Suppose we want to know the longitude and latitude of the place where a phasis of 3 digits will be seen, when the sun is rising, the place being at that moment situated under the horary circle *P IX* place one point of the compass in *e* where the horary circle *P IX* cuts the circle

a L M b, and with an opening equal to the distance of $\frac{a}{3}\frac{b}{3}$ from *A B*, mark that point of the two, in which the line *A B* would be intersected by a complete circle, which is nearer to *Q*; which point is marked *f* in the figure, and the time corresponding to it, which should be $= 2^h\,44^m$, is the horary angle of the sun at Greenwich, at the instant of the phasis. This horary angle being subtracted from the angle of the horary circle *P IX* (augmented by 24^h when required) gives the east longitude in time of the place from Greenwich $= 6^h\ 16^m$. To the radius, *C e*, apply a perpendicular passing through the projection *P* of the pole of the earth, and the arc $e\,g = 64^\circ\ 5'$ comprehended between *e* and the point *g*, where that perpendicular intersects the circle *a L M b*, is the co latitude of the place.

10. In the same manner may be found several longitudes and latitudes of the places of *the phasis at the horizon*, by employing successively the different horary angles and the different distances of the lines *a b*, $\frac{a}{3}\frac{b}{3}$, &c from *A B*. These places being put down in the map will serve to trace the *lines of the phasis at the horizon*, which shall join the two *lines of the greatest phasis* of the same denomination, one on the side where the sun is rising, and the other on the side where the sun is setting. Now, we may observe, that for the places where the sun is seen rising, the obscuration is decreasing; so that their corresponding phases on the east side of the sun will be the greatest that remain to be seen at those places: on the contrary, for the places where the sun is setting, the obscuration is increasing, and the phases inspected on the west side of the sun at the horizon, will be the greatest that can possibly be seen at those places. The curves re-entering into themselves, thus formed by the union of the two *lines of the greatest phasis*, and of the two *lines* of the phasis at the *horizon* of the same denomination, will comprehend all the places in which their respective phases will be the greatest as seen there; and by them we may form an idea of the magnitude of the eclipse for any other place on the earth, because the greatest phasis, that will be seen in a given place, comprehended between two of these re-entering curves, will be in proportion to the distance of that place from the same curves Having now given a specimen of the process we intend to follow for computing a general eclipse, we may proceed to explain the rules by which the lines represented in the projection may be expressed in numbers.

11. *To find the inclination of the relative orbit of the moon to the ecliptic, and her hourly motion in the orbit,*

Rule I. Let Λ denote the moon's hourly motion from the sun in longitude, λ her hourly motion in latitude, we shall have the proportion,
as $\Lambda : \lambda ::$ radius : tang (inclin. of the relative orbit), or as $27'\,36''.2 : 2'\,54''.8 :: 1 : \text{tang}\ \chi$.

$$\log 2'.\,54''.8 = 2.24254$$
$$\text{Ar. co. log } 27\ 36.2 = 6.78089$$
$$\text{Log tang (incl of the ☽'s relative orbit} = \chi - 6^\circ\ 1'.5) = 9.02343$$

This angle is that represented by *q Q r* in Plate XXX: the angle *E C X* has been made equal to it, and it passes over to the north of that side of the ecliptic in which the signs are increasing, viz the eastern side, when the hourly motion of the moon augments her northern, or diminishes her southern latitude, and it passes under, or to the south of the eastern side of the ecliptic, in the contrary cases The hourly motion of the moon in her relative orbit is deduced by the following proportion;

as cos χ radius :: A ☽'s hourly motion in her relative orbit or, as cos 6°. 1′5 · 1 27′. 36″.2 ☽'s hourly motion in the orbit.

log 27′. 36.″2 = 3 .21911
Ar Co. log. cos 6°. 1′ 5 = 0 .00241

Log (☽'s hourly motion in the relative orbit = 27′. 45″ 4) = 3 .22152
Ar. Co. log (difference of the ☽'s and ☉'s parallaxes = 54′ 17″.2) = 6 48716

Log (☽'s hour. mot. in the relative orbit where the radius *CM* = 1) = 9 .70868 = log k

The number 0 5113, corresponding to this last logarithm, expresses the length of the line which in Plate XXX is represented by *Q q*, on the supposition that the radius *CM* is the unit.

12 *To find the nearest distance of the moon's orbit from the sun, and the time of the nearest approach.*

Rule II. In the same plate, the nearest distance is represented by *CI*, and the time of the nearest approach is that in which the moon is projected on the point *I*.

Multiply the latitude of the moon, by the cosine, and also by the sine of the inclination of the relative orbit: the first product will be the nearest distance, and the second product reduced into time, in the ratio of the hourly motion in the relative orbit, will give the time of the nearest approach, counted from the moment of the true conjunction.

Log (☽ 's true latitude = 25′ 57″ 7) = 3 .19248 . . . . . 3 .19248
log cos 6° 1′.5 = 9 .99759 . log sin = 9 .02103

2 .21351

Log (nearest distance = 25 49 1) = 3 .19007
Ar. Co. log (*CM* = 54′ 17″.2) . 6 .48716

Log (*CI* = 0 .47559) . . . . . = 9 .67723

Ar. Co. log (☽ 's hourly motion in the relative orbit = *k*) = 6 .77848
sum = 8 .99199
Log (1^h expressed in seconds of time) . . . . 3 .53630

Log 5^m 53ˢ .4 = 2 .52829
Time of the true conjunction . . . 2^h 10^m 28ˢ .1

Time of the nearest approach . . . 2^h 4^m 34ˢ .7 = T.

The time of the nearest approach from the true conjunction, viz. 5^m 53ˢ .4, has been subtracted from that of the true conjunction, because the inclination raises the moon's orbit towards the north, at the eastern side of the ecliptic, which time ought to be added, when the inclination is in a contrary direction. By the quantities determined from these two rules, we may now know the position of the relative orbit on the plane of projection, the time at which the moon will be projected into the point *I.*, and, by means of her hourly motion, the points into which the moon will be projected at every successive instant

13. *To determine whether an Eclipse will be visible, and whether it will be either Annular or Total.*

Rule III. To the difference of the parallaxes of the moon and sun, add the sum of their semi-diameters, and from the resulting number subtract the nearest distance, found by the preceding rule, and if the remainder be positive, there will be an eclipse Subtract again the moon's diameter from this remainder, and if the difference is still positive, the eclipse will be seen either annular or total. It will be annular, if the semi diameter of the sun is greater than that of the moon, but it will be total, if less. Thus we have

Difference of the ☽'s and ⊙'s parallaxes		54′	17″.2
Sum of the ⊙'s and ☽'s semi-diameters		30	41 .5
Sum	=1°	24	58 .7
Nearest distance		25	49 .1
Remainder		=59	9 .6

As this remainder is positive, there will be an eclipse.

Diameter of the moon	29	42 .4
Difference	29	27 .2

Since this difference is again positive, the eclipse will be seen either annular or total.

⊙'s semi-diameter	15	50 .3
☽'s semi-diameter	14	51 .2
Difference	0	59 .1

The semi-diameter of the sun being here greater than that of the moon, the eclipse will be annular.

The difference 59″.1 being divided by the difference of the parallaxes of the sun and moon, viz. 54′ 17″.2 gives the distance of the two straight lines $\overset{a}{12}\,\overset{b}{12}$, and $\overset{c}{12}\,\overset{d}{12}$ from the straight line *AB*, expressed in parts of the radius *CM*

Log . . . 59″.1 = 1.77159
Ar. Co. Log 54′ 17″.2 = 6.48716

Log (*I.* 12 = 0.0181) = 8.25875

14. *To trace the Lines for the greatest Phasis of the Eclipse of May* 15, 1836.

Rule IV Divide the sun's diameter $= 31' \, 40''.6$ by the difference of the parallaxes of the moon and sun.

Log . . 31′ 40″.6 = 3.27889
Ar. Co. Log 54 17.2 = 6.48716
Log . (*D* = 0.5835) = 9.76605

Divide the value of the sun's semi-diameter $= 0.5835$, into four equal parts, and the value of one of these parts will be $= 0.145875$. Now, the distances between the lines $\overset{a}{12}\,\overset{b}{12}$ and $\overset{a}{9}\,\overset{b}{9}$, $\overset{a}{9}\,\overset{b}{9}$, and $\overset{a}{6}\,\overset{b}{6}$, as well as between $\overset{c}{12}\,\overset{d}{12}$ and $\overset{c}{9}\,\overset{d}{9}$, $\overset{c}{9}\,\overset{d}{9}$ and $\overset{c}{6}\,\overset{d}{6}$, being one-fourth of the sun's diameter, if we subtract the value of *I*. 12, found in the preceding rule, from 0.145875, and if to the difference we add successively this latter number, we shall obtain the distances of $\overset{a}{9}\,\overset{b}{9}$, $\overset{a}{6}\,\overset{b}{6}$, or $\overset{c}{9}\,\overset{d}{9}$, $\overset{c}{6}\,\overset{d}{6}$, &c., from *AB*; we shall then have

Distance of $\overset{a}{12}\,\overset{b}{12}$ or $\overset{c}{12}\,\overset{d}{12}$ from *AB* = 0.0181
0.145875

Distance of $\overset{a}{9}\,\overset{b}{9}$ or $\overset{c}{9}\,\overset{d}{9}$ from *AB* . = 0.127775
0.145875

Distance of $\overset{a}{6}\,\overset{b}{6}$ or $\overset{c}{6}\,\overset{d}{6}$ from *AB* . = 0.273650
0.145875

Distance of $\overset{a}{3}\,\overset{b}{3}$ or $\overset{c}{3}\,\overset{d}{3}$ from *AB* . = 0.419525
0.145875

Distance of *ab* from *AB* . . . = 0.565400

As the lines $\overset{a}{12}\,\overset{b}{12}$, $\overset{c}{9}\,\overset{d}{9}$, $\overset{c}{6}\,\overset{d}{6}$, $\overset{c}{3}\,\overset{d}{3}$ stand over *AB*, and the lines $\overset{c}{12}\,\overset{d}{12}$, $\overset{a}{9}\,\overset{b}{9}$, $\overset{a}{6}\,\overset{b}{6}$, $\overset{a}{3}\,\overset{b}{3}$, $a\,b$ under it, to distinguish their situation relatively to the line *AB*, we shall consider the distances of the former lines as positive, and give them the sign +, and those of the latter lines as negative, and prefix to them the sign −.

Let us take for the axis of the abscissæ x the straight line *XX* passing through the point *C* and parallel to *AB*, and for the axis of the ordinates y the perpendicular *YY*, the positive abscissæ being taken toward the east, and the positive ordinates towards the north pole. The distance *CI* between the lines *XX* and *AB* having been found, in Rule II., equal to 0.47559, if we add this number to the distances of the lines $a\ b$, $\underset{3}{a}\ \underset{3}{b}$, $\underset{6}{a}\ \underset{6}{b}$, $\underset{12}{c}\ \underset{12}{d}$, $\underset{9}{c}\ \underset{9}{d}$, &c., from AB, taking these distances with their algebraical sign before determined, we shall obtain the following Table of the ordinates of the straight lines for the greatest phasis —

TABLE I.

	Lines	Ordinates ζ	log ζ
Observations at the northern side of the sun	Line for the contact of the limbs, or $a\ b$	−0.0898	$\bar{8}$.95328
	Line for the phasis of 3 digits, or $\underset{3}{a}\ \underset{3}{b}$	0.0561	8.71890
	Line for the phasis of 6 digits, or $\underset{6}{a}\ \underset{6}{b}$	0.2019	9.30514
	Line for the phasis of 9 digits or $\underset{9}{a}\ \underset{9}{b}$	0.3478	9.54133
	Line for the annular eclipse, or $\underset{12}{c}\ \underset{12}{d}$	0.4575	9.66039
Observations at the southern side of the sun	Line for the annular eclipse, or $\underset{12}{a}\ \underset{12}{b}$	0.4937	9.69348
	Line for the phasis of 9 digits, or $\underset{9}{c}\ \underset{9}{d}$	0.6031	9.78061
	Line for the phasis of 6 digits, or $\underset{6}{c}\ \underset{6}{d}$	0.7492	9.87460
	Line for the phasis of 3 digits, or $\underset{3}{c}\ \underset{3}{d}$	0.8951	9.95187

The first five lines comprehend the projections of the places where the observers will see the corresponding phases when the moon appears more northward than the sun, and the latter four lines those where the moon will appear more southward.

15. *To find the angle that the projection of the circle of declination passing through the sun, or as it is called the first meridian, makes with the axis* XX *of the abscissæ on its eastern side*, viz. *the angle represented by* ECM *in Plate XXX, and which we will hereafter call* v.

RULE V. Take the angle ECX that the axis XX makes with the projection of the ecliptic, as it has been computed by Rule I. This angle, subtracted from the angle ECM, made by the first meridian with the ecliptic and given in the elements, or added to it, accordingly as CX is to the north or south of the east side of the ecliptic, will give the angle v.

Angle of the first meridian with the ecliptic $= 75°\ 55'.1$
Angle ECX by Rule I $\chi = 6\ \ 1'.5$ to the north

$v = 69\ \ 53'.6$

After having computed all the quantities under the preceding five rules, the first operation that we have to undertake is, to determine the places in which the greatest phasis will be observed at the horizon, and to trace the line of these places on a map. These places will be projected on a, $\overset{a}{3}\ \overset{a}{6}\ \overset{a}{9}\ \overset{c}{12}\ \overset{a}{12}\ \overset{c}{9}$, &c. and on b, $\overset{b}{3}\ \overset{b}{6}\ \overset{b}{9}\ \overset{d}{12}\ \overset{b}{12}\ \overset{d}{9}$ &c. of the circumference of the projection of the earth, at the moments that the centre of the moon shall be in the perpendiculars drawn from those points on the projection AB of the orbit The horary angles, h, of the sun, and the latitudes, φ, of such places are given by the following formulæ,

$$\cot h = -\sin \delta \cot (\upsilon - \gamma)\,, \quad \sin \varphi = \cos \delta \cos (\upsilon - \gamma)\,, \quad \cos \varphi = \frac{\sin (\upsilon - \gamma)}{\sin H}$$

where δ denotes the declination of the sun, and γ the angle, the sine of which is represented by the numbers, which, in Table I. give the values of ζ, or of the ordinates corresponding to the different phases The declination δ must be considered as negative when it is southern, and among the two angles h and $180 + h$ given by the first formula, we must, according to the third equation, choose that, the sine of which has the same sign, as the sine of the angle $\upsilon - \gamma$. As we may take for the angles, γ corresponding to the sines expressed by the numbers in Table I, either the angles themselves, or their supplements, we shall have, according to the two suppositions, and by computation from the first two of the preceding formulæ, the following Table.

TABLE II.

Phasis	γ		$\upsilon-\gamma$		log cot$(\upsilon-\gamma)$	log cot h	h		log cos$(\upsilon-\gamma)$	log sin φ	φ	
$\upsilon=$	69° 53′ 6			log sin $\delta=$	9 51181			log cos $\delta=$	9 97577			
SUN RISING												
	°	′	°	′			°	′			°	′
Contact	105	9 0	−115	15 6	9 67373	$\bar{9}$ 18556	278	43 0	$\bar{9}$ 63010	$\bar{9}$ 60587	−23	47 9
3 digits	176	47 0	−106	53 6	9 48235	$\bar{8}$ 99418	275	38 1	$\bar{9}$ 46320	$\bar{9}$ 43897	−15	56 9
6 digits	168	20 8	− 98	27 2	9 17207	$\bar{8}$ 68390	272	45 9	$\bar{9}$ 16733	$\bar{9}$ 14310	− 7	59 0
9 digits	159	38 6	− 89	45 0	$\bar{7}$ 63982	7 15165	269	55 1	7 63982	7 61559	0	11 2
Annular	152	46 4	− 82	52 8	$\bar{9}$ 09660	8 60813	267	41 7	9 09321	9 06901	6	43 9
Central	151	36 1	− 81	42 5	$\bar{9}$ 16357	8 67510	267	17 3	9 15900	9 13477	7	50 3
Annular	150	25 0	− 80	31 4	$\bar{9}$ 22255	8 73436	266	53 7	9 21658	9 19235	8	57 6
9 digits	142	53 1	− 72	59 5	$\bar{9}$ 48556	8 99739	261	19 4	9 46614	9 44191	16	3 6
6 digits	131	28 7	− 61	35 1	9 73323	9 24500	260	1 7	9 67747	9 65324	26	44 7
3 digits	116	28.7	− 46	35 1	$\bar{9}$ 97598	9 48779	252	51 5	9 83713	9 81290	40	32 4
$\gamma=$	90	0 0	− 20	6 4	0 43612	9 94825	228	21 3	9 97269	9 94846	62	38 1
SUN SETTING												
$\gamma=\upsilon=$	69	53 6	0	0 0	∞	∞	180	0 0	0 00000	9 97577	71	2 2
3 digits	63	31 3	6	22 3	0 95207	$\bar{0}$ 46390	161	2 1	9 99731	9 97308	70	2 0
6 digits	48	31 3	21	22 3	0 40716	$\bar{9}$ 91929	120	42.4	9 96906	9 94483	61	43 6
9 digits	37	6 9	32	46 7	0 19117	$\bar{9}$ 70300	116	46 7	9 92468	9 90045	52	40 2
Annular	29	35 0	40	18 6	0 07112	$\bar{9}$ 58325	110	57 6	9 88227	9,85804	46	9 1
Central	28	23 9	41	29 7	0 05327	$\bar{9}$ 56510	110	10 3	9 87419	9 85026	45	6 1
Annular	27	13 6	42	40 0	0 03511	$\bar{9}$ 54721	109	25 3	9 86647	9 84224	44	3 6
9 digits	20	21 4	49	32 2	9 93091	$\bar{9}$ 44277	105	20 6	9 81222	9 78799	37	51 7
6 digits	11	39 2	58	14 4	9 79173	$\bar{9}$ 30356	101	22 5	9 72128	9 69705	29	51 3
3 digits	3	13 0	66	40 6	9 63463	$\bar{9}$ 14646	97	58 5	9 59761	9 57338	21	59.4
Contact	− 5	9 0	75	2 6	9 42671	$\bar{8}$ 93857	94	57 7	9 41177	9 38754	14	7 6

The last column of this table gives the latitudes of several points of the line which passe through the places in which the greatest phasis is seen at the horizon In order to obtain th longitudes of the same points, we may observe, that the times counted from that of the neare approach in which these phases will be seen, are expressed by the cosines of the angles γ, r duced into parts of an hour by the relative hourly motion of the moon in her orbit. The times will then be as noted in the fourth column of the following table.

TABLE III.

Phasis log $\frac{60'}{\lambda}$ =	log cos γ 2 06947	log *t*	*t* *T* = 2ʰ 4 58		*T* — *t* sun rising		*T* + *t* sun setting		*T* — *t* in space		*T* + *t* in space	
			h	m	h	m			°	′	°	′
Contact . ..	9 99824	2 06771	1	56 87	0	7 71	4	1 45	1	55 6	60	21 7
3 digits . .	9 99932	2 06879	1	57 16	0	7 42	4	1 74	1	51 3	60	26 1
6 digits	9 99096	2 06043	1	54 93	0	9 65	3	59 51	2	24 7	59	52 6
9 digits. .	9 97200	2 04147	1	50 02	0	14 56	3	54 60	3	38 4	58	39 0
Annular .	9 94900	2 .01847	1	44 34	0	20 24	3	48 92	5	3 6	57	13 8
Central ..	9 94432	2 01379	1	43 23	0	21 25	3	47 81	5	18 7	56	57 1
Annular ..	9 93934	2 00881	1	42 05	0	22 53	3	46 63	5	37 9	56	39 1
9 digits . .	9 90169	1 97116	1	33 58	0	31 00	3	38 16	7	45 0	54	32 4
6 digits ..	9 82108	1 89055	1	17 72	0	46 86	3	22 30	11	42 9	50	34 5
3 digits	9 64021	1 71868	0	52 32	1	12 26	2	56 90	18	3 9	44	13 5
γ = 90° . .	— ∞	— ∞	0	0 0	2	4 58		,	31	8 7		
υ — γ = 0	9 53627	1 60574	0	40 34	..	..	2	44 92	.		41	13 8

The fifth and sixth columns of the preceding table contain the time *t*, subtracted from and added to that of the nearest approach, and the seventh and eighth columns the same quantities reduced into space, and, therefore, these last quantities will represent the horary angles of the sun at Greenwich, at the moments in which the phasis, noted in the first column, will be seen at the horizon in the corresponding places. If we subtract these horary angles from those noted in column 6 of Table II., we shall obtain the longitudes of the same places, counted eastwards from Greenwich. Collecting into one table the longitudes thus obtained, and the times and latitudes already found, we shall have the following data for tracing the line of the places, where the observers will see the greatest phasis at the horizon.

TABLE IV.

LONGITUDES AND LATITUDES OF THE PLACES OF THE LINE OF THE GREATEST PHASIS AT THE HORIZON

SUN RISING. SUN SETTING.

Phasis	Time at Greenwich		Longitude of the place		Latitude of the place			Phasis	Time at Greenwich		Longitude of the place		Latitude of the place		
	h	m	°	′	°	′			h	m	°	′	°	′	
Contact .	0	7 7	276	47 4	—23	47 0	South	υ — γ = 0	2	44 9	138	40 2	71	2 2	North
3 digits ...	0	7 4	273	46 8	—15	56 9		3 digits	2	56 9	116	48 6	70	2 0	
6 digits	0	9 6	270	21 2	— 7	59 0		6 digits	3	22 3	79	7 9	61	43 6	
9 digits	0	11 6	266	16 7	0	14 2	North	9 digits	3	38 2	62	14 3	52	40 2	
Annular	0	20 2	262	38 1	6	43 9		Annular	3	46 6	54	18 5	46	9 1	
Central . .	0	21 2	261	58 3	7	50 3		Central	3	47 8	53	13 2	45	6 1	
Annular	0	22 5	261	15 8	8	57 5		Annular	3	48 9	52	11 5	44	3 6	
9 digits ...	0	31 0	256	31 4	16	3 6		9 digits .	3	54 6	46	50 6	37	51 7	
6 digits . .	0	46 9	248	18 8	26	44 7		6 digits	3	59 5	41	29 9	29	51 3	
3 digits ...	1	12 3	234	50 6	40	32 4		3 digits	4	1 7	37	32 4	21	59 4	
γ = 90° .	2	4 6	197	15 6	62	38 1		Contact	4	1 4	34	36 0	14	7 6	

The longitudes and latitudes above computed being traced upon a terrestrial globe, we shall see that the line of the greatest phasis at the horizon begins in the Pacific Ocean, not far from the coasts of Peru, and that, passing along this sea, it rises into the northern hemisphere, where it passes through the Gulf of California; then touches the banks of New Albion in North America, and, crossing the Pacific Sea near the Behring's Straits, goes throughout the Russian Empire to the Arctic Sea. From thence it descends again towards the south into Siberia, to the Caspian Sea, crosses America, the Red Sea, &c. and ends at the boundaries between Sennaar and Abyssinia. The line that passes through all the above-mentioned places, being stereographically projected, gives that dotted line of the greatest phasis at the horizon, which we have traced on the map, represented by fig 3, of Plate XXXI.

16. From the line of the greatest phasis at the horizon we now proceed to the lines of the greatest phasis in general. In determining these, the ordinate ν and the abscissa μ of any point whatever of the semi ellipses, that represent the projection of the parallels of latitude of the earth are given by the following formulæ, in which the radius $C\,M$ is taken for unity.

$$\mu=\cos\varphi\{\ \sin v.\sin h-\sin\delta.\cos v.\cos h\}+\sin\varphi.\cos\delta.\cos v$$
$$\nu=\cos\varphi\{-\cos v.\sin h-\sin\delta.\sin v.\cos h\}+\sin\varphi.\cos\delta.\sin v$$

Where the angles v, h, δ, φ, have the same signification as before.

If we put

$$(1)\quad \cos\delta.\sin v=\cos\tau \qquad\qquad (1)'\quad \cos\delta.\cos v=\cos\tau'$$
$$(2)\quad \sin\delta.\tan v=-\cot u \qquad\qquad (2)'\quad \sin\delta.\cot v=-\cot u'$$

and also

$$(3)\quad \text{tang}\, w=\tan\tau.\cos(h+u) \qquad\qquad (3)'\quad \tan w'=\tan\tau'.\cos(h-u')$$

we shall have

$$\nu=\frac{\cos\tau}{\cos w}\sin(\varphi+w) \qquad\qquad \mu=\frac{\cos\tau'}{\cos w'}\sin(\varphi+w')$$ [*]

Now by approximation we may take the moment in which the centres of the sun and moon are seen in the same perpendicular to the relative orbit, for that of the greatest phasis. Then, as ν and μ may also be considered as the coordinates of the point in which the centre of the sun is projected by an observer corresponding to the same coordinates, if we represent by t the time in minutes, from the moment of the nearest approach to that of the greatest phasis belonging to any of the lines of Table I, we shall have, at the latter moment,

$$\nu=\zeta \qquad\qquad \mu=\frac{k\,t}{60^{\text{m}}}$$

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[*] Those who are acquainted with the formulæ of spherical trigonometry will see that the auxiliary quantities τ and τ' represent the arcs which on the sphere unite the zenith of the observer with the poles or points, where the axis Y or X of the coordinates intersect the sphere, and that u and u' are the angles that the said arcs make with the first meridian *MPC*.

By comparing the above values of ν and μ with the preceding ones, we shall deduce

$$(4)\quad \sin(\varphi+w)=\frac{\cos w}{\cos\tau}\,\zeta \qquad\qquad (5)\quad t=\frac{60^{m}}{k}\,\frac{\cos\tau'}{\cos w'}\,\sin(\varphi+w')$$

The first of these equations will give, for every assumed value of h, the latitude of the place for which the greatest phasis corresponds to the value of ζ, taken from Table I, and the second equation will then give in minutes the time t, to be added to that of the nearest approach; and when their sum has been converted into space, and subtracted from the horary angle h, it will determine the east longitude of the place, as counted from Greenwich

The computation of formulæ (1), (1)$'$, (2), (2)$'$, (3), (3)$'$, may be performed in the following manner,

$\delta=18^\circ\ 57'.8$	$\log\cos\delta=9.97577$	. . . . 9.97577
$v=69\ \ 53\ \ 6$	$\log\sin v=9.97269$	$\log\cos v=9.53627$
	$\log\cos\tau=9.94846$	$\log\cos\tau'=9.51204$
	$\tau=27^\circ\ 21'.9$	$\tau'=71^\circ\ 1'.6$
	$\log\sin\delta=9.51183$	9.51183
	$\log\tan v=0.43642$	$\log\cot v=9.56358$
	$\log\cot u=\bar{9}.94825$	$\log\cot u=\bar{9}.07541$
	$u=131^\circ\ 35'.7$	$u'=96^\circ\ 47'.1$

Having thus prepared the constant angles τ, τ', u and u' we are enabled to construct the two following auxiliary Tables, the contents of which are fully explained by the preceding formulæ.

TABLE V.

Horary angle h	$h+u$ $u = 131°\ 35'.7$ log tan τ =	log cos $(h+u)$ 9 71398	log tan ω	ω At Co log cos τ =	log cos ω 0 05154	log $\frac{\cos\omega}{\cos\tau}$
180°	311° 35′ 7	9 82208	9 53600	18° 57′ 8	9 97577	0 .02731
195	326 35 7	9 92158	9 63550	23 22 .0	9 96283	0 .01437
210	341 35 7	9 97720	9 69118	26 9 .4	9 95308	0 00462
225	356 35 7	9 99923	9 71321	27 10 4	9 91862	0 00016
240	11 35 7	9 99105	9 70503	26 53 .2	9 95032	0 00186
255	26 35 7	9 95143	9 66541	24 50 1	9 .95785	0 00039
270	41 35 7	9 .87382	9 58780	21 9 .6	9 96968	0 02122
285	56 35 7	9 74080	9 45478	15 54 .1	9 98305	0 03159
300	71 35 7	9 49932	9 21330	9 10 9	9 99428	0 01582
315	86 35 7	8 .77374	8 48772	1 42 6	9 99981	0 05135
330	101 35 7	$\bar{9}$ .30318	$\bar{9}$ 01716	− 5 56 3	9 99706	0 04920
345	116 35 7	$\bar{9}$ .65097	$\bar{9}$ 30495	−13 2 .7	9 98864	0 04018
0	131 35 .7	$\bar{9}$ 82208	$\bar{9}$ 53600	−18 57 8	9 97577	0 .02731
15	146 35 7	$\bar{9}$ .92158	$\bar{9}$ 63550	−23 22 .0	9 96283	0 01437
30	161 35 .7	$\bar{9}$ .97720	$\bar{9}$ .69118	−26 9 4	9 95308	0 00462
45	176 35 7	$\bar{9}$ .99923	$\bar{9}$ 71321	−27 10 4	9 91862	0 .00016
60	191 35 7	$\bar{9}$ 99105	$\bar{9}$ 70503	−26 53 2	9 95032	0 00186
75	206 35 7	$\bar{9}$ .95143	$\bar{9}$ 66541	−24 50 1	9 95785	0 00039
90	221 35 .7	$\bar{9}$ 87382	$\bar{9}$ 58780	−21 9 6	9 96968	0 02122
105	236 35 7	$\bar{9}$ 71080	$\bar{9}$ .45478	−15 54 1	9 98305	0 03159
120	251 35 7	$\bar{9}$ 49932	$\bar{9}$ 21330	− 9 10 9	9 99428	0 04582
135	266 35 7	$\bar{8}$ 77374	$\bar{8}$ 48772	− 1 42 6	9 99981	0 05135
150	281 35 7	9 .30318	9 01716	5 56 3	9 .99706	0 04920
165	296 35 7	9 65097	9 30495	13 2 7	9 98864	0 .04018

TABLE VI.

Horary angle h	$h-u'$ $u'=96^\circ\ 47'\ 1$	log cos $(h-u')$	log tan ω'	ω'	Ar Co log cos ω'	$\log \frac{60^m \cos \tau'}{h \cos \omega'}$
	log tan τ' =	0 46369		$\log \frac{60^m}{h} \cos \tau'$ =	1 58151	
180°	83° 12′ 9	9 07241	9 53610	18° 57′ 9	0 02124	1 60575
195	98 12 9	$\bar{9}$ 15499	$\bar{9}$ 61868	−22 34 1	0 03460	1 61611
210	113 12 9	$\bar{9}$ 59570	$\bar{0}$ 05939	−48 54 3	0 18223	1 70374
225	128 12 9	$\bar{9}$ 79142	$\bar{0}$ 25511	−60 56 2	0 31356	1 89507
240	143 12 9	$\bar{9}$ 90357	$\bar{0}$ 36726	−66 46 0	0 40399	1 98550
255	158 12 9	$\bar{9}$ 96782	$\bar{0}$ 43151	−69 41 0	0 45911	2 04092
270	173 12 9	$\bar{9}$ 99695	$\bar{0}$ 46064	−70 54 ,2	0 48523	2 06074
285	188 12 9	$\bar{9}$ 99552	$\bar{0}$ 45921	−70 50 7	0 48395	2 06546
300	203 12 9	$\bar{9}$ 96333	$\bar{0}$ 42702	−69 29 3	0 45545	2 03696
315	218 12 9	$\bar{9}$,89525	$\bar{0}$ 35894	−66 22 0	0 39698	1 97849
330	233 12 9	$\bar{9}$ 77729	0 21098	−60 80 3	0 30287	1 88438
345	248 12 9	$\bar{9}$ 56952	$\bar{0}$ 03321	−47 10 3	0 16776	1 74927
0	263 12 9	$\bar{9}$ 07241	$\bar{9}$ 53610	−18 57 9	0 02121	1 60575
15	278 12 9	9 15499	9 61808	22 31 1	0 03460	1 ,61611
30	293 12 9	9 59570	0 05939	48 54 3	0 18223	1 70374
45	308 12 9	9 79142	0 25511	60 56 2	0 31356	1 89507
60	323 12 9	9 90357	0 36726	66 46 0	0 40399	1 98550
75	338 12 9	9 96782	0 43151	69 41 0	0 45941	2 04092
90	353 12 9	9 99695	0 46064	70 54 2	0 48523	2 06074
105	8 12 9	9 99552	0 45921	70 50 ,7	0 18395	2 06546
120	23 12 9	9 96333	0 42702	69 29 3	0 45545	2 03696
135	38 12 9	9 89525	0 35894	66 22 0	0 39698	1 97849
150	53 12 9	9 77729	0 24098	60 8 3	0 30287	1 88438
165	68 12 9	9 56952	0 03321	47 10 3	0 16776	1 74927

By means of the quantities in these two auxiliary tables, (after twelve terms are reproduced, with the exception that some of them have a different sign,) the computation is reduced to only twelve terms. By the means of the angles ω and ω' and of the quantities in column seven of both tables we may easily compute the values of φ and t given by equations (4) and (5), and therefore determine the places situated in the lines of the greatest phasis.

17 Let us, for instance, begin by determining the latitudes and longitudes of the places in which an observer will see only a contact of the limbs for the greatest phasis By Table II we see that the horary angles of the two places, where only a contact of the limbs is seen at the horizon, are 278° 43′ 0 and 94° 57′.7, and by Table IV we may easily infer that the line of a simple contact of the limbs for the greatest phasis will begin at the former and end at the latter place. For this reason we took from Table V only the quantities in the last column from $h=285^\circ$ to $h=90^\circ$, and by adding to them the log ζ, taken from Table I, and corresponding to the contact of the limbs, we formed the second column in the following table, and by it the other columns according to formulæ (4) and (5).

TABLE VII.

Horary angle h log ζ	log sin $(\phi+\omega)$ $=\bar{8}$ 55328	$\phi+\omega$	ϕ or Latitude	$\phi+\omega'$	log sin $(\phi+\omega')$	log t T=	t 2h 4m 58	T+t	T+t in space	Longitude.
285	$\bar{8}$ 98787	−5° 31′ 8	−21° 28′ 9	−92° 19′ 6	$\bar{9}$ 99961	$\bar{2}$ 06510	−1h 56m 17	0h 8m 41	2° 6 1	282° 53′ 9
300	$\bar{8}$ 99910	−5 43 6	−15 0 5	−84 29 8	$\bar{9}$ 99799	$\bar{2}$ 03195	−1 48 38	0 16 20	4 3 0	295 57 0
315	$\bar{9}$ 00163	−5 18 1	−7 50 7	−70 52 7	$\bar{9}$ 98258	$\bar{1}$ 96107	−1 31 43	0 33 15	8 17 2	306 42 8
330	$\bar{9}$ 00243	−5 16 3	0 10 0	−60 18 3	$\bar{9}$ 93886	$\bar{1}$ 82321	−1 6 56	0 58 02	14 30 3	315 29 7
345	8 99346	−5 39 2	7 23 5	−39 46 8	$\bar{9}$ 80607	$\bar{1}$,55531	−0 35 92	1 28 66	22 9 9	322 50 1
0	$\bar{8}$ 98059	−5 29 3	13 28 5	−5 29 1	$\bar{8}$,98078	0 58653	−0 3 86	2 0 72	30 10 8	329 49,2
15	8 96765	−5 19 6	18 2 4	10 36 5	9 81350	1 42961	0 26 89	2 31 47	37 52 0	337 8 0
30	$\bar{8}$ 95790	−5 12 4	20 57 0	69 51 3	9 97258	1 73632	0 34 19	2 59 07	44 15 8	345 14,2
45	$\bar{8}$ 95314	−5 9 2	22 10 2	83 6 1	9 99685	1 89192	1 17 97	3 22 55	50 30 2	354 21 8
60	$\bar{8}$ 95511	−5 10 5	21 42 7	88 28 9	9 99985	1 98535	1 36 68	3 41 26	55 18 9	4 41 1
75	$\bar{8}$ 96267	−5 15 9	19 34 2	89 15 2	9 99996	2 01088	1 49 87	3 51 45	58 36 7	16 23 3
90	$\bar{8}$ 97450	−5 24 7	15 11 9	86 39 1	9 99926	2 06600	1 56 41	4 0 99	60 11 8	29 15 2

The fourth column of this table is made by subtracting the angle ω, taken from Table V, from the angles of the third column corresponding to the same argument h. The fifth column is got by adding the angles ω', taken from Table VI, to the angles of the fourth column corresponding to the same argument h, and the seventh column is obtained by adding the logarithms, taken from the last column in Table VI, to the logarithms in the sixth column under the same argument h In order to make these additions easy, it will be found convenient to copy the fifth and seventh columns, of Tables V and VI, upon lists of paper to be put under the columns of Table VII, from which corresponding numbers must be added. The first column of the above table gives the horary angles of the sun counted westwards, at the moment that a contact of the limbs is seen at the corresponding places, and the tenth column gives the horary angles of the sun at Greenwich for the same moments. The differences of the horary angles of the two said columns, contained in the eleventh column, represent then the longitudes of the places from Greenwich counted eastwards, and the fourth column gives the latitudes of the same places. The values of h in the first column, being converted into time, will show the hours and parts counted at the places when a contact of the limbs is seen, and the ninth column gives the hours and parts corresponding to the same moments, as reckoned at Greenwich *

18 A similar computation to that we have detailed, for finding the line of a simple con-

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* The places given by the method we have just explained are found on the supposition, that the greatest phasis is observed when the centre of the moon and the place of the observer, or, what is the same, the centre of the sun, are projected on the same perpendicular to the relative orbit, which, as we have said before, is but an approximation Du Sejour, who has treated the theory of eclipses with great skill, has reproached astronomers for this inaccuracy The reproach was not perhaps quite just, because astronomers in these computations profess to acquire only an approximate knowledge of the circumstances of the eclipse, and it would be a waste of time to keep a minute account of all the small quantities However as it may be sometimes useful, to undertake a more accurate computation for those places, where the eclipse will be visible under particular circumstances, we subjoin in this note a method of correction.

Let $\varphi_{,}$ and $\varphi'_{,}$ denote the values of ω and ω', which in Tables V and VI correspond to the horary angle next to that which has given the place, of which we want to correct the elements compute $\nu_{,}$ $\mu_{,}$ and μ by the formulæ

$$\nu_{,}=\frac{\cos\ \tau}{\cos\ \varphi_{,}}\sin\ (\phi+\varphi_{,}),\qquad \mu_{,}=\frac{\cos\ \tau'}{\cos\ \omega}\sin\ (\phi+\varphi'_{,}),\qquad \mu_{,}=\frac{k\ t}{60^{m}},$$ in which

tact for each phasis, by employing successively the corresponding values of ζ, given in Table I, and the limits of h deduced from Table II, will produce the nine following tables of the times,

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in which ϕ and t are the same quantities that have been denoted by these letters in the first approximation,

Then put

$$\text{tang } s = \frac{\nu_{,}-\zeta}{k-(\mu_{,}-\mu)}$$

and we shall have with an inconsiderable error the new values $\phi_{,}$ and $t_{,}$ by the equations

$$\sin(\phi_{,}+\omega) = \frac{\cos\omega}{\cos\tau}\ \zeta\cos s \qquad\qquad t_{,} = t + \frac{60^m}{h}\ \zeta\sin s$$

Let us, for instance, find the corrected latitude and longitude of the place where only a contact of the limbs will be seen for the greatest phasis, at the moment that the horary angle of the sun is 285°, viz the corrections of the longitude and latitude of the first place in Table VII This table gives

	$\phi = -21^\circ\ 28'\ 9$		$\phi = -21^\circ\ 28'\ 9$	
Tab V, arg 300°	$\ldots\ldots\ldots\ \psi = 9\ \ 16\ \ 9$	Tab VI, arg 300°	$\psi' = -69\ \ 20\ \ 3$	
	$\phi+\psi = -12\ \ 12\ \ 0$		$\phi+\psi' = -90\ \ 58\ \ 2$	
	$\log\sin(\phi+\psi) = -\bar{9}\ 32495$		$\log\sin(\phi+\psi') = -\bar{9}\ 99994$	
Tab V, arg 300°	$\log\frac{\cos\tau}{\cos\omega} = 9\ 95418$	Tab VI,	$\log\frac{60^m}{k}\frac{\cos\tau'}{\cos\psi'_{,}} = 2\ 03696$	$\log t = \bar{2}\ 06510$
	$\log\nu_{,} = -\bar{9}\ 27913$		$\log\frac{k}{60^m} = 7\ 93053$	$-7\ .93053$
			$\log\mu_{,} = \bar{9}\ 96743$	$\log\mu = \bar{9}\ 99563$
	$\nu_{,} = -0\ 1902$		$\mu_{,} = -0\ 9277$	
	$\zeta = -0\ 0898$		$\mu = -0\ 9900$	
	$\nu_{,}-\zeta = -0\ 1004$		$\mu_{,}-\mu = 0\ .0623$	
	$\log(\nu_{,}-\zeta) = \bar{9}\ 00173$		$k = 0\ 5113$	
	Ar co $\log(k-\mu_{,}-\nu) = 0\ 34775$		$k-(\mu_{,}-\mu) = 0\ 4490$	
	$\log\text{tang } s = 9\ 34948$		$s = -12^\circ\ 36\ 2$	
	$\log\cos s = 9\ 98941$		$\log\sin s = \bar{9}\ 3388$	
Tab VII	$\log\frac{\cos\tau}{\cos\omega} = \bar{8}\ 98787$		$\log\zeta = 8\ 9533$	
	$\log\sin(\phi_{,}+\omega) = \bar{8}\ 97728$		$\log\frac{60^m}{h} = 2\ .0695$	
			$\log\frac{60^m}{k}\zeta\sin s = \bar{0}\ 3616$	
	$\phi_{,}+\omega\ \ -\ 5^\circ\ 26'\ 7$		number $= -\ \ 2^m\ 30$	
Tab V, arg 285°.	$\omega\ \ 15\ \ 54\ \ 1$	Tab. VII,	$t = -1^h\ \ 56\ \ 17$	
	$\phi_{,}\ \ -21\ \ 20\ \ 8$		$t_{,} = -1\ \ 58\ \ 47$	

The corrected southern latitude will then be 21° 20′ 8, and the time $t_{,}$ subtracted from that of the nearest approach, gives $0^h\ 6^m\ 11$, and this time converted into space, and subtracted from the horary angle 285°, gives the corrected longitude of the place = 283° 28′ 4 east from Greenwich longitudes, and latitudes of the places belonging to the lines of the greatest phasis, to which we have added also the line of a simple contact already found.

Places

On the line of the contact of limbs the contact being at the northern limb of the sun

Time at the Place	Time at Greenwich	Longitude of the Place	Latitude of the Place
19h	0h 8m 11	282° 39′ 9	−21° 28′ 9 S
20	0 16 20	295 57 0	−15 0 3
21	0 33 15	306 12 8	−7 30 7
22	0 58 02	315 29 7	0 10 0 N
23	1 28 66	322 50 1	7 23 5
0	2 0 88	329 19 9	13 28 5
1	2 31 17	337 8 0	18 2 1
2	2 59 07	345 14 2	20 57 0
3	3 22 55	354 21 8	22 10 2
4	3 41 26	4 41 1	21 12 7
5	3 51 15	16 23 3	19 51 2
6	4 0 99	29 45 2	15 14 9

Places

On the line of the phasis of 3 digits the sun being obscured on his northern side

Time at the Place	Time at Greenwich	Longitude of the Place	Latitude of the Place.
19h	0h 9m 11	282° 45′ 4	−12° 26′ 1 S.
20	0 19 31	295 11 4	−5 42 4
21	0 38 71	305 19 4	1 54 6 N
22	1 5 37	313 39 5	9 32 4
23	1 36 00	321 0 0	16 34 1
0	2 6 82	328 17 7	22 23 3
1	2 35 88	336 1 8	26 11 5
2	3 2 21	344 26 9	29 24 1
3	3 23 09	354 13 7	30 32 4
4	3 40 60	4 57 0	30 0 8
5	3 51 14	16 30 4	28 7 3
6	4 0 67	29 50 0	24 32 2

Places

On the line of the phasis of 6 digits the sun being obscured on his northern side.

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
19	0 12 75	280 18 8	−3 16 4 S
20	0 25 79	293 33 2	3 41 0 N
21	0 46 60	303 19 8	11 25 5
22	0 11 18	311 27 3	19 0 4
23	1 44 31	318 55 1	25 50 1
0	2 13 25	326 41 3	31 22 8
1	2 39 61	335 5 9	35 21 7
2	3 2 53	344 22 1	37 53 9
3	3 21 91	354 30 9	38 58 6
4	3 37 81	5 32 1	38 35 1
5	3 49 98	17 30 3	36 14 5
6	3 57 58	30 36 3	33 24 0

Places

On the line of the phasis of 9 digits the sun being obscured on his northern side

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
19	0 20 12	279 53 7	6 13 4 N
20	0 34 28	291 25 8	13 27 3
21	0 57 25	300 11 3	21 20 1
22	1 21 80	308 48 0	28 51.8
23	1 53 20	316 42 0	35 28 3
0	2 19 52	325 7 2	40 12 1
1	2 42 61	334 20 9	44 26 2
2	3 2 31	344 21 9	46 41 2
3	3 19 01	355 14 9	47 11 0
4	3 32 87	6 17 0	47 19 8
5	3 43 89	19 1 7	45 39 2
6	3 51 53	32 7 1	42 34 8

Places

On the line of the annular eclipse the ring being formed on the northern limb of the sun

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
19	0 27 02	278 14 7	13 47 8 N
20	0 43 10	289 9.0	21 16.6
21	1 7 19	298 12 2	29 16 9
22	1 34 17	306 27 5	36 45 6
23	2 0 66	314 50 1	43 10 0
0	2 24 23	323 58 6	48 7 2
1	2 44.33	333 55 1	51 35 3
2	3 1 22	344 41 7	53 44.9
3	3 15 17	356 8 0	54 33 6
4	3 27 18	8 7 8	54 14 3
5	3 37 36	20 39 6	52 42.4
6	3 44 77	33 40 5	49 52 3
7	3 48 68	47 19 8	45 36 0

Places

On the line of the annular eclipse the ring being formed on the southern limb of the sun

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
19	0 30 01	277 29 9	16 25 0 N
20	0 46 92	288 16 2	23 59 5
21	1 12 30	296 55 5	32 2 8
22	1 37 58	305 36 3	39 30 3
23	2 2 95	314 15 8	45 30 1
0	2 25 78	323 33 3	50 40 6
1	2 44 77	333 18 5	54 3 1
2	3 0 65	344 50 3	56 6 3
3	3 14 02	356 29 7	56 55 2
4	3 25 33	8 10 1	56 36 6
5	3 34 77	21 18 5	55 9 0
6	3 42 06	34 29 1	52 23 2
7	3 46 21	48 26 9	48 13 2

Places

On the line of the phasis of 9 digits the sun being obscured on his southern side

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
18	0 31 76	262 3 6	18 9 5 N
19	0 41 05	274 14 3	24 51 9
20	0 59 57	285 6 5	32 19 7
21	1 23 91	294 1 4	41 4.0
22	1 49 03	302 14 6	48 27 2
23	2 11 73	311 4 1	54 29.3
0	2 30 50	322 12 5	58 56 8
1	2 45 71	333 34 9	61 57.2
2	2 58 15	345 27 8	63 41.2
3	3 8 60	357 51 0	64 37 2
4	3 17 62	10 35 7	64 11 2
5	3 25 47	23 37 0	62 51 4
6	3 32 08	36 58.8	60 28 7
7	3 36 76	50 48 6	56 12 1

Places

On the line of the phasis of 6 digits the sun being obscured on his southern side

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
18	0 49 34	257 39 9	30 43 2 N
19	1 2 07	269 29 0	38 19 4
20	1 23 08	279 13 8	47 4 9
21	1 47 08	288 13 8	55 16 4
22	2 8.38	297 51 8	62 58.8
23	2 24 83	308 47 6	68 18 0
0	2 36.76	320 18 6	71 53 2
1	2 45 61	333 35 9	74 7 1
2	2 52 51	346 52 9	75 22 6
3	2 58 34	0 24 9	75 52 1
4	3 3 51	14 7.4	75 41 2
5	3 8.41	27 53 9	74 48.1
6	3 13 24	41 56 4	73 2.4
7	3 17.79	55 48 2	70 7.6
8	3 21 10	69 39 0	65 38 7

PLACES

On the line of the phasis of 3 digits the sun being obscured on his southern side

Time at the place	Time at Greenwich	Longitude of the place	Latitude of the place
h	h m	° ′	° ′
18	1 20 85	249 17 3	18 52 7 N
19	1 39 43	260 8 6	59 51 9
20	2 11 68	266 19 8	74 48 8
20	2 36 66	260 20 1	86 37 1
18	2 40 12	229 53 7	88 18 7
16	2 41 32	199 40 2	89 5 7
14	2 12 01	169 29 9	89 3 8
12	2 42 11	139 23 9	88 38 2

The computation of the places on the line of the phasis of three digits, when the obscuration is on the southern side of the sun, presents some peculiarities, which are worthy of notice. The straight line $\frac{c\ d}{3\ 3}$, which in Plate XXX corresponds to that phasis, stands further from *AB* than the projection *P* of the pole of the earth, and in this case it happens, that some horary circles cut the said line twice on the same side, before reaching the pole, whilst others do not cut it at all These circumstances are also indicated by formula (4). If, after having computed the longitudes and latitudes corresponding to the horary angles 18^h, 19^h, and 20^h, we go on to the horary angle 21^h it will come out log sin $(\varphi+\omega)=0\ 00322$, which logarithm, corresponding to a number greater than the unit, or radius, shows an absurdity, and therefore that no angle $(\varphi+\omega)$ corresponds to the assumed horary angle, the same will happen for some of the successive horary angles. But in this case, the value of sin $(\varphi+\omega)$, corresponding to the horary circle 20^h, gives both the angle and its supplement such, that, subtracting from it the angle ω, the difference is for both angles less than 90°. We should then from this horary circle begin a new series, going back with the values of the horary angles as far as the limit given by Table II, and taking the supplements of the angles belonging to sin $(\varphi+\omega)$ compute by the same formulæ the longitudes and latitudes of the corresponding places. In this way have been computed the last five places on the line of the phasis of 3 digits, and this example shows how we must proceed in similar cases. We have in this instance assumed intervals of two hours, since the places corresponding to them are sufficiently near, to enable us to trace their line with the same degree of accuracy with which the other lines may be described.

All the places corresponding to the longitudes and latitudes given in the nine preceding tables being stereographically projected, and joined together, give the lines that bear the same name in the map contained in fig 3 of PLATE XXXI.

19. To complete the computation of the elements for the drawing of a general eclipse, it remains to find the places where the observers shall see a contact, a phasis of 3 digits, of 6 digits, &c on the east side of the sun, when he is rising, or on the west side when he is setting. These lines will unite themselves with those of the greatest phasis, in the points where they cut the dotted line denoting the middle of the eclipse at the horizon, and will form with them curves returning into themselves. The greatest phasis visible in a place comprehended between two of these returning curves will be limited by the number of digits corresponding to them, so that by their means we shall be able to form an idea of the magnitude of the eclipse for any given place When Tables II, III, and IV, have been constructed, the most easy way for finding the longitudes and latitudes of the places on the lines of which we are speaking, will be the following.—Those places will all be projected on the circumference of the circle *aLKMb*, when the sun is at the horizon, viz. when the phasis required will happen. The co-ordinates, μ and ν, of each of them, will then be respectively represented by the cosines and sines of the angles comprehended between the axis *XX* and the straight line that goes from

the centre C to the point of projection of the place. Thus, if we take for μ and ν the cosines and sines of the angles γ, employed in the construction of Table II., the horary angles of the sun and the latitudes of the places will be the same as those noted in that Table, and we shall only want to compute the times, reckoned at Greenwich, in which the corresponding phases will be seen, in order to obtain the longitudes of the same places. Now the time t, as counted at Greenwich, at which the phasis f is seen in the place, corresponding to the co-ordinates μ and ν, is given in general by the formula

$$(6) \qquad t = T + \frac{60^{m}}{k}\left\{\mu \pm \sqrt{(D+d-f)^2 - (\nu-\Delta)^2}\right\}$$

the upper sign taking place when the phasis happens on the east side of the sun, and the under one when on the west side. In our case we must adopt the upper sign for the places in which the sun is rising, and the under sign for those in which he is setting. According to the preceding supposition, the places of which we are going to determine the longitudes, are those projected on the points $\frac{a}{3}$, $\frac{a}{6}$, $\frac{a}{9}$, &c. so that the values of $\nu-\Delta$ corresponding to them will be given by the distances of the lines $\frac{a}{3}\frac{b}{3}$, $\frac{a}{6}\frac{b}{6}$, $\frac{a}{9}\frac{b}{9}$, &c. from AB, we shall then have (See No. 14, Rule IV.)

for $\frac{a}{3}$ and $\frac{b}{3}$ . . . $\nu-\Delta = -0.4195$ . . . $\log(\nu-\Delta) = \bar{9}.62273$
$\frac{a}{6}$ and $\frac{b}{6}$ . . . $= -0.2736$ . . . $= \bar{9}.43712$
$\frac{a}{9}$ and $\frac{b}{9}$ . . . $= -0.1278$ . . . $= \bar{9}.10653$
o and o' . . . $= 0.0000$ . . . $= -\infty$
$\frac{c}{9}$ and $\frac{d}{9}$ . . $= 0.1278$ . . . $= 9.10653$
$\frac{c}{6}$ and $\frac{d}{6}$ . . . $= 0.2736$ . . . $= 9.43712$
$\frac{c}{3}$ and $\frac{d}{3}$ . . . $= 0.4195$ . . . $= 9.62273$
k . . . $= 0.4635$ . . . $= 9.66605$
M . . . $= 0.5244$ . . . $= 9.71966$

As for the values of $D+d-f$ we have evidently

For a simple contact . . . $D+d-f = D+d = 0.5654$ ar. co. $\log = 0.24764$
For the phasis of 3 digits . . $D+d-f = \frac{1}{2}D+d = 0.4195$ $= 0.37727$
For the phasis of 6 digits . . $D+d-f = d = 0.2736$ $= 0.56288$
For the phasis of 9 digits . . $D+d-f = -\frac{1}{2}D+d = 0.1278$ $= 0.89347$
For the annular eclipse . . $D+d-f = D-d = 0.0181$ $\log = 8.25875$

In order to perform the computation of the preceding formula in an easier manner, let us put

$$(7)\quad \frac{\nu-\Delta}{D+d-f}=\sin\beta \qquad\qquad \mu=\cos\gamma$$

and that formula will become

$$t=T+\frac{60^m}{k}.(\cos\gamma\pm(D+d-f).\cos\beta)$$

The first two terms of this equation give the numbers which stand in the fifth and sixth columns of Table III, and since the horary angles of the sun at the respective places are the same as those given in Table II, we shall obtain the times in which the phasis will be seen, and the longitudes of the places of observation, by only adding to, or subtracting from the times and longitudes given in Table IV, the last term of the former equation, computed either in time or in space, which corresponds to the different phases. Now by an easy computation we may compose the following

AUXILIARY TABLE

Containing the values of $\frac{60^m}{k}.(D+d-f).\cos\beta$ in time and space corresponding to several points of the circumference $aKMb$, and to the different phases.

Points	Phase,										Points
	Contact		3 Digits		6 Digits		9 Digits		Annular		
	In Time	In Space	In Time	In Space	In Time	In Space	In Time	In Space	In Time	In Space	
0	$1^h\ 6^m.3$	16° 35′.2	$0^h\ 49^m.2$	12° 18′.1	$0^h\ 32^m.1$	8° 1′.6	$0^h\ 13^m.0$	3° 15′.0	$0^h\ 2^m.1$	0° 31′.9	$0'$
a_9 or c_9	1 4.6	16 9.4	0 46.9	11 43.3	0 28.4	7 5.8					b_9 or d_9
a_6 or c_6	0 58.1	14 30.9	0 37.3	9 19.8							b_6 or d_6
a_3 or c_3	0 44.5	11 7.2									b_3 or d_3
k	0 28.8	6 12.1									
M	0 38.0	9 30.0									M

In computing the numbers in this table, the angles β were first found by formula (7), and then the values of $\frac{60^m}{k}.(D+d-f).\cos\beta$ were deduced. These values result the same for all the points that are marked by letters having the same index underneath. According to what we have before said, we have only to add to, or to subtract from, the numbers in columns 2 and 3 of Tables IV, respectively, the values of $\frac{60^m}{k}.(D+d-f).\cos\beta$ of the above Table, and the sums, or differences, will be the times in which the respective phases will be seen, and the longitudes of the places where they will be observed. We shall then have the following

TABLE

OF LONGITUDES AND LATITUDES OF PLACES ON THE LINES OF THE PHASES AT THE HORIZON, AND THEIR CORRESPONDING TIMES.

ON THE EAST SIDE, SUN RISING						ON THE WEST SIDE, SUN SETTING					
CONTACT											
Point of projection	Time from Table IV	Time at Greenwich	Longitude from Table IV	Longitude of the place	Latitude of the place	Point of projection	Time from Table IV	Time at Greenwich	Longitude from Table IV	Longitude of the place	Latitude of the place
	h m	h m	° ′	° ′	° ′		h m	h m	° ′	° ′	° ′
a_3	0 7.4	0 51.9	273 46.8	262 39.6	−15 56.9	M	2 44.9	2 6.9	138 46.2	148 16.2	71 2.2
a_6	0 9.6	0 7.7	270 21.2	255 50.3	−7 59.0	d_9	2 56.9	2 12.4	116 48.6	127 53.8	70 2.0
a_9	0 14.6	1 19.2	266 16.7	250 7.3	0 14.2	d_6	3 22.3	2 24.2	79 7.9	93 38.8	61 10.6
o	0 21.2	1 27.5	261 58.3	245 23.1	7 50.3	d_3	3 38.2	2 33.6	62 14.3	78 23.7	52 40.2
c_9	0 31.0	1 35.6	256 34.1	240 15.0	16 3.6	o'	3 47.8	2 41.5	53 13.2	69 48.4	45 6.1
c_6	0 46.9	1 45.0	248 18.8	233 47.9	26 44.7	b_3	3 54.6	2 50.0	46 50.6	63 0.0	37 51.7
c_3	1 12.3	1 56.8	234 50.6	223 43.4	40 32.4	b_6	3 59.5	3 1.4	41 29.9	56 0.8	29 51.3
k	2 4.6	2 29.4	197 15.6	191 3.5	62 38.1	b_9	4 1.7	3 17.2	37 32.4	48 39.6	21 59.4
M	2 44.9	3 22.9	138 16.2	129 16.2	71 2.2						
3 DIGITS.											
a_6	0 9.6	0 16.9	270 21.2	261 1.4	−7 59.0	d_6	3 22.3	2 15.0	79 7.9	88 27.7	61 43.6
a_9	0 14.6	1 1.5	266 16.7	254 33.4	0 14.2	d_9	3 38.2	2 51.3	62 14.3	73 57.6	52 40.2
o	0 21.2	1 10.4	261 58.3	249 39.9	7 50.3	o'	3 47.8	2 58.6	53 13.2	65 31.6	45 6.1
c_9	0 31.0	1 17.9	256 34.4	244 51.1	16 3.6	b_9	3 54.6	3 7.7	46 50.6	58 33.9	37 51.7
c_6	0 46.9	1 26.1	248 18.8	238 59.0	26 44.7	b_6	3 59.5	3 22.2	41 29.9	50 49.7	29 51.3
6 DIGITS											
a_9	0 14.6	0 43.0	266 16.7	259 10.9	0 14.2	d_9	3 38.2	3 9.8	62 14.2	69 20.1	52 40.2
o	0 21.1	0 53.3	261 38.3	253 56.7	7 50.3	o'	3 47.8	3 15.7	53 13.2	61 14.8	45 6.1
c_9	0 46.9	0 59.4	256 34.4	249 28.6	16 3.6	b_9	3 54.6	3 26.2	46 50.6	53 56.4	37 51.7
9 DIGITS											
o	0 21.2	0 36.3	261 58.3	258 13.3	7 50.3	o	3 47.8	3 32.8	53 13.2	56 58.2	45 6.1
ANNULAR											
o	0 21.2	0 23.3	261 58.3	261 26.4	7 50.3	o'	3 47.8	3 45.7	53 13.2	58 45.1	45 6.1

The numbers of the columns 1 and 3, both for the phases on the east side and on the west, are taken from Table IV, column 2, for the phases on the east side, is made by adding to the numbers in the first column the respective numbers in time taken from the preceding Table, and column 4, by subtracting from the numbers in column 3 the respective numbers in space, taken likewise from the said Table. Vice versâ, column 2 for the phasis on the west side is the difference of the numbers in column 1, and of the respective numbers in time of the preceding Table, and column 4, the sum of the numbers in column 3, taken with the respective numbers in space from the above Table. Columns 2 of both sides give the times in which the corresponding phases will be seen at the horizon, columns 4 the longitudes of the places where they will be seen, and columns 5 the latitudes of the places, these last being taken from Table IV.

The maps commonly constructed for a general eclipse instead of the *lines of the phasis at the horizon* contain the two lines of a simple contact on the west side of the sun, when he is rising, or on the east side when he is setting, which joined with the two lines for a simple contact, given by the above Table, make a curve returning into itself that has the appearance of figure 8. We have left out these two lines, which seem to us of very little use, but if any one would like to trace them, he has only to change the additions made in the construction of the former part of the preceding Table into subtractions, and vice versâ, and he will have the respective times, and the longitudes of the places for tracing the two required lines, and the latitudes of the places will be still the same for the other lines.

The longitudes and latitudes given in this Table, being stereographically projected and joined together, produce the lines which unite, in fig 3 of Plate XXXI, both on the side where the sun is rising, and on that in which he is setting, as the corresponding lines of the greatest phasis.

Other problems are sometimes added to the theory of a general eclipse, the object of which is, to determine the circumstances of it for particular places. As these problems will never present any difficulty to those persons who have understood the spirit of the general method already laid down, we will not dwell upon them, but will limit ourselves to the following problem.

20. *To find the places where the first and last contact of the limbs will be seen, and the two times in which they will happen, so that the interval between them shall give the duration of the eclipse for the whole earth.*

As the two required places must have the sun at the horizon, the letters μ and ν, in formula (6), will represent the cosine and sine of an angle; and when $f = o$, that formula will give for the value of t either a minimum or maximum, when

$$\nu = \sin\gamma = \frac{\Delta}{1 + D + d}, \qquad \mu = \cos\gamma$$

or when

$$t = T + \frac{60^{m}}{k} \cdot (1 + D + d) \cdot \cos\gamma$$

In which formulæ the supplement of the angle γ ought to be used for the place where the first contact is seen, and the angle itself for the place where the last contact will be observed.

In our case we have

$$\nu = \sin\gamma = \frac{0\ 4756}{1+0\ 5654}, \ \gamma = 162^\circ\ \ 18'\,8, \text{ or } \gamma = 17^\circ\ \ 41'.2$$

therefore

$$t = 2^h\ \ 4^m.58 + \frac{60^m}{h}\ (1+0.5654).\cos 162^\circ\ \ 18'.8\ \ = -0^h\ \ 50^m.48$$

or

$$t = 2^h\ \ 1^m.38 + \frac{60^m}{h}.(1+0.5654).\cos\ \ 17^\circ\ 41'\,2\ \ =\ \ 4^h\ \ 59^m.61$$

The eclipse will then begin at $11^h\ 9^m.6$ in the morning, and will end at $4^h\ 59^m.6$ in the afternoon, both the times being counted at Greenwich, and the duration of the eclipse for the whole earth will be $5^h\ 50^m$.

In order to find the longitudes and latitudes of the places where the beginning and the end of the eclipse will be seen, we have only to make use of the formulæ given under Rule V, and we shall have for the former place

$$\begin{aligned} \cot h &= -\sin(18^\circ\ 57'.8).\cot(69^\circ\ 53'\,6 - 162^\circ\ 18'.8),\ h = 269^\circ\ 12'.8 \\ \sin\varphi &= \ \ \cos(18^\circ\ 57'\,8)\ \cos(69^\circ\ 53'.6 - 162^\circ\ 18'.8),\ \varphi = -2^\circ\ 17'.3 \end{aligned}$$

and for the latter place

$$\begin{aligned} \cot h &= -\sin(18^\circ\ 57'\,8).\cot(69^\circ\ 53'\,6 - \ 17^\circ\ 41'.2),\ h = 104^\circ\ \ 8'.7 \\ \sin\varphi &= \ \ \cos(18^\circ\ 57'\,8)\ \cos(69^\circ\ 53'.6 - \ 17^\circ\ 41'\,2),\ \varphi = \ 35^\circ\ 25'.2 \end{aligned}$$

The two values of φ give the latitudes of the two places, the former of them being southern because negative. For obtaining their longitudes we must subtract from their horary angles h, the respective times t, reduced into space, and we shall have

$$\begin{aligned} &\text{Longitude of the former place} = 269^\circ\ \ 12'.8 + 12^\circ\ \ 36'\,4\ \ = 281^\circ\ \ 49'\,2 \\ &\text{Longitude of the latter place} = 104^\circ\ \ \ 8'.7 - 74^\circ\ \ 54'.1\ \ = \ 29^\circ\ \ 14'.6 \end{aligned}$$

The former place is situated at a short distance from Quito, and the latter place lies in the Mediterranean sea, about five degrees to the east of Candia.

21. When the moments of commencement and of termination of a solar eclipse are the sole objects of observation, a good telescope and a clock, or chronometer, with a known error and rate, will be competent to give the observation, but when the diameter of the sun, or of the phasis, is required to be measured, some one of the micrometers that measure a large angle must be used in conjunction with the telescope, which for this purpose ought not to have a magnifying power of more than 80, lest the field of view should be too small for the object. As the sun has an apparent motion during the eclipse, as well as at other times, a double image micrometer, with a scale large enough to comprehend his whole diameter, will be the most convenient, such as Rochon's, Brewster's, and Dollond's divided object glass, or Jones's dioptric micrometer, all which have been described in their places. When the two images of the unobscured line are brought side to side with the convex curve of one in contact with the con-

cave curve of the other, the measure subtracted from the sun's whole diameter, will give the breadth of the phasis at any moment, which may be converted into digits and parts, by dividing it by one-twelfth part of the whole diameter. When one of the double image micrometers is used, the magnifying power may indeed be considerably increased, because if the contact of the two images is seen at the centre of the field of view, it is not necessary that the whole diameter of the sun should be included.

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§ C ON THE APPLICATION OF THIRD AND FOURTH DIFFERENCES

1. When great precision is required in obtaining the moon's place by interpolation, particularly in right ascension and declination, from the Nautical Almanac, Mr. Henderson has computed Tables of third and fourth differences, to give the small quantities which second differences alone will not reach in many cases, and which we will therefore subjoin, after having made some modification in their arrangement. His rule is this. "Take out of the ephemeris the three right ascensions or declinations preceding, and the three following the given time, and take their first, second, third, and fourth differences, and apply their signs in the usual way, in order to obtain the mean of the two second differences standing nearest the given time, and the mean of the fourth differences. Then to the proportional part of the middle first difference, corrected by the equation of mean second difference, apply the correction, found in the first Table here given, answering to the middle third difference, and also the correction in the second Table, answering to the mean fourth difference, and the result will be the moon's true place."

When the third difference is positive, and the hour from noon or midnight less than 6, the correction must be taken positive, but negative when the third difference is negative under the same circumstances. With respect to the fourth difference, the equation has always the same sign as the mean of the two fourth differences.

2. TABLE I.

EQUATION OF THIRD DIFFERENCE

Time after Noon or Midnight	0' 10"	0' 20"	0' 30"	0' 10"	0' 50"	1' 0"	2' 0"	3' 0"	1' 0"	5' 0"	6' 0"	7' 0"	8' 0"	9' 0"	10' 0"	Time after Noon or Midnight
+																—
0h 0m	0" 0	0' 0	0" 0	0" 0	0' 0	0" 0	0" 0	0" 0	0' 0	0" 0	0" 0	0' 0	0" 0	0" 0	0".0	12h 0m
0 30	0 0	0 1	0 1	0 1	0 2	0 2	0 4	0 5	0.7	0 9	1 1	1 3	1 5	1 6	1 8	11 30
1 0	0 1	0 1	0 2	0 2	0 3	0 3	0 6	1 0	1 3	1 5	1 9	2 2	2 5	2 9	3.2	11 0
1 30	0 1	0 1	0 2	0 3	0 3	0.4	0 8	1 2	1 6	2 1	2 5	2 9	3.3	3.7	4.1	10 30
2 0	0 1	0 2	0 2	0 3	0 4	0 5	0 9	1 4	1 9	2 3	2 8	3 2	3 7	4.2	4 6	10 0
2 30	0 1	0 2	0 2	0 3	0 4	0 5	1 0	1 4	1 9	2 4	2 9	3 4	3 8	4 3	4 8	9 30
3 0	0 1	0 2	0 2	0 3	0 4	0 5	0 9	1 4	1 9	2 3	2 8	3 3	3 7	4 2	4 7	9 0
3 30	0 1	0 1	0 2	0 3	0 4	0 4	0 9	1 3	1 7	2 2	2 6	3 0	3 4	3 9	4 3	8 30
4 0	0 1	0 1	0 2	0 2	0 3	0 4	0 7	1 1	1 5	1 9	2 2	2 6	3 0	3 3	3 7	8 0
4 30	0 0	0 1	0 1	0 2	0 2	0 3	0 6	0 9	1 2	1 5	1 8	2 1	2 3	2 6	2.9	7 30
5 0	0 0	0 1	0 1	0 1	0 2	0 2	0 4	0 6	0 8	1 0	1 2	1 4	1 6	1 8	2.0	7 0
5 30	0 0	0 0	0 1	0 1	0 1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0.8	0.9	1 0	6 30
6 0	0 0	0 0	0 0	0 0	0 0	0 1	0 0	0 0	0 0	0 0	0 0	0 0	0.0	0.0	0 0	6 0
+																—

3

TABLE II.

EQUATION OF FOURTH DIFFERENCE

Time from Noon or Midnight		0′ 10″	0′ 20″	0′ 30″	0′ 10″	0′ 50″	1′ 0″	2 0″	3′ 0″	4′ 0″	5′ 0″	Time from Noon or Midnight	
0h	0m	0″ 0	0″ 0	0″ 0	0′ 0	0″ 0	0′ 0	0″ 0	0″.0	0″ 0	0″ 0	12h	0m
0	30	0 0	0 1	0 1	0.1	0 2	0 2	0 4	0 6	0 8	1 0	11	30
1	0	0 1	0 1	0 2	0 3	0.3	0.4	0 8	1 2	1 6	2 0	11	0
1	30	0 1	0 2	0 3	0 4	0 5	0.6	1 2	1 7	2.3	2 9	10	30
2	0	0 1	0 2	0 4	0 5	0 6	0 7	1 5	2 2	3 0	3 7	10	0
2	30	0 1	0 3	0 4	0 6	0 7	0 9	1.8	2 7	3 6	4 5	9	30
3	0	0.2	0 3	0 5	0 7	0 9	1 0	2.1	3 1	4 1	5 1	9	0
3	30	0 2	0 4	0 6	0 8	0 9	1 1	2.3	3 4	4 6	5 7	8	30
4	0	0 2	0 4	0 6	0.8	1 0	1.2	2 5	3.7	4 9	6 2	8	0
4	30	0 2	0 4	0 7	0 9	1 1	1 3	2.6	3.9	5.2	6 5	7	30
5	0	0 2	0 5	0 7	0 9	1 1	1.4	2.7	4 1	5 4	6 8	7	0
5	30	0 2	0 5	0 7	0 9	1 2	1 4	2 8	4 2	5 6	7.0	6	30
6	0	0 2	0.5	0 7	0 9	1 2	1 4	2 8	4 2	5 6	7 0	6	0

4. *Example.*—Let it be required to find the moon's declination on the 5th of December, 1824, at 17h 44m, or 5h 44m past midnight, Greenwich time, by using the third and fourth differences in addition to the work given at page 610?

					1st diff.	2nd diff.	3rd diff.	4th diff.
Midnight	Dec. 4.	22°	0′	46″	+58′ 17″	−17′ 53″	− 2′ 33″	+ 1′ 24″
Noon	5.	23	0	3	+40 24	−20 26	− 1 9	+ 0 39
Midnight	5.	23	40	27	+19 58	−21 35	− 0 35	Mean +0′ 59″
Noon	6.	24	0	25	− 1 37	−22 10		
Midnight	6.	23	58	48	−23 47			
Noon	7.	23	35	1				
					+19 58 Middle	−21 0.5 Mean	− 1 9 Middle	

Moon's declination at midnight Dec. 5	+23°	40′	27″	
P p. as 12h · 19′ 58″ 5h 44m	+	9	32.38	
Correction from Tab. of 2nd diff. (−21′ 0″.5) . .	+	2	37.15	
Correction for 3rd diff. . . . . (− 1 9) . .	−		0.10	
Correction for 4th diff. . . . . (+ 0 59) . .	+		1.40	
Correct declination at the given time	23	52	37.73	

5. The same method will apply to the determination of the correct right ascension, and also of the correct longitude and latitude of the moon, in all which cases the first differences have the sign +, when the quantities are increasing, and − when decreasing; and then as the upper difference is always algebraically subtracted from the next lower, in all the denominations, the signs of the other differences will be derived from the process of the successive subtractions

§ CI TO DETERMINE THE OBLIQUITY OF THE ECLIPTIC BY OBSERVATION

1. The angle that the ecliptic makes with the equinoctial, commonly called the obliquity of the ecliptic, has frequently been determined by observing the zenith distances of the solstitial points of summer and winter, and a succession of such observations, made in different years, has confirmed the theory of an annual diminution of this angle, which therefore affects the sun's declination, and the latitudes of all the heavenly bodies, as they are referable to the ecliptic. The determination of the obliquity requires an instrument of the first order to be used, and the application of various solar corrections, which will be best understood from a few examples. If we call the meridian zenith distance of the sun's centre taken at the summer solstice, or reduced to it, Z, and the zenith distance taken and reduced in a similar manner, at the winter solstice, Z', one half of the difference of the arcs, or $\frac{Z'-Z}{2}$ will be $= \omega$, the obliquity required, for the middle of the included half year This determination has no reference to the latitude of the place of observation; but if the sun be observed only at one solstice, then it is necessary to know the latitude with the greatest exactness, as being an element of computation. Since the zenith distance of the equinoctial line is always equal to the latitude of the place, $\frac{Z'+Z}{2}$ will give L, the latitude, when not previously known, and consequently we have $L - Z$ and $Z' - L$ each $= \omega$, accordingly as the observation is made at the summer or winter solstice. We will take, as our first example, the solstitial observations of the sun's upper limb, at noon, on December 6, 9, 10, and 16, 1824, and of his lower limb on the 7th of the same month, in latitude 52° 25′ 51″ N with one of Troughton's altitude and azimuth circles, and will arrange the observed zenith distances (Z') with their corresponding corrections in a tabular form, which will exhibit the whole process of the computations, in a manner more intelligible than a verbal description alone could render it.

2

Example 1.

1824 Day	Limb's observed Z'	Coll	Level	Bar	Ther	Refrac	Paral	Semid	Reduced Z' of Centre	$Z' - L$, or observed δ	$i =$ Reduction to Solstice
Dec 6	74° 38′ 51″ 33	+11″ 17	0 0	29 15	33	3′ 36′ 1	−8″ 61	+16′ 16′ 2	74° 58′ 46′ 16	22° 32′ 55″ 16	51 48′ 45
7	75 18 16 33	+11 47	− 1 5	29 0	40	3 36 5	−8 94	−16 16 3	75 5 37 56	22 39 16 56	47 57 77
9	74 58 14 33	+17 20	−11 30	29 20	12	3 32 2	−8 61	+16 16 5	75 18 0 29	22 52 9 29	35 38 20
10	75 3 15 33	+23 53	+ 5 0	29 50	41	3 36 2	−8 64	+16 16 6	75 23 28 02	22 57 37 02	30 7 53
16	75 26 41 0	+15 00	+ 4 0	29 50	46	3 39 7	−8 60	+16 17 3	75 16 51 40	23 21 0 40	6 42 50

Latitude (L.) = 52° 25′ 51″ N.

1824 Day	Apparent Obliquity $= \delta + r$	Lunar Equat	Solar Equat	Sun's Latitude	Reduction to Jan 0	Mean Obliquity
Dec 6	23° 27′ 43″ 61	+0″.17	−0″ 40	+0″ 08	−0 03	23° 27 43′ 42
7	23 27 44 33	+0 16	−0 41	−0 07	−0 03	23 27 43 98
9	23 27 47 49	+0 13	−0 42	−0 35	−0 02	23 27 46 83
10	23 27 44 55	+0 11	−0 42	−0 49	−0 02	23 27 43 73
16	23 27 42 90	+0 07	−0 43	−0 59	−0 02	23 27 41 93

Mean obliquity for Jan 0, 1825	23 27 43 .98	
According to Bessel	23 27 43 .38	Mean of the three 23° 27′ 43″.87.
According to Dr. Brinkley . .	23 27 45 .29	
According to Mr Pond . .	23 27 42 .93	

Mr Pond's Determination was the result of 15 summer and 15 winter solstitial observations, from 1812 to 1826 inclusive, for the year 1820, which has here been reduced to 1825 by the annual diminution − 0″.43 His latitude was taken only at 51° 28′ 39″.

3. In this example the observations were taken on the meridian, and in one position of the instrument, and therefore require no reduction to the meridian; but it was necessary to know the error of collimation in the existing state of the level, and also to apply the semi-diameter of the sun as given in the Nautical Almanac, on a supposition of its being perfectly correct. The French refractions were used, and the resulting obliquity will be acknowledged to be highly satisfactory. As a second example we will take the observations made after the solstice on the 24th, 26th, and 29th of the same month, which were taken at various intervals before and after noon, with the instrument in the reversed positions agreeably to the plan first practised by Dr. Brinkley with the Dublin circle. In this method, as the upper and lower limbs were observed alternately, the reduced altitudes, when a mean is taken, are those of the sun's *centre*, whatever might be his semi-diameter at the time; and the errors of collimation merge in the reversed observations. The separate reductions to the meridian were made for the respective hour angles, the mean of which and also of the corresponding reductions to the meridian, are those which we have tabulated, to spare the insertion of a multiplicity of computations, which are explained in our first volume. The columns of this example are consequently a little different from those containing the corrections of our first example, but are arranged in such succession as will explain themselves without further comment, except that the difference between the altitude reduced to the meridian and the co-latitude of the place, is here taken for the observed obliquity, instead of the difference between the zenith distance on the meridian and the latitude, which is always the same quantity.

Example 2.

Day	Observed Altitude ☉	Mean of the Hour Angles	Mean Reduc to Merid	Level	Bar	Ther.	Refrac	Paral	Reduced Alt of Centre	Co lat — Alt or observed δ	r = Reduction to Solstice.
											+
Dec 24	14° 8 58″ 33	11m 48s 5	2′ 40′ 2	0″	29 05	45	−3 42′ 3	+8″ 00	14° 8′ 4″ 83	23° 26′ 4″ 17	1′ 40″ 69
26	14 14 25 25	5 38 0	0 42 64	0	29 40	46	−3 43 67	+8 60	14 11 32 82	23 22 36.18	5 8 26
29	14 23 27 0	4 55 7	0 30 96	−7 0	29 65	45	−3 43 62	+8 60	14 20 15.94	23 13 53 06	13 51 34

Co-latitude = 37° 34′ 9″.

Day	Apparent Obliq = δ + r	Lunar Equat	Solar Equat	Sun's Latitude	Reduc to Jan 0	Mean Obliquity.
Dec 24	23° 27′ 44″ 86	−0″ 00	−0′ 42	+0″ 51	−0 01	23 27 44.78
26	23 27 44 44	−0 03	−0 41	+0 66	−0 00	23 27 44 22
29	23 27 44 40	−0 06	−0 38	+0 64	−0 00	23 27 44 24
Mean Obliquity from these Observations . . .						23 27 44 41

4. As a third example, to illustrate the method of deducing the obliquity of the ecliptic from solstitial observations without reference to the latitude of the place, we will avail ourselves of Mr Groombridge's observations made at Blackheath in the years 1818 and 1819, in the months of June and December in each year, which, when reduced, will stand thus (*Phil. Trans. of London*, 1820.)

Example 3.

June, 1818 Zen. dist of the observed
solstitial point . . . 28° 0′ 9″ 99
Nutation + 7″.40, Parallax − 4 11, Sun's lat − 0 .52 . . . + 2 77
$Z = 28\ \ 0\ \ 12.76$

December, 1818 . . . = 74 56 5 07
Nutation − 8″ 64, Parallax − 8 45, Sun's lat − 0 49 . . . − 17 58
$Z'_1 = 74\ \ 55\ \ 47\ \ 49$
$Z\ \ \ 28\ \ 0\ \ 12.76$
$Z - Z = 46\ \ 55\ \ 34\ \ 73$
At the vernal equinox $\frac{Z - Z}{2} = 23\ \ 27\ \ 47\ \ 36 = \omega$
$\frac{Z' + Z}{2} = 51\ \ 28\ \ \ 0\ \ 12 = L$

June, 1819 . . . = 28° 0′ 9″ 19
Nutation + 9″.23, Parallax − 4 11, Sun's lat − 0 50 . . . + 4.62
$Z = 28\ \ 0\ \ 13\ \ 81$

December, 1819 . . . . 74 55 47.22
Nutation − 9″.60, Parallax − 8 .45, Sun's lat − 0 53 . . . − 18 58
$Z' = 74\ \ 55\ \ 47.22$
$Z = 28\ \ 0\ \ 13.81$
$Z' - Z = 46\ \ 55\ \ 33.41$
$\frac{Z' - Z}{2} = 23\ \ 27\ \ 46\ \ 70 = \omega$
$\frac{Z' + Z}{2} = 51\ \ 28\ \ \ 0.51 = L$

From the mean of several observed stars the latitude was found = 51 28 2 18

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§ CII RECENT DETERMINATION OF THE CONSTANT OF ABERRATION

1. Since the preceding portion of this Volume was printed, we have been favoured with a copy of the constants of aberration resulting from 4119 Greenwich observations, as deduced by W. Richardson, one of the assistants at the Royal Observatory, in computing which he employed the constants of nutation determined by Dr Brinkley at the Dublin Observatory, and pursued the same process pointed out in the Right Reverend Prelate's communication to the Royal Society of London, in the year 1821. By taking a mean of all the observations by both circles, and assigning to each observation its own weight, the computer finds that, though different stars seem to have their appropriate constants of aberration, the mean of the whole is 20″.5035, or nearly a quarter of a second greater than has been usually adopted.

2 This conclusion accords with the assumption of Mr. Herschel, who in his "*Tables for computing the apparent places of the forty-four principal fixed stars*," adopted 20″.5 : and consequently the factor, which we have given in page 471 of our Appendix to Vol. I., viz. log 0 .00522, or Nat. No. 1 .0121, will be proper for using with our Tables. The subjoined Table, which was at the same time communicated to the Astronomical Society, will explain its own contents.

Names of the Stars	RESULTS BY TROUGHTON'S CIRCLE			RESULTS BY JONES' CIRCLE			Mean of the two Circles
	Constant of Aberration	No of Obs	Const of Aber × No of Obs	Constant of Aberration	No of Obs	Const of Aber × No. of Obs	
γ Cassiopeiæ	20″ 137	80	1610″ 960	20′ 408	80	1632″ 640	20″ 273
Polaris	20 246	160	3239 360	20 213	152	3072 376	20 230
by Refraction	20 390	73	1488 470	20 201	72	1454 472	20 296
S P	20 767	150	3115 050	20 973	150	3145 950	20 870
by Refraction	20 427	70	1420 .890	20 285	62	1257 670	20 356
γ Ursæ Majoris	20 723	80	1657 840	20 594	80	1647 520	20 658
ζ	20 411	80	1632 880	20 575	80	1646 100	20 494
η	21 028	80	1682 240	20 733	80	1658 640	20 880
γ Draconis	21 059	80	1684 720	21 430	80	1714 880	21 247
β Ursæ Minoris	20 924	80	1673 920	21 112	80	1688 960	21 018
ν Draconis	20 577	240	4938 480	20 456	240	4909 .440	20 516
γ Lyræ	20 403	160	3264 480	20 324	160	3251 840	20 364
by Refraction	20 267	35	709 345	20 712	32	662 784	20 490
β	20 146	80	1611 680	20 424	80	1633 920	20 285
δ Draconis	20 597	160	3295 520	20 376	160	3260 .160	20 487
γ Cygni	20 354	123	2503 542	20 112	128	2574 336	20 233
by Refraction	20 .359	80	1628 720	20 732	80	1658 560	20 545
γ Cephei	20 373	137	2791 101	20 205	132	2674 .960	20 319
β	20 386	118	2405 548	20 .358	125	2544 750	20 372
	390 574	2066	42303 746	390 291	2053	42090 .036	389 933
	20 504		20 .505	20 542		20 502	20 523
	Total number of Observations (4119) gives						20 5036

§ CIII AN ALPHABETICAL LIST OF OBSERVATORIES, ETC

Places	Latitudes			Longitudes in Time			Longitudes in Arc.			Astronomers.	Stations
Abo	60°	27′	7″ N	1^h	29^m	10^s E	22°	17′	30″	Walbeck	Observatory
Alexandria	31	13	5 N	1	59	41 E	29	55	15		
Altona	53	52	51 N	0	39	50 E	9	57	30	Schumacher	Observatory
Amsterdam	52	22	17 N	0	19	33 E	4	53	15		
Archangel	64	34	0 N	2	42	52 E	40	43	5		Royal Arsenal
Armagh	54	21	15 N	0	26	30 W.	6	37	30	Robinson	Observatory
Bagdad	33	19	40 N	2	57	39 E	44	24	45		
Barcelona	41	21	44 N	0	8	40 W	2	9	57		Tower of Mountjoy
Batavia	6	9	0 S	7	7	27 E	106	51	45		Observatory
Bedford	52	8	48 N	0	2	49 E	0	27	12	Capt Smyth, R N	Observatory
Bergen	60	23	40 N	0	21	23 E	5	20	45	Bohn	
* Berlin	52	31	45 N.	0	53	29 E	13	22	15	Encke (late Bode).	Observatory
* Berne	46	56	55 N	0	29	45 E	7	26	15	Treschel	
Blackman Street.	51	30	3 N	0	0	21 7W	0	5	26 4	South	Observatory
Blenheim	51	50	25 N	1	21	6 W	0	5	24 .4	Late Duke of Marlborough	
Bologna	44	30	12 N	0	45	26 E.	11	21	30	Caturegli.	
Bordeaux	44	50	14 N	0	2	16 W.	0	33	59		
* Bremen	53	4	38 N	0	35	12 E	8	48	0	Olbers.	Observatory
Breslau	51	6	30 N	1	8	9 E	17	2	18		
* Brest	48	23	14 N	0	17	55 W	4	28	45		
Brunswick	52	16	29 N	0	42	8 E	10	32	0		
Brussels	50	50	59 N	0	17	29 E.	4	22	15		
* Buda	47	29	44 N	1	16	10 E.	19	2	30	Kmeth	Observatory
Bushey Heath	51	37	44 N	0	1	21 W	0	22	15	(Late Col. Beaufoy)	Observatory.

AN ALPHABETICAL LIST OF OBSERVATORIES, ETC.

	Latitudes			Longitudes in Time			Longitudes in Arc			Astronomers	Stations
.... .	30°	32′	0″ N	0^h	25^m	10^s W	0°	17′	30″		Observatory
... ..	30	3	11 N	2	5	15 E	31	18	45	Ruppell	Garden of Aselin.
.. ...	22	31	15 N	5	53	44 E	88	26	0		
.	52	12	50 ±	0	0	23 5E	0	5	53	Airy (late Woodhouse)	Observatory
lHope	33	55	42 N	1	13	32 E	10	23	0	Fallows	Observatory
. . .	51	19	20 S	0	38	21 E	9	35	15		
..	59	51	10 N	0	43	1 E	10	45	15	Hansteen	Small Observatory
....	50	15	17 N	0	43	54 E	10	58	30	Opitz and Gobel	
..	40	12	30 N	0	33	39 W	8	24	42		
ple .	41	1	27 N	1	55	41 E	28	55	15		St Sophia
en .	55	41	4 N	0	50	20 E	12	35	6	Schumacher and Ursin	
.. .	50	3	38 N	1	19	49 E	19	57	9		
. . .	53	52	21 N	0	34	52 E	8	43	1	Tralles (pro tempore).	Light House
.... ..	54	20	48 N	1	14	32 E	18	38	5		
.. . ..	49	40	12 N	0	4	33 E	1	8	15	Nell de Breauté	La Chapelle
... .	47	19	25 N	0	20	8 E	5	2	5	Vallot	
... .	58	22	47 N	1	46	48 E	26	42	0	Struve	Observatory
... .	51	3	38 N	0	54	57 E	13	41	15		Salon Mathematique
.. ..	53	23	13 N	0	25	22 W	6	20	30	Hamilton (late Brinkley)	Observatory
h	55	56	12 N	0	12	41 W	3	10	15	Wallace	Observatory
.. .	45	20	10 N	0	57	41 E.	14	20	15	Bouvard (pro tempore)	Village Steeple
....	43	46	41 N.	0	45	3 E	11	15	45	Inghirami	
	50	7	30 N	0	31	24 E	8	36	0		
.. . ...	46	12	0 N.	0	24	35 E	6	8	45	Gautier	Observatory
.. .	44	25	0 N	0	35	52 E	8	58	0	Late Zach.	
. .	55	51	32 N	0	17	4 W	4	16	0	Ure	Observatory
....	50	56	8 N	0	42	56 E	10	44	0	Hansen	Seeberg Observatory
.. .	51	31	56 N	0	39	17 E	9	56	45	Harding and Gauss	Observatory.
..	51	28	40 N	0	0	0	0	0	0	Pond	Royal Observatory
.. ..	43	7	2 N	0	21	32 E	6	7	55		
.	undetermined			0	0	46 .8W	0	11	42	South	Observatory
. .	51	28	16 N	0	1	10 W	0	17	30		Pagoda
..	51	28	37 N	0	1	3 W	0	15	45	(Olim Bradley)	Village
rg . .	54	42	12 N	1	21	57 E	20	29	15	Bessel	Observatory
	43	33	5 N	0	41	7 E	10	16	15		
. ... .	51	20	16 N	0	49	27 E	12	21	15		
.	52	0	30 N	0	17	57 E	4	20	13	Moll	
	53	0	30 N	0	35	37 E	8	54	15	Shroeter.	
. .	28	42	24 N	0	36	30 W	9	7	30		Observatory
. .	51	30	40 } 31 }	0	0	23 } 32 }	0	5 8	17 } 15 }		St Paul's St James's, Piccadilly
. .	13	4	[illegible] N	5	21	28 E	80	22	0	Taylor Goldingham.	Flagstaff
. .. .	40	21	57 N	0	14	40 W	3	42	15	Bauza	Grand Square
. . . .	35	53	0 N	0	58	2 E	14	30	35		Valetta Observatory.
.	49	20	18 N	0	33	52 E	8	28	0	Nicolai.	Observatory
. . . .	43	54	28 N	0	42	18 E	10	34	27	Pons	Royal Observatory
.. ...	43	17	40 N	0	21	29 E.	5	22	15	Gambart	Observatory
. . .	10	25	45 N	6	36	21 W	99	5	15		
. . .	45	28	2 N	0	36	47 E	9	11	45	Cesaris Oriani. Carlini	Observatory.
... ..	43	5	7 N	0	7	30 E	1	52	26		Observatory
. ...	56	39	6 N	1	34	54 E	23	43	27		
.	44	34	8 N	0	43	41 E.	10	55	15	Amici	
l	44	0	55 N	0	5	23 E.	1	20	45		Observatory
er ..	43	30	16 N.	0	15	31 E.	3	52	40		Observatory
	55	45	45 N	2	30	12 E	37	33	0		Observatory
. .	48	8	26 N.	0	46	26 E	11	36	30	Soldner	Bogenhausen Observ
.	40	50	15 N	0	57 ′	5 E.	14	16	15	Brioschi	Observatory.
. . .	46	59	0 N	2	8	1 E.	32	0	15	Knorre	Observatory
. ...	43	50	16 N	0	17	27 E.	4	21	25	Valz	
. ..	40	26	55 N	0	44	17 E	11	4	15		
. ...	51	45	39 N	0	5	1 W.	1	15	22	Rigaud	Observatory.
.. ...	45	24	2 N.	0	47	26 E.	15	51	32	Santini	Observatory
.. . .	38	6	44 N	0	53	28 E	13	22	0 E.	Cacciatore (late Piazzi)	Observatory
... .	33	48	45 S	10	4	5 E.	15	1	15	Sir T Brisbane Rumker	Observatory
.. ..	48	50	14 N	0	9	21 E.	2	20	15	Arago Bouvard Nicollet	Observatory
......	48	51	31 N	0	9	8 1 E	2	17	2	South	Observatory
....	45	10	47 N	0	36	39 E.	9	9	48	Oriani.	Observatory

Places	Latitudes	Longitudes in Time	Longitudes in Arc	Astronomers	Stations
Pekin . .	39° 54′ 13″ N	7ʰ 45ᵐ 51ˢ E	116° 27′ 45″		Imperial Observatory
* Petersburgh	59 56 23 N	2 1 15 E	30 18 45		Ditto
Philadelphia .	39 56 55 N	5 0 40 W	75 11 30		
Pisa	43 43 11 N	0 41 36 E	10 24 0	Piazzini	
Plymouth .	50 22 20 N	0 16 29 W	4 7 16		New Church.
Portsmouth	50 48 3 N	0 4 21 W	1 5 59	Inman Burnie	
* Prague	50 5 10 N	0 57 41 E	14 25 15	Biela	
Quebec	46 47 30 N	4 44 30 W	71 9 45		
Quito . . .	0 13 17 S	5 15 0 W	78 45 0		
Ratisbon . .	49 0 53 N	0 48 18 E	12 4 30		
Richmond . .	51 28 8 N	0 1 15 W	0 18 45	Demainbry	King's Observatory.
Riga .	56 56 47 N.	1 36 31 E	24 7 45	Keussler	Observatory
* Rome . .	41 53 54 N	0 49 53 E	12 28 15	Ciccolini. Calandrelli Conti	Roman College
* Rosenna	50 17 41 N	0 44 8 E	11 2 0		
Slough	51 30 20 N.	0 2 24 W	0 36 0	Herschel (late Sir W H)	Observatory
South Kilworth	52 25 51 N	0 24 26 W.	1 6 30	Pearson	Rectory House
St Gall. . .	47 25 40 N	0 37 29 E	9 22 15	Scherer	Observatory
* Stockholm . .	59 20 31 N	1 12 14 E.	18 3 30		
* Strasburg .	48 34 56 N.	0 30 57 E	7 44 15	{ Hornschneider Kramp. Schmidt }	
Toulouse . . .	43 35 46 N	0 5 46 E.	1 26 30		
Tubingen .	48 31 10 N	0 36 14 E	9 3 30		Observatory.
Turin .	48 4 0 N.	0 30 41 E	7 40 15	Plana.	
Upsal . .	59 51 50 N	1 10 36 E	17 39 0		
Uraniburg . .	55 54 38 N	0 50 52 E	12 43 0	(Olim) Tycho Brahe	Observatory
Utrecht . .	52 5 31 N.	0 20 29 E	5 7 16		
Venice . .	45 25 32 N	0 49 24 E	12 20 59		St Mark's
Verona . . .	45 26 7 N.	0 44 5 E	11 1 15	Cagnoli.	Observatory
* Vienna .	48 12 40 N	1 5 31 E	16 22 45	Littrow	Observatory
Viviers . .	44 29 2 N	0 18 44 E	4 41 0	Flaugergues	Observatory
* Wardhuus . . .	70 22 36 N.	2 1 28 E	30 22 0		
* Warsaw .	52 14 28 N	1 24 11 E	21 2 45		
Weimar . . .	50 59 12 N	0 45 21 E	11 21 0		
* Wilna . .	54 41 2 N.	1 41 10 E	25 17 30	Slawinski, &c	Observatory
Zurich . .	47 22 33 N	0 34 6 E	8 31 30	Horner	

At those places to which an asterisk is prefixed, the solar eclipse of Sept. 7, 1820, was observed; and several of them had their longitudes corrected thereby as given in this list. *(Annales de Mathematiques, Tome IX. p.* 118 *Zach's Correspondence, Astron. passim)*

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§ CIV CONCLUSION

1. WE have now arrived at that stage of our work, which authorizes us to take leave of our readers, and places us in a situation of imploring their indulgence for the increase of matter, that offered itself to our notice, as we advanced, beyond our original expectation The improvements in Practical Astronomy have lately made rapid strides, imposing upon us the task of attempting to keep pace with them · and we are aware, that many sections might have been added to this ponderous volume, such as the methods of determining by observations the parallaxes of the various planetary bodies, and thence their distances, &c , but as these interesting subjects are treated of in most of the theoretic treatises on Astronomy, the omission will readily be dispensed with.

CONCLUSION

On taking a retrospective view of our successive descriptions of astronomical instru , we cannot help remarking, that the gradual improvements in their construction have accompanied by corresponding discoveries, and increased accuracy in making and re- g the various observations. In no other branch of natural and experimental philosophy, mechanism of human contrivance and accomplishment, is so much ingenuity displayed, he construction of modern horological and astronomical instruments.

. The clock and the chronometer, as now constructed, are each an assemblage of the ingenious inventions, conferring honour on their various inventors and improvers, through succession of years; the telescope penetrates beyond the ordinary limits of space, and its to the human eye the images of objects, concerning the existence of which we must wise have remained in utter ignorance, the reading microscope enlarges and subdivides ities so small, that the human eye, unassisted, could not have appreciated even a large onal part thereof, the limbs of circles have been graduated and subdivided with such teness, and at the same time with such accuracy, that the delicacy and equal distances e dividing strokes can only be rendered visible by optical contrivances, the micrometers, neasure magnified angular distances too diminutive to be referred to a divided limb, are ructed with a precision, that astonishes the inexperienced observer, and a great variety ose delicate instruments, different in principle, as well as dissimilar in construction, ally concur in effecting similar results. The methods too, by which the due positions of arger instruments are ascertained, regulated, directed, and fixed, all depend upon the ingenious contrivances; and the adjustments are not only effected, but watched also, by ianical arbiters both sensible and vigilant.

4. When the practical Astronomer has proved, and can confide in his various and delicate nces, his mind, like his telescope, is pointed to Heaven, and his soul is wrapped up in the emplation of objects, that prove beyond contradiction the immutability of those laws, by h the Omnipotent Creator upholds, actuates, and directs the luminous bodies composing solar system. Whenever he detects a glaring discrepancy in his comparative observations, n the errors of his clock, he arraigns not the heavenly bodies, or the earth on which he ds, as subject to move under the misguidance of capricious laws, but suspects his own im- ect powers, or the tendency of material mechanism to change its position or dimensions, variations of temperature, and a repetition of the observation, stamps conviction of the h of such supposition on his mind in characters indelible. He finds in every failure a of of human impotency; but in the general agreement of all his successful efforts, dis- ers a regulating power infinitely greater than his own, and participates in the feelings of poet, who, considering that a complete Observatory displays the sublimest works of both d and man, thus expresses his admiration—

"Here truths sublime, and sacred science charm,
    Creative arts new faculties supply,
Mechanic powers give more than giant's arm,
    And piercing optics more than eagle's eye,—
Here man explores Creation's wondrous laws,
That teach him to adore the GREAT DESIGNING CAUSE"

THE END.

G Woodfall, Printer,
ngel Court, Skinner Street, London

PLATES

BELONGING

TO THE SECOND VOLUME

OF

AN INTRODUCTION

TO

PRACTICAL ASTRONOMY.

BY THE REV. W. PEARSON, LL.D. F.R.S. ETC.

RECTOR OF SOUTH KILWORTH, LEICESTERSHIRE, AND TREASURER TO THE ASTRONOMICAL
SOCIETY OF LONDON

G WOODFALL, ANGEL COURT, SKINNER STREET, LONDON

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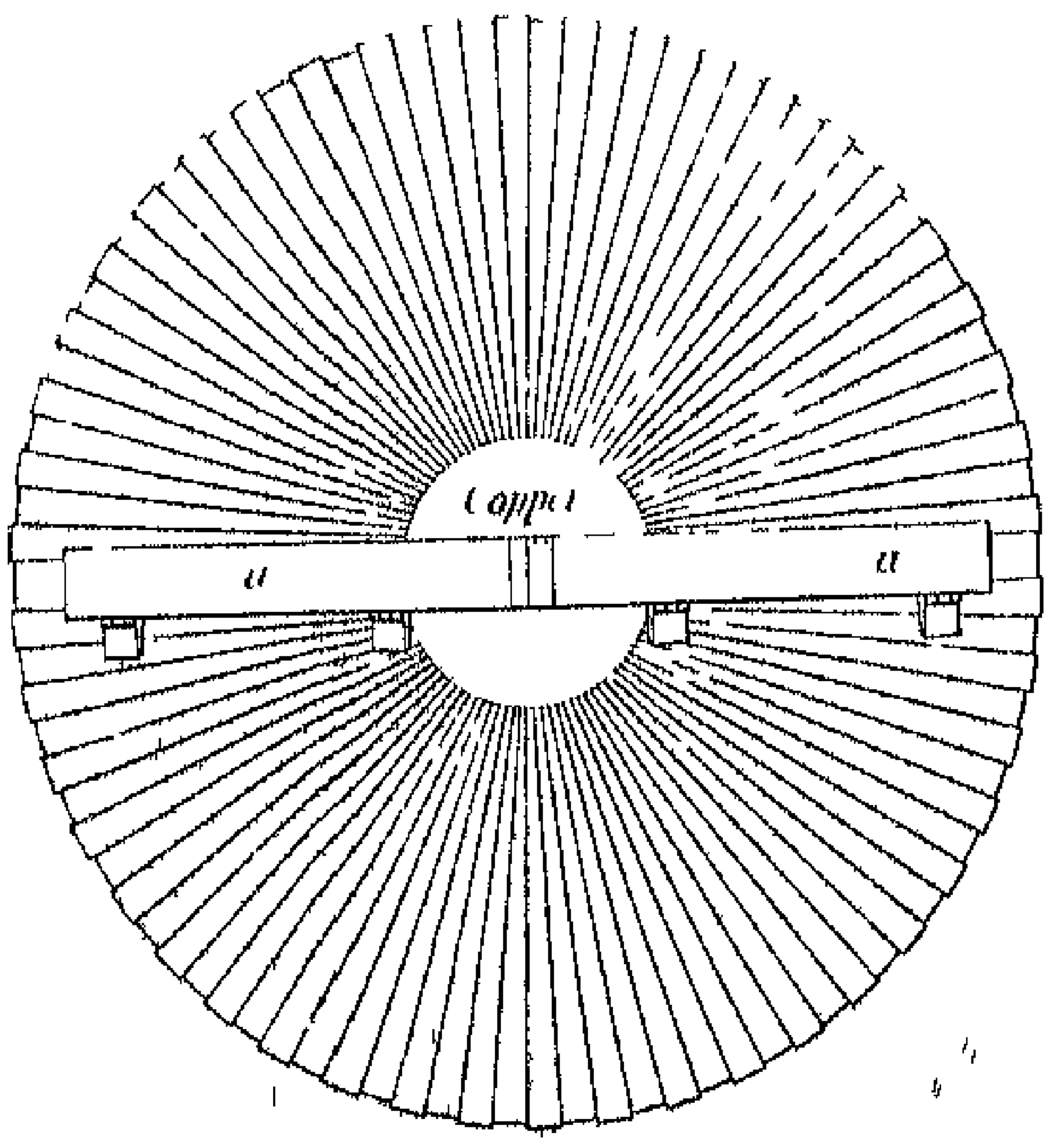
ROTATIVE DOMES AND SMEATON'S BLOCK

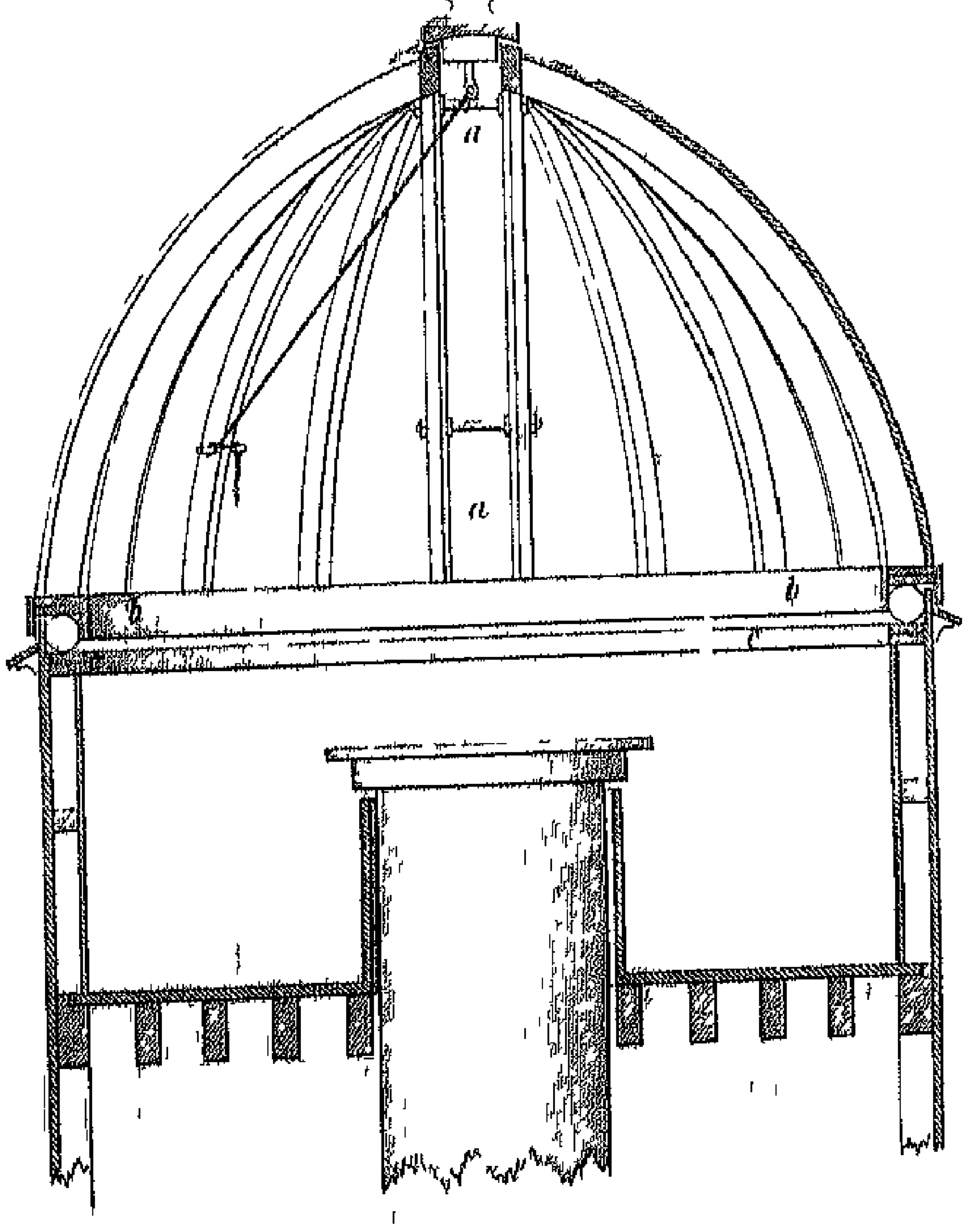
Plate

Fig 1



Pearson's Dome

Fig 5



Smeaton's Dome

Fig 2

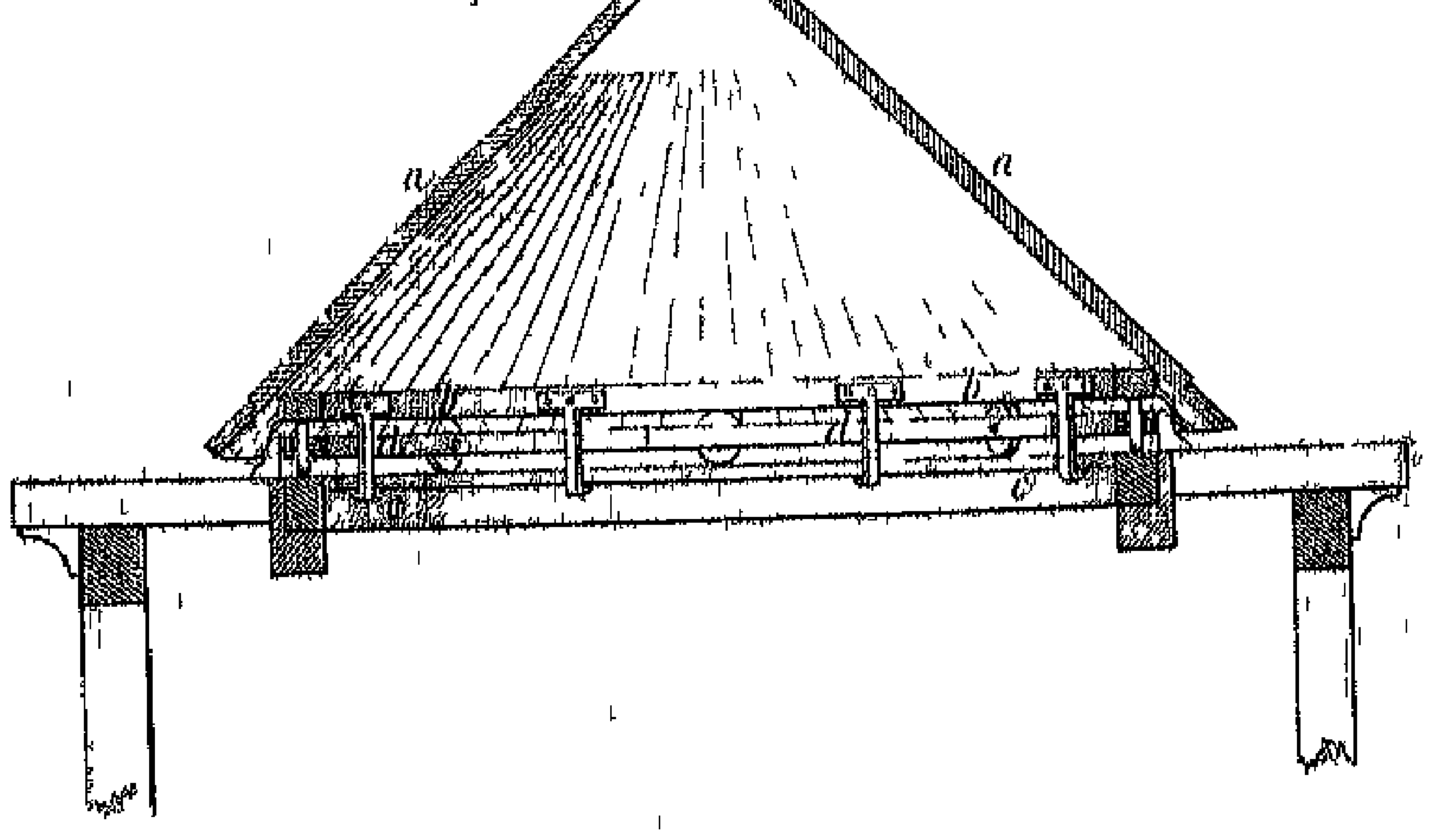


Fig 6

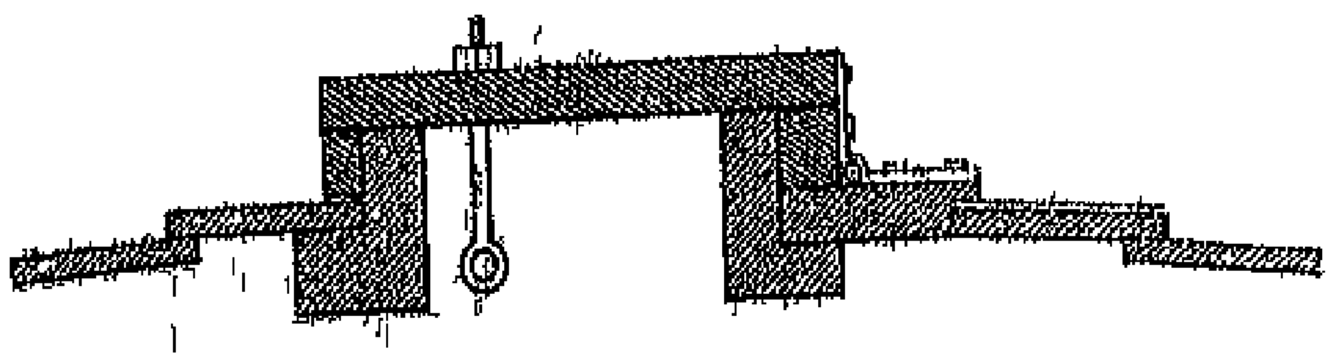


Fig 3

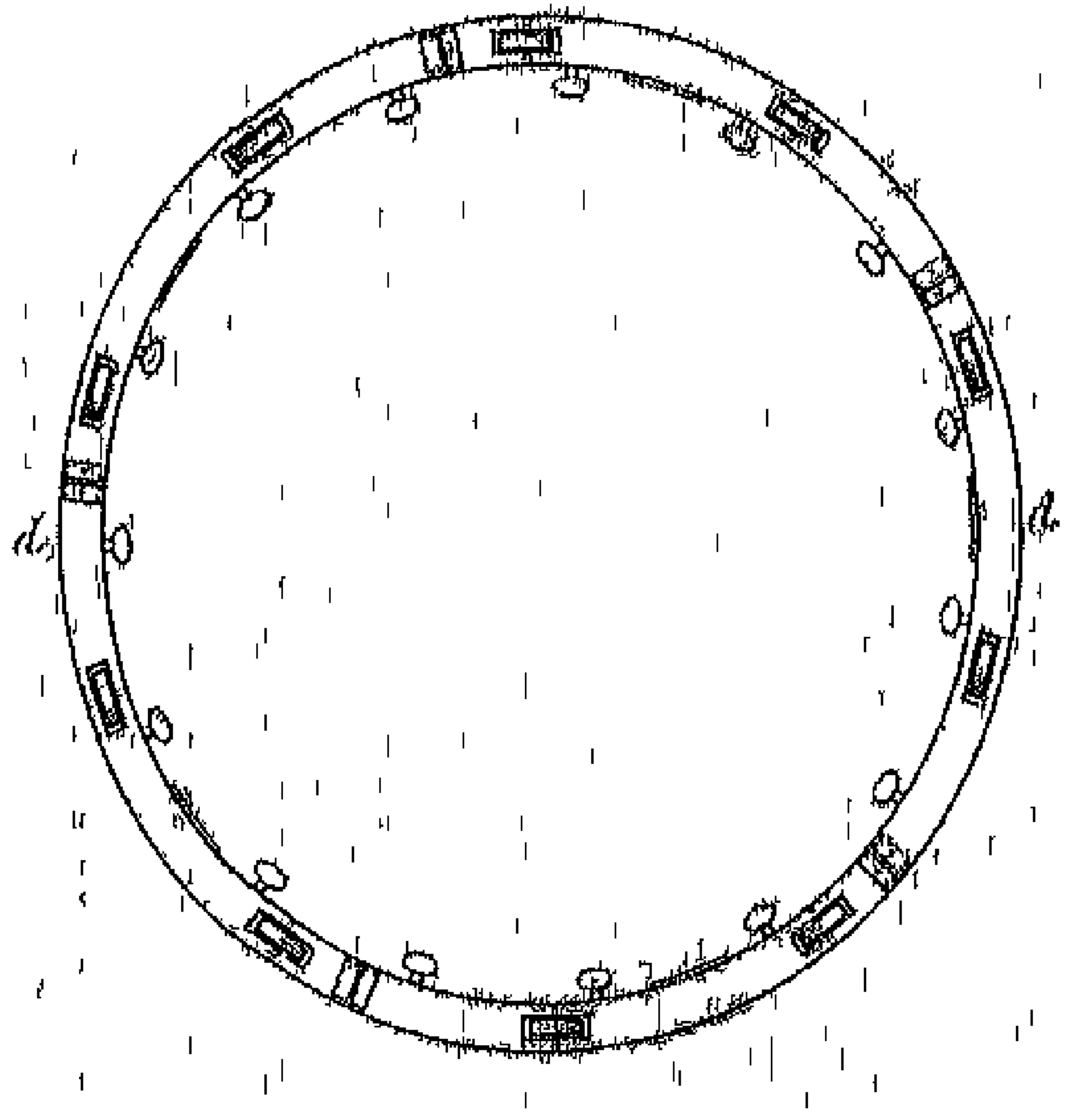


Fig 4

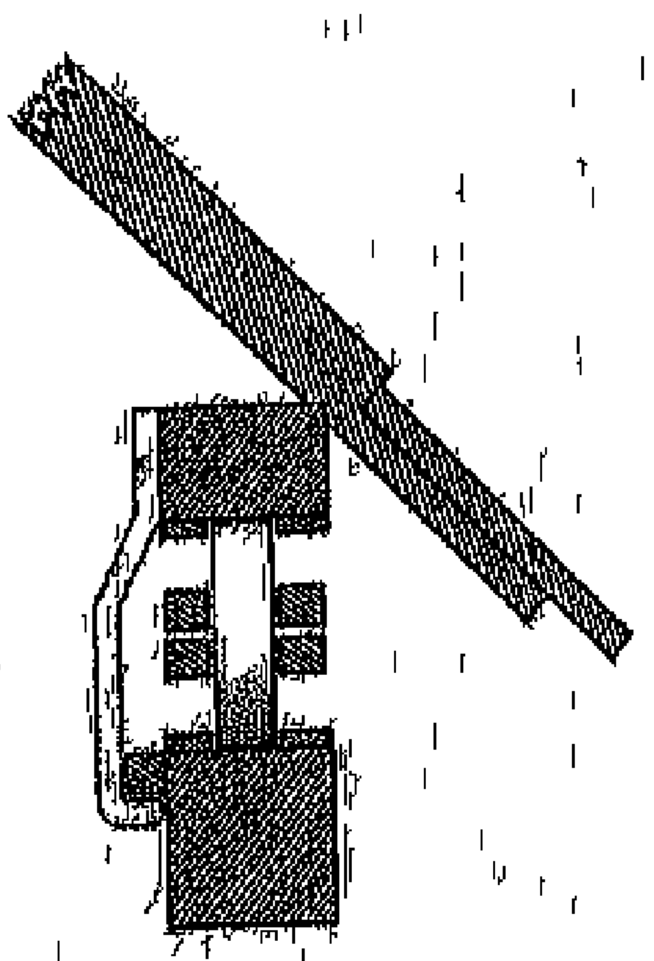
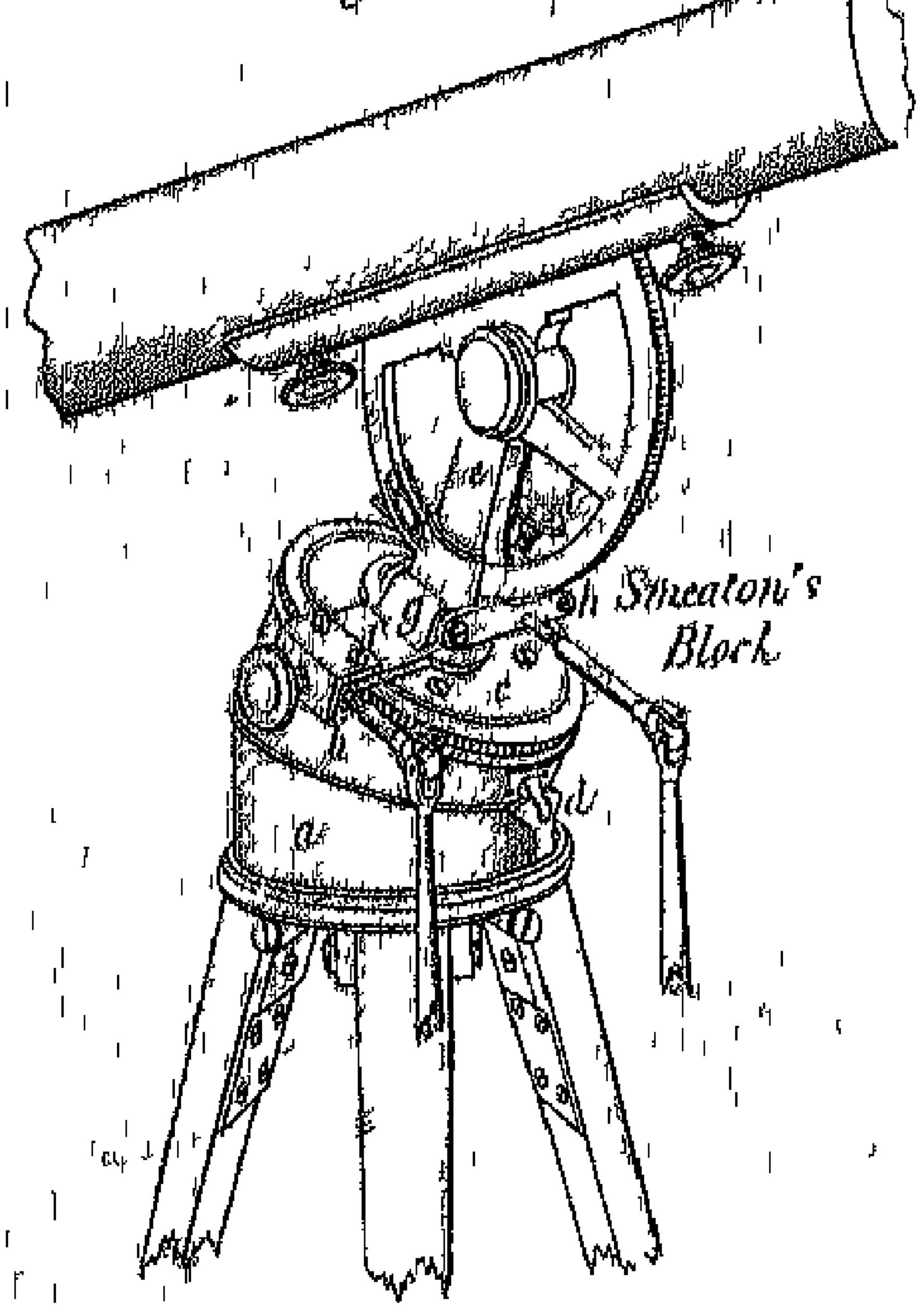
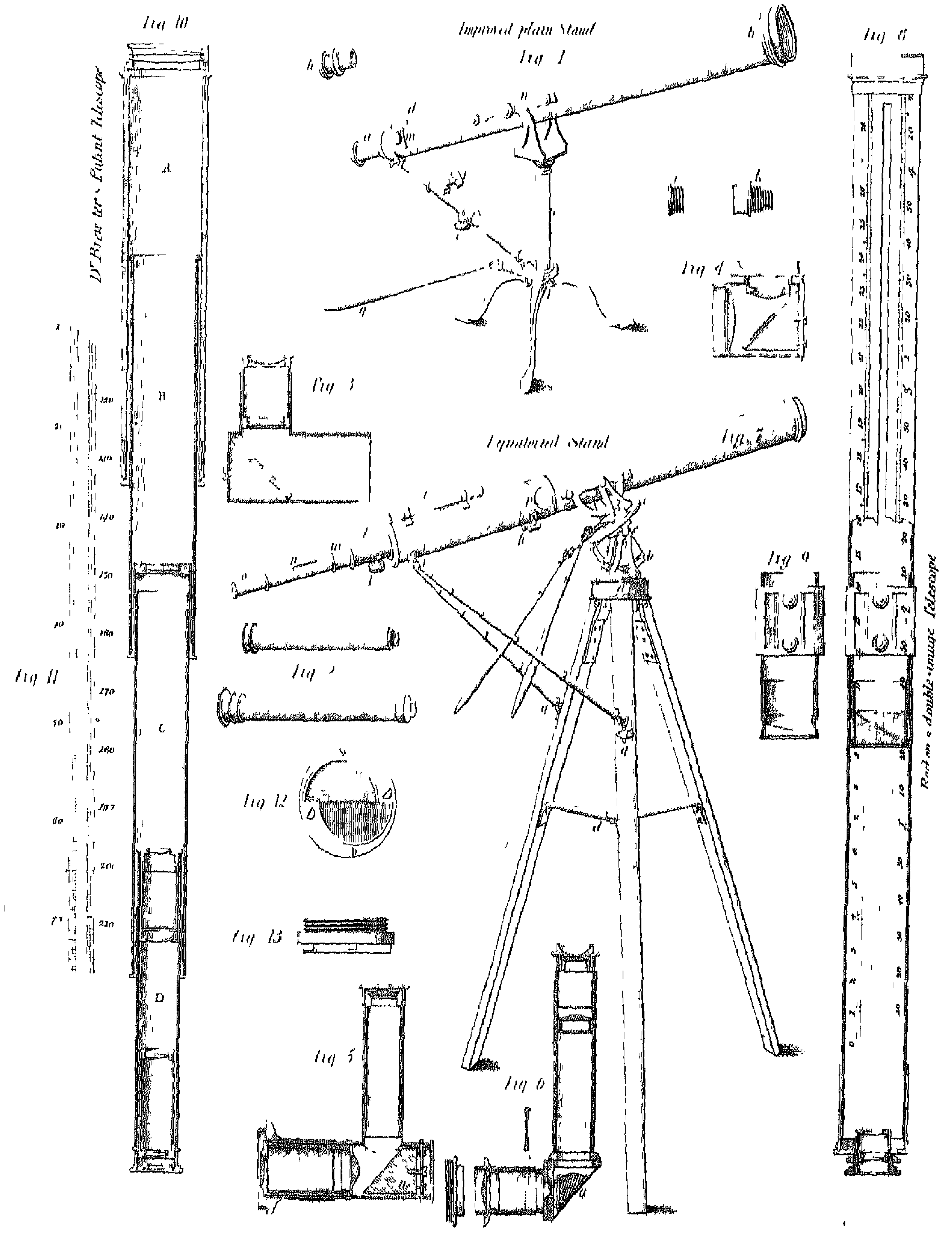


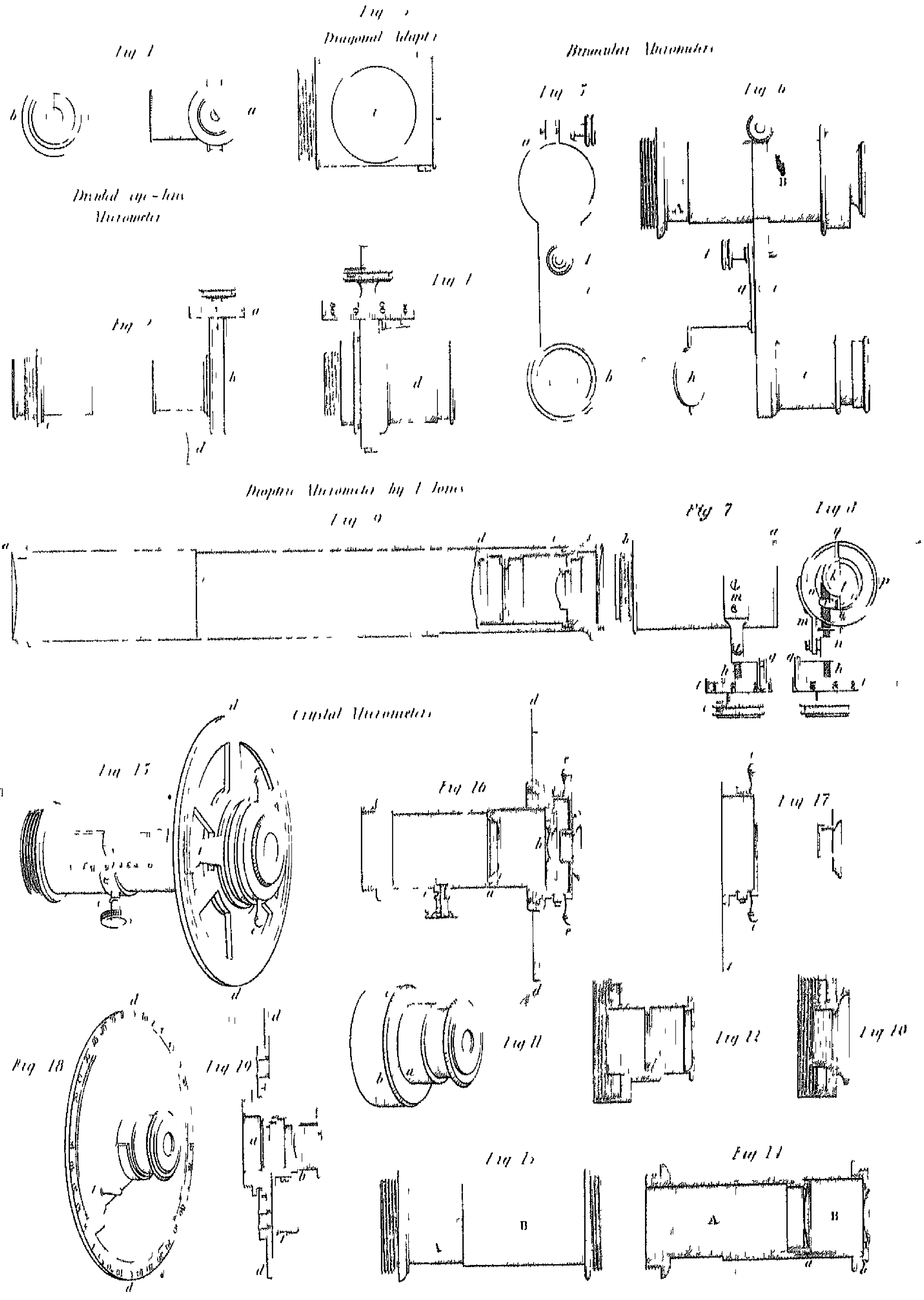
Fig. 7



REFRACTING TELESCOPES AND STANDS

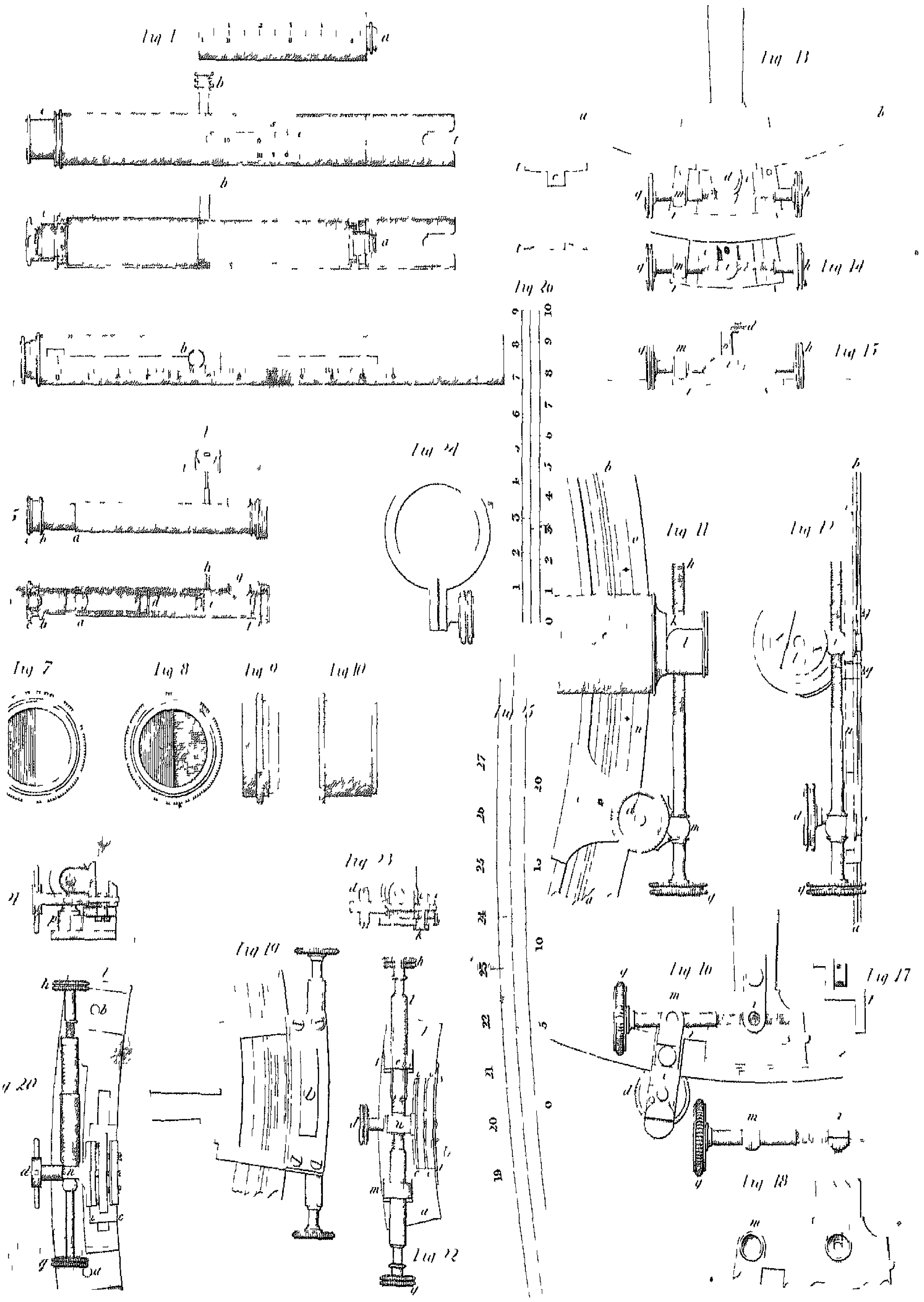


VARIOUS MICROMETERS



PRISMS, CLAMPS, TANGENT SCREWS, AND VERNIERS &c.

Plate IV



DOLLOND'S STANDS FOR ACHROMATIC TELESCOPES, AND MICROMETERS.

Plate

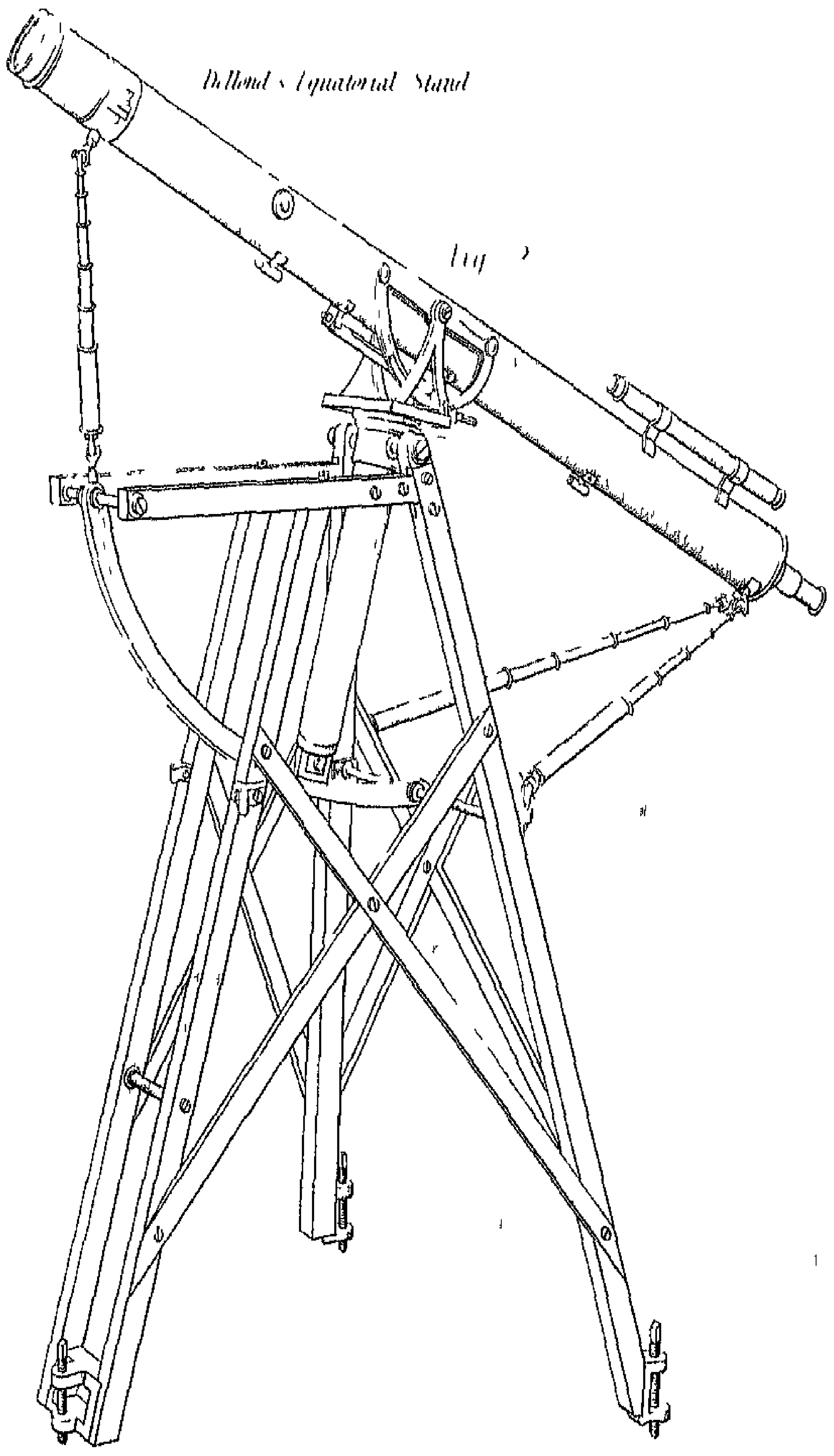

Dollond's Equatorial Stand

Fig 2

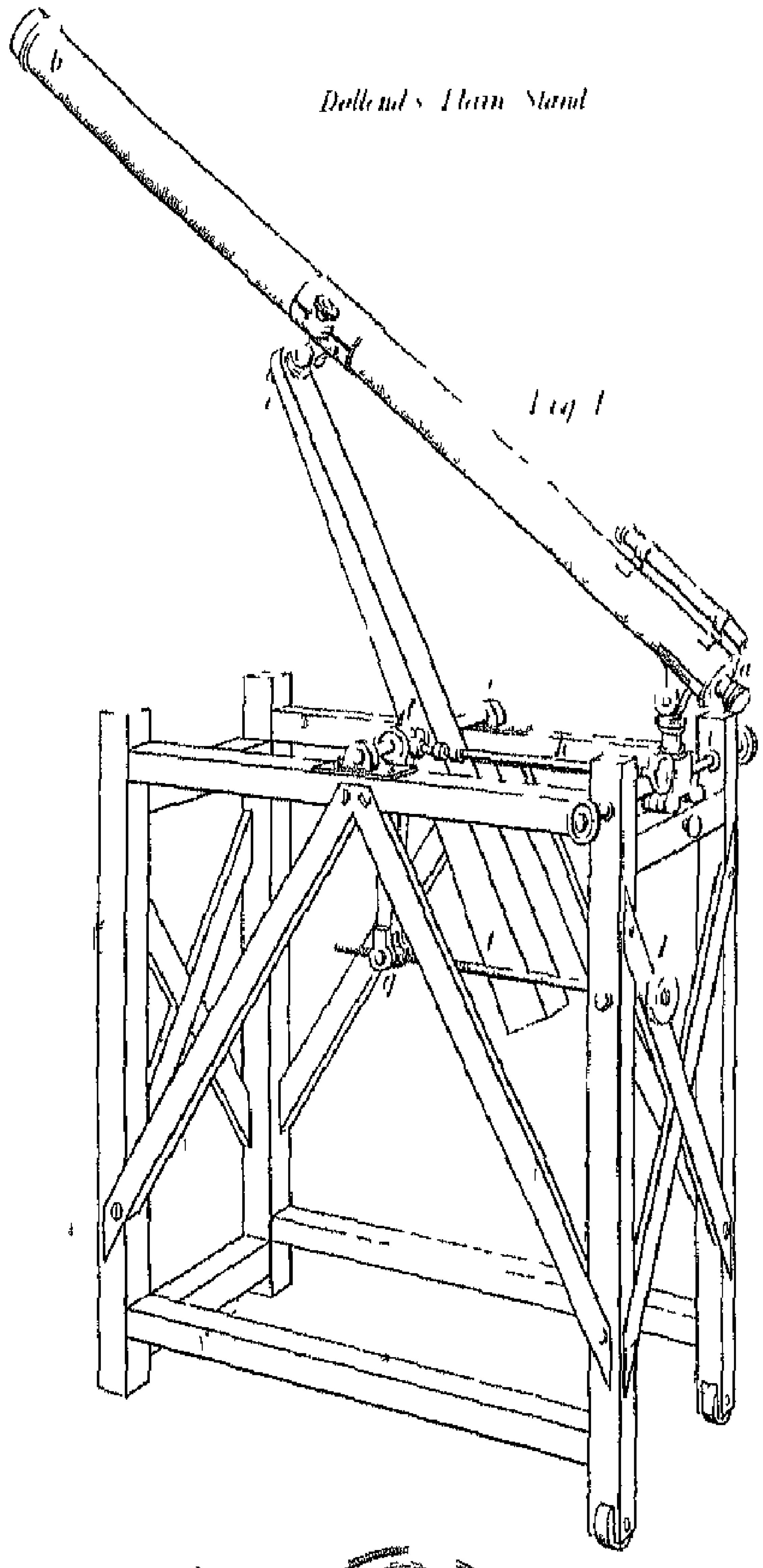

Dollond's Plain Stand

Fig 1

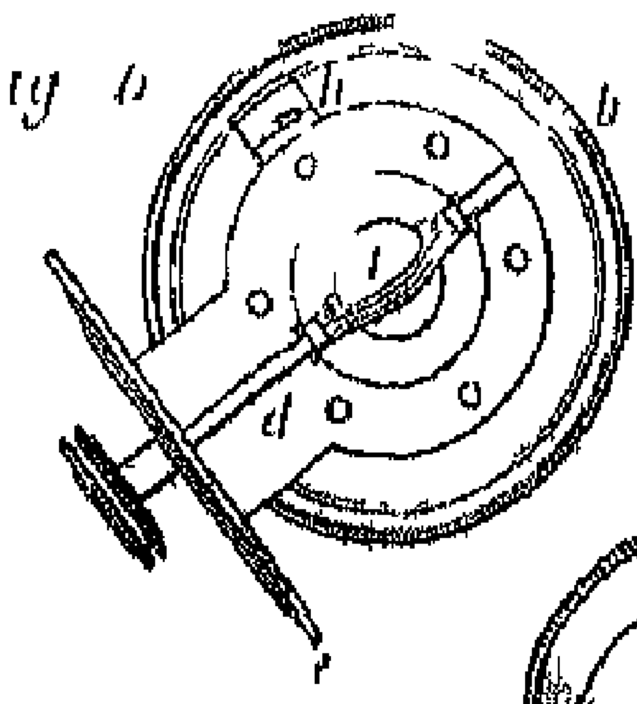


Fig 6

Dollond's crystal Micrometer

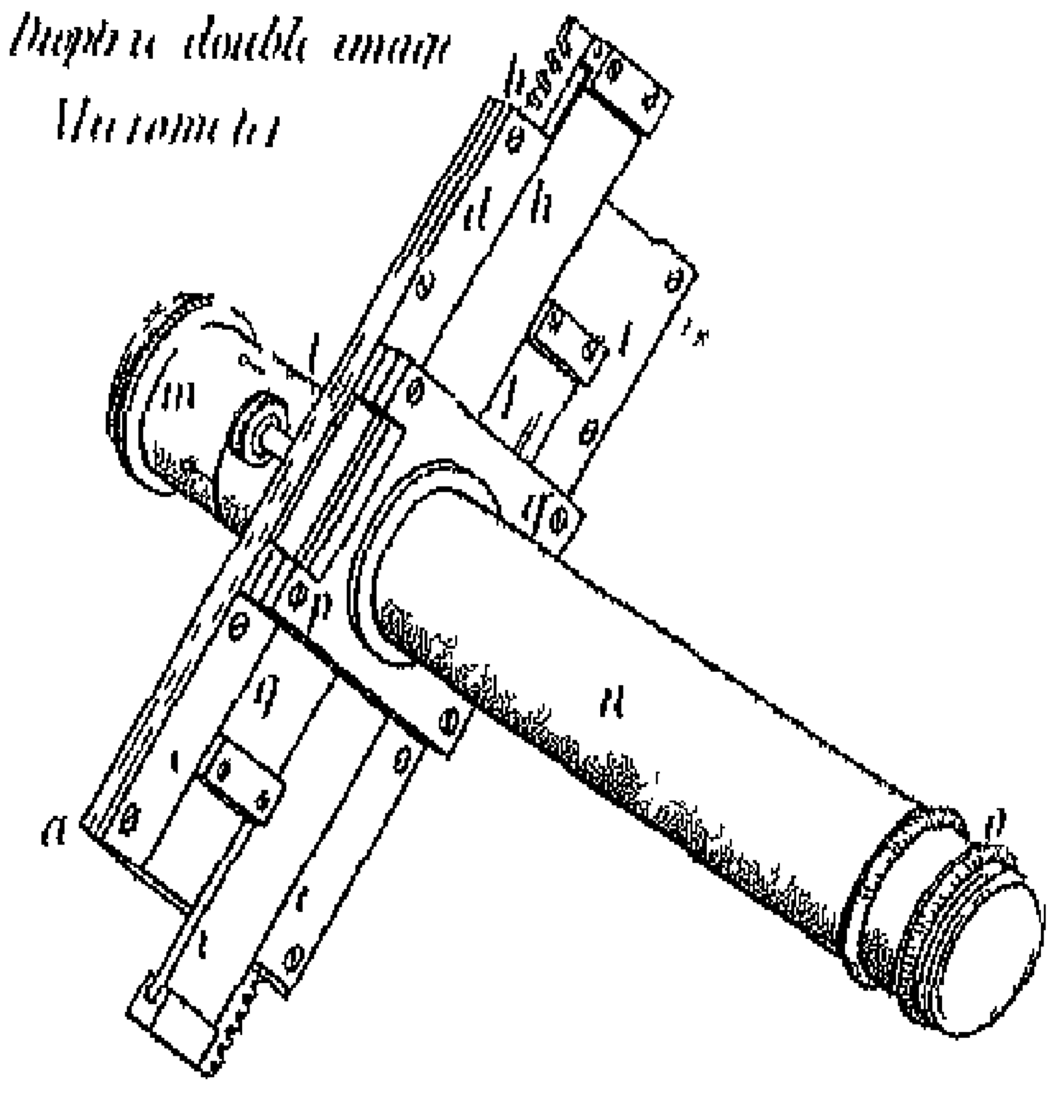


Fig 4

Direct Dioptric double image Micrometer

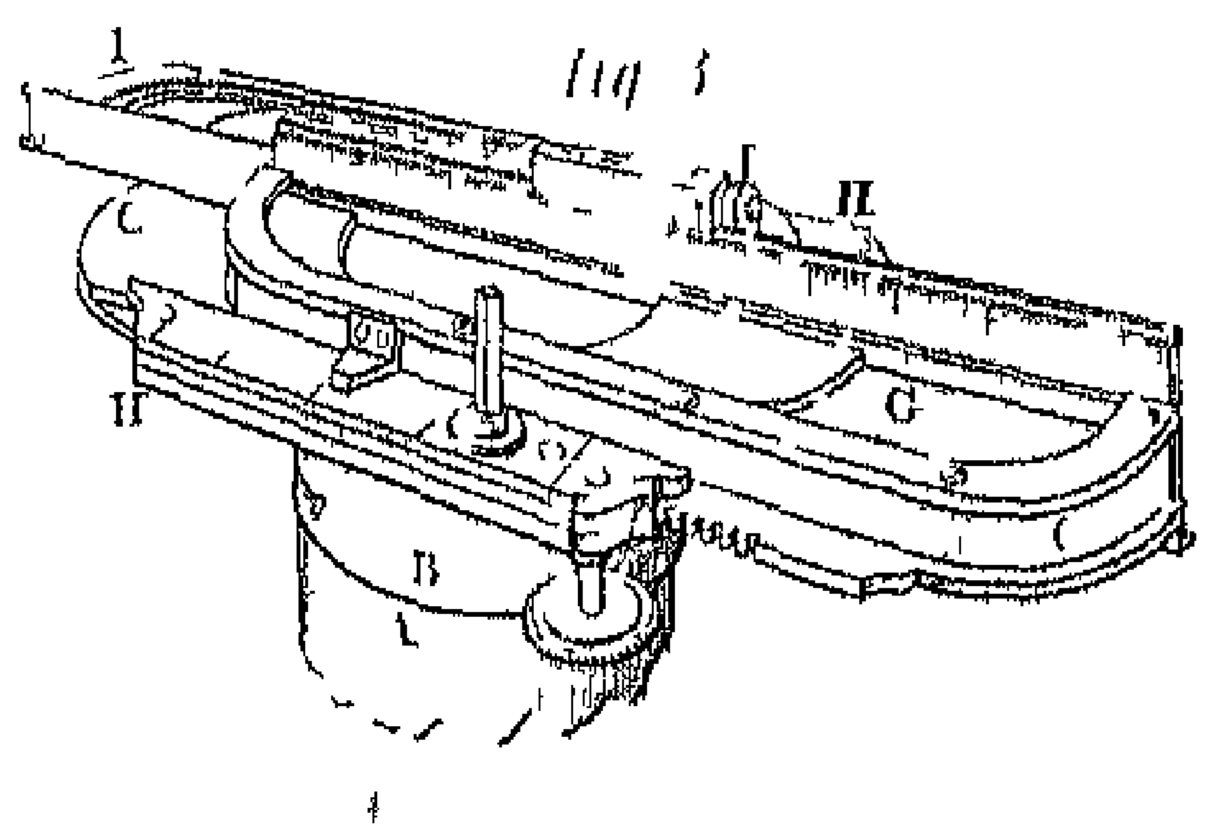

Dollond's improved object glass Micrometer

Fig 3

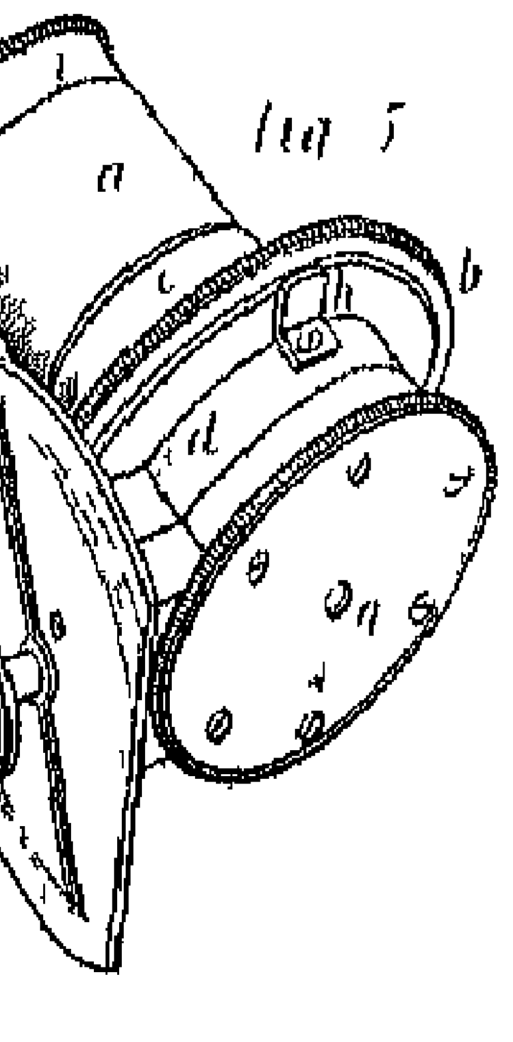
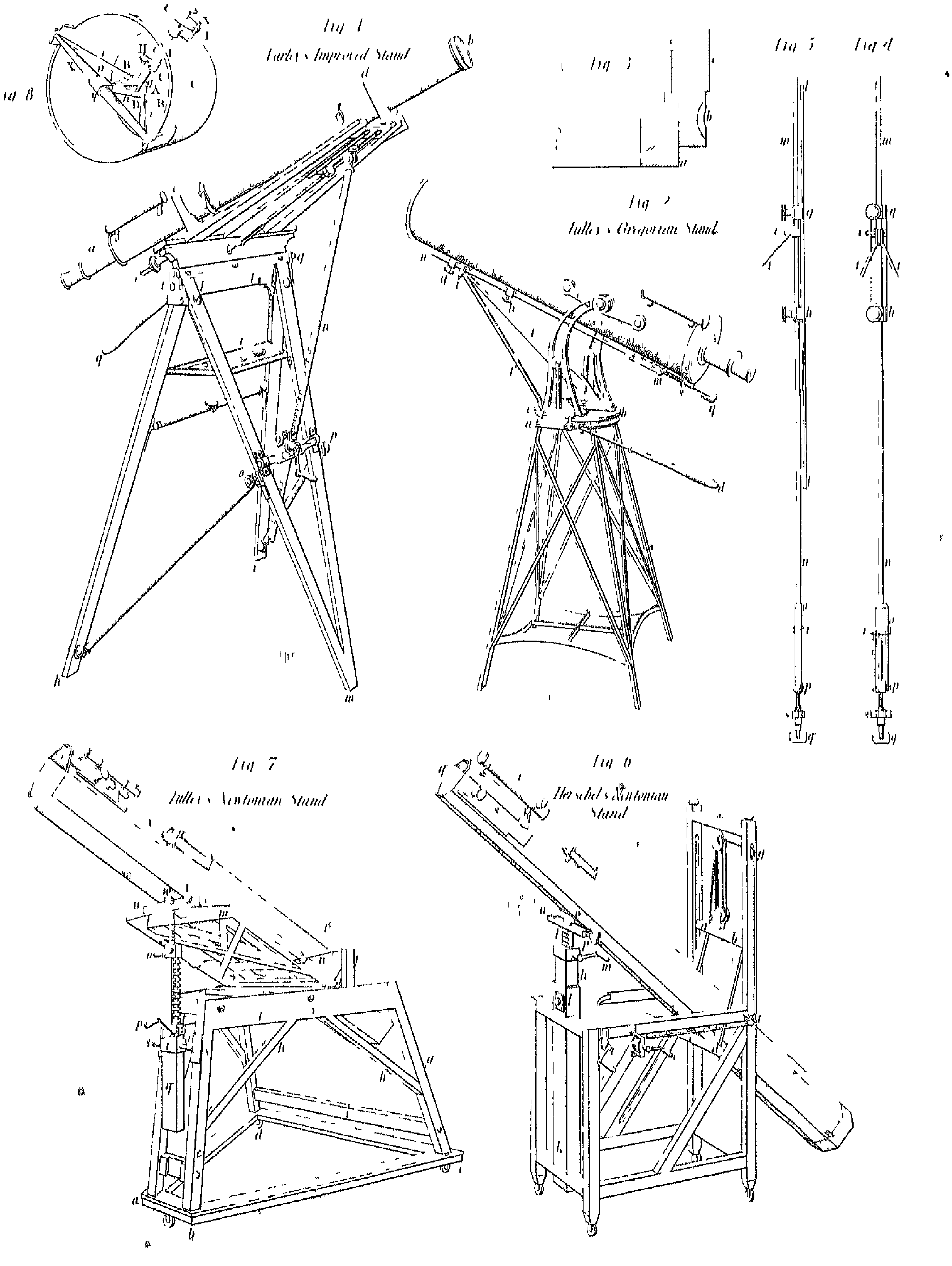


Fig 5

STANDS FOR LARGE TELESCOPES AND A CATOPTRIC MICROMETER. Plate VI

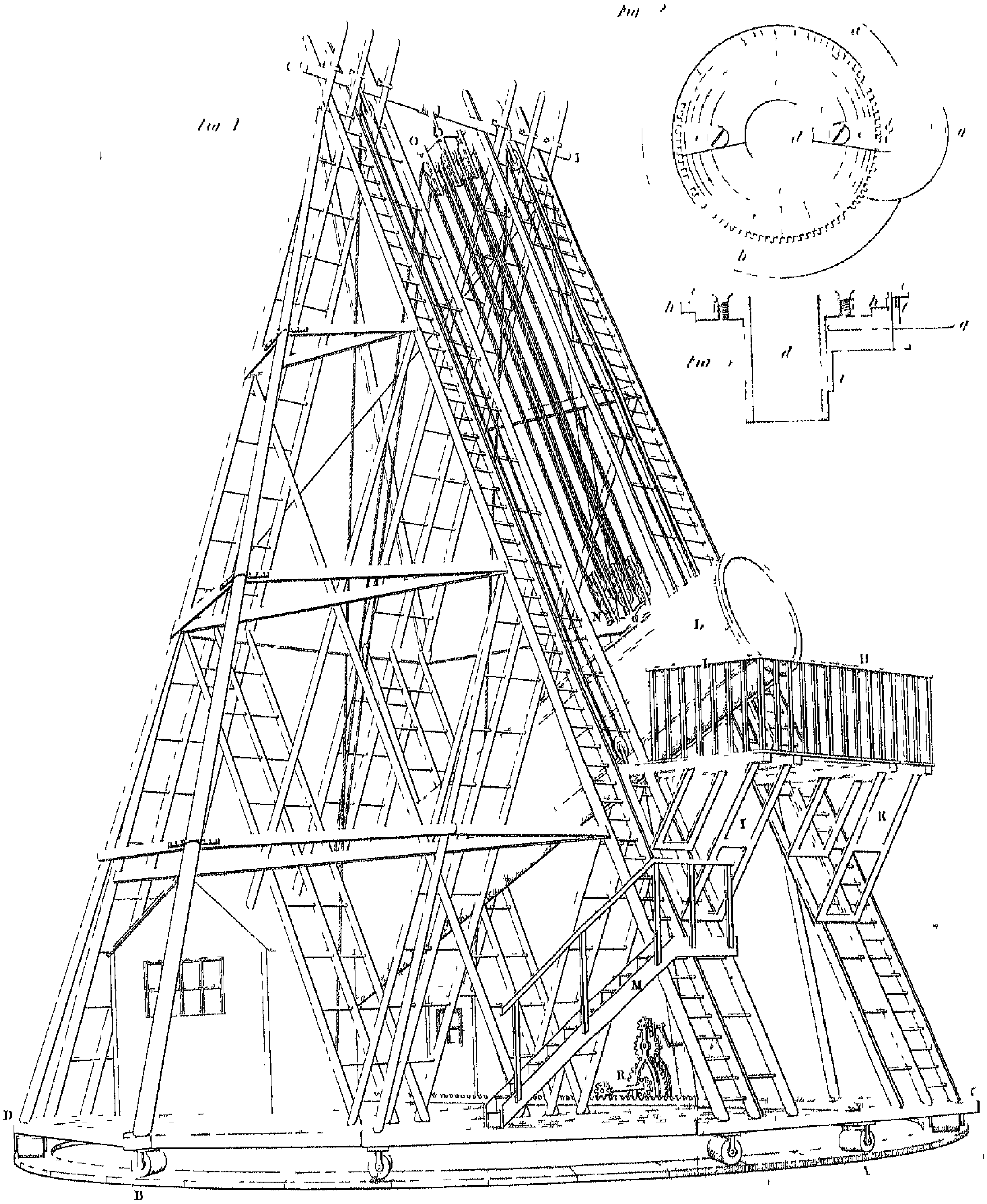


E. Turrell sc.

Plate VIII

SIR W. HERSCHEL'S 10 FEET REFLECTOR

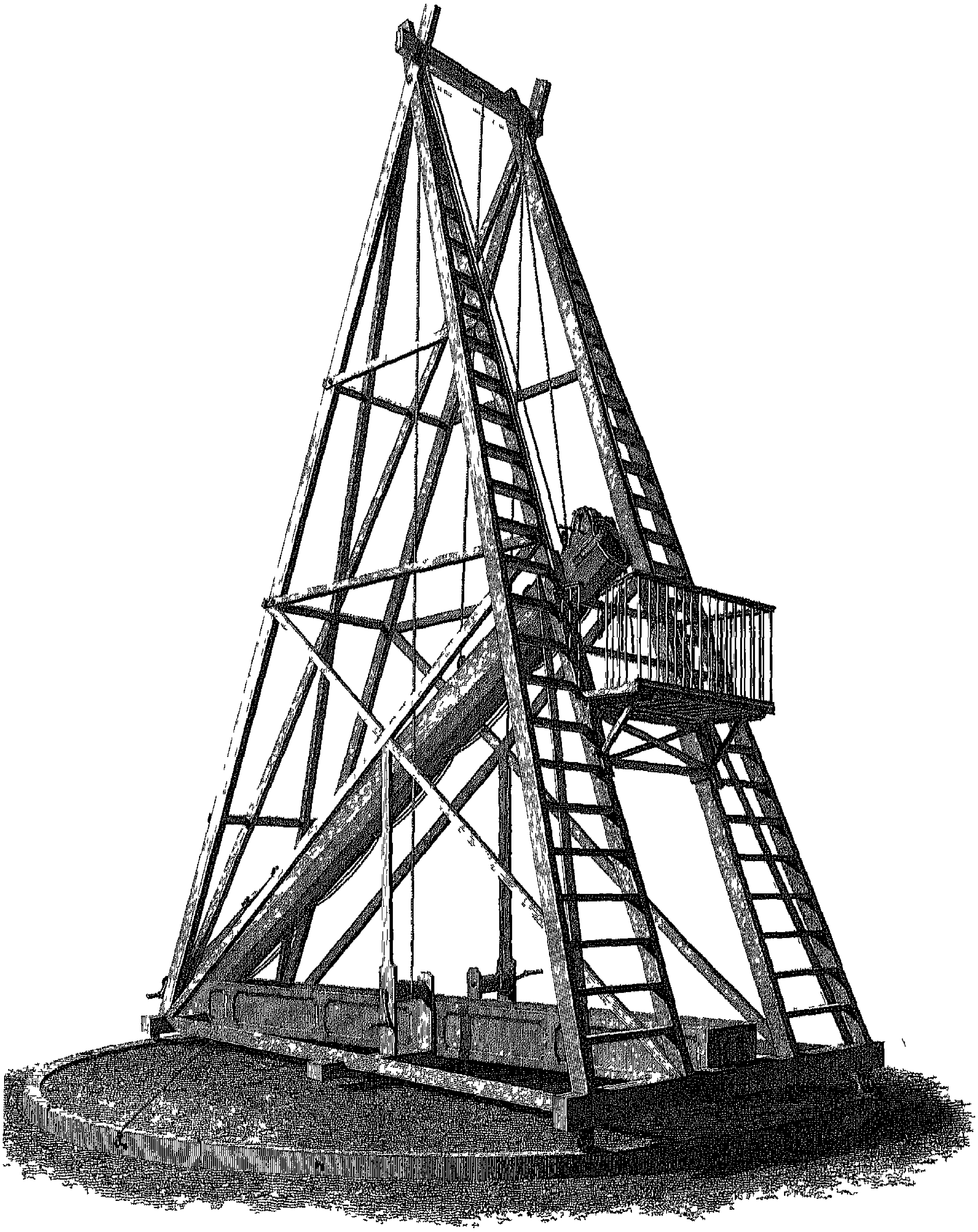
Section of Micrometer by Nairne



London Published by the Author Nov. 1st 1828

J. Farrell sc.

RAMAGE'S TELESCOPE   Plate LI



Perspective View of Mr. Ramage's Reflecting Telescope Erected at the Royal Observatory Greenwich

by J. Bentley

London Published for the Author Nov. 1st 1828

Engraved by J. Farrell

DETAILS OF RAMAGE'S TELESCOPE.

Fig 1

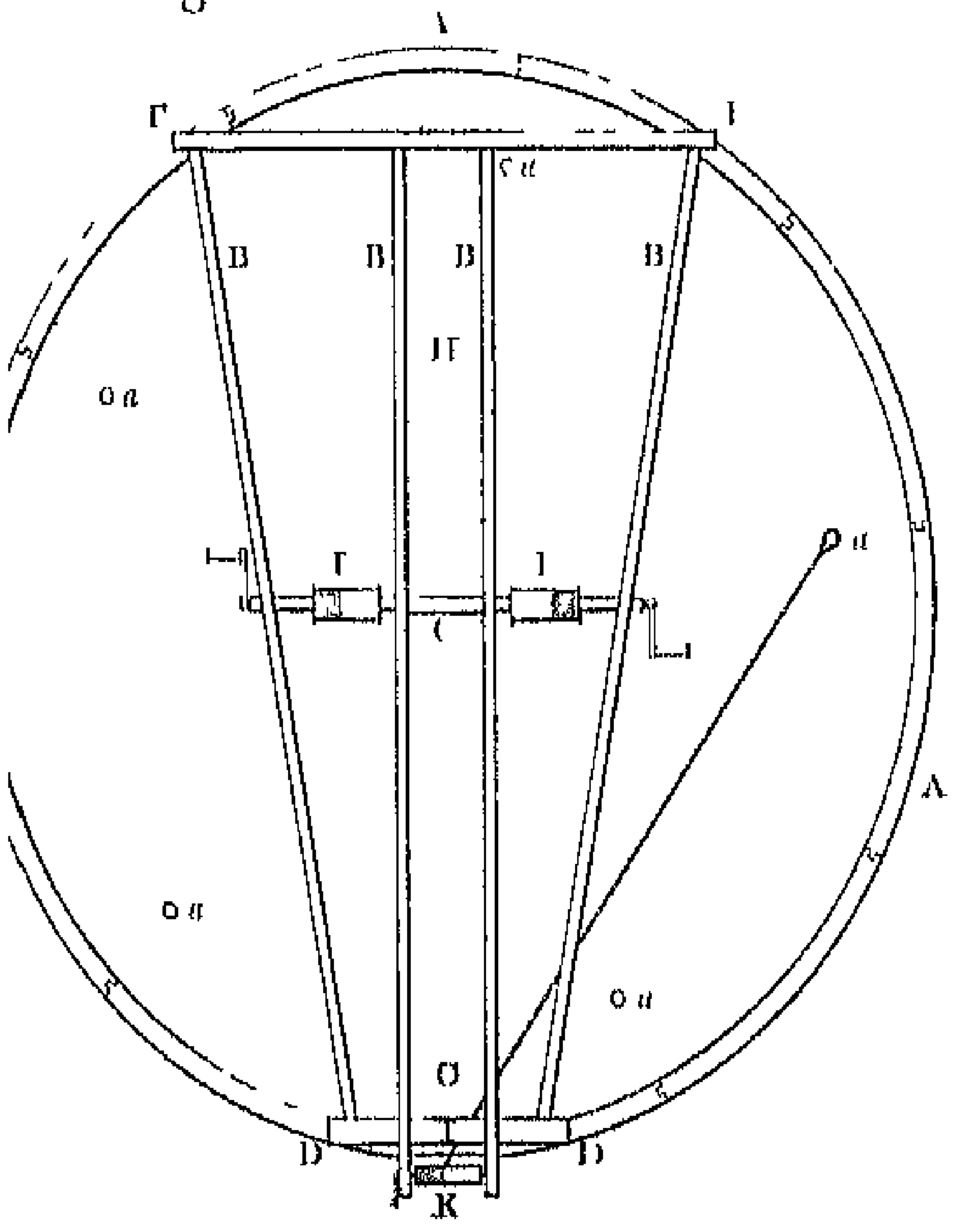


Fig 2

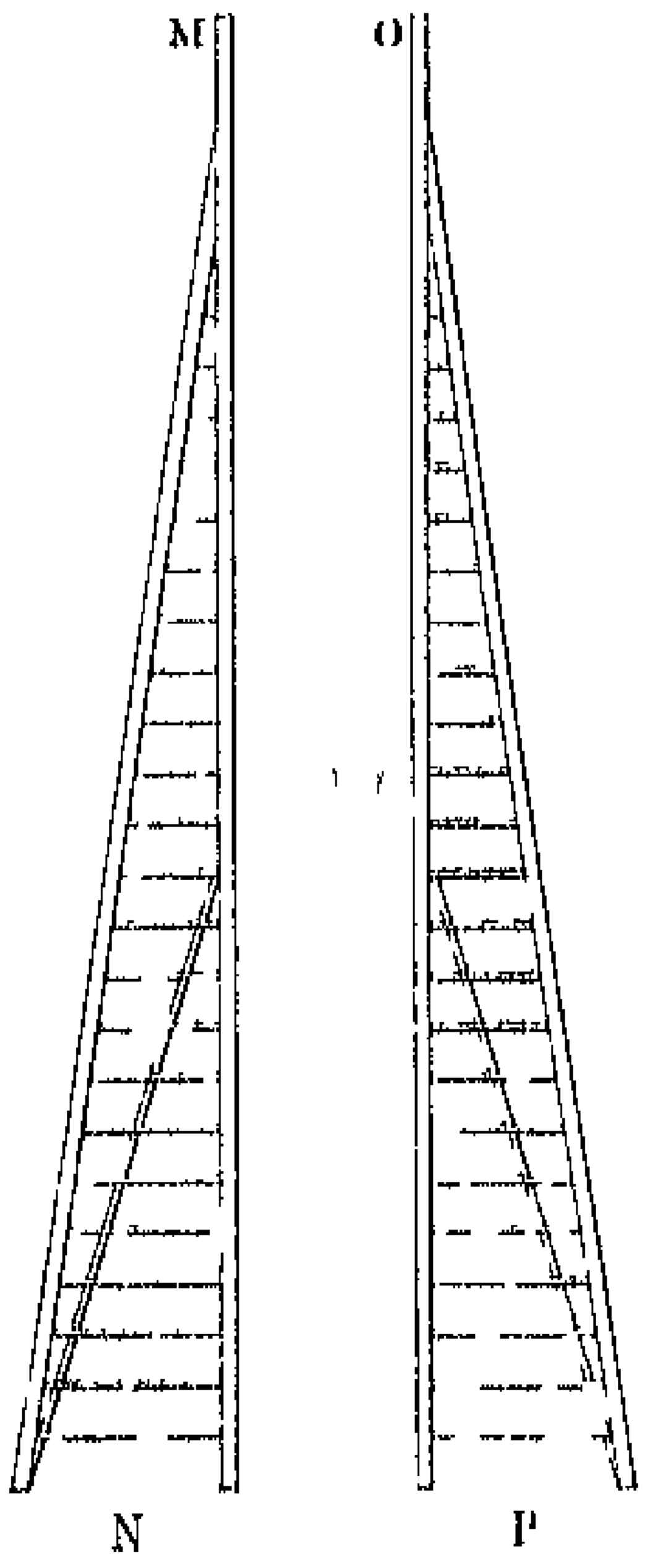


Fig 3

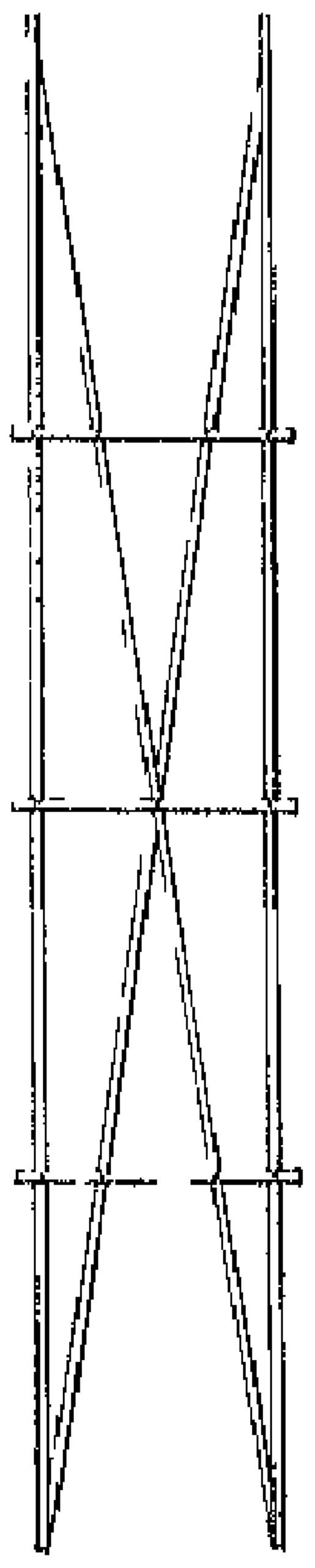


Fig 4

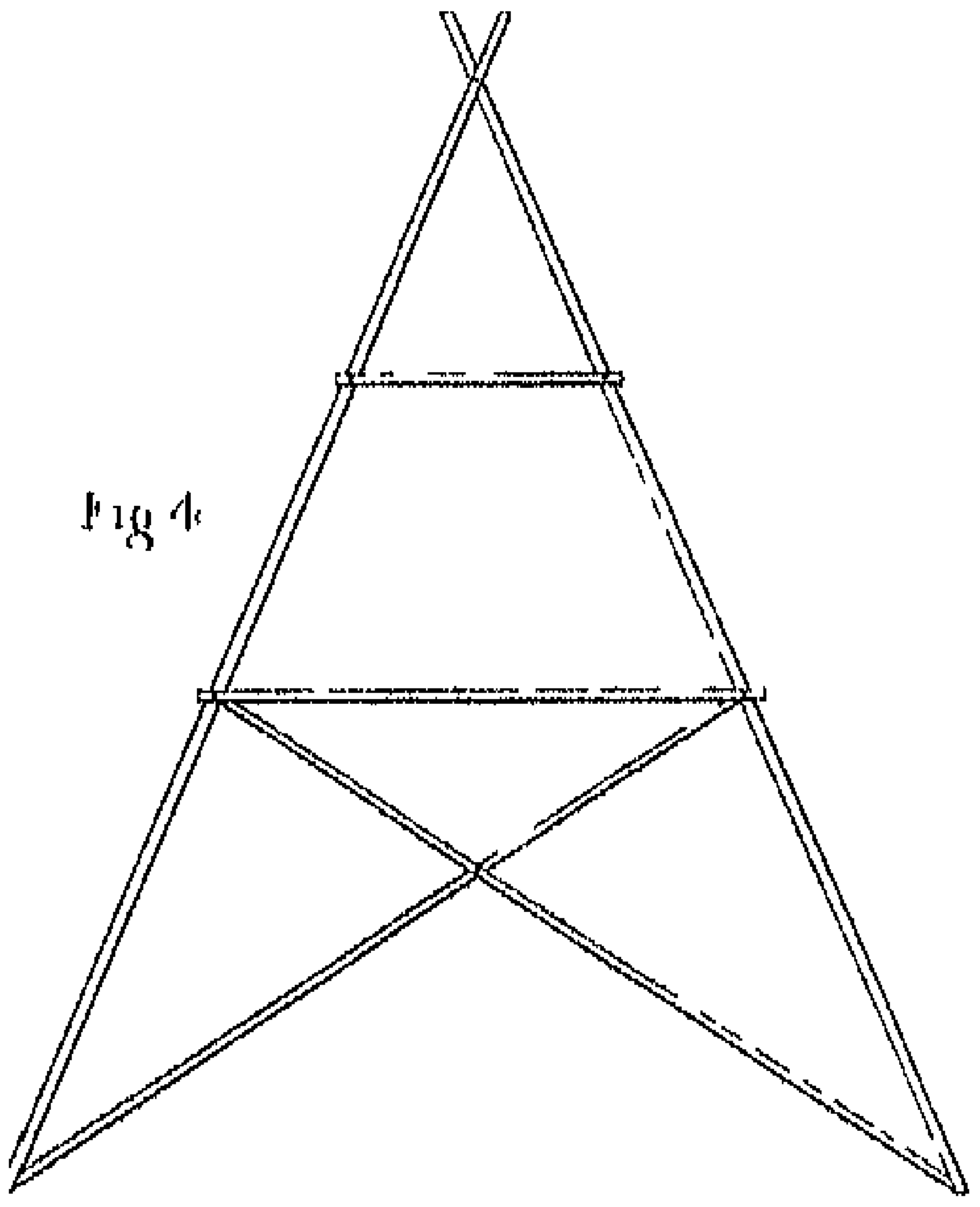


Fig 5

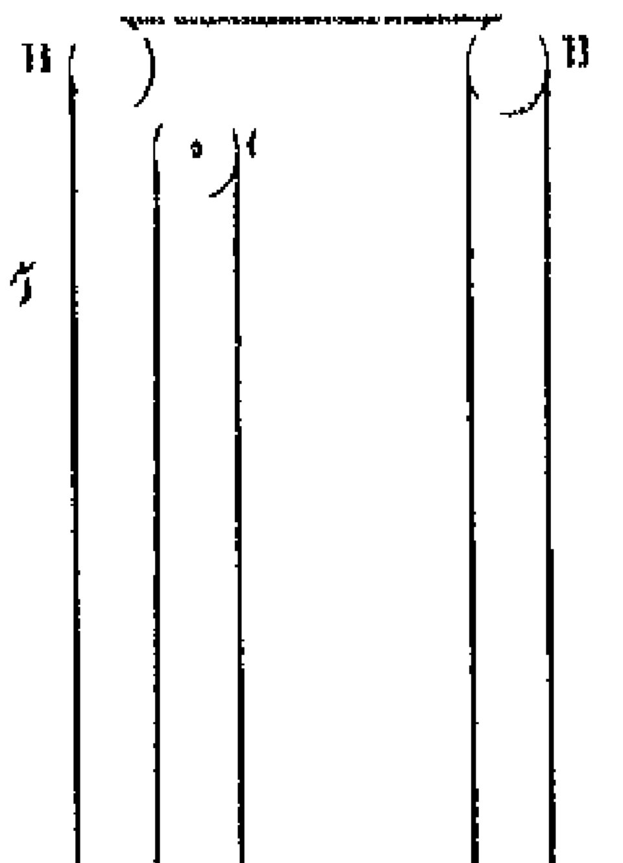


Fig 6

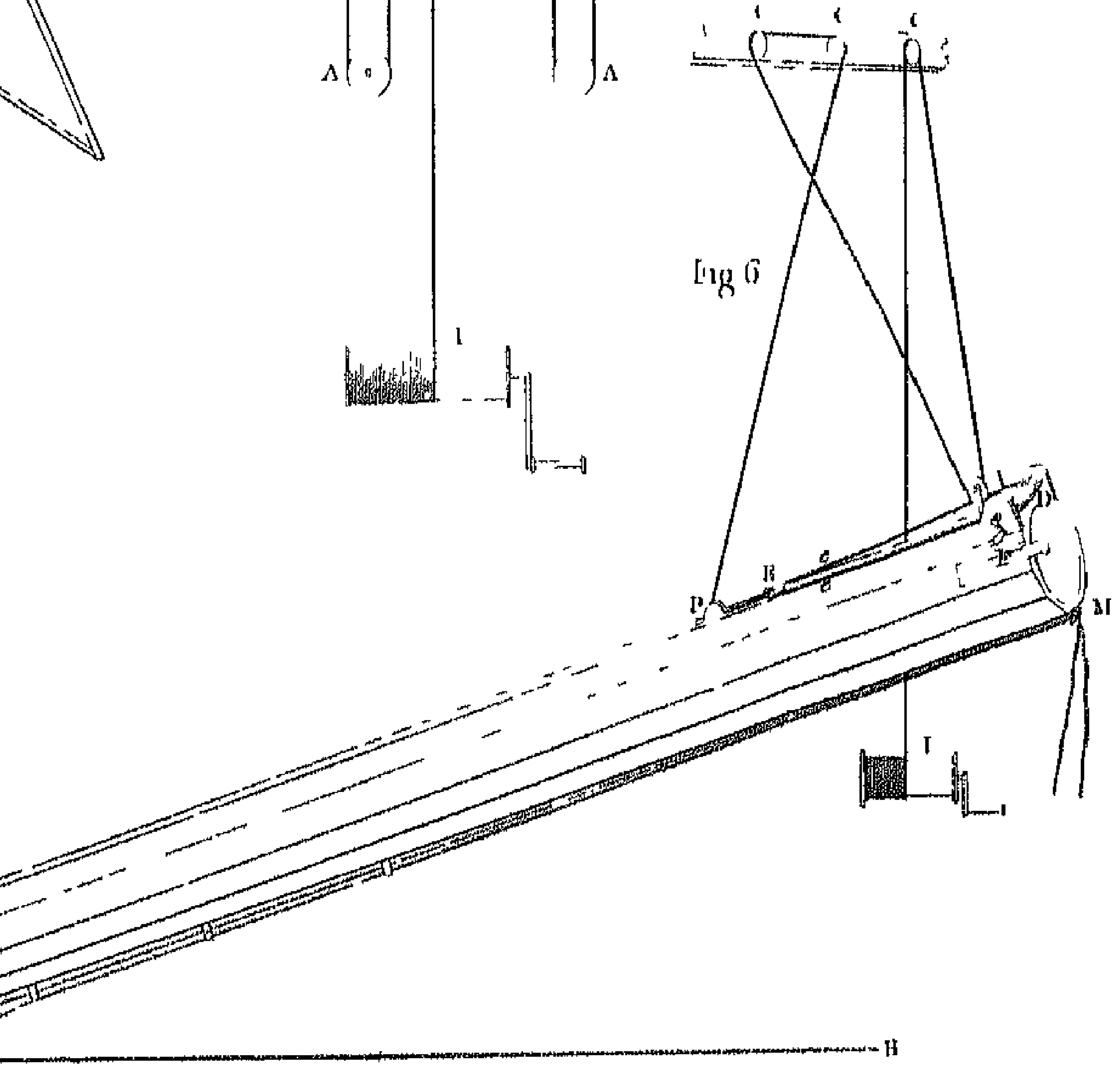
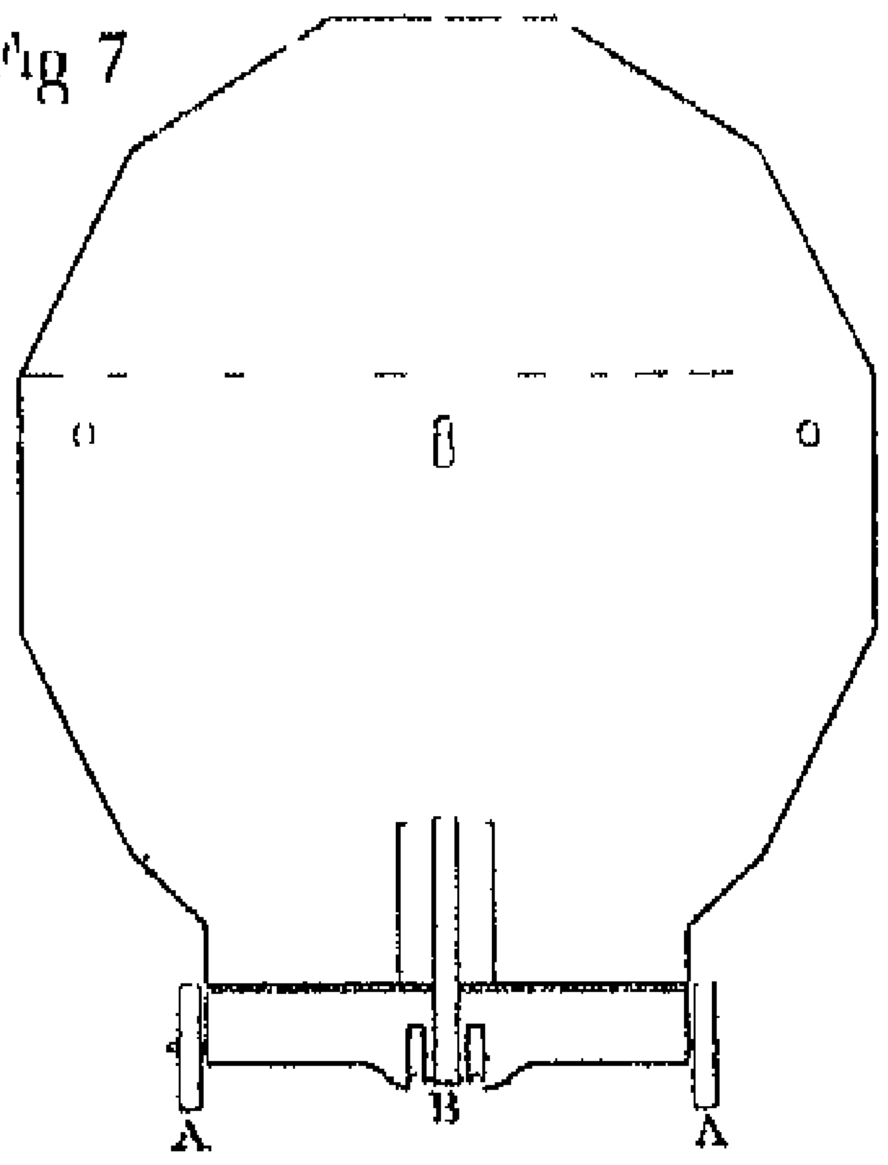
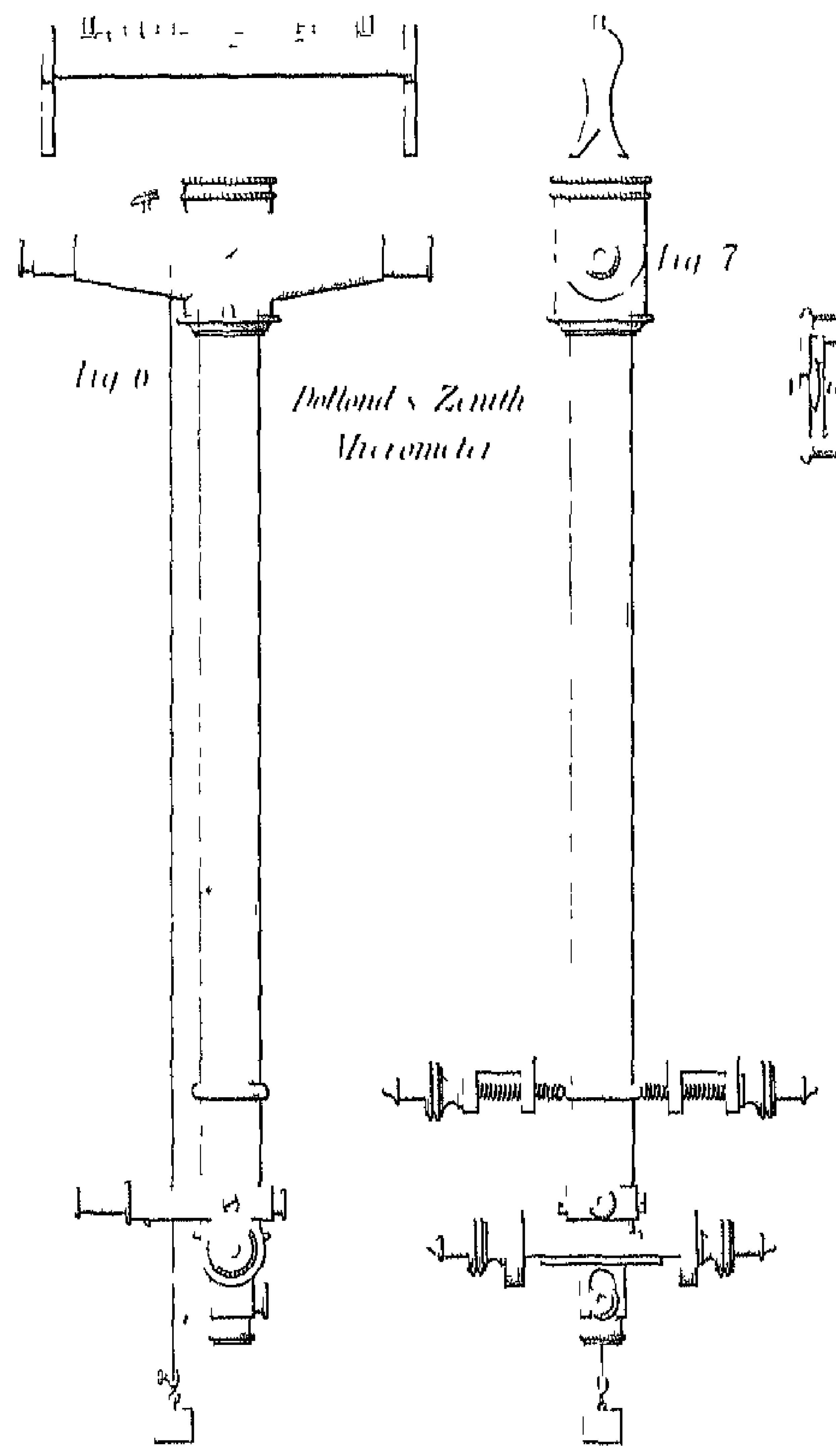
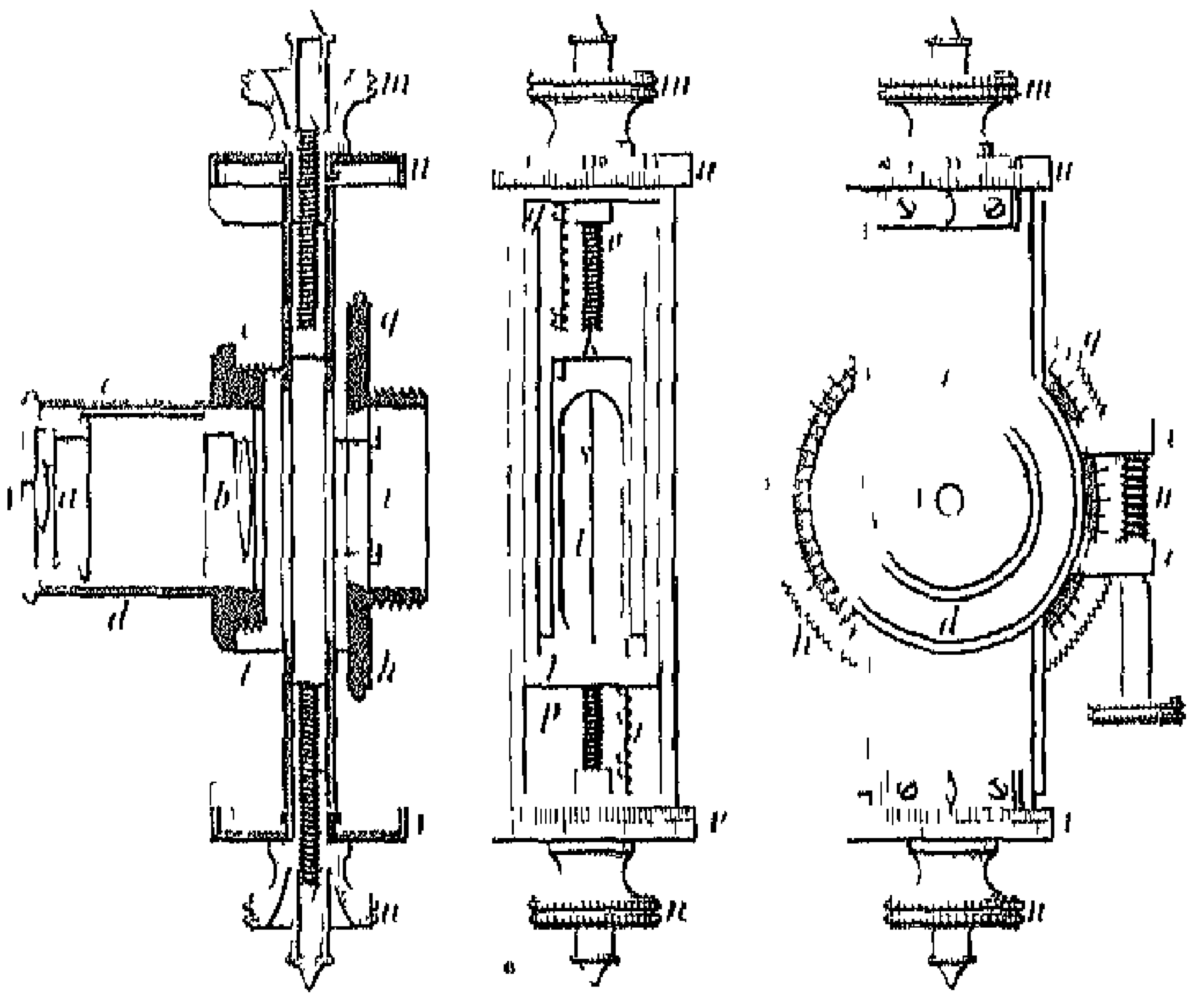


Fig 7

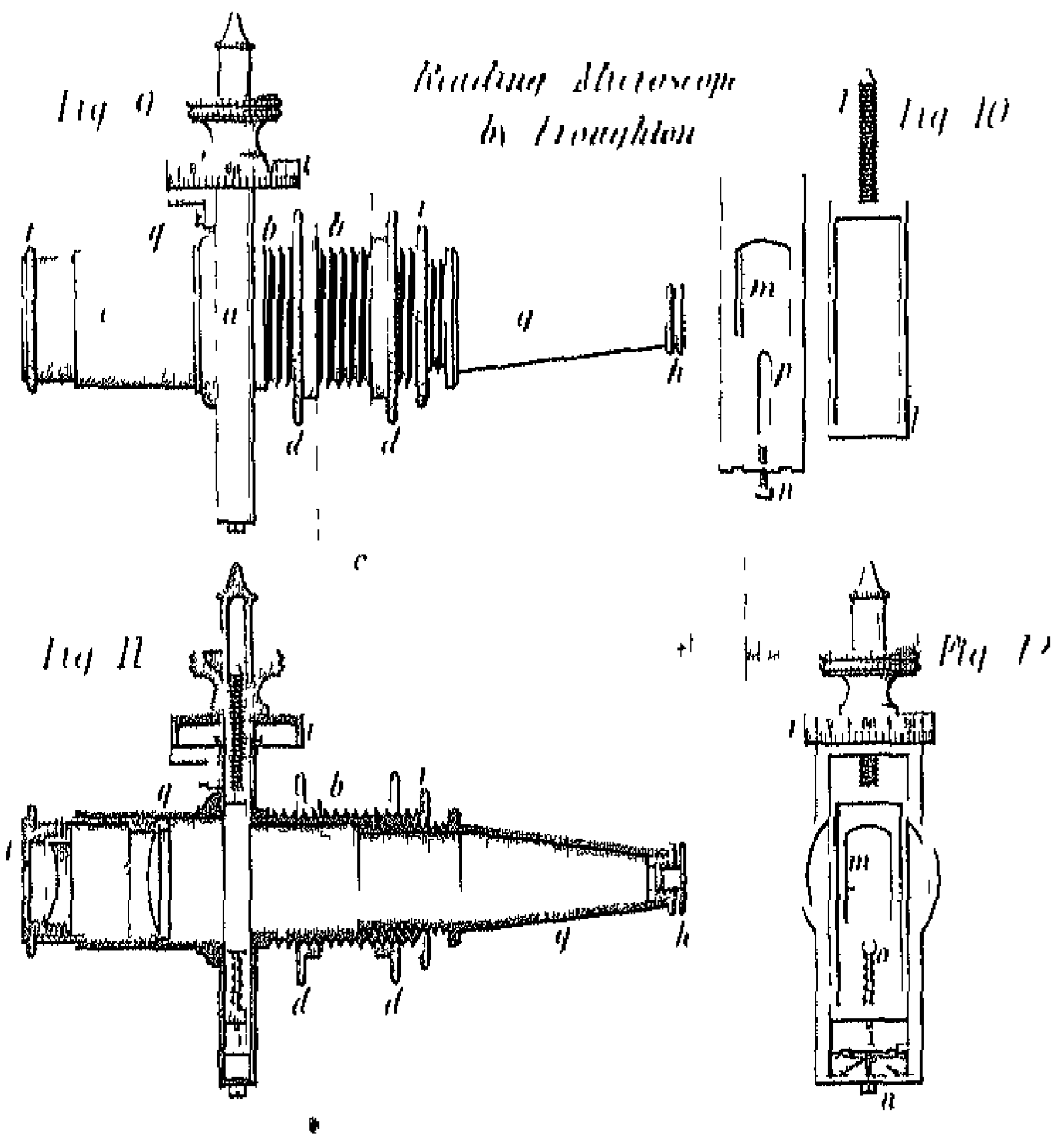


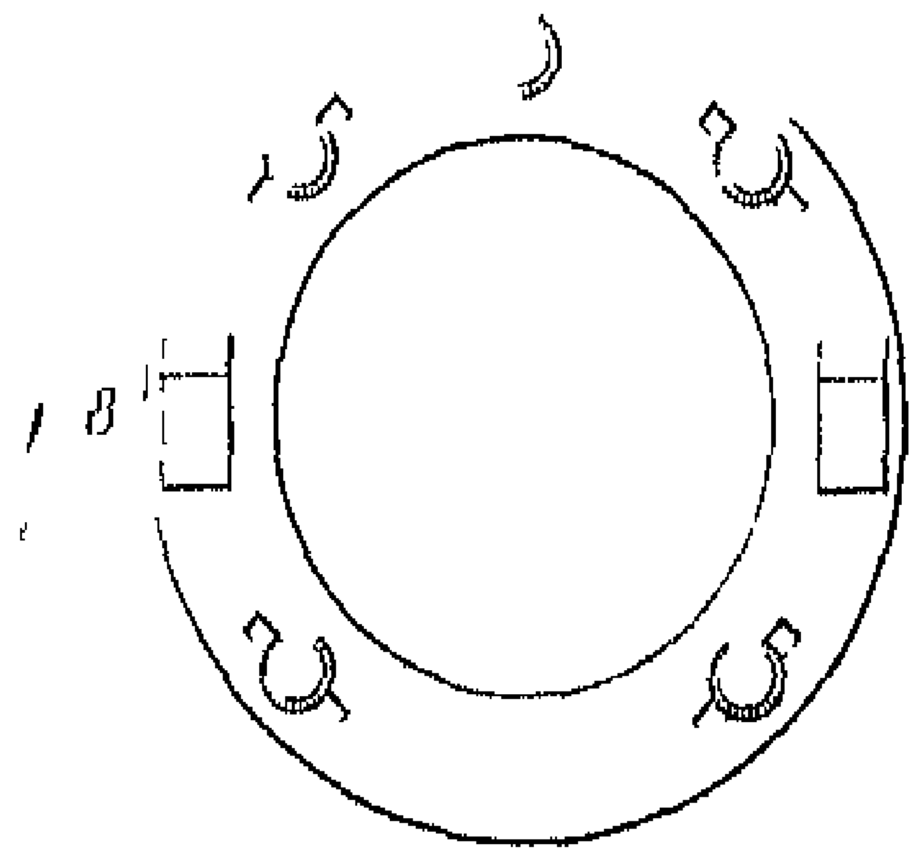


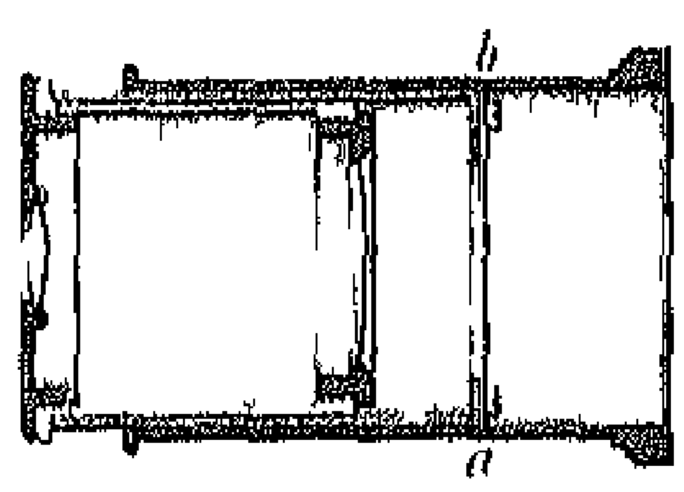
Troughton's Line Micrometer



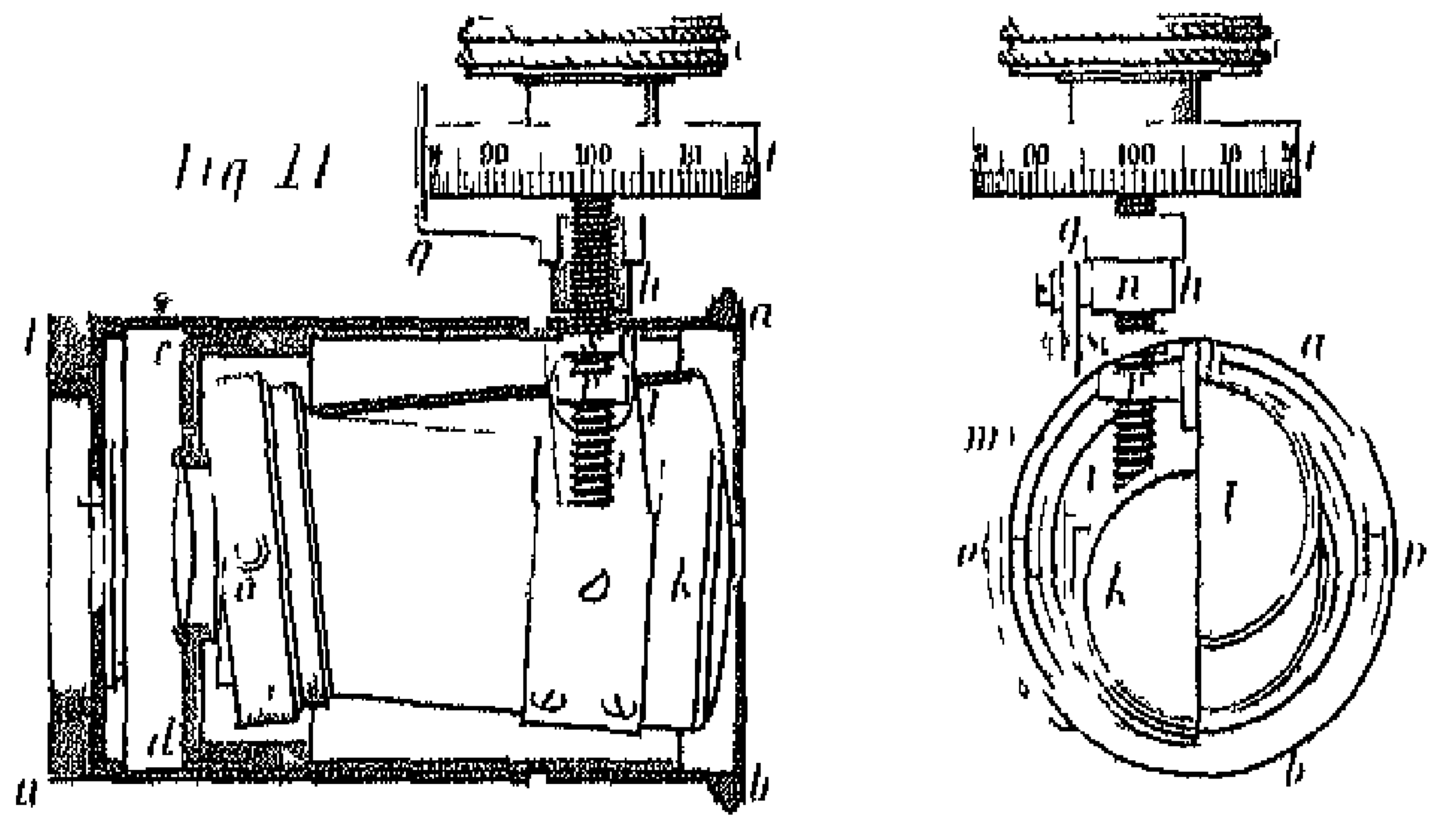
Reading Microscope by Troughton



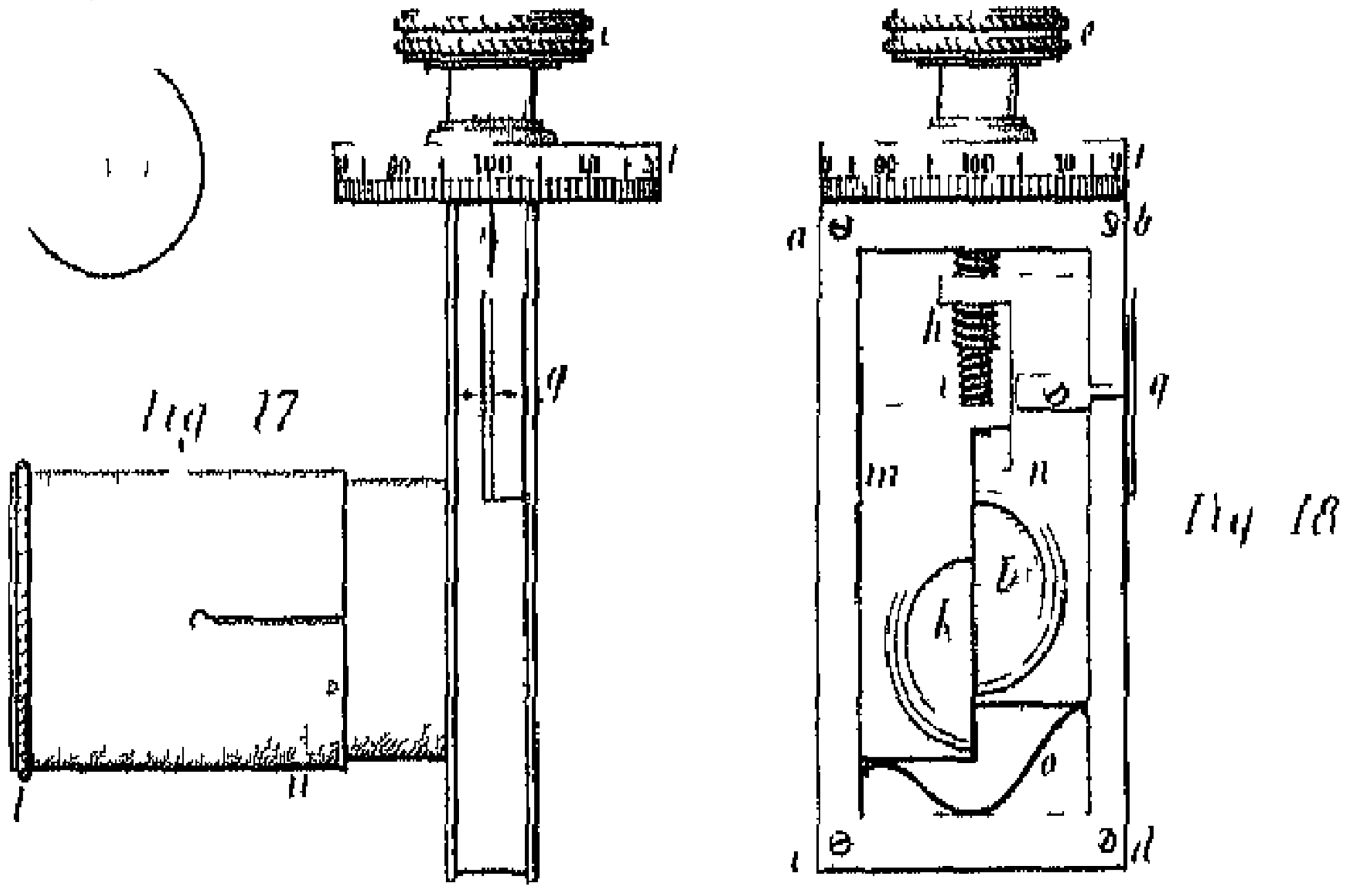




Dynameter by Jones



Dollond's Dynameter



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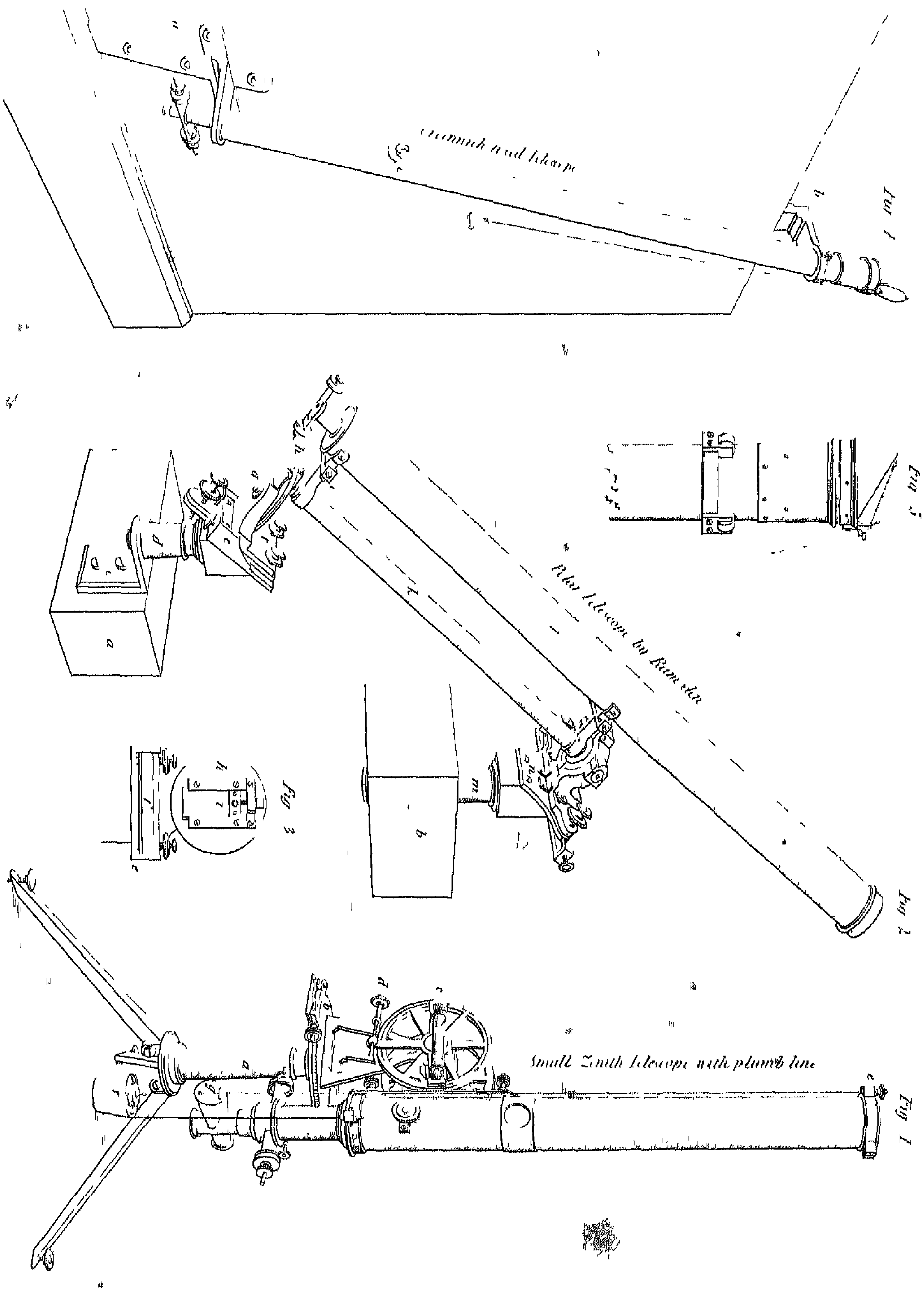
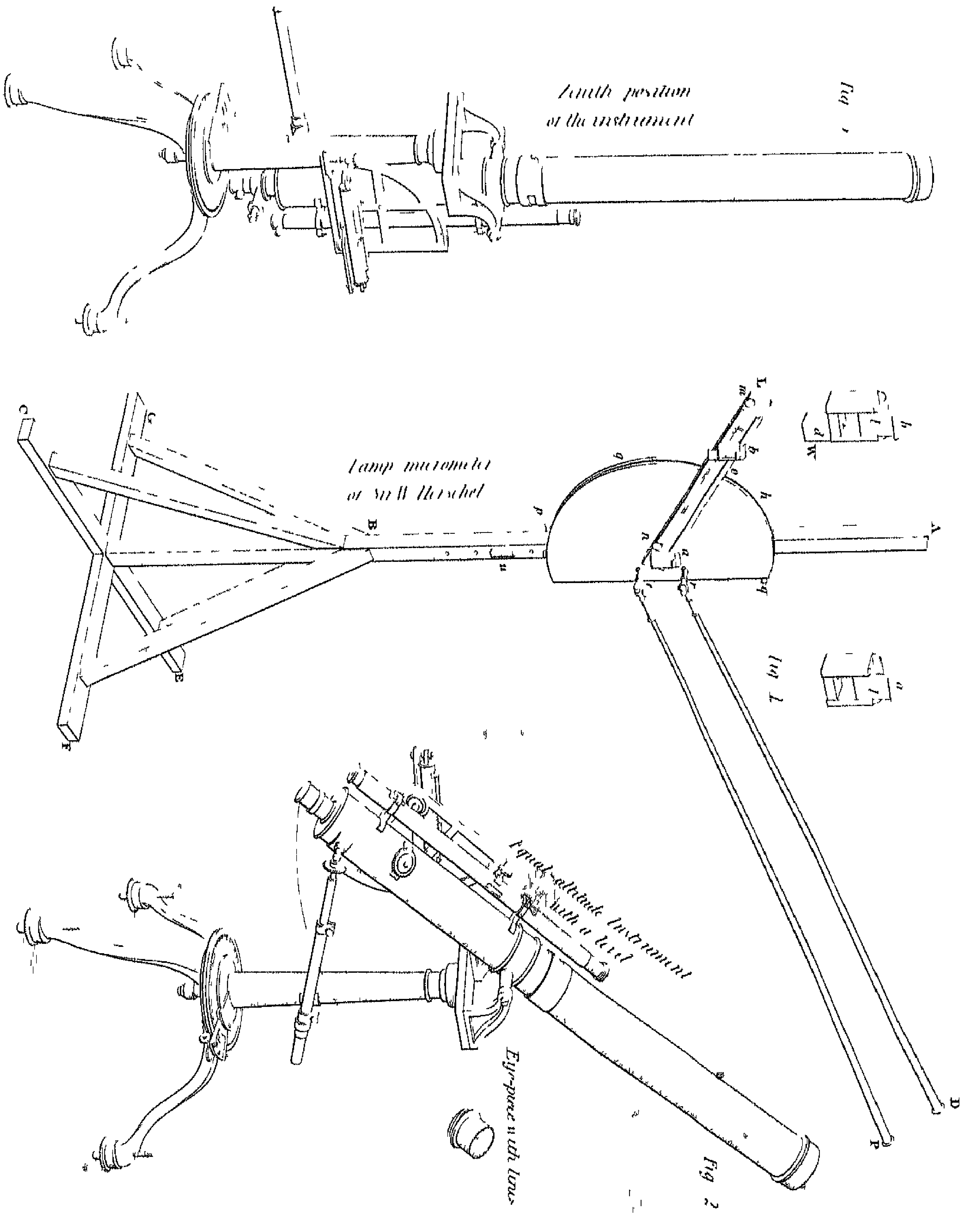
DIFFERENT APPLICATIONS OF A REFRACTING TELESCOPE.

Plate


Azimuth and Telescope
Fig 4
Polar Telescope by Ramsden
Fig 2
Fig 5
Fig 3
Small Zenith Telescope with plumb line
Fig 1
a
b
h
m
d
c

AN EQUAL-ALTITUDE AND ZENITH TELESCOPE, AND LAMP-MICROMETER Plate


Zenith position of the instrument
Fig 1
Lamp micrometer of Sir W Herschel
C
G
B
E
F
A
D
P
L
m
a
b
c
d
e
g
h
n
p
q
r
u
W
Fig 3
Equal-altitude Instrument with a level
Eye-piece with lens
Fig 2

Farey delt

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Plate XIV

PORTABLE TRANSIT INSTRUMENT, &c.

by T. Jones

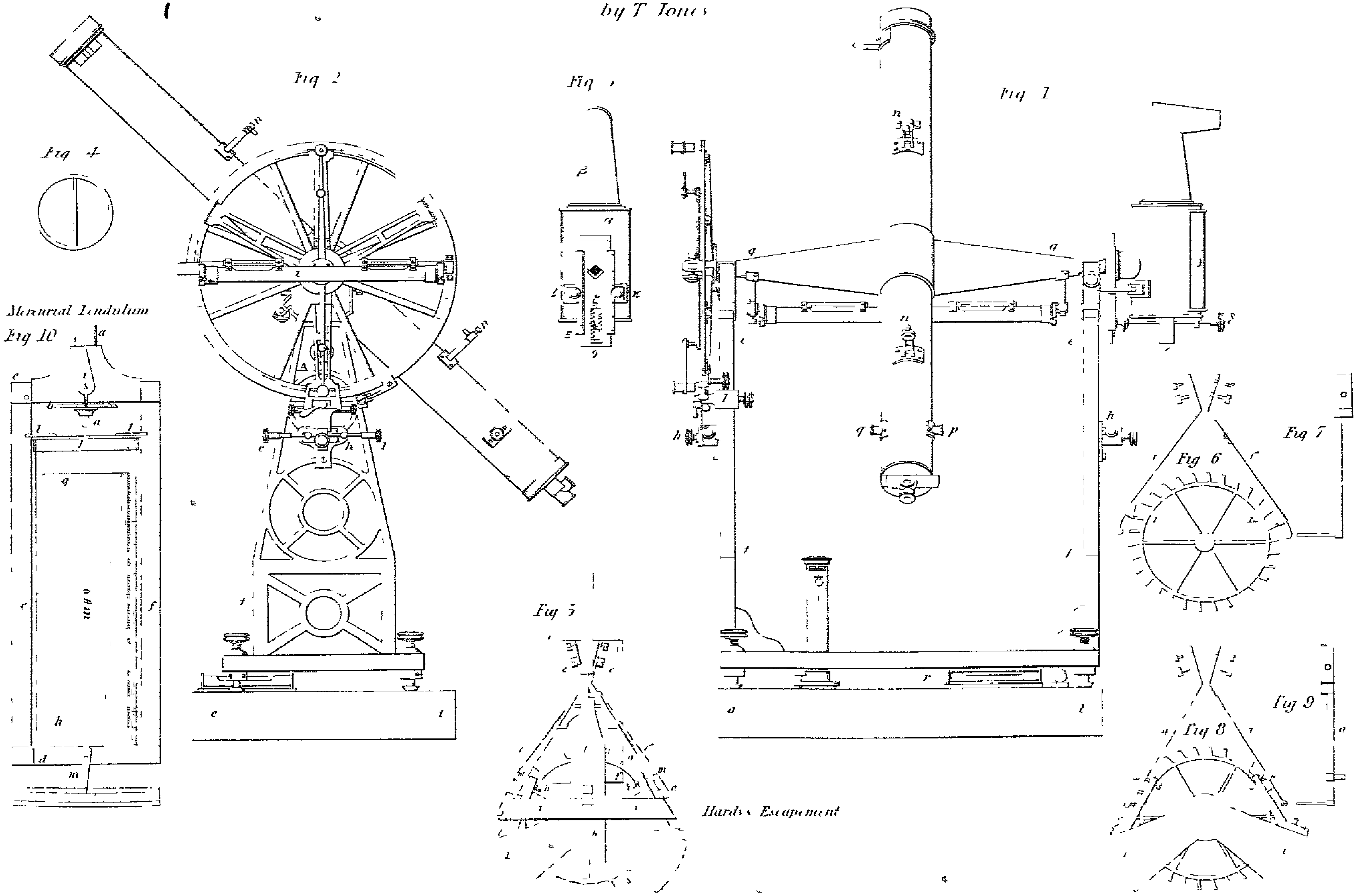


Fig 1

Fig 2

Fig 3

Fig 4

Fig 5

Fig 6

Fig 7

Fig 8

Fig 9

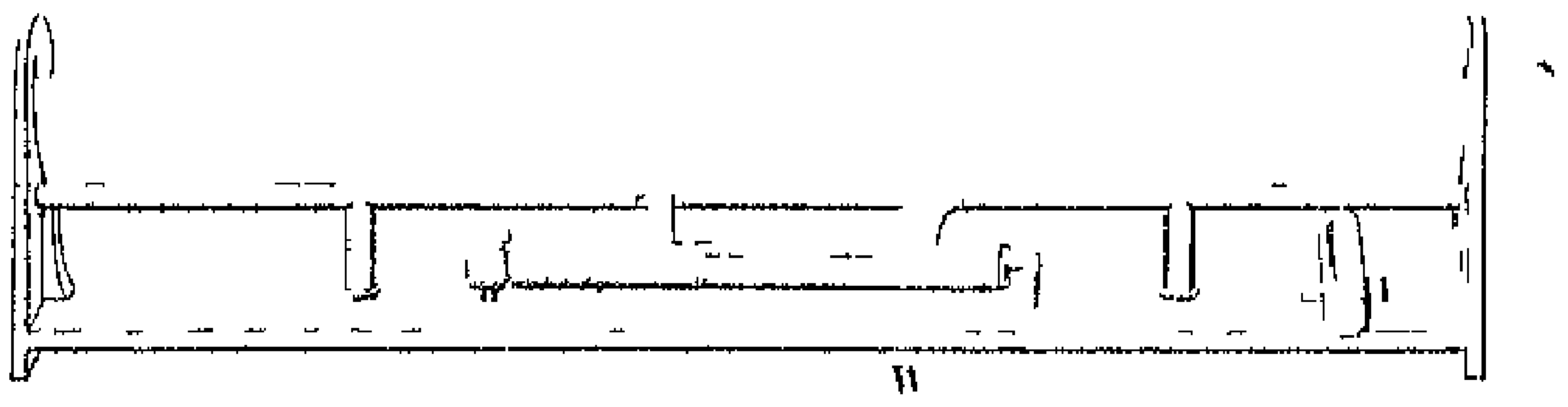
Mercurial Pendulum

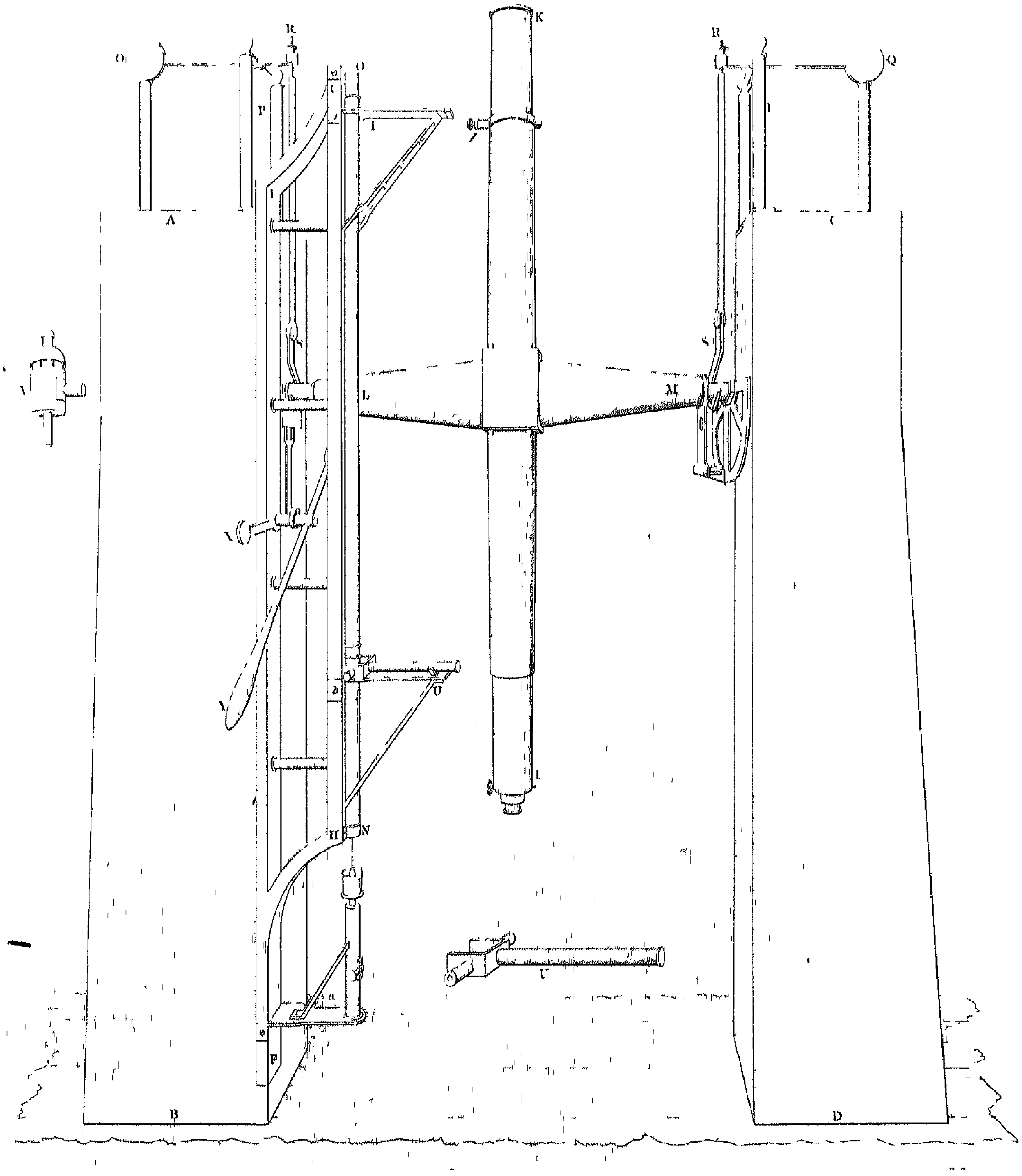
Fig 10

Hardy's Escapement

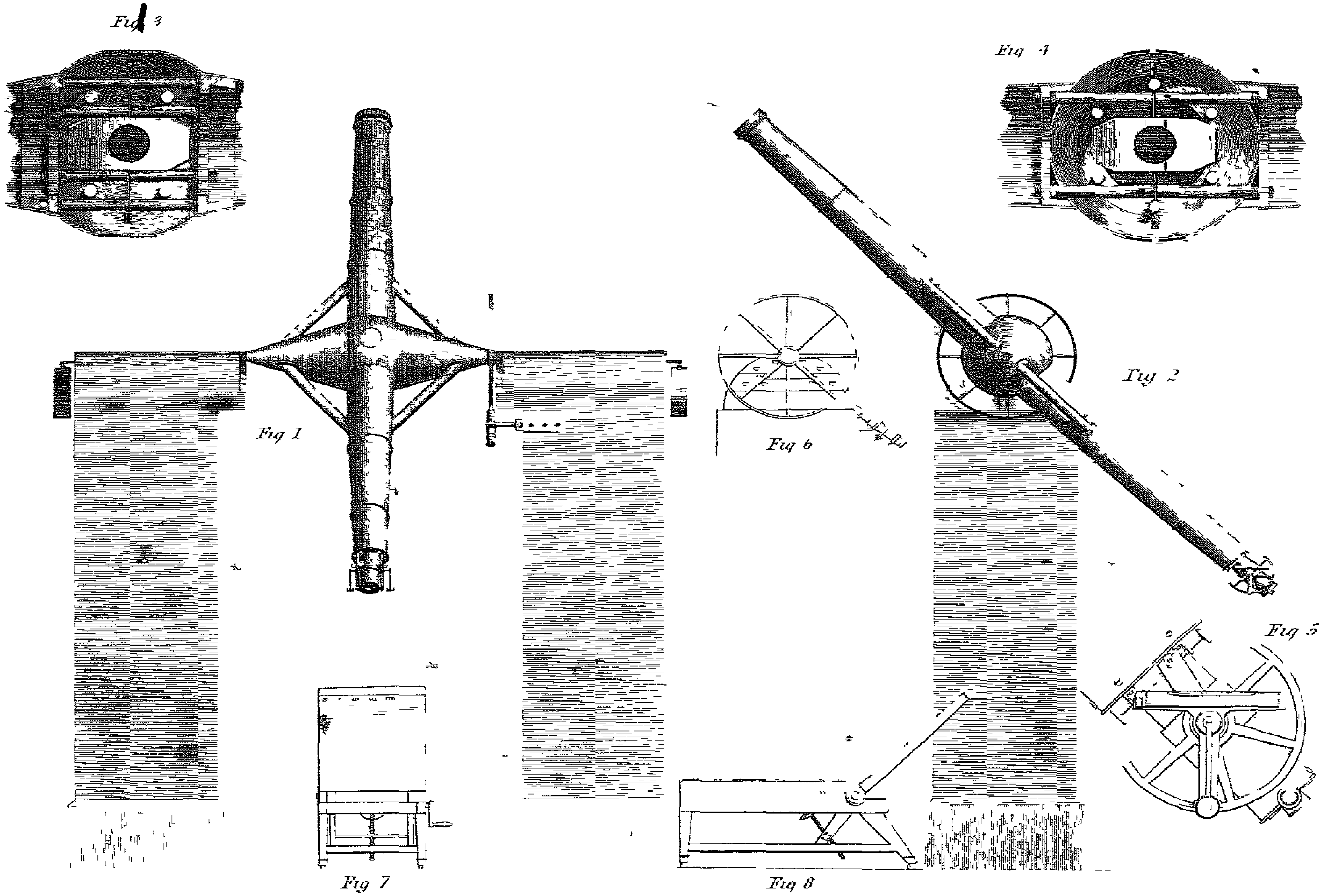
TRANSIT-INSTRUMENT BY CARY
at Moscow

Plate V





GREENWICH TRANSIT INSTRUMENT BY TROUGHTON.



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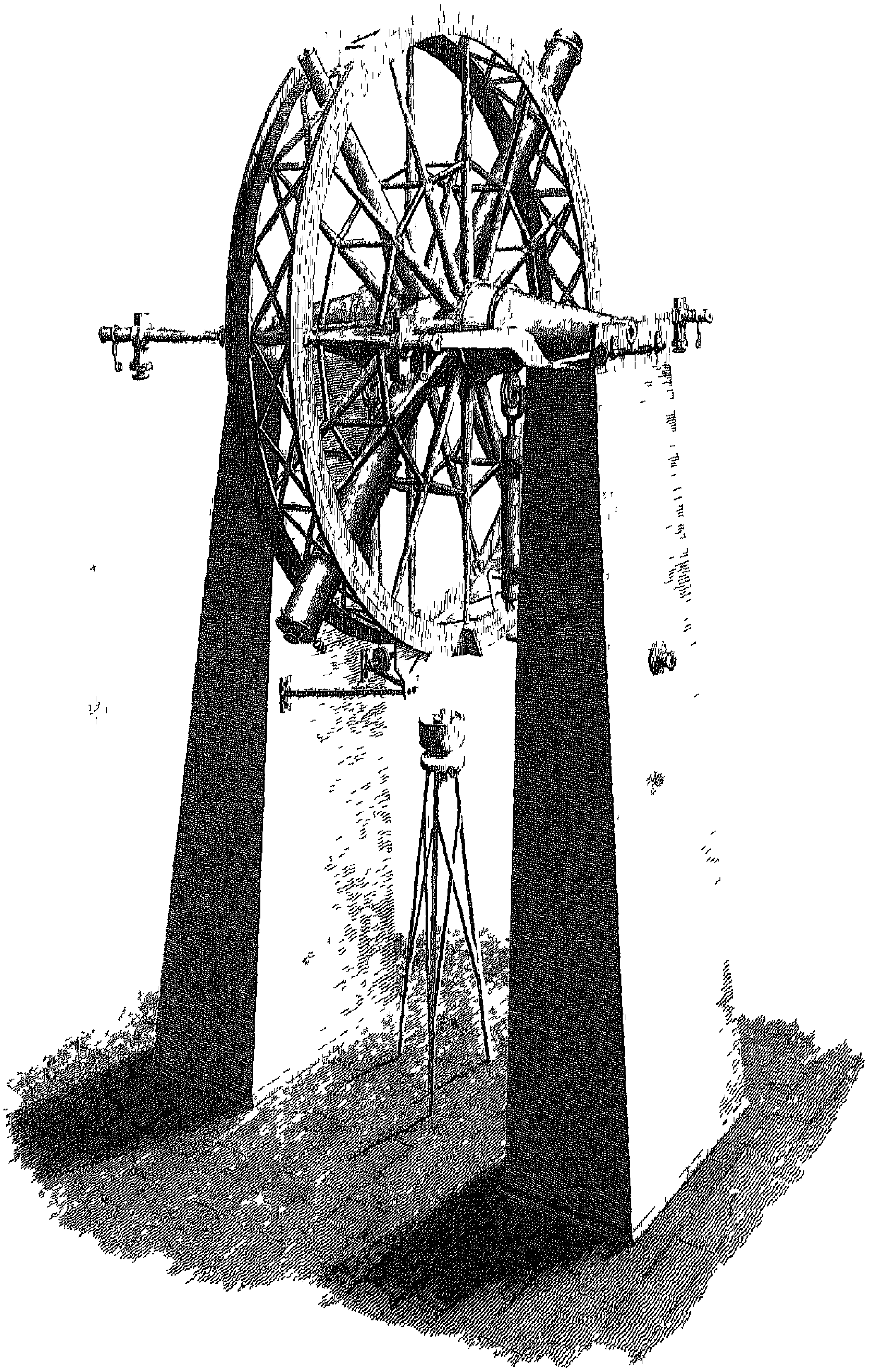
TRANSIT CIRCLE BY TROUGHTON.



J Farey del^t

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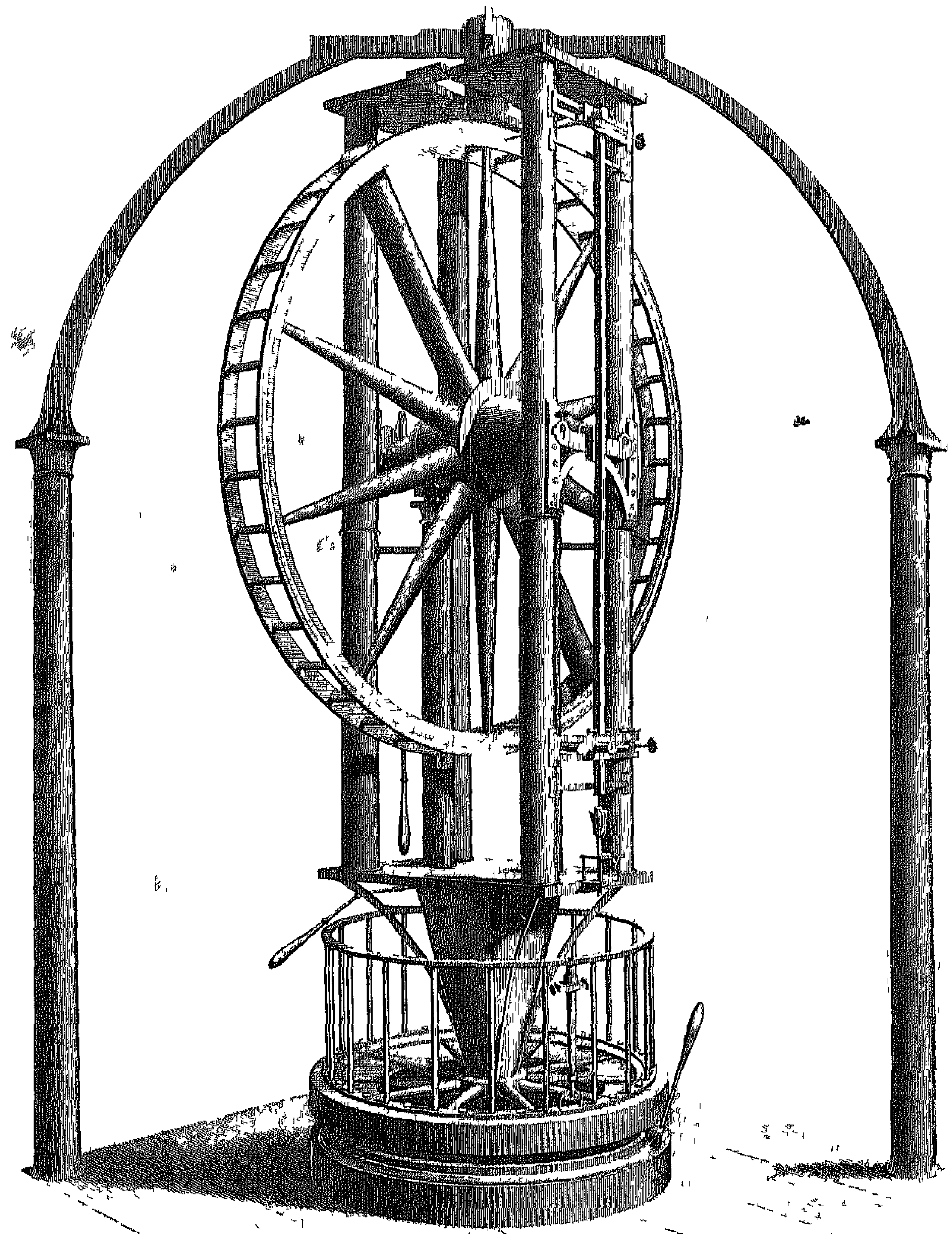
PIAZZI'S ALTITUDE AND AZIMUTH CIRCLE BY RAMSDEN.



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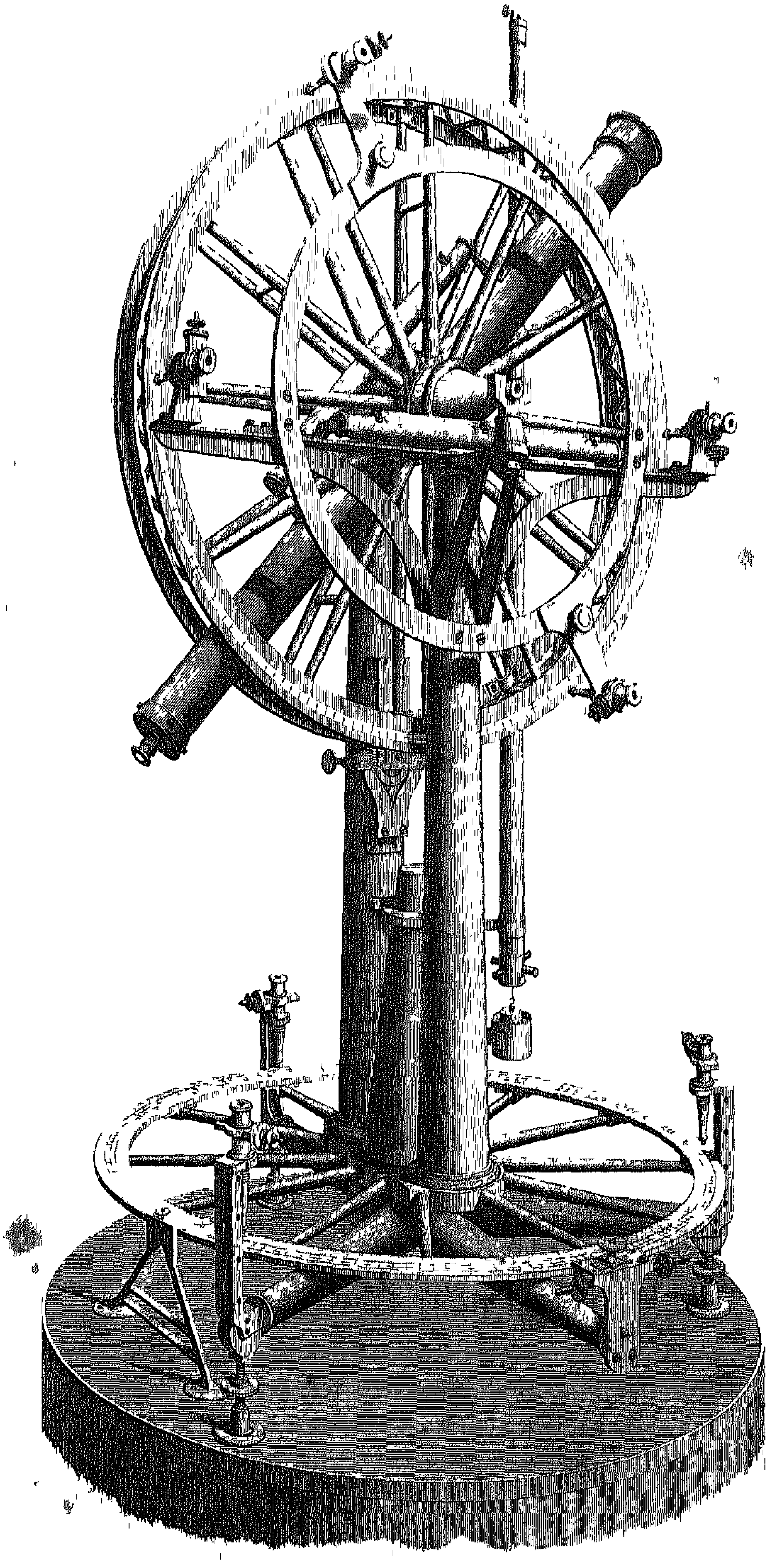
ALTITUDE AND AZIMUTH CIRCLE BY TROUGHTON *Plate XX*



Farey del^t *London. Published for the Author Nov. 1st 1828* [illegible]

GREENWICH MURAL CIRCLE BY TROUGHTON.

Side Elevation of the Circle and Wall

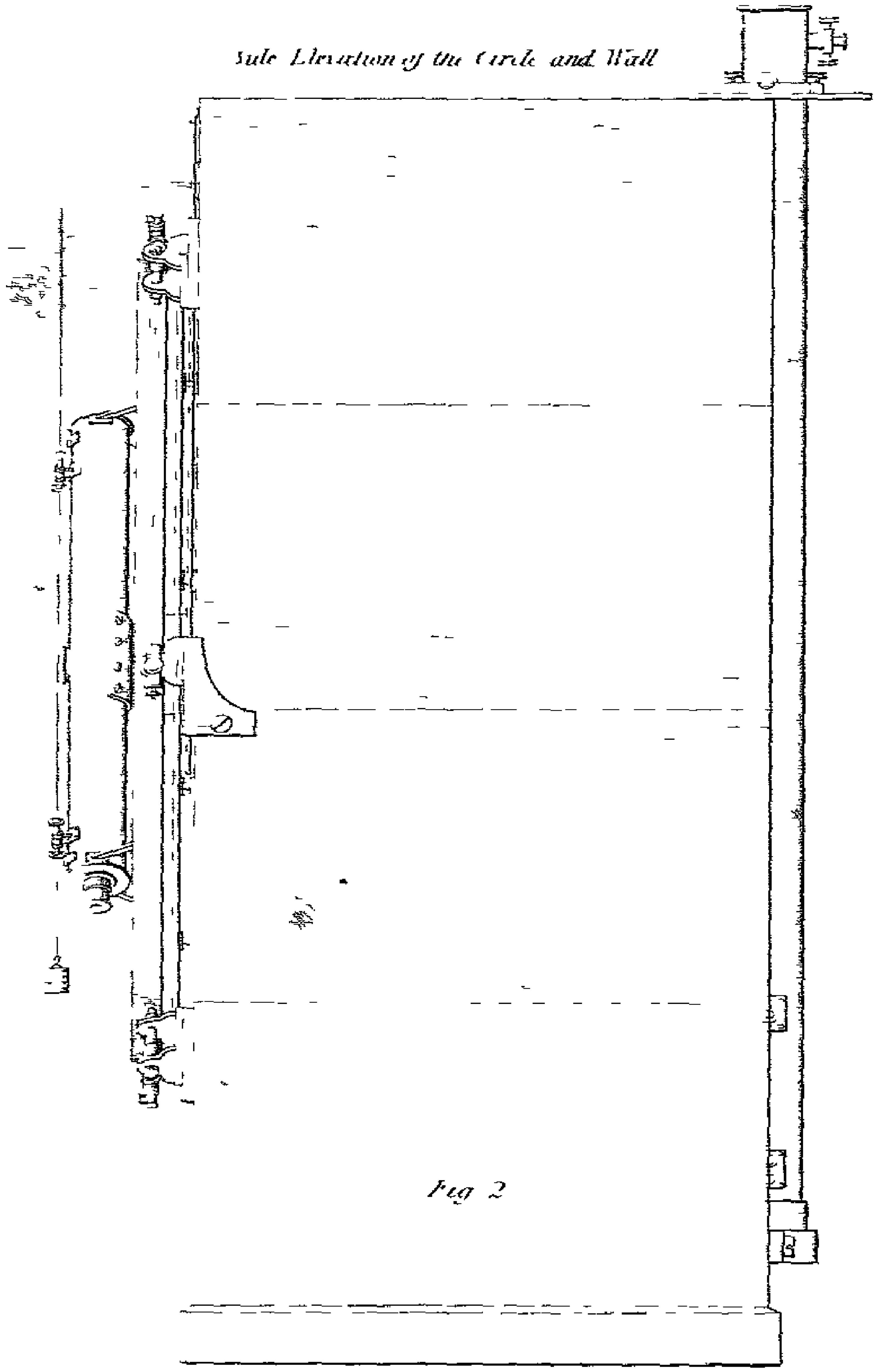


Fig 2

Front Elevation of the Circle

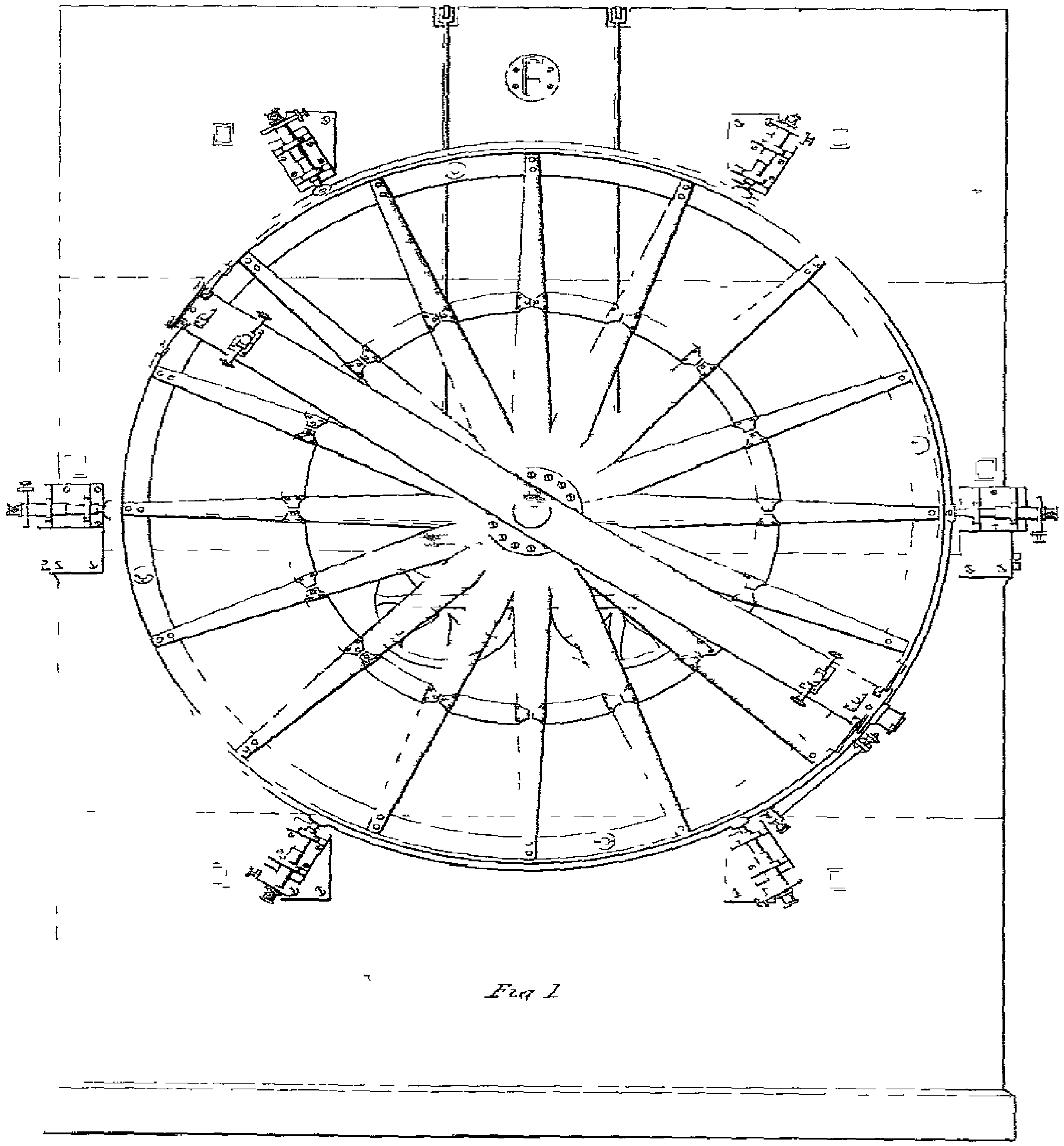


Fig 1

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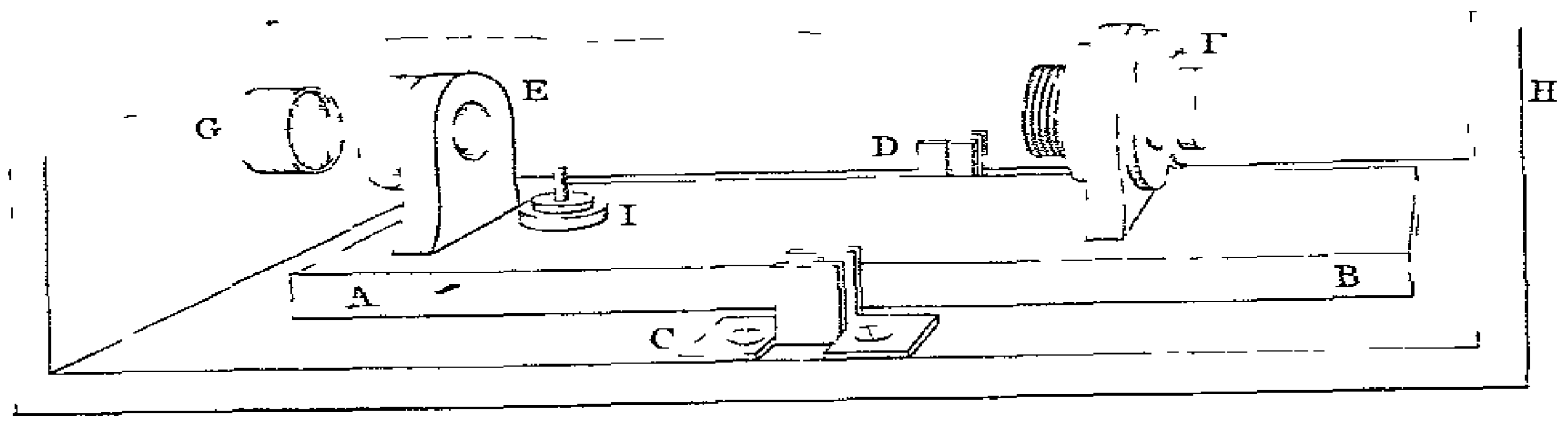
J Farrell

COLLIMATORS AND AMICI'S MICROMETER.

Horizontal Collimator Fig 1



Vertical Collimator

Fig 2

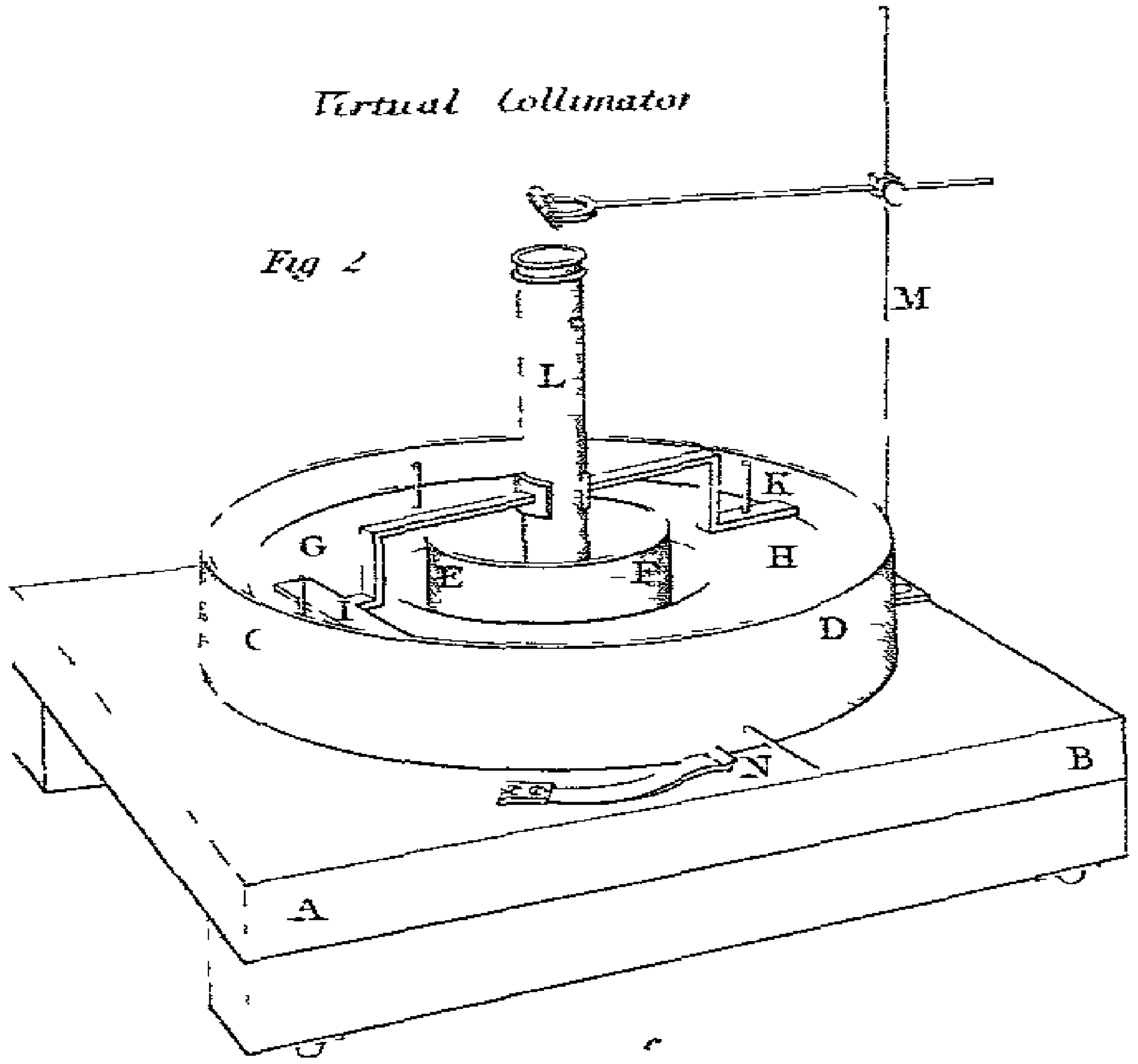


Fig 3

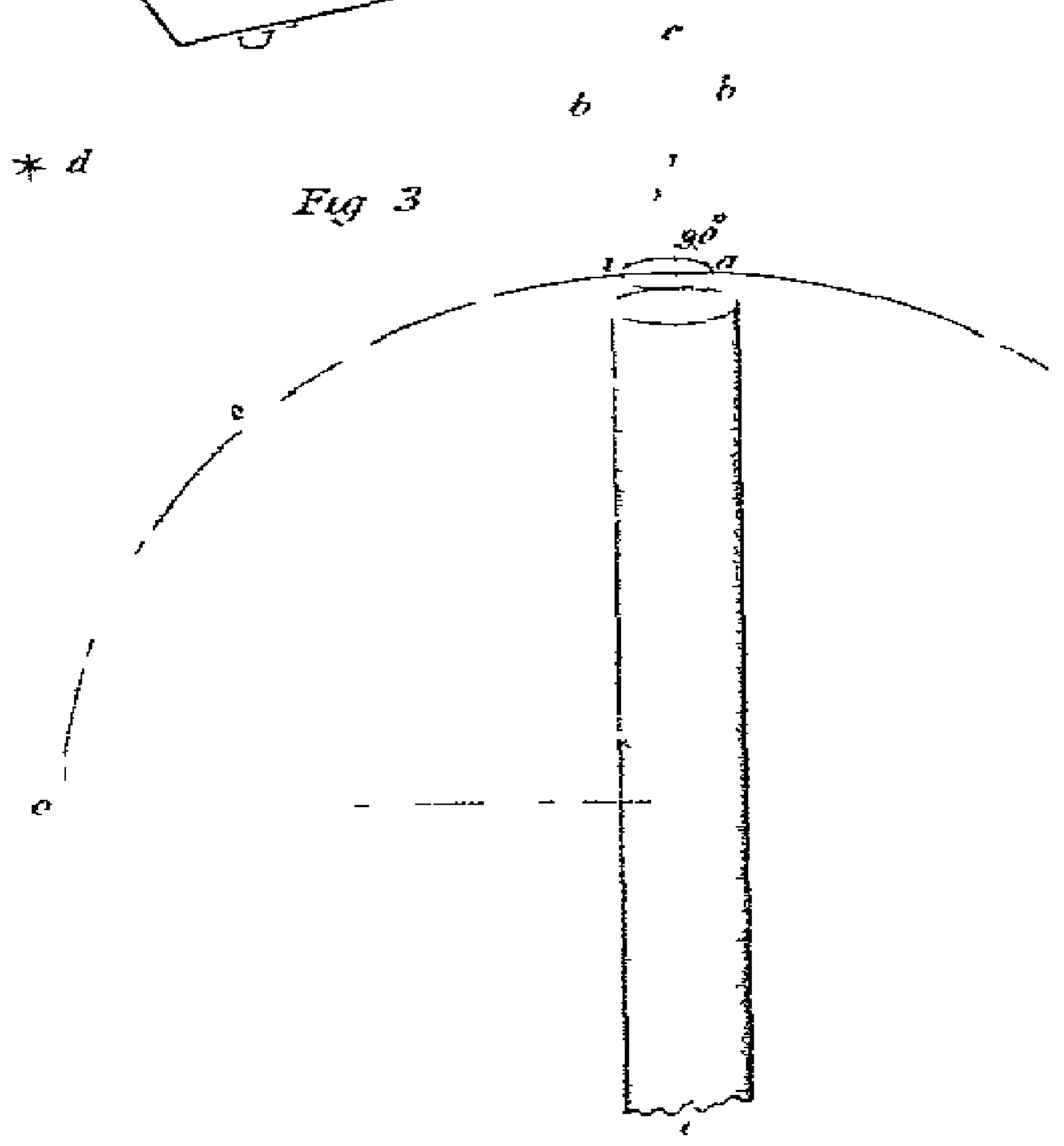
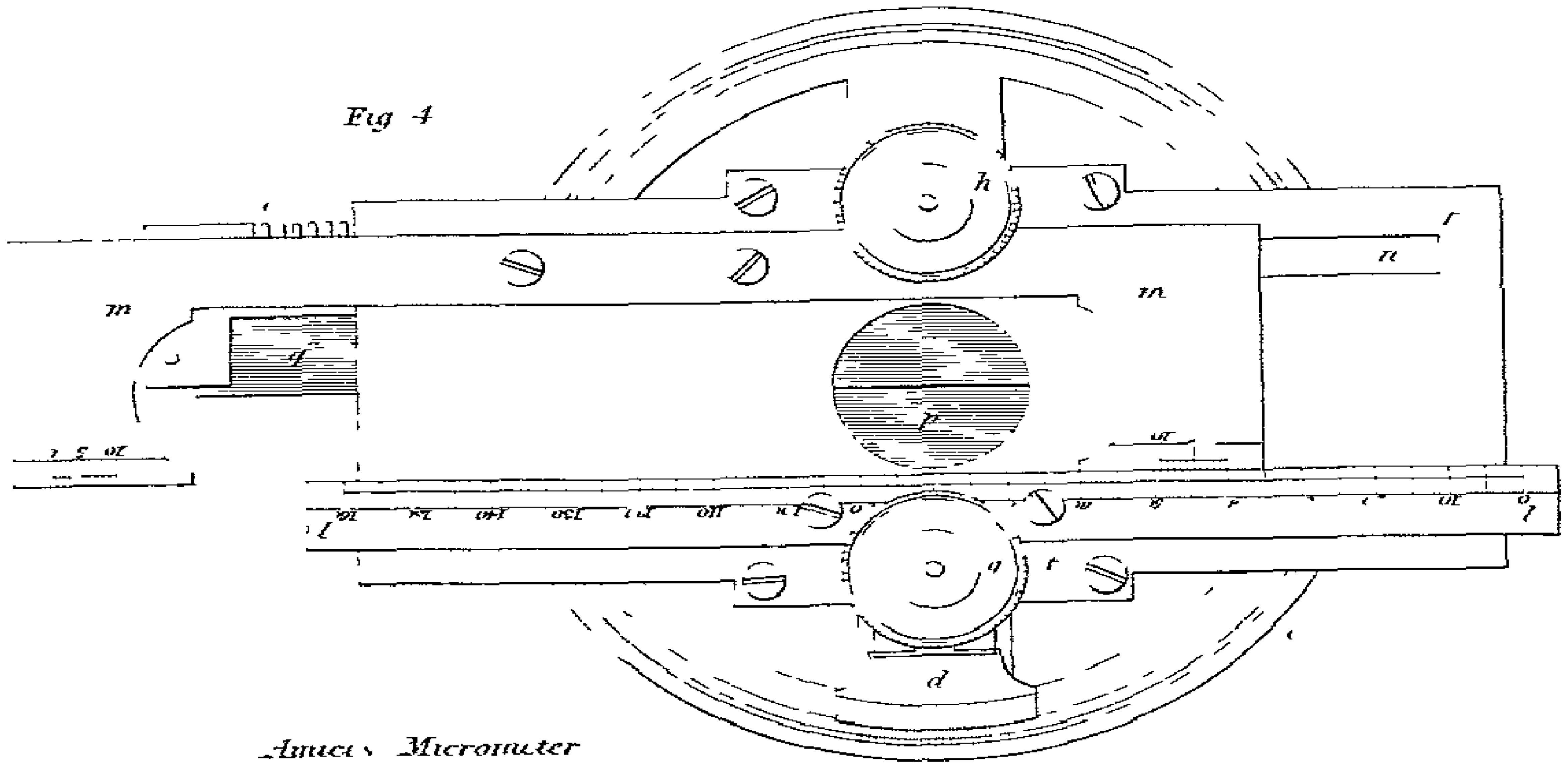
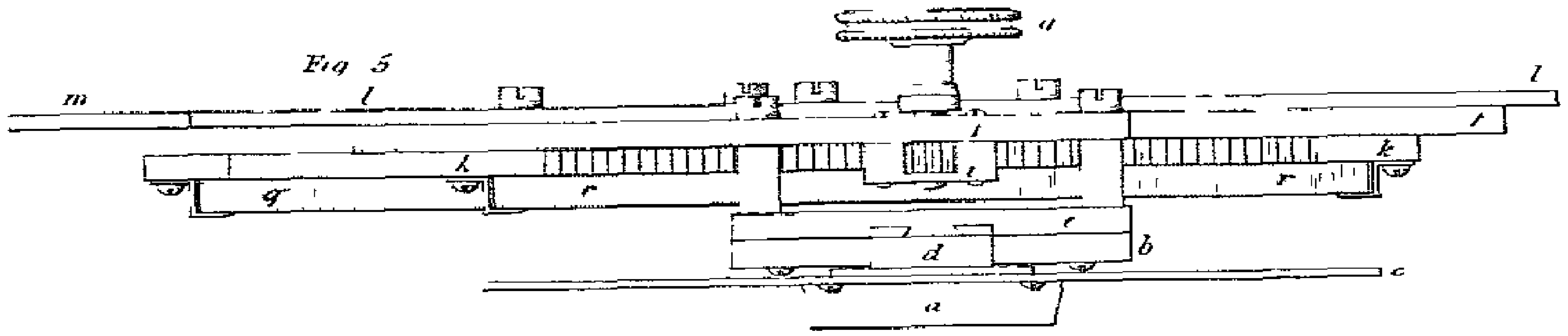


Fig 4

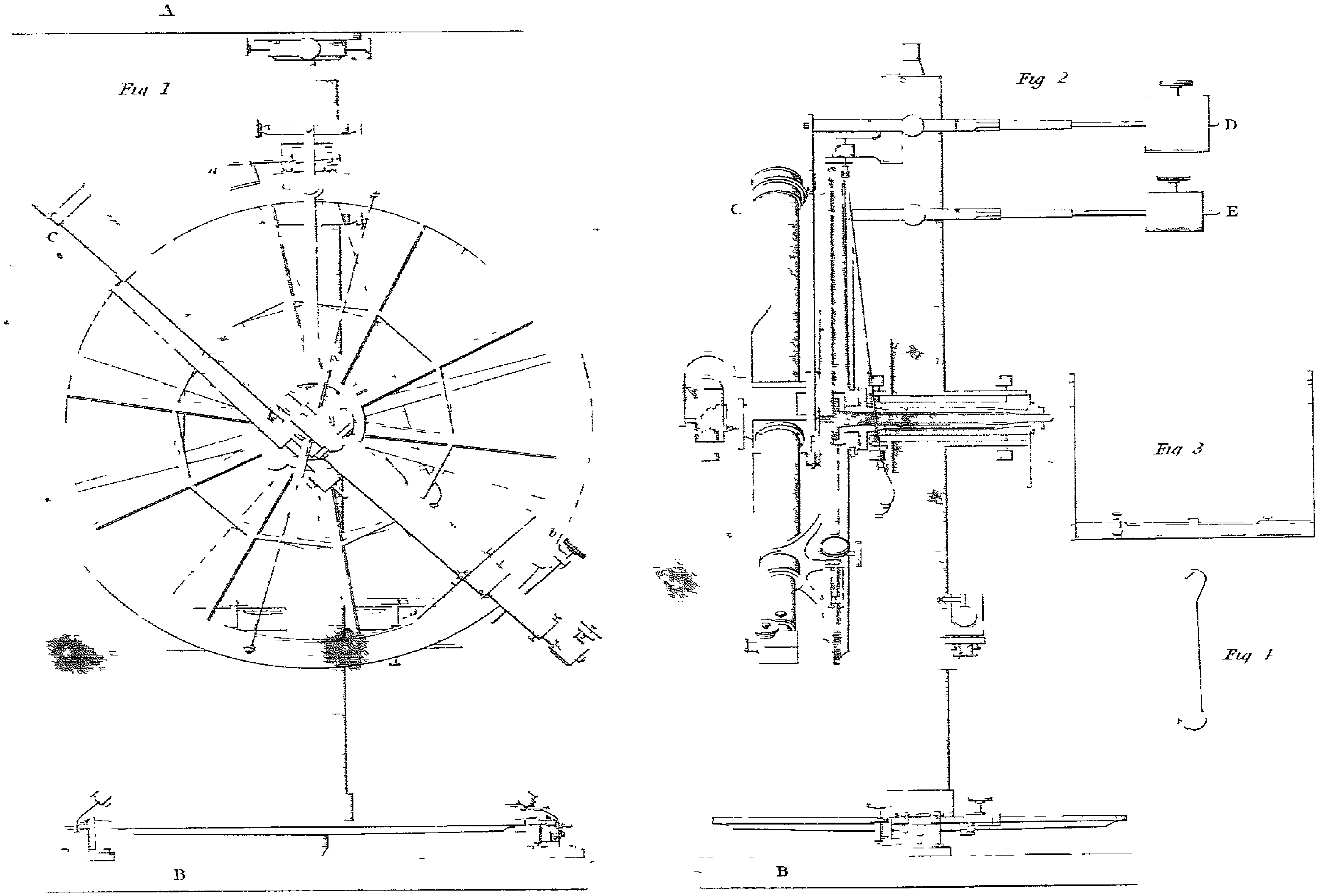


Amici's Micrometer

Fig 5



CIRCULAR INSTRUMENT BY REICHENBACH.



PORTABLE CIRCLES

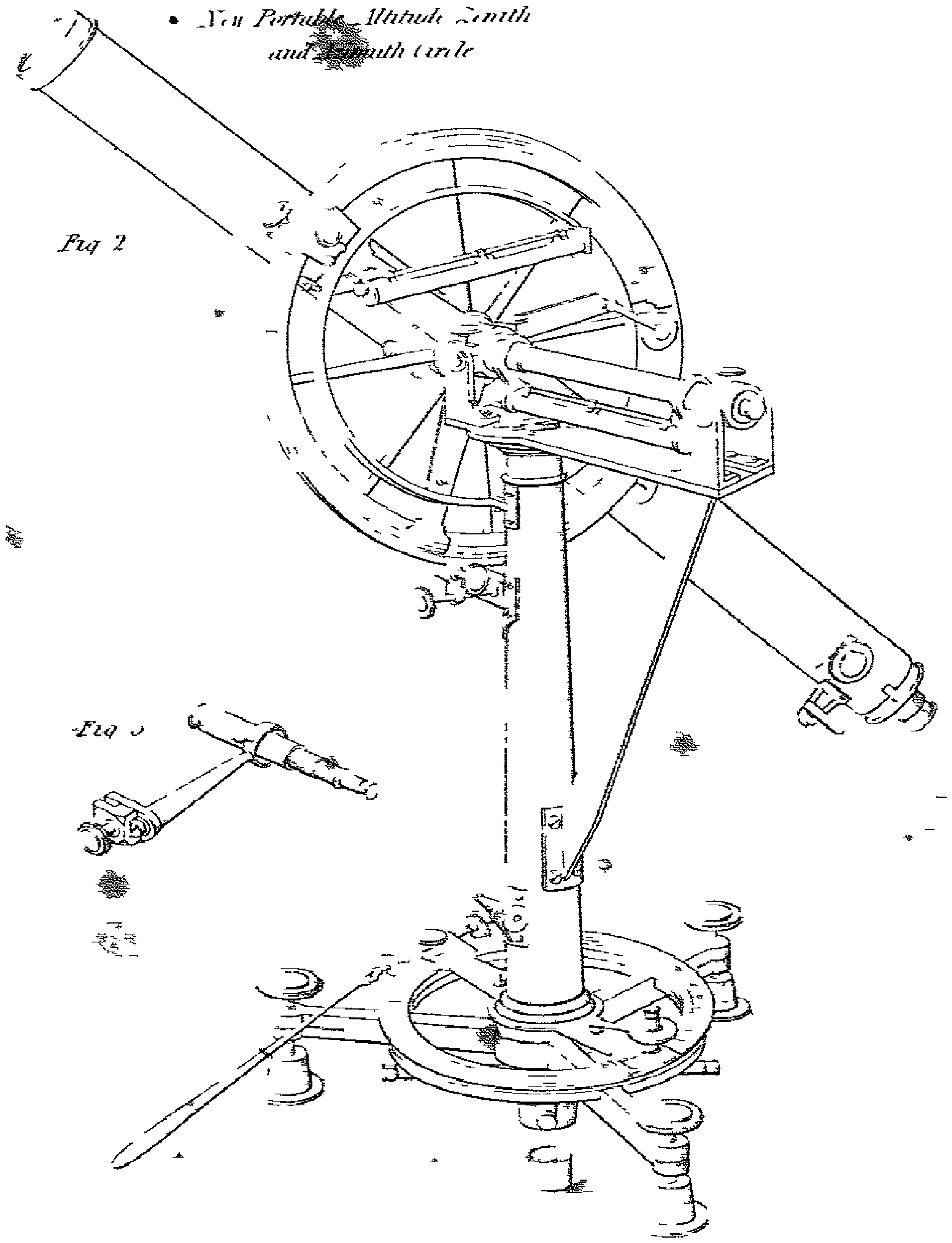

New Portable Altitude Zenith and Azimuth Circle

Fig 2

Fig 3

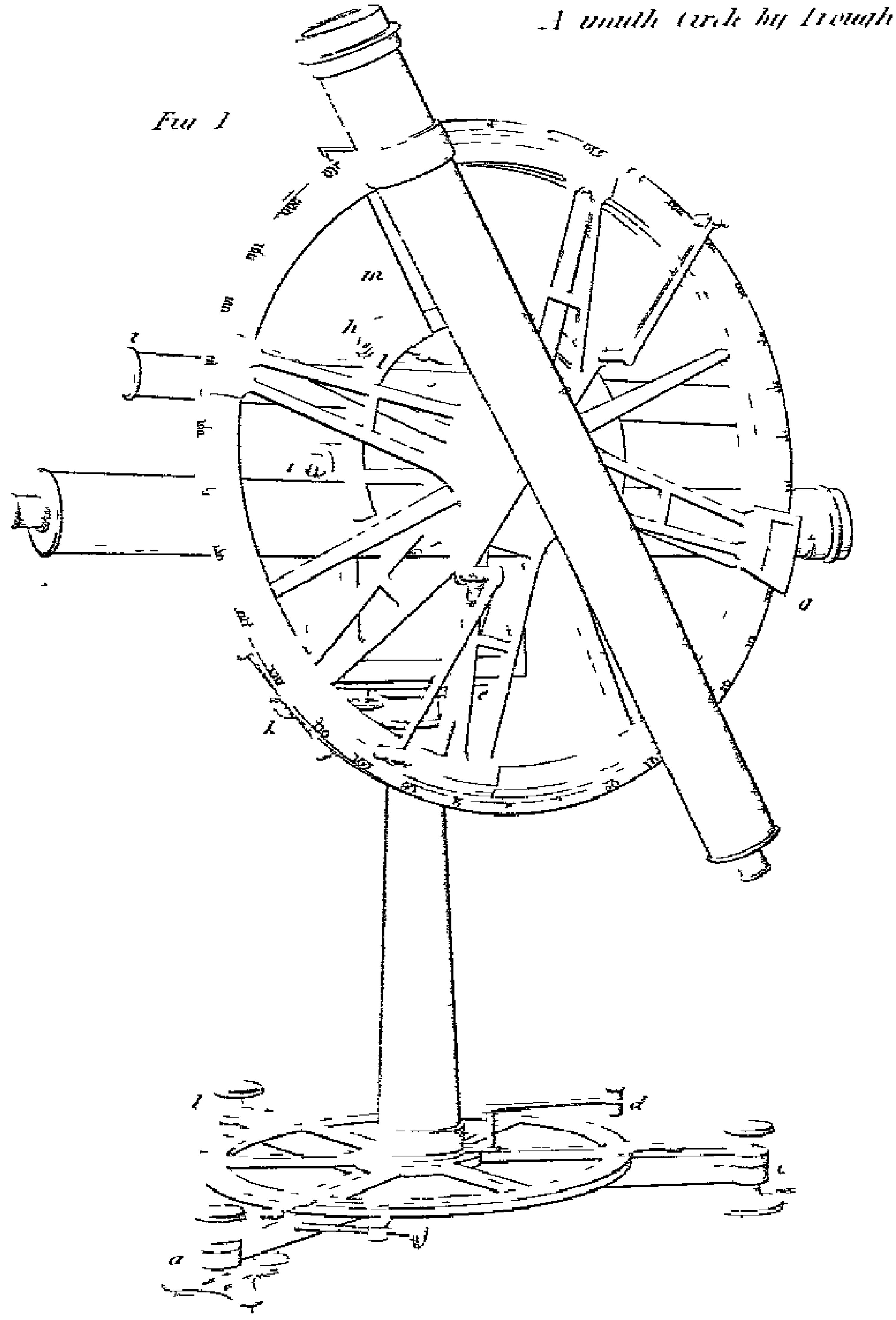

Repeating Altitude and Azimuth Circle by Troughton

Fig 1

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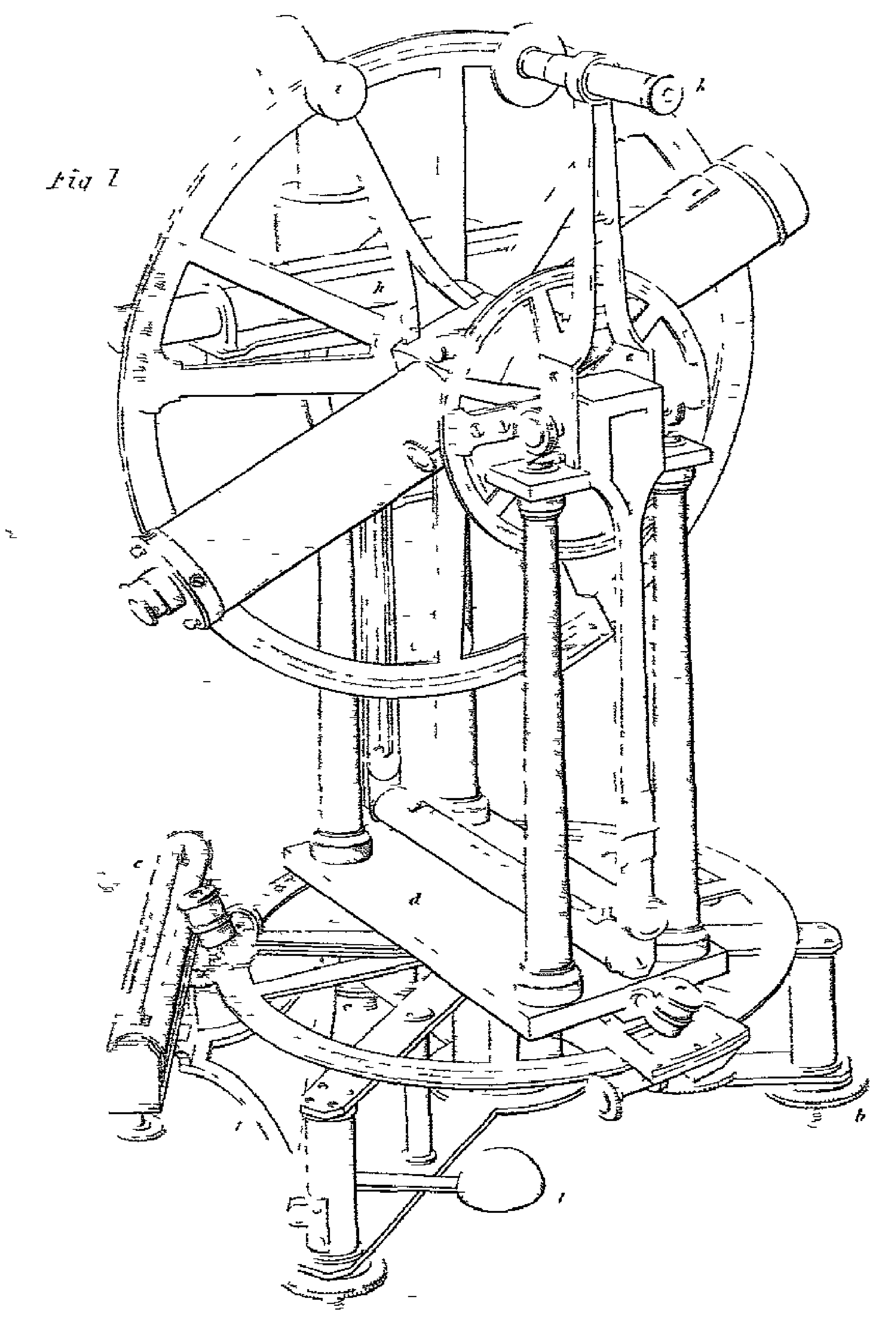
E. Turrell

Plate XXIV

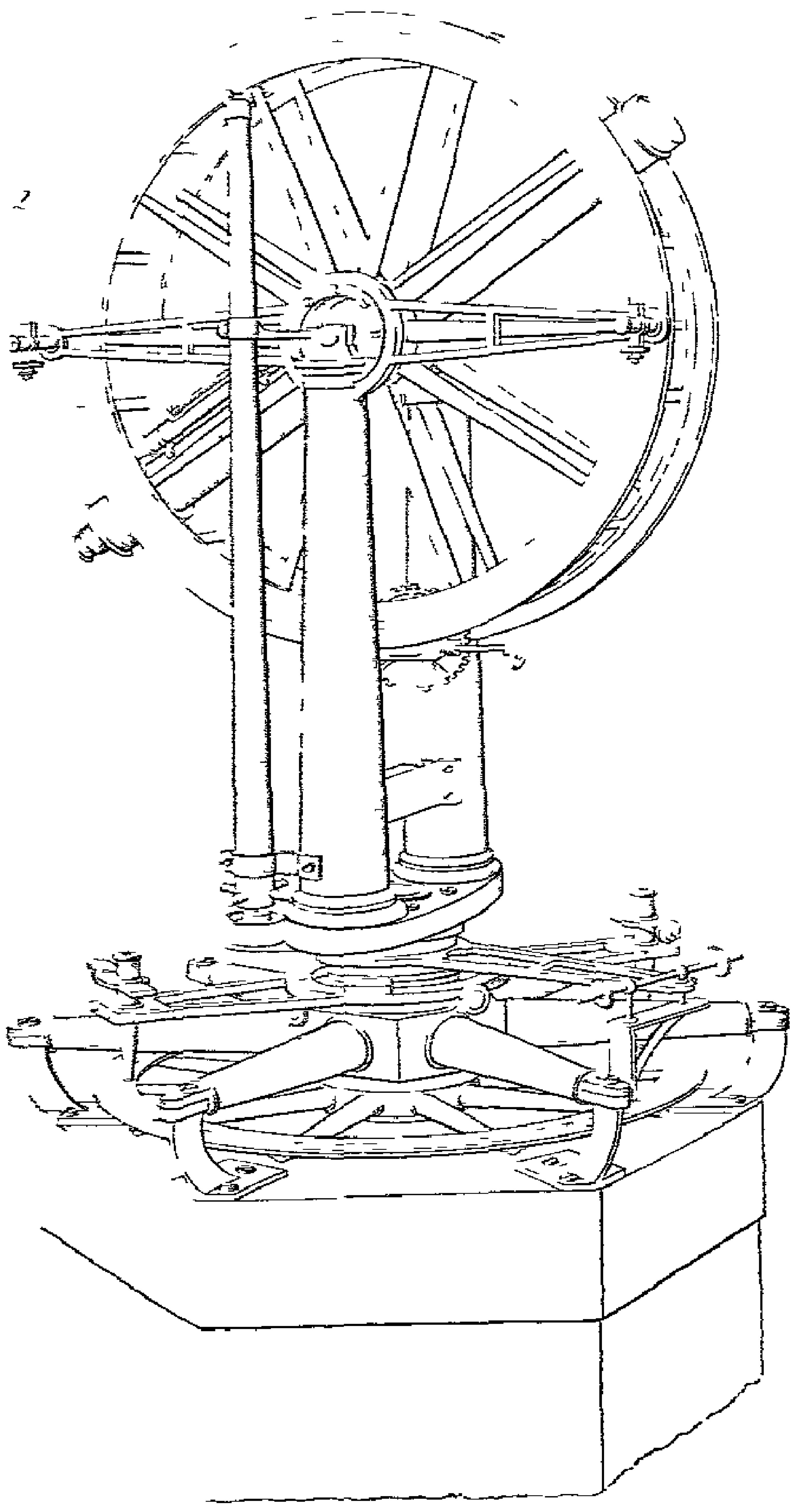
DOLLOND'S REPEATING ALTITUDE AND AZIMUTH CIRCLE

Fig 1



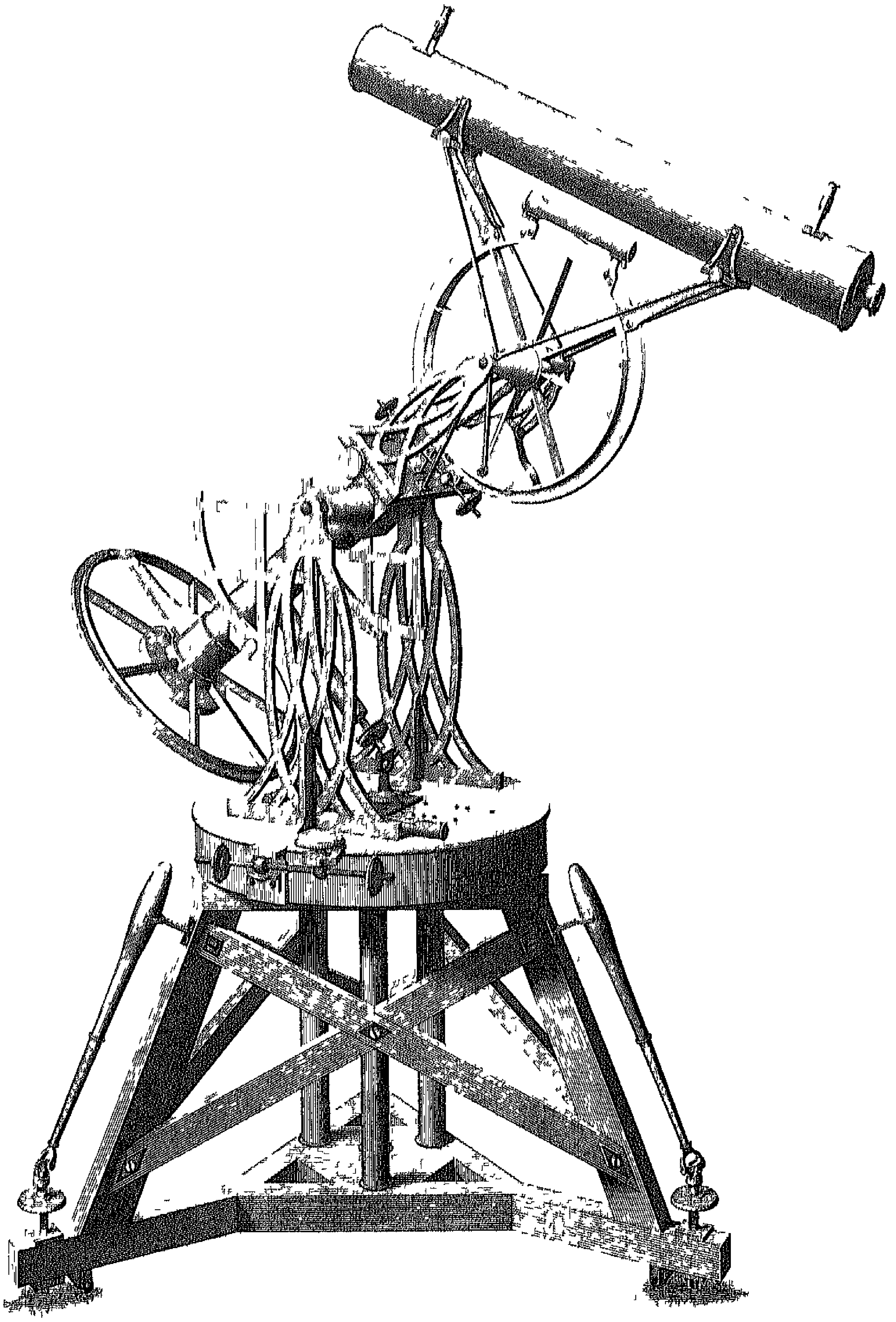
WESTBURY CIRCLE BY TROUGHTON.

Fig 2



AN EQUATORIAL INSTRUMENT BY TROUGHTON

Plate XII



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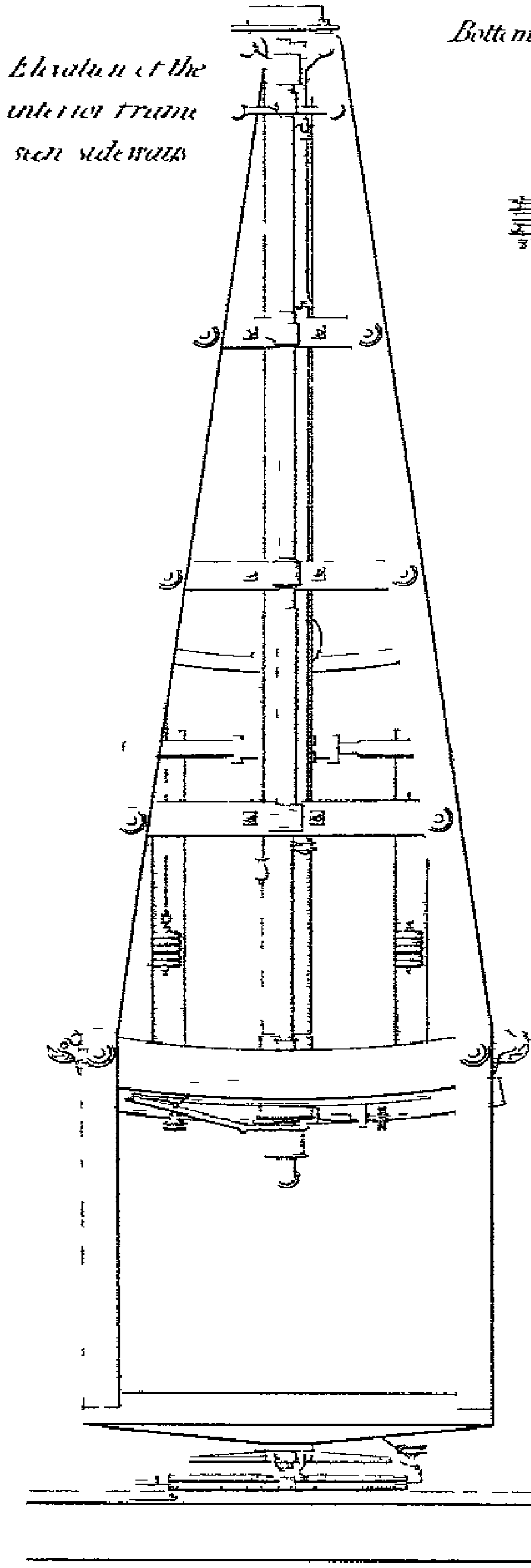
RAMSDEN'S ZENITH SECTOR IN PARTS

Plate XXVI

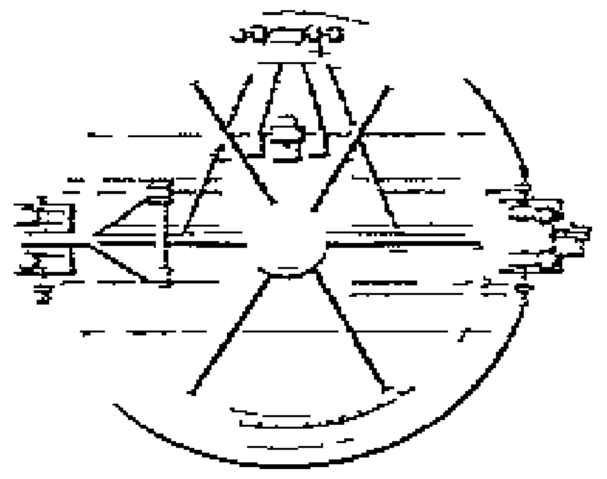
Fig 2

Elevation of the interior frame seen sideways



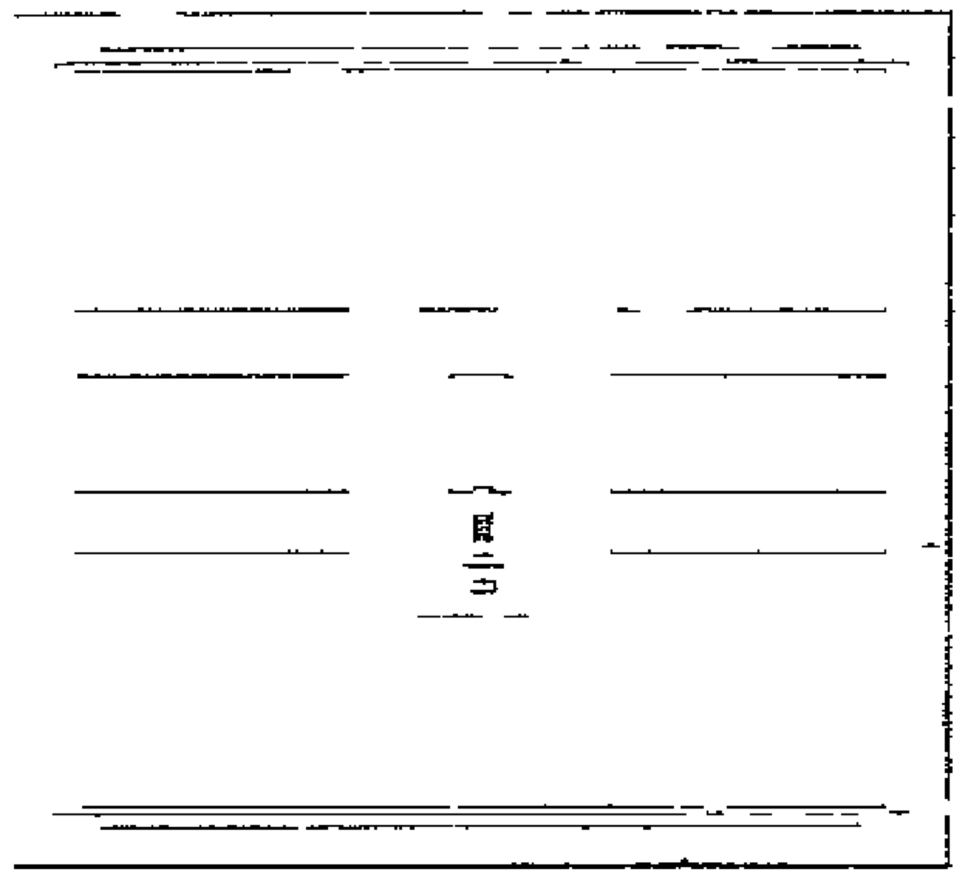
Bottom of the internal frame

Fig 6



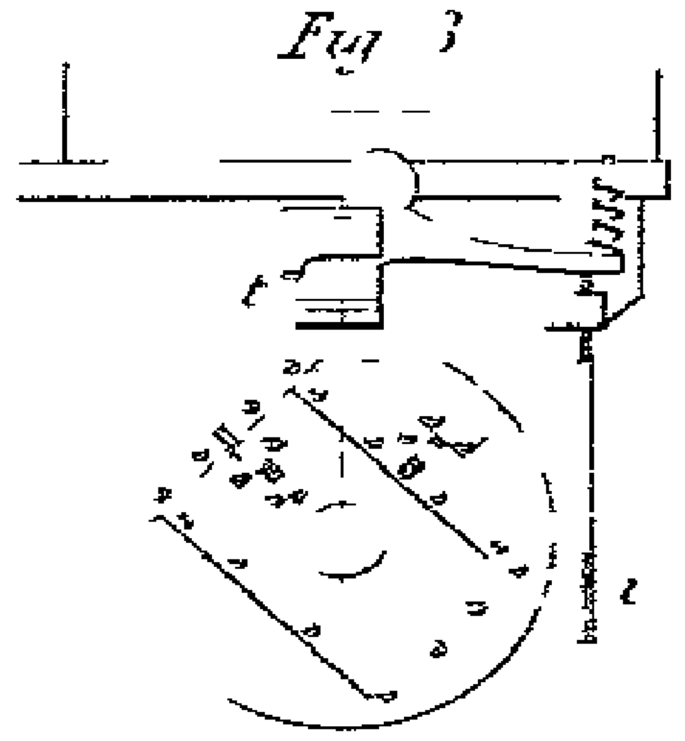
Top of the external stand

Fig 7



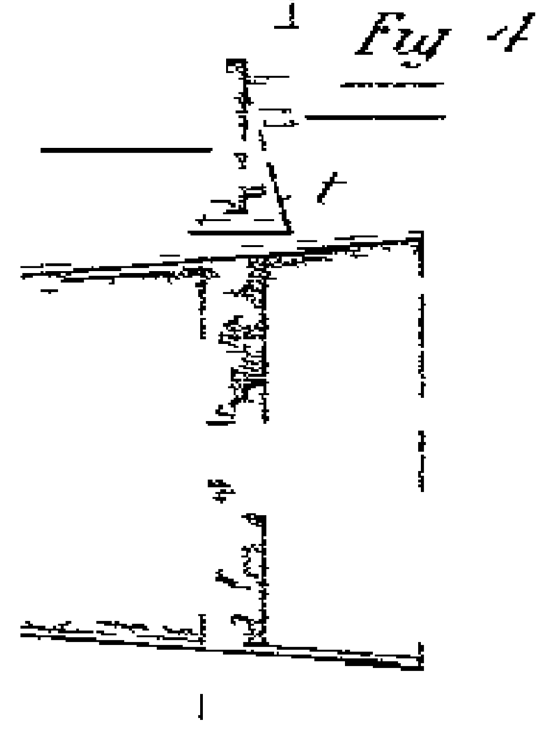
Section of the diaphragm

Fig 3



Section of the axis

Fig 4



Front and view of the end of the Telescope and micrometer

Fig 5

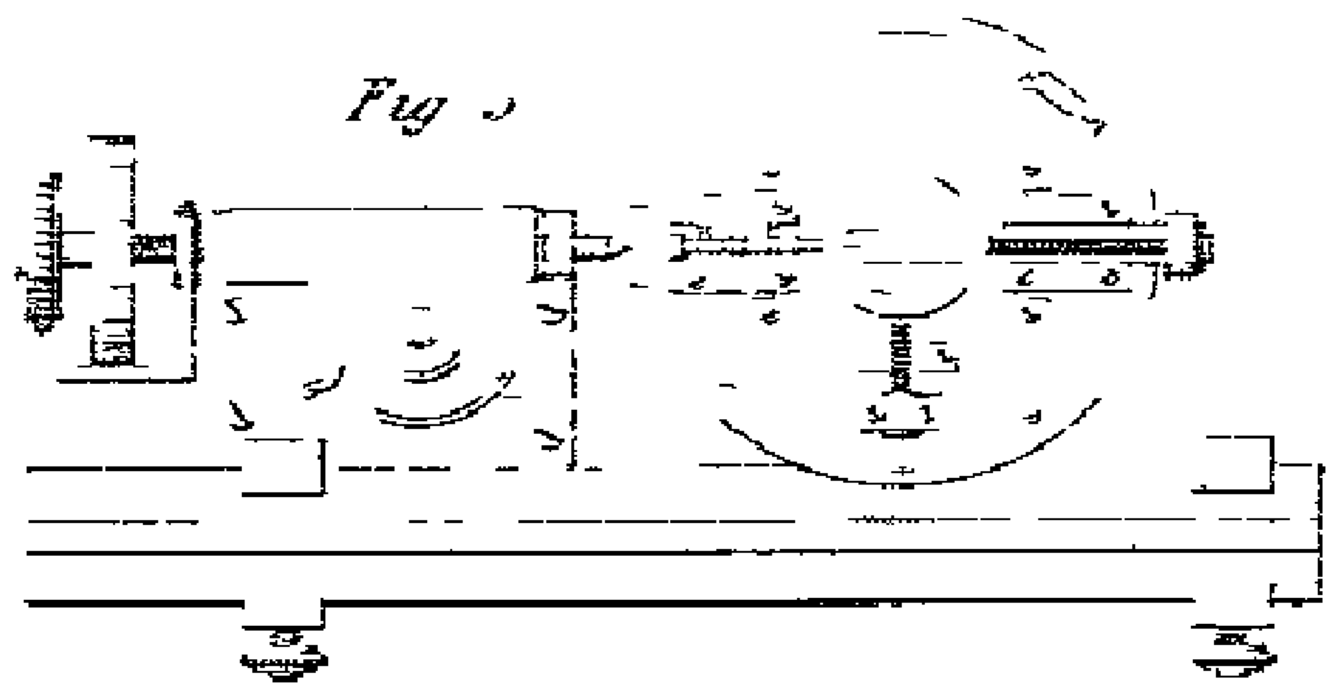
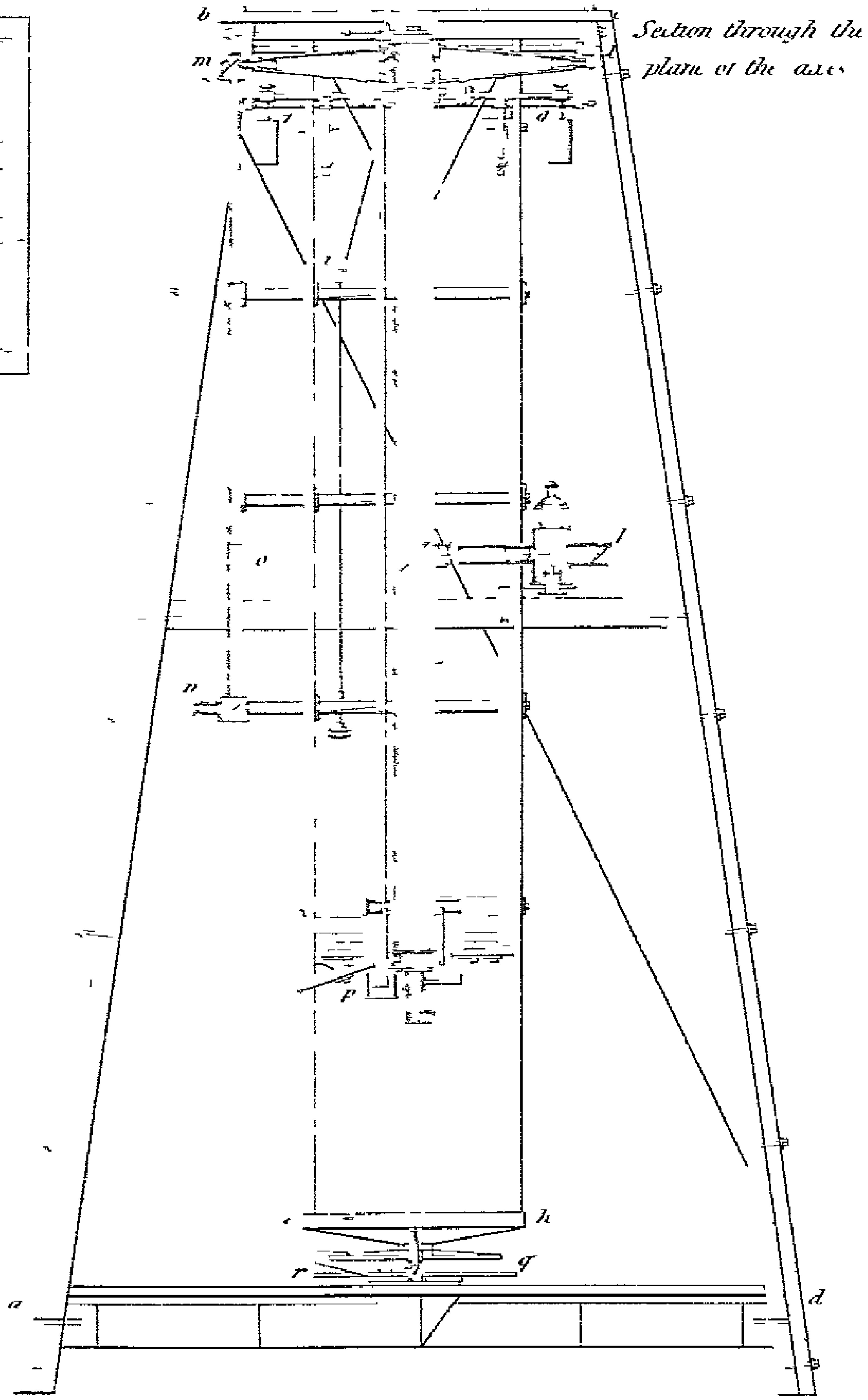


Fig 1

Section through the plane of the axes



ZENITH SECTORS BY GRAHAM, RAMSDEN, AND TROUGHTON.

Fig 1 A general view

Ramsden's

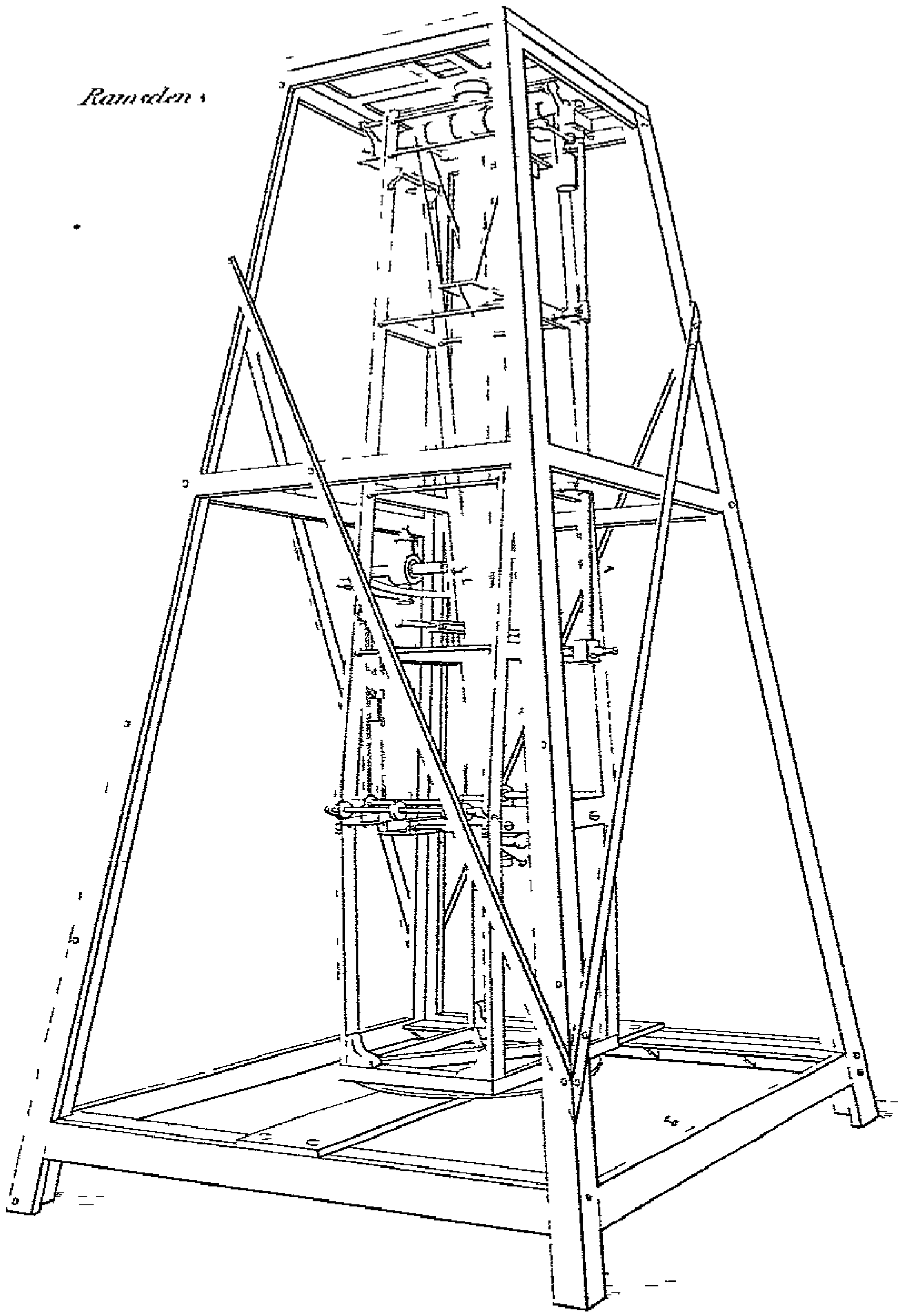


Fig 2

Graham's

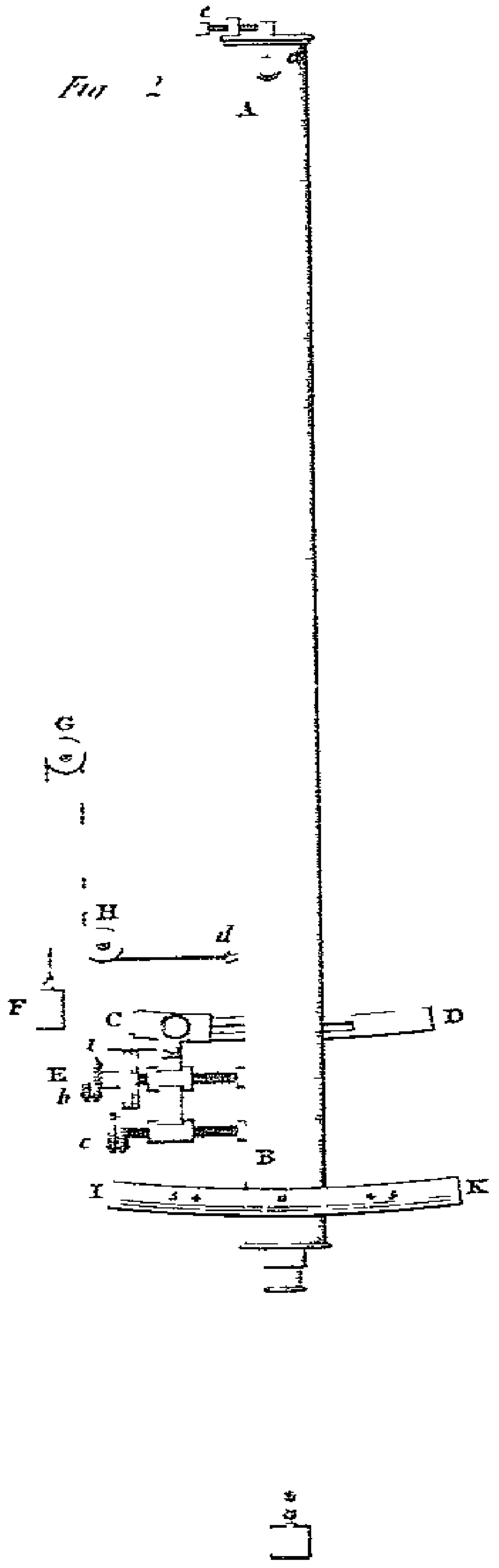
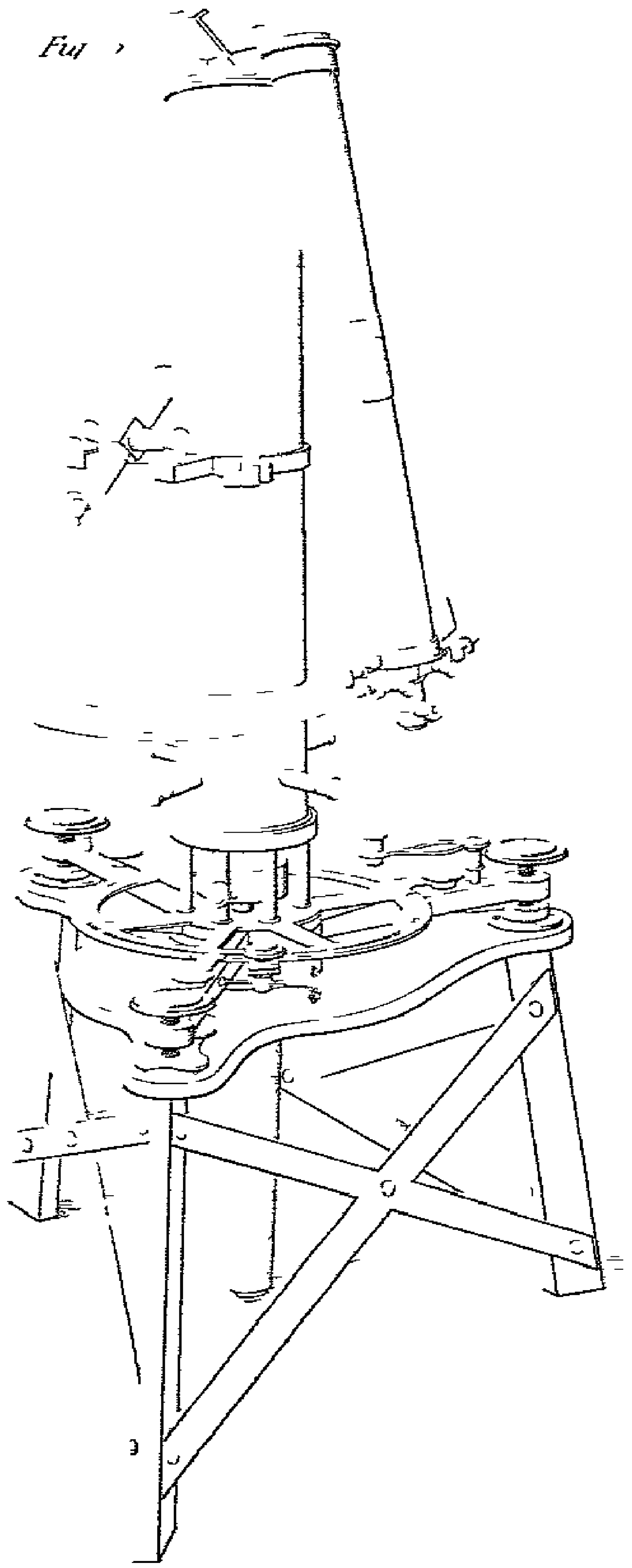


Fig 3

Troughton's



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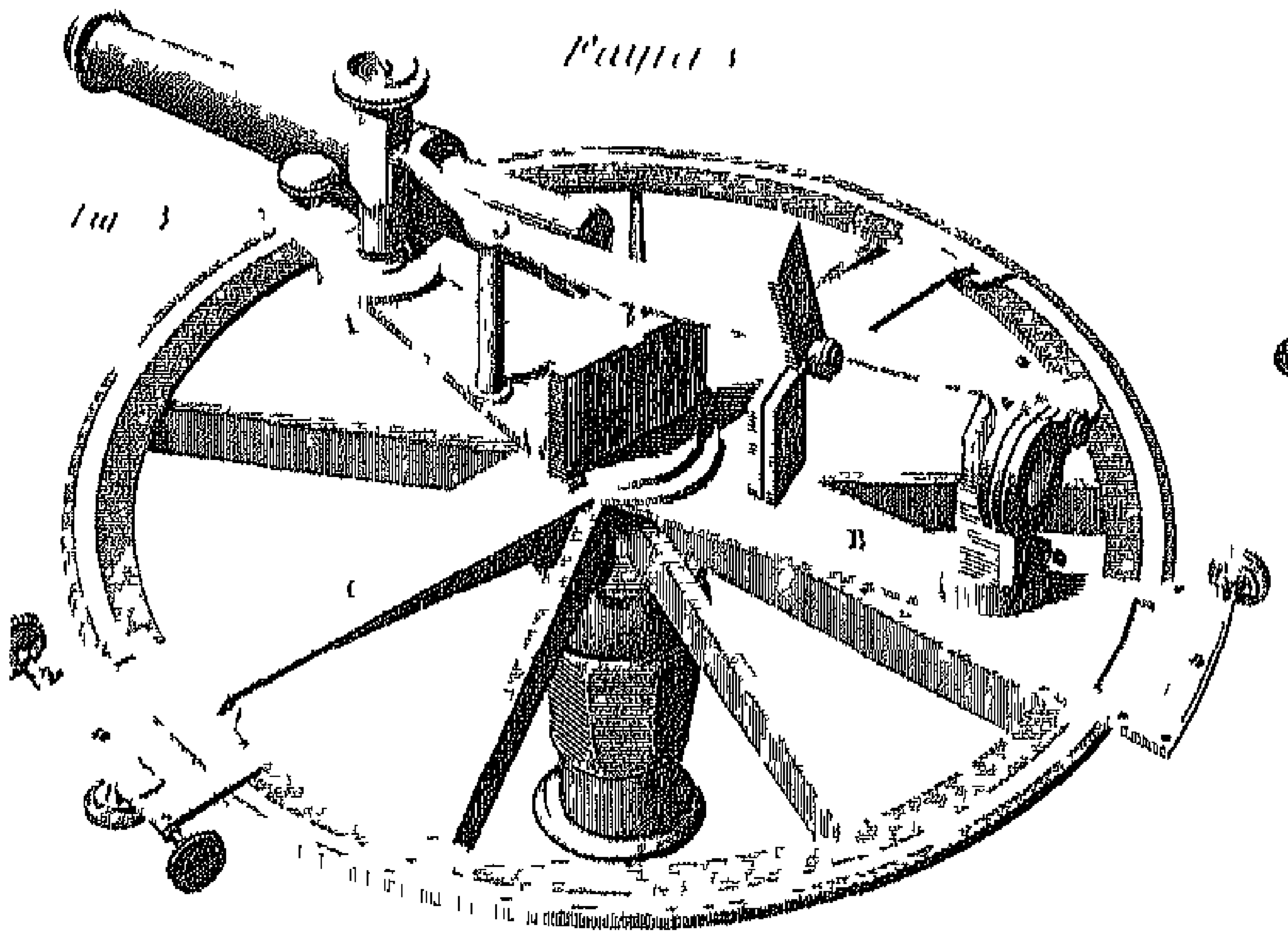
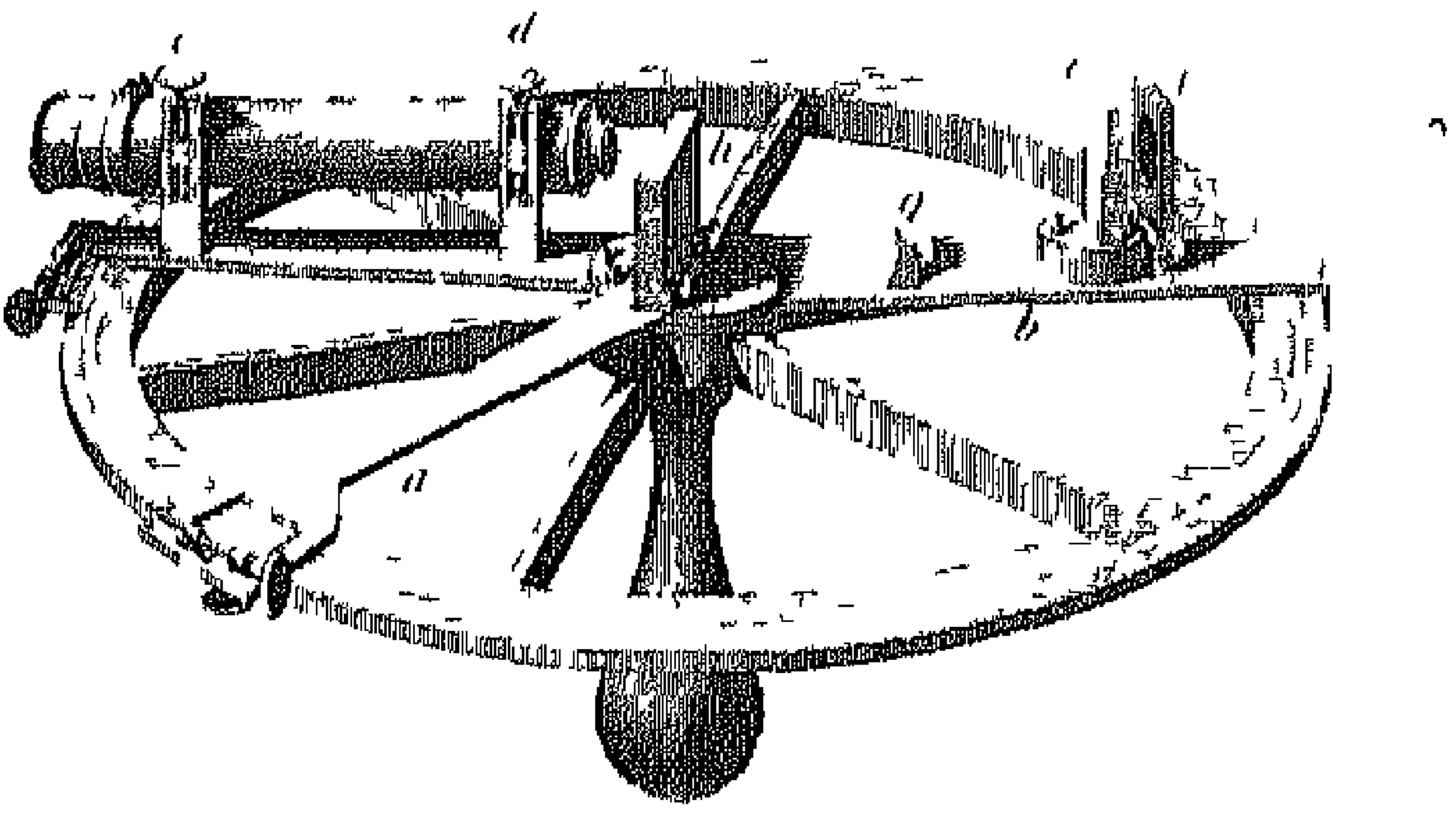
Fayrer's

Fig. 3



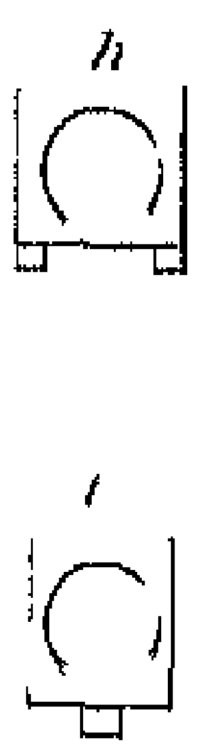
Borda's

Fig. 1



Mendoza's

Fig. 2



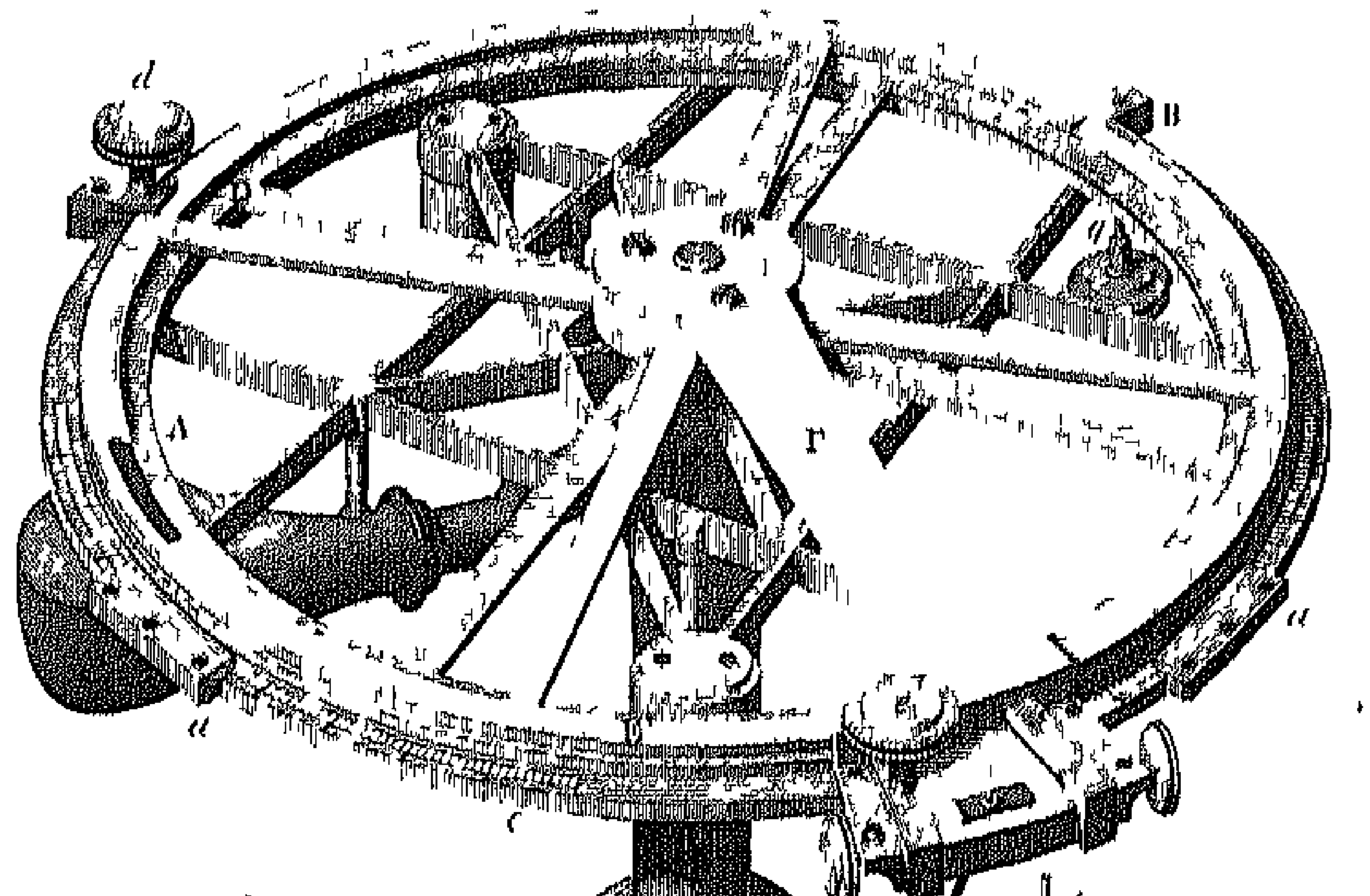
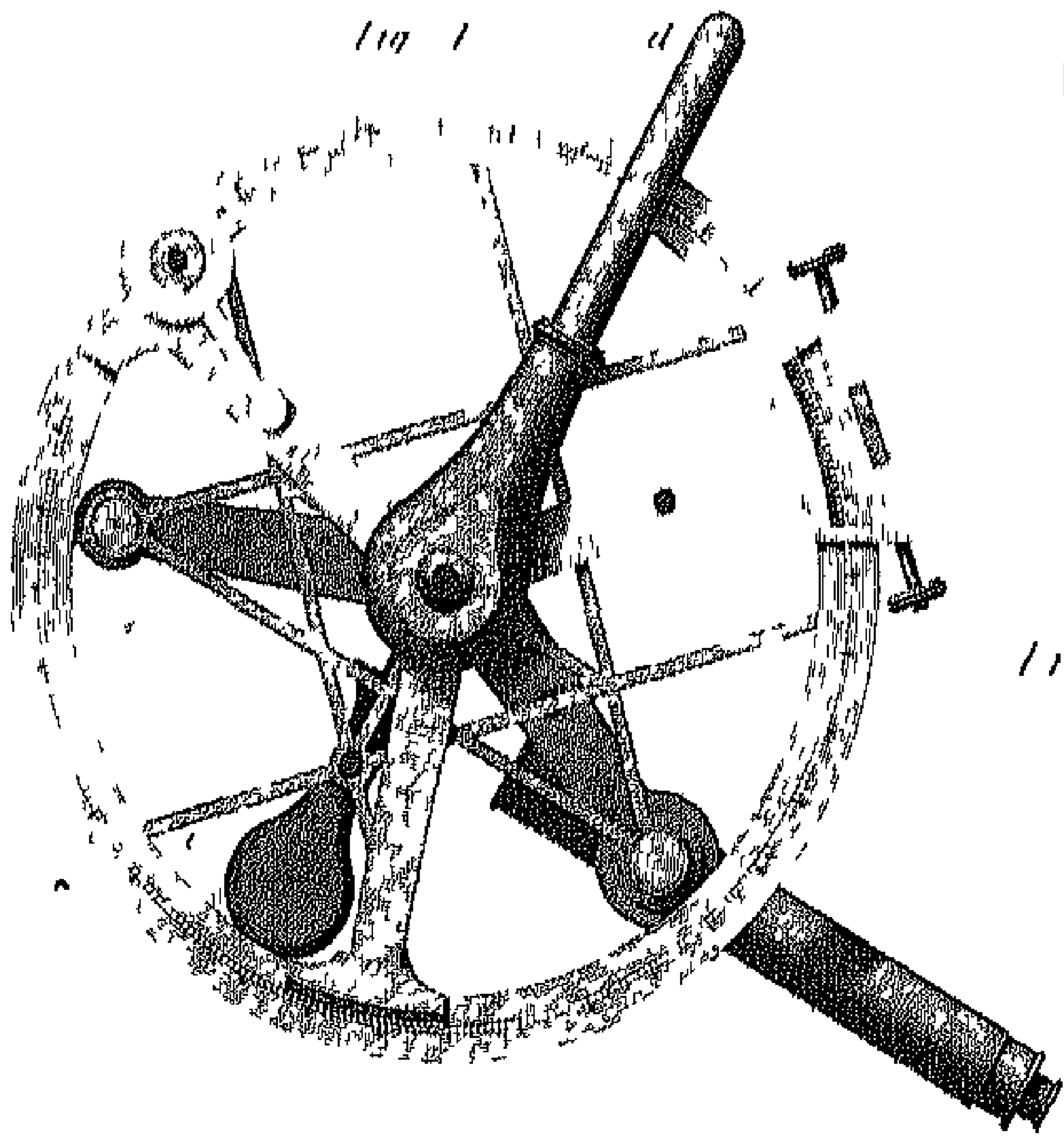
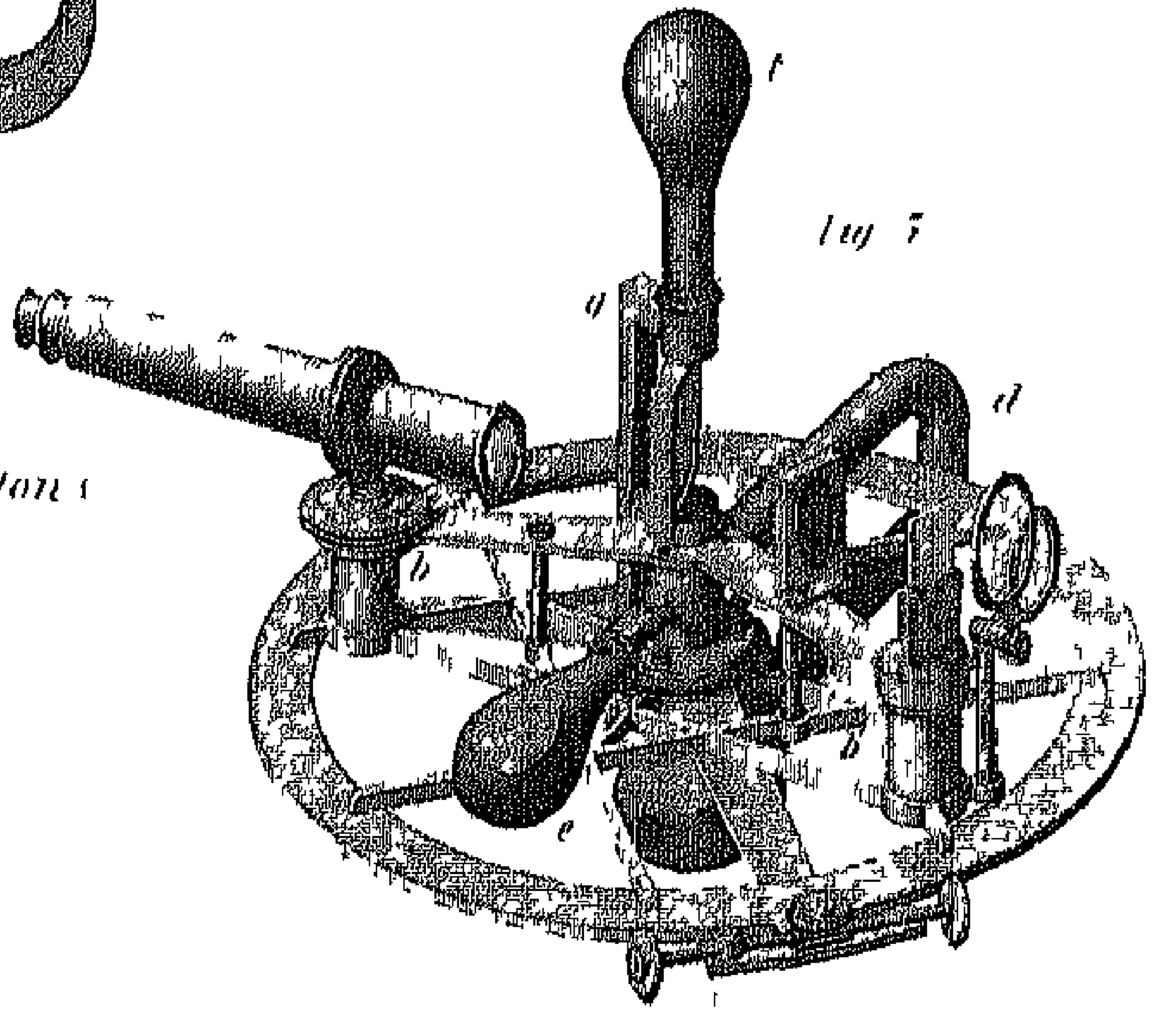


Fig. 4



Troughton's

Fig. 5



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ASTRONOMICAL QUADRANT, SEXTANT, &c.

Plate XXIV

Dollond's Object Glass Micrometer

Fig. 1

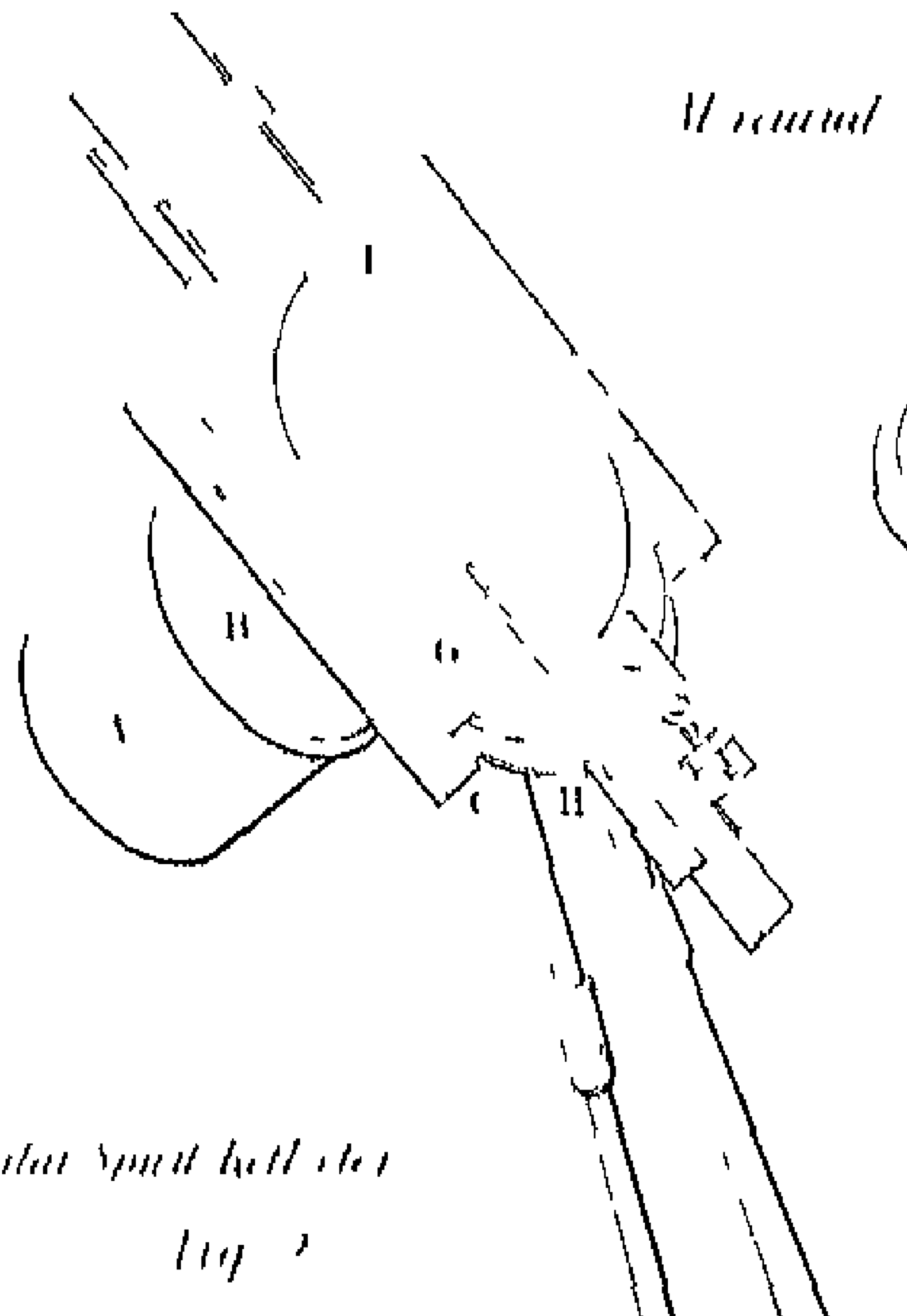
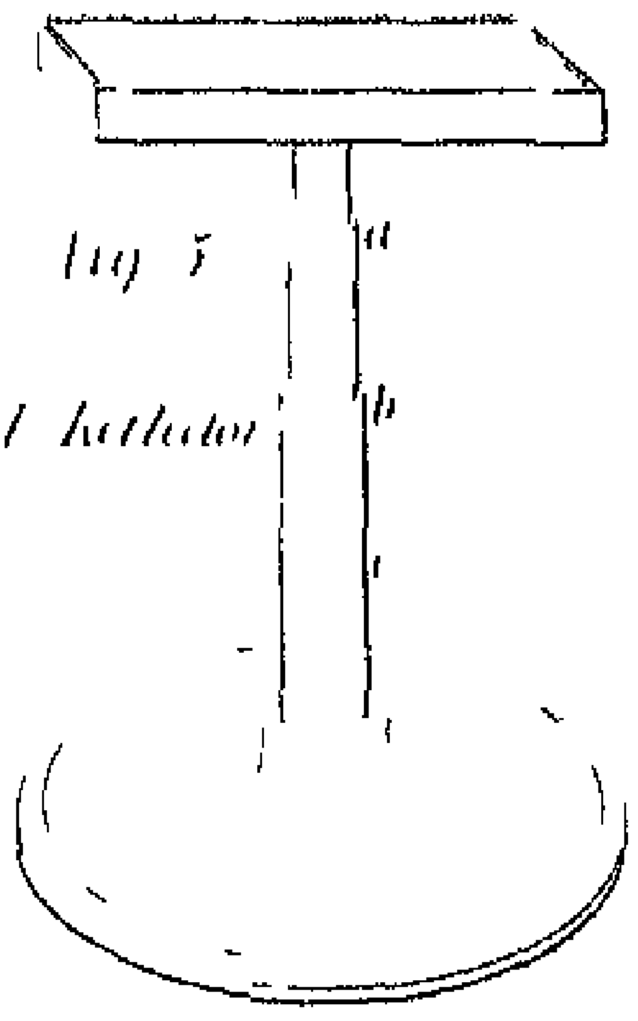


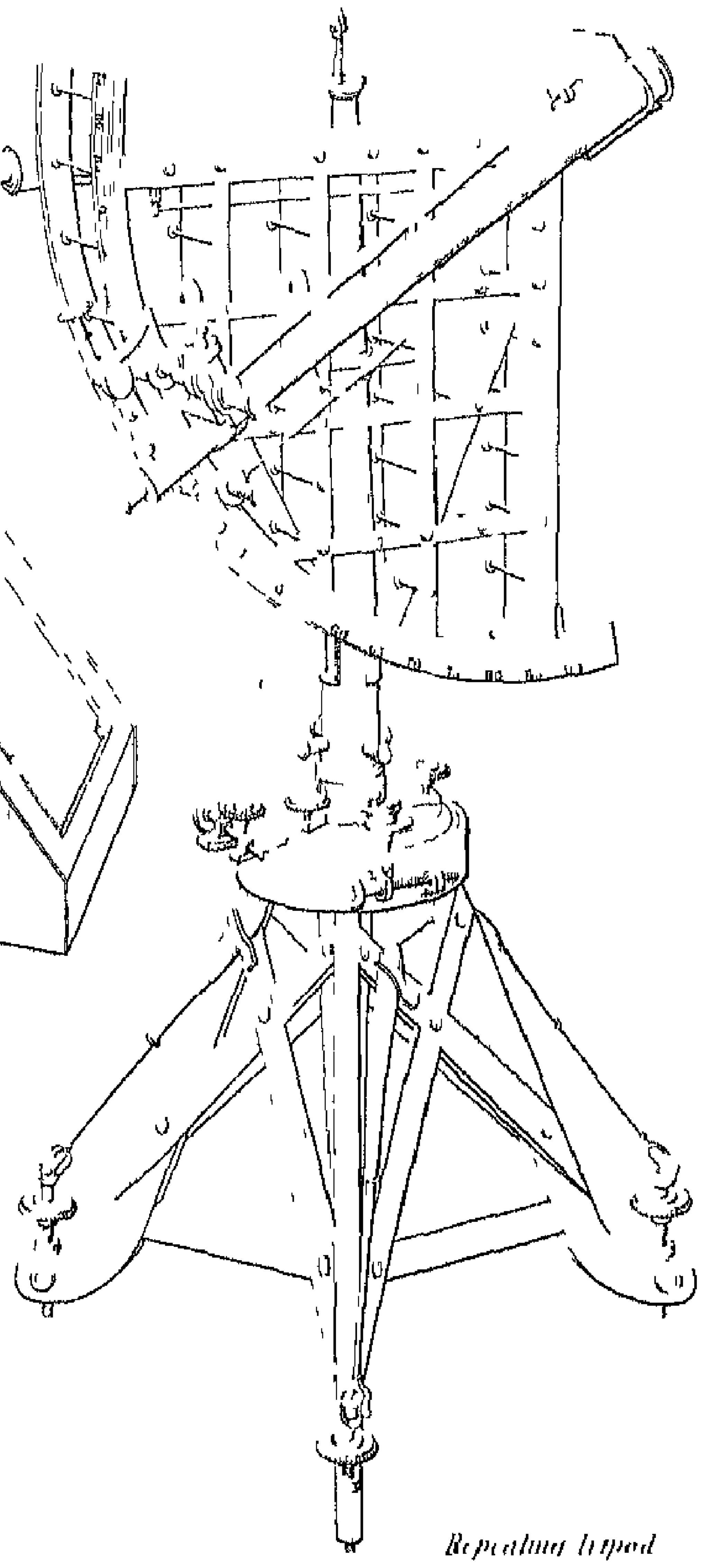
Fig. 5

Mercurial Reflector



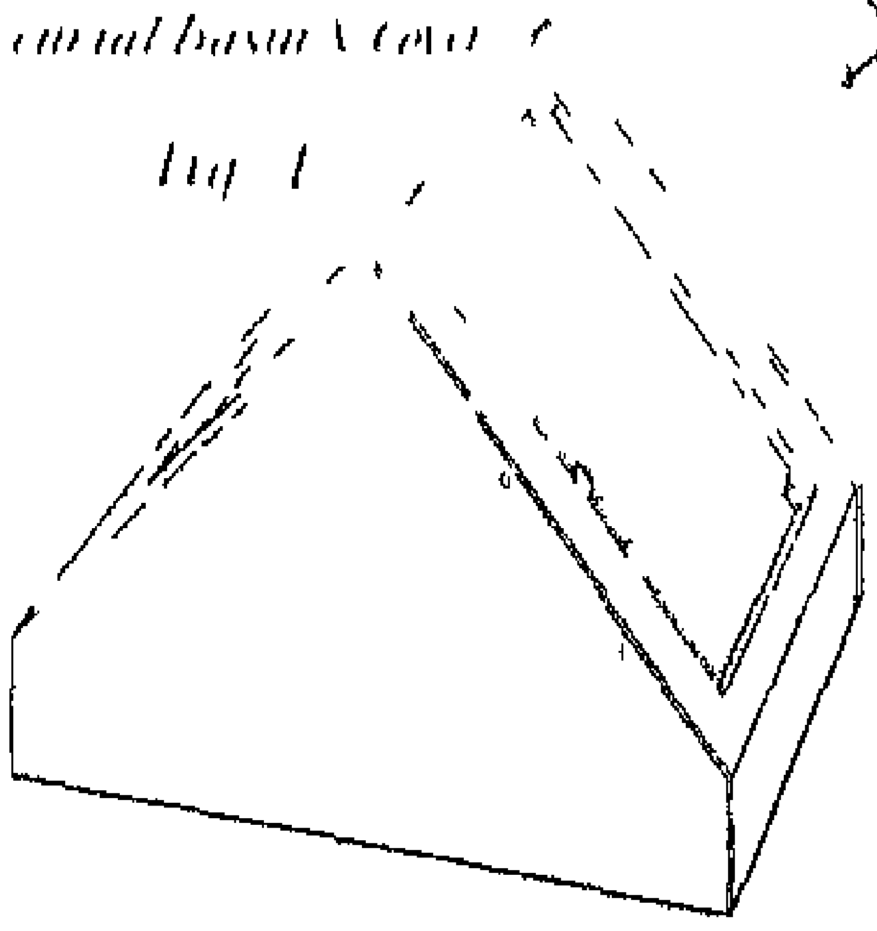
Troughton's Astronomical Quadrant

Fig. 6



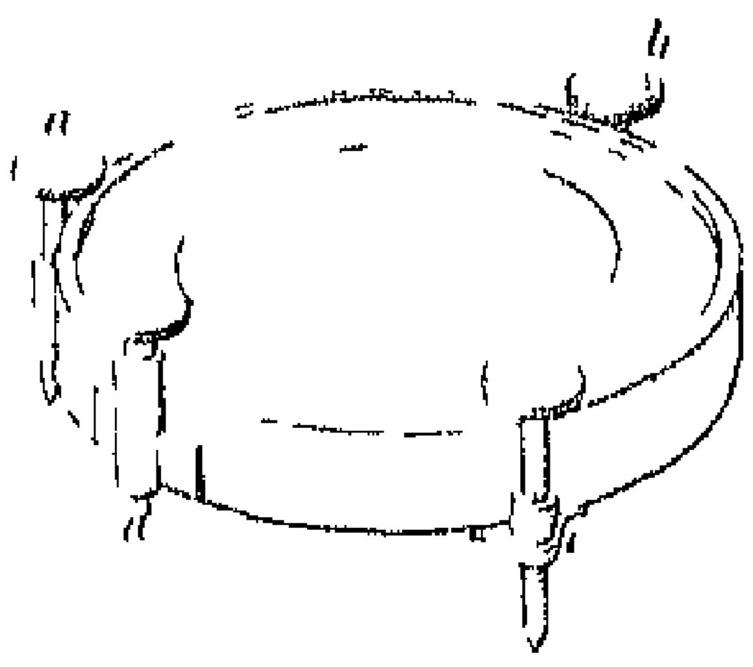
Mercurial basin & Cover

Fig. 4



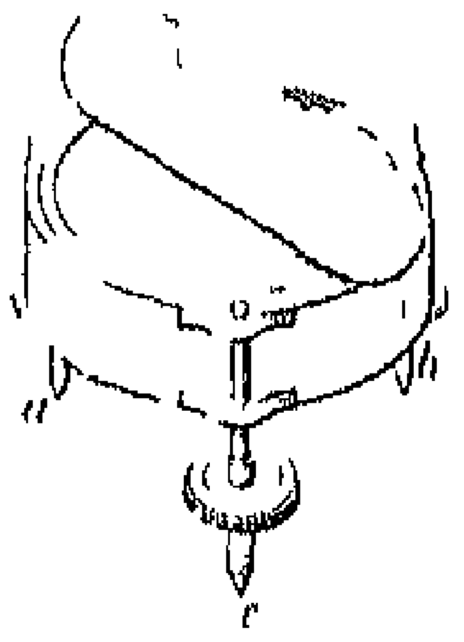
Circular Spirit Reflector

Fig. 2



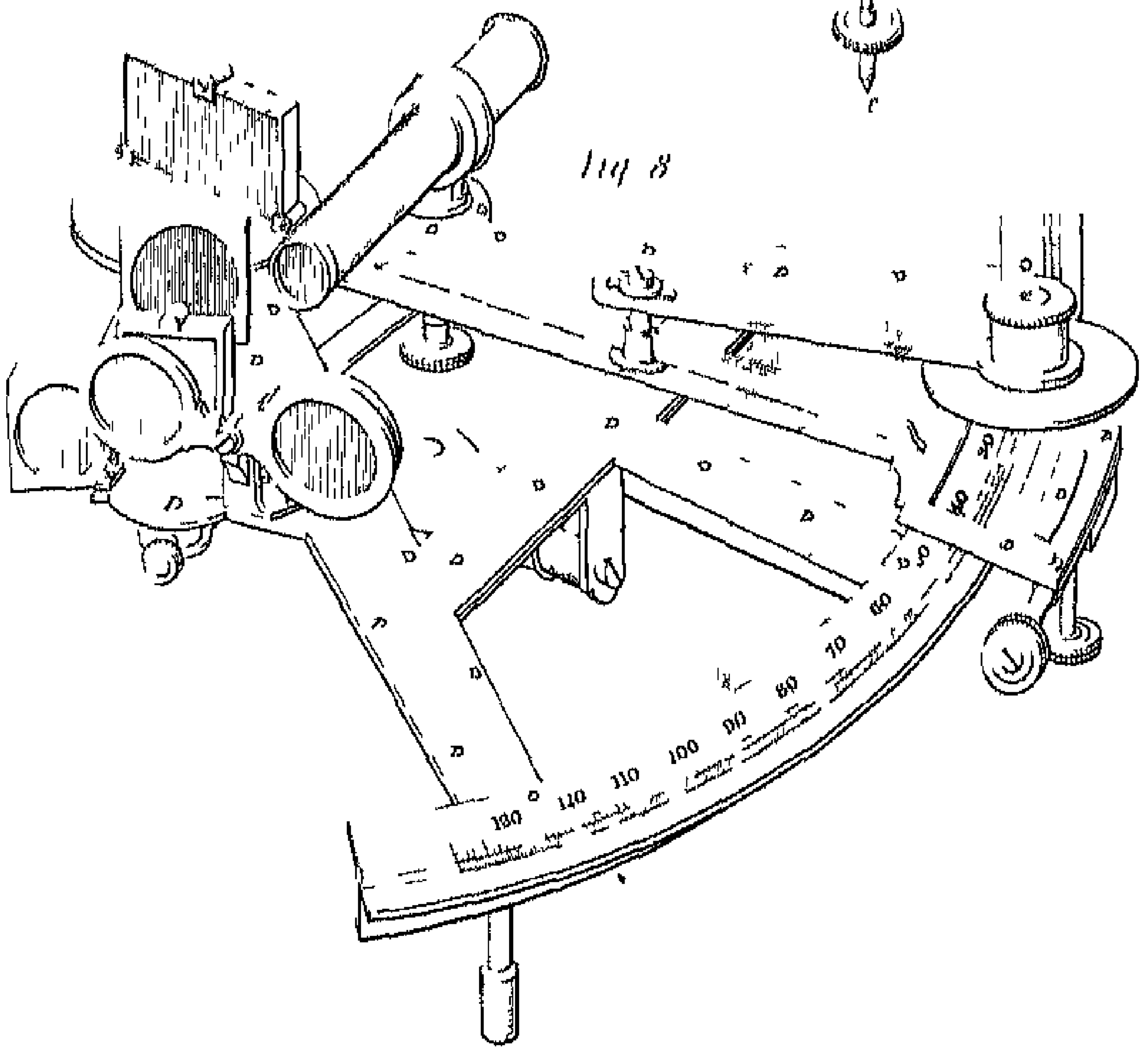
Dark Glass Reflector with small Fig. 1

Fig. 3



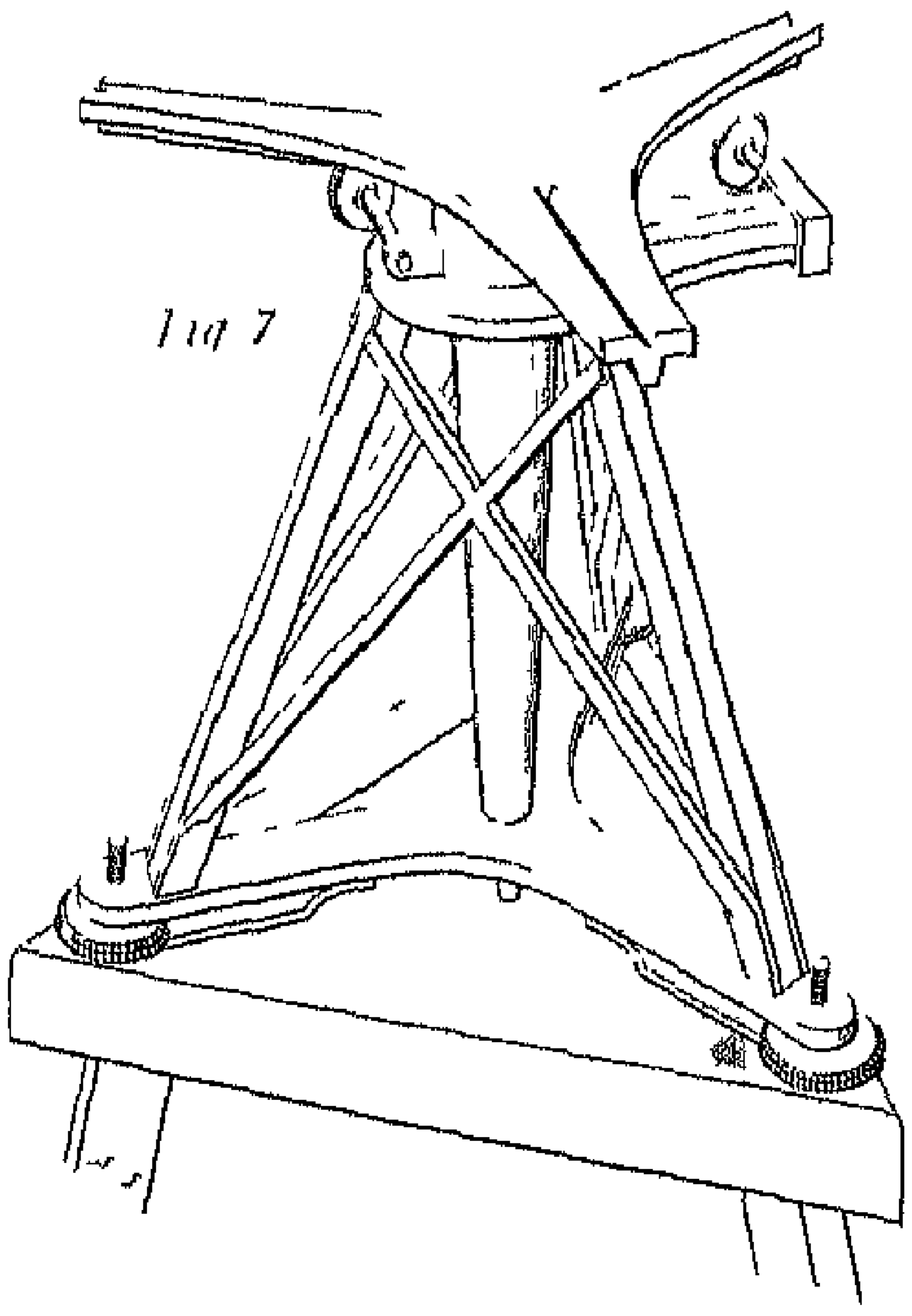
Ramsden's Sextant

Fig. 8



Repeating Tripod

Fig. 7



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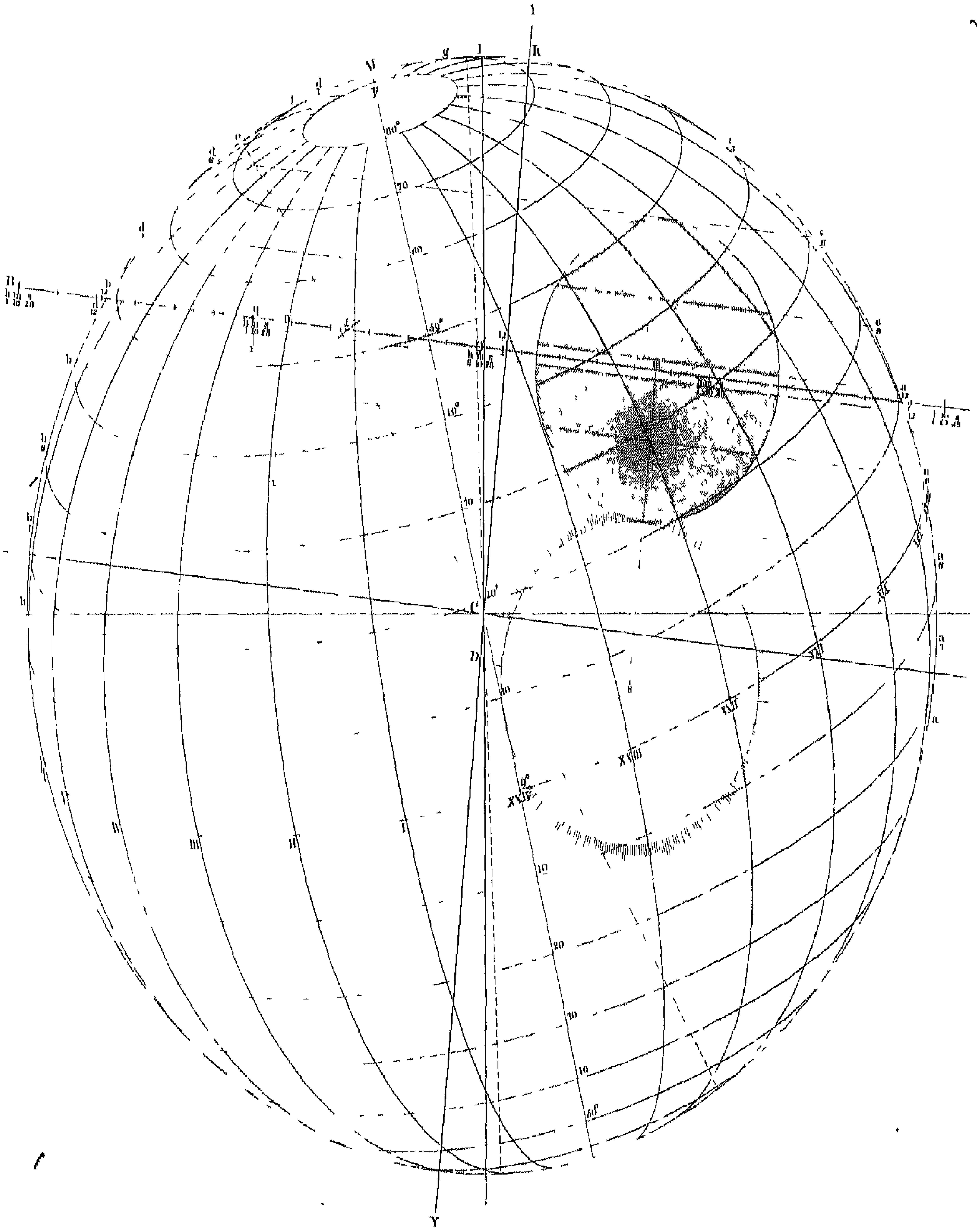
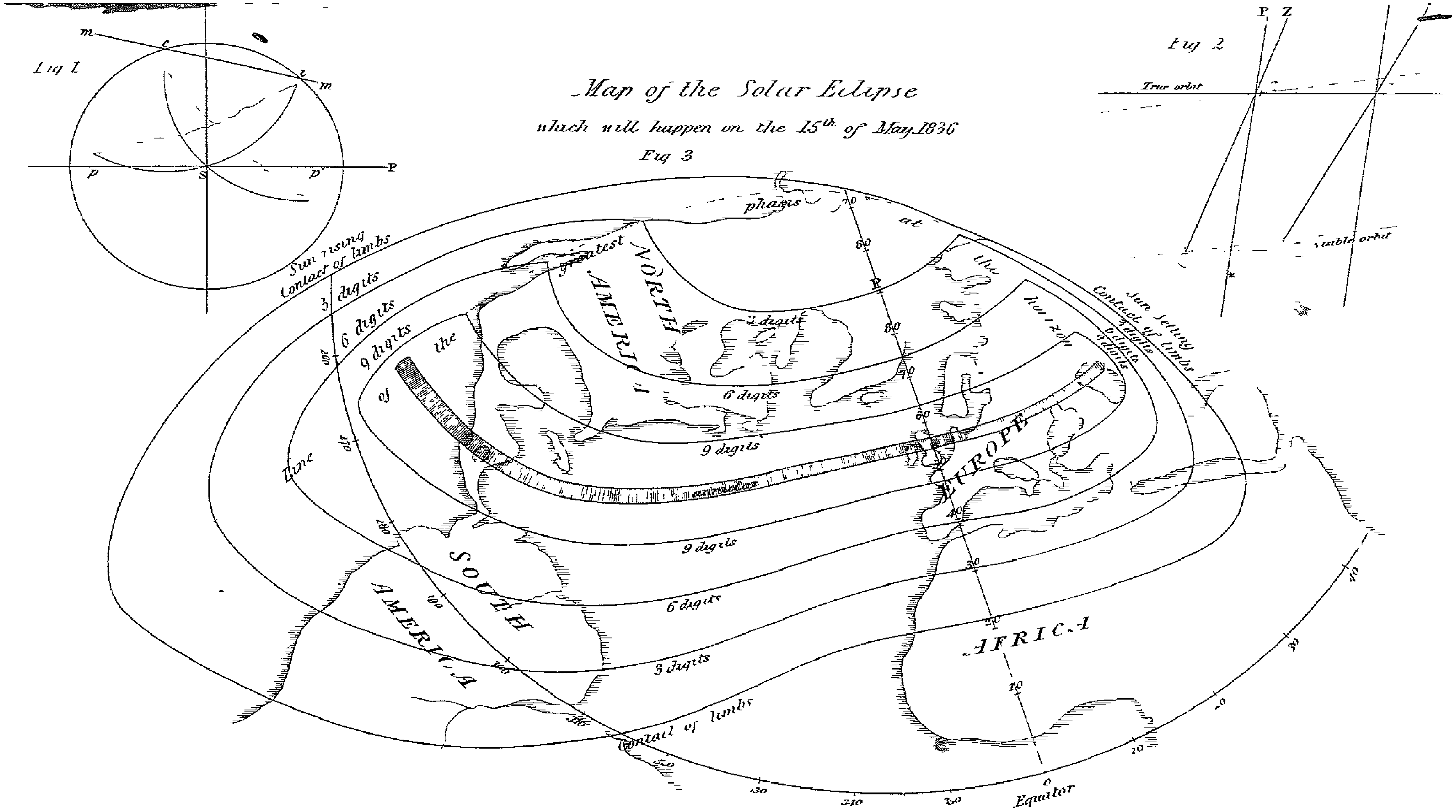
[illegible]

PROJECTION OF THE SOLAR ECLIPSE OF MAY 15, 1836

Plate III





Map of the Solar Eclipse
which will happen on the 15th of May 1836
Fig 3

F M'Ssolli delt

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